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Random number sensitivity in simulation of wind loads

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Abstract. Recently, an efficient and practical method has been developed for the generation of univariate non-Gaussian wind pressure time histories on low building roofs; this methodology requires intermittent exponential random numbers for the simulation. On the other hand, the conventional spectral representation scheme with random phase is found suitable for the generation of univariate Gaussian wind pressure time histories on low building roofs; this simulation scheme requires uniform random numbers. The dependency of these simulation methodologies on the random number generator is one of the items affecting the accuracy of the simultion result; therefore, an attempt has been made to investigate the issue. This note presents the observed sensitivity of random number sets in repetitive simulations of Gaussian and non-Gaussian wind pressures.

Key words: Gaussian; non-Gaussian; random number; simulation; wind loads.

1. Introduction

Recently, based on the characteristics of several wind tunnel measured pressures on various low building roofs, a general approach for representing Gaussian as well as non-Gaussian wind pressure characteristics using FFT (Fast Fourier Transform) algorithm has been suggested (Suresh Kumar 1997, 1999, Suresh Kumar and Stathopoulos 1997, 1999). This approach uses the conventional spectral representation scheme with random phase (Rice 1954, Shinozuka and Deodatis 1991) for simulating Gaussian pressures, and the spectral representation scheme with a new stochastic model representing phase for simulating non-Gaussian pressures. The accuracy of these simulation methods depend on: (1) the approximate representation of the power spectral density of the time series, (2) the number of samples considered in analysis, (3) the random number generator and (4) the precision of the numerical technique (Grigoriu 1986). The errors in simulation results caused by approximate representation of the spectrum have already been evaluated (Grigoriu 1986). Since the above mentioned simulation methods are dependent on random numbers, it is of interest to study the influence of the random number generator on the simulation results. This note presents the results of a study with the main objective to investigate the sensitivity of random number data sets in repetitive simulations of Gaussian and non-Gaussian wind pressures.

2. Simulation methodology

Univariate Gaussian and non-Gaussian zero-mean wind pressure time series can be generated by

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inverting the properly selected Fourier coefficients with the help of the FFT algorithm. The Discrete Fourier Transform (DFT) equation (Suresh Kumar 1997, Suresh Kumar and Stathopoulos 1997) used for such simulation is described by:

$$Z_t = n^{-1}\sum_{k=0}^{n-1}\sqrt{I_k}e^{i\phi_k}e^{i2\pi kt/n}, \quad t = 0, 1, \ldots\ldots n-1 \tag{1}$$

Where, Z_t corresponds to time series, n corresponds to time series length, $\sqrt{I_k}$ corresponds to Fourier amplitude, ϕ_k corresponds to Fourier phase and the tem $2\pi k/n$ is the integer multiple of the fundamental frequency $2\pi/n$ known as Fourier frequency. The Fourier amplitude required for this simulation is taken as the amplitude part of the DFT of the known (measured or target) sample Gaussian or non-Gaussian time series (X_t) which is in the form

$$\sqrt{I_k} = \left|\sum_{t=0}^{n-1}X_t e^{-i2\pi kt/n}\right| \tag{2}$$

By using the amplitude of the specified sample, the method ensures the reproduction of the various second order characteristics of the given sample. For convenience, the zero frequency component of the amplitude part(mean) is kept zero in all the simulations. Later, the mean of the corresponding time series can be added separately to the simulated zero-mean time series. The phase part of the Fourier coefficient of a Gaussian time series can be represented by independent uniform random numbers (u) ranging between $-\pi$ and π (Rice 1954, Shinozuka and Deodatis 1991):

$$\phi_k = \begin{cases} 0, & k = 0 \\ u_k, & 1 \le k \le \frac{n}{2} \\ -u_{n-k}, & \frac{n}{2}+1 \le k \le n-1 \end{cases} \tag{3}$$

However, in the case of non-Gaussian time series, the phase part cannot be represented by independent uniform random numbers. After an extensive investigation, the Exponential Peak Generation (EPG) model is proposed for the generation of skeleton time series from which the required phase can be drawn (Suresh Kumar 1997, 1999, Suresh Kumar and Stathopoulos 1997, 1999). The EPG model takes the form.

$$Y_t = \begin{cases} 0 & \text{, with probability } b \\ e_t & \text{, with probablity } 1-b \end{cases} \quad 0 \le b < 1 \tag{4}$$

Where, Y_t corresponds to skeleton time series, b is the probability parameter which controls the intensity as well as the frequency of spikes in the skeleton time series, and e_t is the exponential random number. The skeleton time series, Y_t, consists of intermittent exponential random numbers. The Fourier phase (ϕ_k) required for the non-Gaussian simulation can be obtained by taking the phase part of the DFT of skeleton time series (Y_t) by

$$\phi_k = \arctan\left[\frac{-\sum_{t=0}^{n-1} Y_t \sin(2\pi kt/n)}{\sum_{t=0}^{n-1} Y_t \cos(2\pi kt/n)}\right] \tag{5}$$

Result of the mathematical operation arctan representing four-quadrant inverse tangent will lie in the interval $-\pi$ to π which is the same for phase angles of a time series. This is in contrast with the result of simple inverse tangent which is limited to the interval $-\pi/2$ to $\pi/2$ (MATLAB 1992).

A new parametric estimation procedure has been introduced in this study; the computation of parameter b is accomplished by minimizing the sum of the squared errors in higher order statistics such as skewness and kurtosis (Suresh Kumar 1997, 1999, Suresh Kumar and Stathopoulos 1997, 1999). Further, stationarity of the simulated non-Gaussian time series is justified for $b \leq 0.9$; values of $b > 0.9$ are not obtained even when modeling highly non-Gaussian pressure fluctuations (Suresh Kumar 1997, Suresh Kumar and Stathopoulos 1999). Simplicity and effectiveness of this methodology have been demonstrated using several wind tunnel measured pressures on low building roofs (Suresh Kumar 1997, 1999, Suresh Kumar and Stathopoulos 1997, 1999).

3. Sensitivity of random numbers

Wind tunnel measured Gaussian and non-Gaussian pressure time series ($n = 8192$) on a monoslope roof of a low building have been used for the demonstration of random number sensitivity in corresponding simulations. Fig. 1 shows the selected time histories and their statistics. Appearance as well as statistics reveals that sample S22 is Gaussian and sample S1 is non-Gaussian. Note also that sample S1 is negatively skewed due to the presence of many negatively going spikes; this is

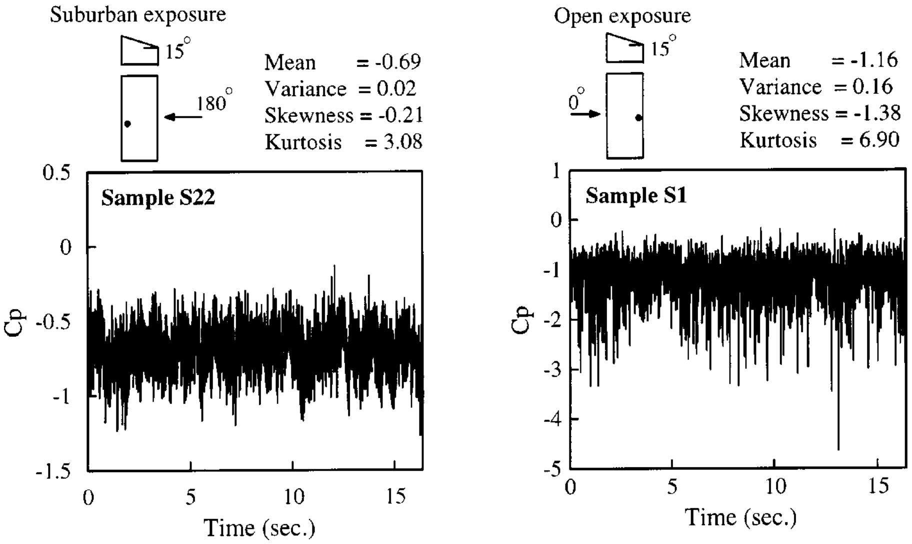

Fig. 1 Measured (Target) pressure time series on a monoslope roof

typical for the case of non-Gaussian pressures on roofs (Suresh Kumar 1997).

In order to show the sensitivity of random numbers in repetitive simulations, the sample time histories are reconstructed using the previously described simulation methodology. In the following Gaussian simulations, sample S22 is reconstructed using the amplitude part of the DFT of their target counterpart (Eq. 2) and phase represented by uniform random numbers (Eq. 3). On the other hand, in non-Gaussian simulations, sample S1 is reconstructed using the amplitude part of the DFT of their target counterpart (Eq. 2) and phase generated using the EPG model (Eqs. 4 and 5).

3.1. Gaussian simulations

For the simulation of Gaussian time series, uniform random numbers (u) ranging between $-\pi$ and π are required to represent the phase part of the Fourier coefficients. Though various algorithms are available and listed by Knuth (1981) good random number generators are hard to find. Park and Miller (1988) presented the inadequacy of the many available random number generators along with the discussion of practical and theoretical issues concerning the design, implementation and use of a good, minimal standard random number generator that will port to virtually all systems. They found that the linear congruential generator with proper parmetric values is good in terms of accomplishing full periodicity, randomness and easy implementation. On this basis, this generator is selected for the random number generation in this study. Three quantities, i.e., a multiplier, a modulus and an initial seed value are required to generate uniform random numbers by using this algorithm. The value of multiplier and modulus equal to 7^5 and $2^{31}-1$ respectively, which provide full period, randomness and easy implementation capabilities to the generator (Park and Miller 1988), are used in this study. The initial seed value is adopted to be 931316785, the value set by MATLAB (1992) at the start of any simulation. The basic algorithm is

$$
\begin{aligned}
&u1 = \text{seed}/(2^{31}-1),\\
&u = -\pi + (2\pi * u1),\\
&\text{seed} = (7^5 * \text{seed}) \quad \text{mod}(2^{31}-1),
\end{aligned}
$$

where, $u1$ corresponds to random number whose value is between 0 and 1 and u corresponds to random number whose value is between $-\pi$ and π. Subsequent sets of random numbers are expected to be different due to the change of initial seed value. Therefore, an attempt has been made to examine the sensitivity of uniform random number sequences on simulation results.

For each simulation ($n = 8192$), 4096 uniform random numbers ranging between $-\pi$ and π are required (see Eq. 3). One hundred distinct blocks of 4096 uniform random numbers each have been generated and the variation of their first four moments (mean, variance, skewness, and kurtosis) is displayed in Fig. 2 using boxplot. Boxplot produces a box and whisker for each data set (MATLAB 1994). The box has lines at the lower quartile, median and upper quartile values. The whiskers are lines extending from each end of the box to show the extend of the rest of the data. Mean and skewness of the random number sets are supposed to be zero (see Appendix - A); however, negligible variations up to ±5% are noted. Variations in variance and kurtosis values of the random number sets are also shown in the same figure after they have been normalized with respect to their corresponding theoretical values provided in Appendix - A. Again, negligible variations up to ±5% have been observed. Simultaneously, the same 100 blocks of random numbers have been used to simulate 100 corresponding Gaussian time histories using the amplitude part of the sample S22. Fig. 3 presents the variation of the first four moments of the simulated time histories in a boxplot format.

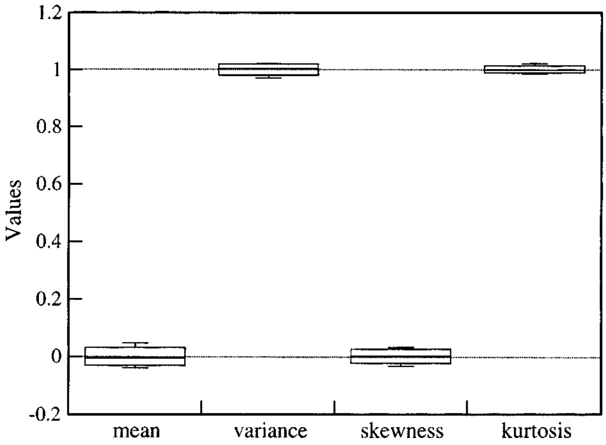

Fig. 2 Variation of the first four moments of the uniform random number data sets

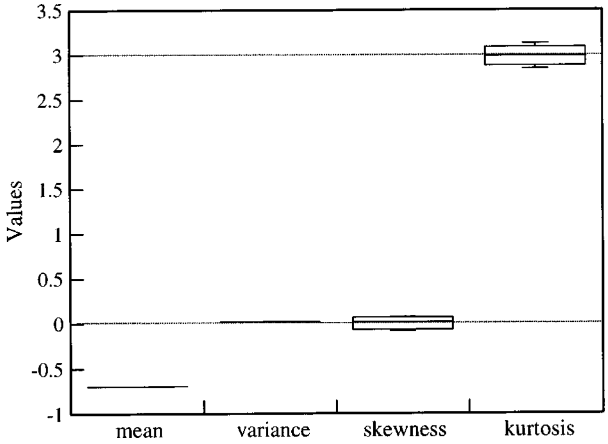

Fig. 3 Variation of the first four moments of the simulated time histories

Note that mean and variance of the simulated time histories are always equal to the corresponding target values of –0.69 and 0.02 respectively. This is due to the employment of the same amplitude part of the DFT of the target signal S22 in all simulations. Further, it is clear from the theory that the different random number sets (phase part) do not have an effect on the simulated means and variances; on the other hand, they do have an effect on skewness and kurtosis values of the simulated time histories (Suresh Kumar 1997). Based on the Gaussian assumption, skewness and kurtosis values of the simulated time histories are supposed to be zero and three respectively; however, due to the varying statistical properties of the random number sets, variations up to ±15% have been observed. Furthermore, since many time histories are required for extreme value and fatigue analysis, the average skewness and kurtosis values among many samples are expected to be close to zero and three respectively. Overall, the performance of the used random number generator is satisfactory and the small variations noted in simulated skewness and kurtosis values can be neglected for practical applications.

3.2. Non-Gaussian simulations

For the simulation of non-Gaussian time series, intermittent exponential random numbers (ψ_t) are required. On the other hand, generation of exponential random numbers (e_t) is essential for the generation of intermittent exponential random numbers. Many algorithms are currently available to generate exponential random numbers (Clark and Holz 1960, Knuth 1981). For the present study, logarithmic transformation of uniform random numbers if employed for the generation of exponential random numbers. This is the most widely used algorithm. The intermittent exponential random number sequence, controlled by the parameter b is generated using the following algorithm:

$$\text{if } (0 < u1(i) < b, \ \psi_t(i) = 0$$
$$\text{if } (b \leq u1(i) \leq 1), \ \psi_t(i) = \log(u2(i))$$

where, $u1$ and $u2$ are two independent sets of uniform random numbers whose values are between 0 and 1. ψ_t represents intermittent exponential random numbers whose upper limit is obviously zero but its lower limit varies. Subsequent sets of intermittent exponential random numbers are expected to be different due to the change of initial seed value used in the generation of uniform random numbers. Therefore, an attempt has been made to examine the sensitivity of intermittent exponential random number sets on simulation results.

For each simulation, 8192 intermittent exponential random numbers are required. One hundred distinct sets of 8192 intermittent exponential random numbers each have been generated using $b = 0.87$ (the parameter estimated for sample S1). For each set, the first four moments (mean, variance, skewness and kurtosis) have been computed and then normalized with respect to their corresponding theoretical values estimated using the equations provided in Appendix - A. The variation of their normalized moments is displayed in Fig. 4 using boxplot. Clearly, the variation of the statistics is higher than those in the case of uniform random numbers shown in Fig. 2. The mean, variance and skewness of the sequences vary up to ±15%, while the kurtosis values vary up to ±25%. It is suspected that high variation in statistics is due to the presence of very small values close to zero in some of the sets of uniform random numbers. This can change the statistics of the exponential random numbers drastically since the logarithm of those values are high. Since this variation in statistics is

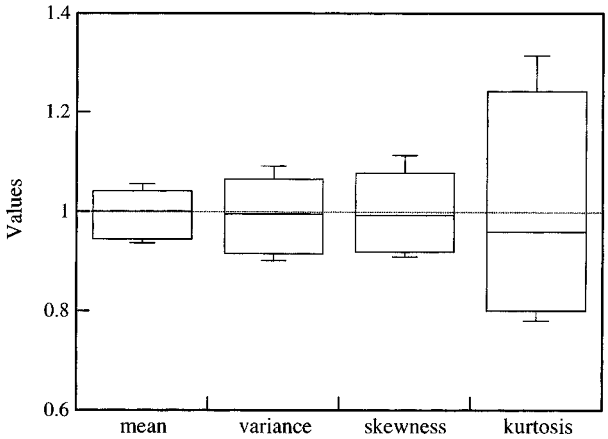

Fig. 4 Variation of the first four moments of the intermittent exponential random numbers

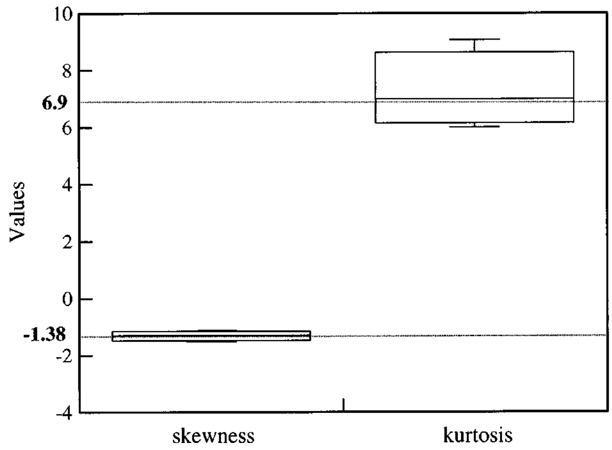

Fig. 5 Variation of skewness and kurtosis values of the simulated time histories

suspected to be due to the transformation of uniform random numbers, other algorithms not using the transformation of uniform random numbers have been attempted. For instance, the algorithm provided by Clark and Holz (1960) and some of the algorithms provided by Knuth (1981) were applied but, no significant improvement over the present method was observed. On this basis, the present method is used in this study. Simultaneously, the same 100 blocks of random numbers have been used to simulate 100 corresponding non-Gaussian time histories using the amplitude part of the sample S1. Fig. 5 presents the variation of skewness and kurtosis of the simulated time histories in a boxplot format. Variations up to ±25% have been observed in both quantities and this high variaiton is due to the highly varying statistical properties of the intermittent exponential random number sets shown in Fig. 4. On the other hand, the noted high variation in skewness and kurtosis of the simulated time series can be reduced by averaging them for a number of samples. Moreover, this scenario seems practical since several time histories are required for carrying out extreme value and fatigue analysis. A typical example provided in Fig. 6 shows the average skewness as well as kurtosis values against the number of simulations. The time histories previously simulated for Fig. 5 have been used for this demonstration. For instance, the skewness value at the 50th number of

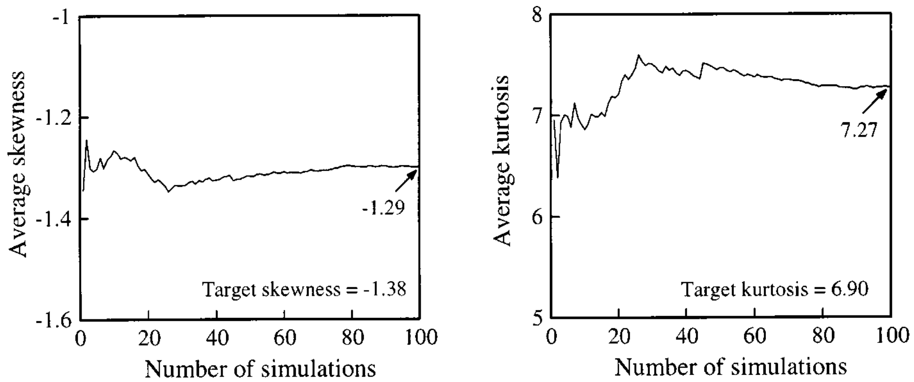

Fig. 6 Variation of average skewness and kurtosis values with respect to number of simulations

simulation represents the average skewness value of the first 50 simulated time histories. As the number of simulations increases, the average skewness as well as kurtosis stabilizes. After 100 consecutive simulations, the target skewness and kurtosis are achieved within 5%. Overall, the performance of the used random number generation is satisfactory. Nevertheless, further research is required to develop a good exponential random number generator that would produce independent random number sets with stable statistics.

4. Conclusions

This note presents the observed sensitivity of random number generator in repetitive simulations of Gaussian and non-Gaussian wind loads. This investigation employed (1) the conventional spectral representation method using random phase for the digital generation of univariate Gaussian wind pressure time histories, (2) the recently suggested simulation methodology for the digital generation of univariate non-Gaussian wind pressure time histories, and (3) several wind tunnel measured pressures on low building roofs. The results show that the performance of the uniform random number generator used in Gaussian simulations is satisfactory. On the other hand, non-Gaussian simulations appear sensitive to the intermittent exponential random number data set produced by the used generator; however, the discrepancy in simulation results can be reduced by considering numerous samples.

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Appendix - A: Properties of random variables

Uniform Random Variable

Let U be a uniform random variable. The probability density function of U is

$$f_U(u) = \frac{1}{2\pi}, \quad -\pi \le u \le \pi \tag{A.1}$$

The first four moments (mean, variance, skewness and kurtosis) of U are derived using the principles of mathematical expectation (Papoulis 1984). The derived moments are

$$\text{mean } (U) = 0 \tag{A.2}$$

$$\text{Variance } (U) = \frac{\pi^2}{3} \tag{A.3}$$

$$\text{Skewness } (U) = 0 \tag{A.4}$$

$$\text{Kurtosis } (U) = 1.8 \tag{A.5}$$

Intermittent Exponential Random Variable

The intermittent exponential random variable (Ψ) required for EPG model has the form,

$$\Psi = IE \tag{A.6}$$

where, I represents discrete random variable and E represents continuous exponential random variable. The probability density function of I is

(A.7)

I	**0**	**1**
P_I (I = i) for $0 \le b < 1$	b	$(1-b)$

The probability density function of E is

$$f_E(e) = \lambda \exp(-\lambda e), \; e > 0 \tag{A.8}$$

where, the parameter λ governs the properties of this distribution. In the present study, $\lambda = -1$ is used which directly generates negatively going spikes observed in non-Gaussian wind pressure time series on roofs. From the perviously discussed properties of I and E, the first four moments (mean, variance, skewness and kurtosis) of Ψ are derived using the principles of mathematical expectation (Papoulis 1984). The derived moments are

$$\text{mean } (\Psi) = \mu = E(\Psi) = (b-1) \tag{A.9}$$

$$\text{variance } (\Psi) = \sigma^2 = E[\Psi - \overline{\Psi})^2] = (1-b^2) \tag{A.10}$$

$$\text{Skewness } (\Psi) = \frac{E[(\Psi - \overline{\Psi})^3]}{\sigma^3} = \frac{2(b^3-1)}{(1-b^2)^{3/2}} \tag{A.11}$$

$$\text{Kurtosis }(\Psi) = \frac{E[(\Psi-\overline{\Psi})^4]}{\sigma^4} = \frac{3(3-2b^2-b^4)}{(1-b^2)^2} \tag{A.12}$$

Since, I and E are independent and identically distributed (*i.i.d.*) sequences, theoretically, Ψ must also be and *i.i.d*. sequence. Therefore, the values of Ψ at two different times are uncorrelated.

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