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On the modeling of dynamic behavior of composite plates using a simple nth-HSDT

I. Klouche Djedid[1], Kada Draiche*[1,2], B. Guenaneche[2], Abdelmoumen Anis Bousahla[3,4], Abdelouahed Tounsi[2,5] and E.A. Adda Bedia[6]

[1]Département de Génie Civil, Université Ibn Khaldoun Tiaret, BP 78Zaaroura, 1400 Tiaret, Algérie
[2]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria
[3]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[4]Centre Universitaire Ahmed Zabana de Relizane, Algeria
[5]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia
[6]Centre of Excellence for Advanced Materials Research, King Abdulaziz University, Jeddah, 21589, Saudi Arabia

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Abstract. In the present paper, a simple refined nth-higher-order shear deformation theory is applied for the free vibration analysis of laminated composite plates. The proposed displacement field is based on a novel kinematic in which include the undetermined integral terms and contains only four unknowns, as against five or more in case of other higher-order theories. The present theory accounts for adequate distribution of the transverse shear strains through the plate thickness and satisfies the shear stress-free boundary conditions on the top and bottom surfaces of the plate, therefore, it does not require problem dependent shear correction factor. The governing equations of motion are derived from Hamilton's principle and solved via Navier-type to obtain closed form solutions. The numerical results of non-dimensional natural frequencies obtained by using the present theory are presented and compared with those of other theories available in the literature to verify the validity of present solutions. It can be concluded that the present refined theory is accurate and efficient in predicting the natural frequencies of isotropic, orthotropic and laminated composite plates.

Keywords: nth-higher-order theory; free vibration; laminated composite; Hamilton's principle

1. Introduction

A composite material is formed by the combination of two or more constituent materials with significantly different physical or chemical properties to form a new material, whose properties and performances are designed such as the result is greater than those of the constituent materials acting independently. Laminated composite, one of the types of composite materials, consists of several orthotropic layers of different materials that are bonded together with adhesive, to give added strength, stiffness, corrosion resistance, or some other benefit, these materials offer definite advantages compared to more traditional materials than steel or aluminum (Sahadat Hossain *et al*. 2017). Laminated composite plates are widely used around the world in many fields such as aerospace, naval, automotive, civil industries, mechanical engineering, biomedical and other structural applications due to their attractive properties and reliability. In view of the increase importance in the application of laminates in engineering structures, a variety of laminated theories have been developed in order to study the static and dynamic behavior of laminated composite plates (Baseri *et al*. 2016, Becheri *et al*. 2016, Javed *et al.* 2016, Chikh *et al*. 2017, Hirwani *et al*. 2017a).

Several studies have been carried out on the bending, vibration and buckling problems of isotropic, orthotropic and laminated composite plates and various researchers have done mathematical modeling of these different structures using classical plate theory and first-order shear deformation theory. It should be noted that the classical plate theory (CPT) developed by Kirchhoff (1850) is the simplest theory applicable for thin laminated composite plates but inaccurate for the thick plate due to the neglect of transverse shear deformation effect. To overcome the limitations of CPT and accurately incorporate the transverse shear deformation effects for moderately thick or thick plates, Reissner (1945) and Mindlin (1951) have been proposed the first-order shear deformation theory (FSDT) in which requires a shear correction factor to correct the unrealistic variation of the shear strain/stress through the thickness and in order to satisfy the free transverse shear stress conditions on the top and bottom surfaces of the structure (Al-Basyouni *et al*. 2015, Madani *et al*. 2016, Bouderba *et al*. 2016, Bellifa *et al*. 2016, Kolahchi 2017, Cherif *et al*. 2018, Draoui *et al*. 2019, Semmah *et al*. 2019, Karami *et al*. 2019a). The limitations of CPT and FSDT forced the development of higher order shear deformation theories (HSDTs) to avoid the use of shear correction factors, to include correct cross sectional warping and to get the realistic variation of the transverse shear strains and

*Corresponding author, Ph.D.
E-mail: kdraiche@yahoo.fr

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stresses through the thickness of structures (Belkorissat *et al*. 2015, Mahi *et al*. 2015, Bousahla *et al*. 2016, Bounouara *et al*. 2016, Houari *et al*. 2016, Kolahchi and Moniri Bidgoli 2016, Beldjelili *et al*. 2016, Kolahchi *et al*. 2017a,b,c, Khetir *et al*. 2017, Hajmohammad *et al*. 2017, Bellifa *et al*. 2017a,b, Hachemi *et al*. 2017, Abdelaziz *et al*. 2017, Kolahchi and Cheraghbak 2017, Mouffoki *et al*. 2017, Sekkal *et al*. 2017a, Selmiand Bisharat 2018, Golabchi *et al*. 2018, Fakhar and Kolahchi 2018, Hosseini and Kolahchi 2018, Hajmohammad *et al*. 2018a,b,c, Bouadi *et al*. 2018, Mokhtar *et al*. 2018, Belabed *et al*. 2018, Kaci *et al*. 2018, Attia *et al*. 2018, Yazid *et al*. 2018, Bakhadda *et al*. 2018, Karami *et al*. 2019b, Bourada *et al*. 2019, Meksi *et al*. 2019).

Therefore, many six variable and five variable plate theories have been developed for the analysis of plates. Reddy (1984a) has developed well-known higher order shear deformation theory considering polynomial functions in-terms of thickness coordinate for the analysis of laminated composite plates. Soldatos (1992) proposed a hyperbolic shear deformation theory for homogenous monoclinic plates. Touratier (1991) has developed a trigonometric shear deformation theory for bending, buckling and vibration analysis of laminated composite and sandwich plates. Shimpi *et al.* (2000) proposed a trigonometric theory for static and free vibration analysis of isotropic, orthotropic and layered composite plates. Zenkour (2004) used a unified theory of cross-ply laminated composite plates to investigate the bending response of laminated plates under a sinusoidally distributed transverse mechanical load and a sinusoidally non-uniform distribution of temperature. Metin (2006) Compared the various shear deformation theories for bending, buckling, and vibration of rectangular symmetric cross-ply plate with simply supported edges conditions. Karama *et al*. (2009) have proposed an exponential shear deformation plate theory to predict the mechanical behaviour of multilayered laminated composite structures. Ghugal and Sayyad (2010 and 2011) have developed trigonometric shear deformation theory considering the effects of transverse shear and normal deformations for static flexure and free vibration analysis of thick isotropic and orthotropic plates. Mantari *et al*. (2012) used the trigonometric function in the new displacement field for analyzing the static behavior of isotropic and composite laminated and sandwich plates, however the results show that the present model is in close agreement with Reddy's and Touratier's shear deformation theories.

In the last few years, a new class of plate theories has been developed by researchers in which displacement field involves only four unknowns without including the thickness-stretching effect and five unknowns in which both shear deformation and thickness stretching effects are included. Thai *et al.* (2010) has developed a two variable refined plate theory for the free vibration analysis of antisymmetric cross-ply and angle-ply laminates, to extend the refined plate theory (RPT) developed by Kim et al. (2009) for static bending and buckling analyses of laminated composite plates. Benachour *et al*. (2011) analyzed the free vibration of rectangular functionally graded plates with arbitrarily varying material properties along the thickness direction by using a four variable refined plate theory. Tounsi *et al*. (2013) developed a refined trigonometric shear deformation theory for the thermoelastic bending analysis of functionally graded sandwich plates. Bouderba *et al*. (2013) presented an analytical solution for the thermo-mechanical bending response of functionally graded plates resting on Winkler-Pasternak elastic foundations, in which the theoretical formulations are based on a refined trigonometric shear deformation theory. Nedri *et al*. (2014) investigated the free vibration of laminated composite plates resting on elastic foundations. Bousahla *et al*. (2014) proposed a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite. Meziane *et al*. (2014) presented an efficient and simple refined shear deformation theory for the bucking and vibration analyses of exponentially graded sandwich supported by elastic foundations with considering various types of boundary conditions. Chattibi *et al*. (2015) studied the thermo-mechanical bending response of antisymmetric cross-ply laminated composite plates by using a simple four variable sinusoidal theory. A novel four variable refined plate theory for the buckling response of isotropic and orthotropic plates is presented and discussed by Bourada et al. (2016). A higher order shear and normal deformation theory for the static flexural analysis of laminated composite plates subjected to uniformly distributed, uniformly varying and concentrated loads has been presented by Draiche *et al*. (2016). Many researches based on refined shear deformation theories are available in the literature for the bending, buckling and free vibration analysis of isotropic, orthotropic, laminated composite and functionally graded structures (Ahmed 2014, Zidi *et al*. 2014, Attia *et al*. 2015, Shinde *et al*. 2015, Sayyad *et al*. 2016, Ahouel *et al*. 2016, Hebali *et al*. 2016, Akavci 2016, Saidi *et al*. 2016, Janghorban 2016, Aldousari 2017, Fahsi *et al*. 2017, Bouazza *et al*. 2017, Kadari *et al*. 2018, Shahsavari *et al*. 2018, Karami *et al*. 2018a,b). Some experimental studies can be also consulted in different works found in literature (Sahoo *et al*. 2016, Hirwani *et al.* 2016, Sharma *et al*. 2017a,b, Hirwani *et al*. 2017bc, Mehar *et al*. 2017, Sahoo *et al*. 2017, 2018, Sharma *et al*. 2018a,b, Bisen *et al*. 2018, Sahoo *et al*. 2019, Tlidji *et al*. 2019, Singh *et al*. 2019).

In the present paper, a simple refined *nth*-higher-order shear deformation theory using undetermined integral terms in the displacement field is applied to develop the analytical solution for the free vibration analysis of isotropic, orthotropic and laminated composite plates. The present theory has only four unknowns and four governing equations, satisfies the shear stress free condition at top and bottom surface of the plates without using shear correction factors. Governing equations of motion are derived from the Hamilton's principle. A closed form solution for simply supported boundary conditions is obtained by employing a double trigonometric series technique developed by Navier. The numerical results of natural frequencies obtained by using the present theory are presented and compared with those of the classical plate theory (CPT), first-order shear deformation plate theory (FSDT) and other higher-order shear deformation plate theories (HSDTs) available in the literature to demonstrate the validity of the theory.

2. Mathematical formulation

Consider a laminated composite plate of the length *a*, width *b* and a constant thickness *h* along the *z*-direction as shown in Fig. 1. The coordinate system (*x*, *y*, *z*) chosen and the co-ordinate parameters are such that $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$. The plate is assumed to be constructed of an arbitrary number, *N*, of linearly elastic orthotropic layers.

2.1 The displacement field

The present higher-order shear deformation theory has a novel displacement field which includes undetermined integral terms and contains only four unknowns, as against five in case of other shear deformation theories. The displacement field at a point located at (*x*, *y*, *z*) in the plate can be written in a simpler form as (Besseghier *et al.* 2017, El-Haina *et al.* 2017, Menasria *et al.* 2017, Zine *et al*. 2018, Bourada *et al*. 2018)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

Where $u_0(x,y)$, $v_0(x,y)$, $w_0(x,y)$ and $\theta(x,y)$ are the four unknown functions of middle surface of the plate. The in-plane displacement field uses the *nth*-order polynomial function in terms of the thickness coordinate to include the transverse shear deformation effect. The constants k_1 and k_2 depends on the geometry. The shape function $f(z)$ is chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate obtained by putting (*n*=3,5,7,…) and is given as (Sayyad and Ghugal 2015, Xiang and Liu 2016, Becheri *et al*. 2017)

$$f(z) = -\frac{1}{n}\left(\frac{2}{h}\right)^{n-1} z^n + z \tag{2}$$

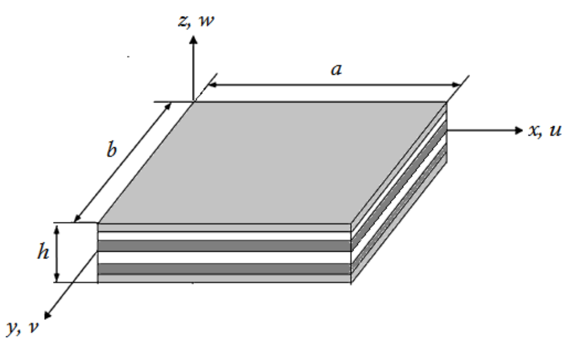

Fig. 1 Coordinate system and geometry of laminated composite plates

The infinitesimal strains associated with the displacement field in Eq. (1) are obtained using strain-displacement relationship from linear theory of elasticity

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix}, \tag{3a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3b}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta dx + k_2\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta dy \\ k_1\int \theta dx \end{Bmatrix} \tag{4b}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{5}$$

The integrals used in the above relations shall be resolved by a Navier solution and can be expressed by

$$\begin{aligned} &\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x \partial y}, \\ &\int \theta dx = A'\frac{\partial \theta}{\partial x}, \quad \int \theta dy = B'\frac{\partial \theta}{\partial y} \end{aligned} \tag{6}$$

where the parameters A' and B' are defined according to the type of solution employed, in this case via Navier. Hence, A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \; B' = -\frac{1}{\beta^2}, \; k_1 = \alpha^2, \; k_2 = \beta^2 \tag{7}$$

where the parameters α and β are defined as

$$\alpha = r\pi / a, \; \beta = s\pi / b \tag{8}$$

2.2 Constitutive relations

Laminated composites are typically constructed from orthotropic layers containing unidirectional fibers embedded in a matrix material. Generally, in a macroscopic sense, the lamina is assumed to behave as a homogeneous orthotropic material. The constitutive relation for a linear elastic orthotropic layer in the local coordinate system is derived from Hooke's law for a plane stress by

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{9}$$

In which, $\{\sigma_x,\sigma_y,\tau_{xy},\tau_{yz},\tau_{xz}\}^{Tr}$ and $\{\varepsilon_x,\varepsilon_y,\gamma_{xy},\gamma_{yz},\gamma_{xz}\}^{Tr}$ are the stresses and the strains vectors with respect to the plate coordinate system. The material constants Q_{ij} are defined in terms of the material properties of the orthotropic layer given as

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}},\ Q_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}},\quad Q_{66}=G_{12},\ Q_{44}=G_{23},\ Q_{55}=G_{13} \tag{10}$$

The laminate is usually made of several orthotropic layers with their material axes oriented arbitrarily with respect to laminate coordinates. Each layer must be transformed into the laminate coordinate system (*x*, *y*, *z*). The stress–strain relations in the laminate coordinates of a k_{th} layer are

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}^{(k)}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \overline{Q}_{44} & 0\\ 0 & 0 & 0 & 0 & \overline{Q}_{55}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^{(k)} \tag{11}$$

where $\overline{Q}_{ij}$ are the transformed material constants, are expressed as

$$\begin{aligned}
\overline{Q}_{11}^{k}&=Q_{11}cos^4\theta_k+2(Q_{12}+2Q_{66})sin^2\theta_k\,cos^2\theta_k\\&\quad+Q_{22}\,sin^4\theta_k\\
\overline{Q}_{12}^{k}&=(Q_{11}+Q_{22}-4Q_{66})sin^2\theta_k\,cos^2\theta_k\\&\quad+Q_{12}\left(sin^4\theta_k+cos^4\theta_k\right)\\
\overline{Q}_{16}^{k}&=(Q_{11}-Q_{12}-2Q_{66})sin\theta_k\,cos^3\theta_k\\&\quad+(Q_{12}-Q_{22}+2Q_{66})sin^3\theta_k\,cos\theta_k\\
\overline{Q}_{22}^{k}&=Q_{11}sin^4\theta_k+2(Q_{12}+2Q_{66})sin^2\theta_k\,cos^2\theta_k\\&\quad+Q_{22}cos^4\theta_k\\
\overline{Q}_{26}^{k}&=(Q_{11}-Q_{12}-2Q_{66})sin\theta_k\,cos^3\theta_k\\&\quad+(Q_{12}-Q_{22}+2Q_{66})cos\theta_k+sin^3\theta_k\\
\overline{Q}_{66}^{k}&=(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})sin^2\theta_k\,cos^2\theta_k\\&\quad+Q_{66}\left(sin^4\theta_k+cos^4\theta_k\right)\\
\overline{Q}_{44}^{k}&=Q_{44}cos^2\theta_k+Q_{55}sin^2\theta_k\\
\overline{Q}_{45}^{k}&=(Q_{55}-Q_{44})cos\theta_k\,sin\theta_k\\
\overline{Q}_{55}^{k}&=Q_{55}cos^2\theta_k+Q_{44}sin^2\theta_k
\end{aligned} \tag{12}$$

where θ_k is the angle of material axes with the reference coordinate axes of each layer and Q_{ij} are the plane stress-reduced stiffnesses coefficients defined in Eq. (10).

2.3 Governing equations

Hamilton's principle is used to find the compatible set of governing equations and boundary conditions for given stresses and strains associated with the present theory. The principle can be stated in an analytical form as (Zemri *et al*. 2015, Arani and Kolahchi 2016, Kolahchi *et al*. 2016a,b, Bilouei *et al*. 2016, Zamanian *et al*. 2017, Zidi *et al*. 2017, Klouche *et al*. 2017, Amnieh *et al*. 2018, Youcef *et al*. 2018, Adda Bedia *et al*. 2019, Chaabane *et al*. 2019, Karami *et al*. 2019c).

$$0=\int_0^T(\delta U+\delta V-\delta K)dt \tag{13}$$

Where δU is the variation of strain energy; δV is the variation of work done by external load applied to the plate; and δK is the variation of kinetic energy. They can be expressed as

$$\delta U=\int_V\begin{Bmatrix}\sigma_x\delta\,\varepsilon_x+\sigma_y\delta\,\varepsilon_y+\tau_{xy}\delta\,\gamma_{xy}+\tau_{yz}\delta\,\gamma_{yz}\\ +\tau_{xz}\delta\,\gamma_{xz}\end{Bmatrix}dV \tag{14}$$

$$\delta V=-\int_A q\delta\,w\,dA \tag{15}$$

$$\delta K=\int_V\rho(z)(\dot{u}\delta\,\dot{u}+\dot{v}\delta\,\dot{v}+\dot{w}\delta\,\dot{w})dV \tag{16}$$

Where A is the top surface and q is the distributed transverse load. Substituting Eqs. (1), (3) and (11) into Eq. (13) and integrating through the thickness of the plate, Eq. (13) can be rewritten as

$$\begin{aligned}
&\int_A\Big\{N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta\varepsilon_x^1+M_y^b\delta\varepsilon_y^1\\
&+M_{xy}^b\delta\gamma_{xy}^1+M_x^s\delta\varepsilon_x^2+M_y^s\delta\varepsilon_y^2+M_{xy}^s\delta\gamma_{xy}^2+S_{yz}^s\delta\gamma_{yz}^0\\
&+S_{xz}^s\delta\gamma_{xz}^0-q\delta w_0-I_0(\dot{u}_0\delta\dot{u}_0+\dot{v}_0\delta\dot{v}_0+\dot{w}_0\delta\dot{w}_0)\\
&+I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x}+\frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0+\dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y}+\frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right)\\
&-I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}+\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\\
&-I_3\left[k_1A'\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x}+\frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right)+k_2B'\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y}+\frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right]\\
&+I_4k_1A'\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right)\\
&+I_4k_2B'\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}+\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\\
&-I_5\left((k_1A')^2\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+(k_2B')^2\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\Big\}dA=0
\end{aligned} \tag{17}$$

where the stress resultants $(N,\ M^b,\ M^s,\ S^s)$ and the inertia constants $I_i\ (i = 0,1,2,3,4,5)$ are defined by the following equations

$$
\begin{aligned}
&(N_x, N_y, N_{xy}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\,dz,\\
&(M_x^b, M_y^b, M_{xy}^b) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\,z\,dz,\\
&(M_x^s, M_y^s, M_{xy}^s) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\,f(z)\,dz,\\
&(S_{xz}^s, S_{yz}^s) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\tau_{xz}, \tau_{yz})\,g(z)\,dz\\
&(I_0, I_1, I_2) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^{(k)} (1, z, z^2)\,dz\\
&(I_3, I_4, I_5) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^{(k)} \left(f(z), z\,f(z), [f(z)]^2\right) dz
\end{aligned}
\tag{18}
$$

Substituting expressions for stresses and strains of the present theory into the Hamilton's principle and integrating Eq. (17) by parts and collecting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$, the governing differential equations in terms of stress resultants are obtained as follows

$$
\begin{aligned}
&\delta u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A' I_3 \frac{\partial \ddot{\theta}}{\partial x}\\
&\delta v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + k_2 B' I_3 \frac{\partial \ddot{\theta}}{\partial y}\\
&\delta w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2}\\
&+ N_y^0 \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + q = I_0 \ddot{w}_0 + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right)\\
&- I_2 \nabla^2 \ddot{w}_0 + k_1 A' I_4 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' I_4 \frac{\partial^2 \ddot{\theta}}{\partial y^2}\\
&\delta \theta: \quad k_1 A' \frac{\partial^2 M_x^s}{\partial x^2} + (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_2 B' \frac{\partial^2 M_y^s}{\partial y^2}\\
&- k_1 A' \frac{\partial S_{xz}^s}{\partial x} - k_2 B' \frac{\partial S_{yz}^s}{\partial y} = -I_3 \left(k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + k_2 B' \frac{\partial \ddot{v}_0}{\partial y} \right)\\
&+ I_4 \left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right)\\
&- I_5 \left((k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right)
\end{aligned}
\tag{19}
$$

Substituting stress–strain relations from Eq. (11) into the Eq. (18) and integrating through the thickness, the following equations are obtained

$$
\left\{ \begin{array}{c} \left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array} \right\} \\ \left\{ \begin{array}{c} M_x^b \\ M_y^b \\ M_{xy}^b \end{array} \right\} \\ \left\{ \begin{array}{c} M_x^s \\ M_y^s \\ M_{xy}^s \end{array} \right\} \end{array} \right\}
=
\begin{bmatrix}
\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} &
\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} &
\begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{21} & E_{22} & E_{26} \\ E_{61} & E_{62} & E_{66} \end{bmatrix} \\
\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & A_{62} & B_{66} \end{bmatrix} &
\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} &
\begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{21} & F_{22} & F_{26} \\ F_{61} & F_{62} & F_{66} \end{bmatrix} \\
\begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{21} & E_{22} & E_{26} \\ E_{61} & E_{62} & E_{66} \end{bmatrix} &
\begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{21} & F_{22} & F_{26} \\ F_{61} & F_{62} & F_{66} \end{bmatrix} &
\begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{21} & H_{22} & H_{26} \\ H_{61} & H_{62} & H_{66} \end{bmatrix}
\end{bmatrix}
\left\{ \begin{array}{c} \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} \\ \left\{ \begin{array}{c} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{array} \right\} \\ \left\{ \begin{array}{c} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{array} \right\} \end{array} \right\}
\tag{20}
$$

and

$$
\left\{ \begin{array}{c} S_{yz}^s \\ S_{xz}^s \end{array} \right\} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \left\{ \begin{array}{c} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{array} \right\}
\tag{21}
$$

where $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ and A_{ij}^s are the plate stiffness defined by

$$
\begin{aligned}
&(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} (1, z, z^2)\,dz, \quad i, j = 1,2,6\\
&(E_{ij}, F_{ij}, H_{ij}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} \left(f(z), z\,f(z), [f(z)]^2\right) dz
\end{aligned}
\tag{22a}
$$

$$
A_{ij}^s = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} [g(z)]^2\,dz, \quad i, j = 4,5
\tag{22b}
$$

3. Boundary conditions and Navier's solution

The Navier method is applicable to obtain the closed-form solutions of the partial differential equations in Eq. (19) for simply supported rectangular plates. Two different types of antisymmetric laminated plates are considered, cross-ply $(0°/90°)_n$ and angle-ply $(45°/-45°)_n$.

- The boundary conditions along the edges of the plate for antisymmetric cross-ply laminates can be expressed as

$$
\text{at edges } (y = 0, b): u_0 = w_0 = N_y = M_y^b = M_y^s = \theta = 0 \tag{23a}
$$

$$
\text{at edges } (x = 0, a): v_0 = w_0 = N_x = M_x^b = M_x^s = \theta = 0 \tag{23b}
$$

$$
\text{at edges } (x = 0, a): v_0 = w_0 = N_x = M_x^b = M_x^s = \theta = 0
$$

The boundary conditions in Eq. (23) are satisfied by the following expansions

$$
\left\{ \begin{array}{c} u_0 \\ v_0 \\ w_0 \\ \theta \end{array} \right\} = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \left\{ \begin{array}{l} U_{rs} \cos(\alpha\, x) \sin(\beta\, y)\, e^{i\,\omega\, t} \\ V_{rs} \sin(\alpha\, x) \cos(\beta\, y)\, e^{i\,\omega\, t} \\ W_{rs} \sin(\alpha\, x) \sin(\beta\, y)\, e^{i\,\omega\, t} \\ \Phi_{rs} \sin(\alpha\, x) \sin(\beta\, y)\, e^{i\,\omega\, t} \end{array} \right\}
\tag{24}
$$

Table 1 Comparison of non-dimensional natural frequencies $\bar{\varpi}$ of simply supported isotropic square plate (a=b)

h/b	Theory	Mode (1,1)	(2,1)	(3,1)	(1,2)	(2,2)	(3,2)	(1,3)	(2,3)	(3,3)
0.1	FSDT [(a)]	1.9317	4.6084	8.6162	4.6084	7.0716	10.8093	8.6162	10.8093	14.1908
	HSDT [(a)]	1.9317	4.6088	8.6188	4.6088	7.0732	10.8145	8.6188	10.8145	14.2022
	Present n = 3	1.9317	4.6088	8.6188	4.6088	7.0732	10.8145	8.6188	10.8145	14.2022
	Present n = 5	1.9327	4.6140	8.6346	4.6140	7.0844	10.8376	8.6346	10.8376	14.2376
	Present n = 7	1.9339	4.6204	8.6551	4.6204	7.0987	10.8683	8.6551	10.8683	14.2869
	Present n = 9	1.9349	4.6254	8.6712	4.6254	7.1099	10.8925	8.6712	10.8925	14.3260
0.2	FSDT [(a)]	1.7679	3.8656	6.6006	3.8656	5.5879	7.9737	6.6006	7.9737	9.9802
	HSDT [(a)]	1.7683	3.8693	6.6176	3.8693	5.5984	8.0030	6.6176	8.0030	10.0362
	Present n = 3	1.7683	3.8693	6.6176	3.8693	5.5984	8.0030	6.6176	8.0030	10.0362
	Present n = 5	1.7711	3.8793	6.6372	3.8793	5.6149	8.0255	6.6372	8.0255	10.0589
	Present n = 7	1.7747	3.8936	6.6710	3.8936	5.6409	8.0702	6.6710	8.0702	10.1197
	Present n = 9	1.7775	3.9049	6.6985	3.9049	5.6620	8.1072	6.6985	8.1072	10.1715
0.3	FSDT [(a)]	1.5768	3.1962	5.1426	3.1962	4.4356	6.0836	5.1426	6.0836	7.4342
	HSDT [(a)]	1.5780	3.2059	5.1807	3.2059	4.4605	6.1456	5.1807	6.1456	7.5452
	Present n = 3	1.5780	3.2059	5.1807	3.2059	4.4605	6.1456	5.1807	6.1456	7.5452
	Present n = 5	1.5819	3.2153	5.1889	3.2153	4.4706	6.1477	5.1889	6.1477	7.5298
	Present n = 7	1.5874	3.2324	5.2214	3.2324	4.4977	6.1869	5.2214	6.1869	7.5765
	Present n = 9	1.5918	3.2464	5.2497	3.2464	4.5207	6.2224	5.2497	6.2224	7.6218
0.4	FSDT [(a)]	1.3970	2.6771	4.1505	2.6771	3.6199	4.8521	4.1505	4.8521	5.8537
	HSDT[(a)]	1.3996	2.6942	4.2116	2.6942	3.6609	4.9482	4.2116	4.9482	6.0192
	Present n = 3	1.3996	2.6942	4.2116	2.6942	3.6609	4.9482	4.2116	4.9482	6.0192
	Present n = 5	1.4037	2.6995	4.2034	2.6995	3.6601	4.9252	4.2034	4.9252	5.9630
	Present n = 7	1.4102	2.7162	4.2296	2.7162	3.6834	4.9537	4.2296	4.9537	5.9912
	Present n = 9	1.4155	2.7305	4.2548	2.7305	3.7048	4.9835	4.2548	4.9835	6.0262

[(a)] Taken from Shufrin and Eisenberger (2005)

- The boundary conditions along the edges of the plate for antisymmetric angle-ply laminates can be expressed as

$$\text{at edges } (y=0,b): v_0 = w_0 = N_{xy} = M_y^b = M_y^s = \theta = 0 \quad (25a)$$

$$\text{at edges } (x=0,a): u_0 = w_0 = N_{xy} = M_x^b = M_x^s = \theta = 0 \quad (25b)$$

The boundary conditions in Eq. (25) are satisfied by the following expansions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \begin{Bmatrix} U_{rs}\sin(\alpha\, x)\cos(\beta\, y)\, e^{i\,\omega\, t} \\ V_{rs}\cos(\alpha\, x)\sin(\beta\, y)\, e^{i\,\omega\, t} \\ W_{rs}\sin(\alpha\, x)\sin(\beta\, y)\, e^{i\,\omega\, t} \\ \Phi_{rs}\sin(\alpha\, x)\sin(\beta\, y)\, e^{i\,\omega\, t} \end{Bmatrix} \quad (26)$$

Where α and β are defined in Eq. (8), U_{rs}, V_{rs}, W_{rs} and Φ_{rs} are the unknown coefficients of the respective Fourier expansions, and ω is the natural frequency of the system. Substituting this form of solution given by Eq. (24) and Eq. (26) and setting transverse load q equal to zero into the governing differential equations Eq. (19) results into a system of the algebraic equations which can be written in matrix form as follows

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = \{0\} \quad (27)$$

where $[K]$, $[M]$ and $\{\Delta\}$ are stiffness matrix, mass matrix and amplitude vector, respectively.

4. Numerical results and discussions

In this section, various numerical examples are presented to verify the validity and efficacy of the present "*nth*-HSDT" in predicting the free vibration responses of the simply supported isotropic, orthotropic and laminated composite plates. The numerical results obtained for natural frequencies will be compared and discussed with those obtained by the CPT, FSDT and HSDTs available in literature and exact elasticity solution provided by Srinivas *et al*. (1970) wherever applicable. The following material properties are considered for the various examples in the present study.

Table 2 Comparison of non-dimensional natural frequencies ϖ of simply supported isotropic rectangular plate (*a/b=1.5*)

h/b	Theory	Mode								
		(1,1)	(2,1)	(3,1)	(1,2)	(2,2)	(3,2)	(1,3)	(2,3)	(3,3)
0.1	FSDT [a]	1.4082	2.6491	4.6084	4.1303	5.2656	7.0716	8.1942	9.1982	10.8093
	HSDT [a]	1.4082	2.6491	4.6088	4.1306	5.2662	7.0732	8.1965	9.2015	10.8145
	Present n = 3	1.4082	2.6491	4.6088	4.1306	5.2662	7.0732	8.1965	9.2015	10.8145
	Present n = 5	1.4087	2.6510	4.6140	4.1348	5.2728	7.0844	8.2110	9.2191	10.8376
	Present n = 7	1.4094	2.6532	4.6204	4.1400	5.2811	7.0987	8.2297	9.2422	10.8683
	Present n = 9	1.4099	2.6549	4.6254	4.1441	5.2875	7.1099	8.2444	9.2603	10.8925
0.2	FSDT [a]	1.3164	2.3612	3.8656	3.5117	4.3405	5.5879	6.3282	6.9717	7.9737
	HSDT [a]	1.3166	2.3620	3.8693	3.5145	4.3457	5.5984	6.3433	6.9917	8.0030
	Present n = 3	1.3165	2.3620	3.8693	3.5145	4.3456	5.5984	6.3432	6.9916	8.0030
	Present n = 5	1.3182	2.3666	3.8793	3.5232	4.3575	5.6149	6.3621	7.0122	8.0255
	Present n = 7	1.3203	2.3727	3.8936	3.5353	4.3748	5.6409	6.3937	7.0489	8.0702
	Present n = 9	1.3219	2.3774	3.9049	3.5449	4.3887	5.6620	6.4195	7.0789	8.1072
0.3	FSDT [a]	1.2010	2.0526	3.1962	2.9336	3.5439	4.4356	4.9536	5.3986	6.0836
	HSDT [a]	1.2016	2.0553	3.2059	2.9412	3.5569	4.4605	4.9879	5.4425	6.1456
	Present n = 3	1.2016	2.0553	3.2059	2.9412	3.5569	4.4605	4.9879	5.4425	6.1456
	Present n = 5	1.2042	2.0610	3.2153	2.9499	3.5669	4.4706	4.9967	5.4495	6.1477
	Present n = 7	1.2076	2.0696	3.2324	2.9649	3.5867	4.4977	5.0278	5.4839	6.1869
	Present n = 9	1.2103	2.0764	3.2464	2.9771	3.6032	4.5207	5.0547	5.5142	6.2224
0.4	FSDT [a]	1.0851	1.7818	2.6771	2.4742	2.9436	3.6199	4.0090	4.3419	4.8521
	HSDT[a]	1.0864	1.7871	2.6942	2.4879	2.9662	3.6609	4.0643	4.4115	4.9482
	Present n = 3	1.0864	1.7871	2.6942	2.4879	2.9662	3.6609	4.0643	4.4115	4.9482
	Present n = 5	1.0894	1.7924	2.6995	2.4936	2.9705	3.6601	4.0584	4.3999	4.9252
	Present n = 7	1.0937	1.8018	2.7162	2.5086	2.9892	3.6834	4.0839	4.4269	4.9537
	Present n = 9	1.0972	1.8096	2.7305	2.5214	3.0055	3.7048	4.1081	4.4534	4.9835

[a] Taken from Shufrin and Eisenberger (2005)

- Material 1: Isotropic plate

$E_1 / E_2 = 1, \quad \nu = 0.3$

- Material 2: Orthotropic plate

$$E_1 = 20.83\, Pa,\ E_2 = 10.94\, Pa,\ E_3 = 10\, Pa,\ G_{12} = 6.10\, Pa,$$
$$G_{13} = 3.71\, Pa,\ G_{23} = 6.19\, Pa,\ \nu_{12} = \nu_{13} = 0.44,\ \nu_{23} = 0.23$$

- Material 3: Laminated composite plate

$$E_1 = 40E_2,\ G_{12} = G_{13} = 0.6E_2,\ G_{23} = 0.5E_2,\ \nu_{12} = 0.25$$

The numerical results according to the present study are presented and discussed in Tables 1 through 10 as non-dimensional terms of natural frequencies.

In the first part of the analysis, a simply supported isotropic square and rectangular plates were examined with thickness-to-width ratios *h/b* = 0.1, 0.2, 0.3, 0.4 and for different values of the modes of vibration (*r*, *s*). The results obtained by using the present "*nth*-HSDT" theory in Tables 1–3 are compared with the corresponding results based on FSDT and HSDT theory provided by Shufrin and Eisenberger (2005). In this example the non-dimensional terms of natural frequencies of the isotropic plates is defined by $\varpi = (\omega b^2 / \pi^2) / \sqrt{\rho h / D}$, where the constant appeared here can be determined as $D = Eh^3 / 12(1-\nu^2)$. It should be noted that the current results are in excellent agreement with the HSDT solutions, especially when the parameter nth-order of the shear strain shape function $f(z)$ proposed in this analysis is equal to 3.

In the second part, a comparison of the non-dimensional natural frequencies for simply supported orthotropic square plates is presented in Table 4 for thickness ratio10 and for all modes of free vibration. The non-dimensional form used while presenting numerical result of natural frequencies is denoted by $\varpi = \omega h / \sqrt{\rho / Q_{11}}$. The analytical solutions obtained from the present theory are compared with the FSDT and HSDT presented by Reddy (1984b) and the mixed first-order transverse shear deformation plate theory MFPT given by Zenkour (2001), and exact elasticity solutions given by Srinivas *et al*. (1970) for free vibration of orthotropic plates. It can be seen that the frequency values obtained using present theory are in good agreement with those obtained by other theories available in the literature.

Table 3 Comparison of non-dimensional natural frequencies ϖ of simply supported isotropic rectangular plate (*a/b=2*)

h/b	Theory	Mode								
		(1,1)	(2,1)	(3,1)	(1,2)	(2,2)	(3,2)	(1,3)	(2,3)	(3,3)
0.1	FSDT [(a)]	1.2227	1.9317	3.0762	3.9611	4.6084	5.6580	8.0453	8.6162	9.5468
	HSDT [(a)]	1.2227	1.9317	3.0763	3.9614	4.6088	5.6588	8.0475	8.6188	9.5505
	Present n = 3	1.2227	1.9317	3.0763	3.9614	4.6088	5.6588	8.0475	8.6188	9.5505
	Present n = 5	1.2231	1.9327	3.0788	3.9653	4.6140	5.6663	8.0615	8.6346	9.5692
	Present n = 7	1.2236	1.9339	3.0818	3.9702	4.6204	5.6758	8.0796	8.6551	9.5939
	Present n = 9	1.2240	1.9349	3.0841	3.9739	4.6254	5.6831	8.0938	8.6712	9.6133
0.2	FSDT [(a)]	1.1521	1.7679	2.7023	3.3847	3.8656	4.6183	6.2313	6.6006	7.1916
	HSDT [(a)]	1.1522	1.7683	2.7036	3.3872	3.8693	4.6244	6.2457	6.6176	7.2134
	Present n = 3	1.1522	1.7683	2.7036	3.3872	3.8693	4.6244	6.2457	6.6176	7.2134
	Present n = 5	1.1535	1.7711	2.7094	3.3954	3.8793	4.6373	6.2642	6.6372	7.2344
	Present n = 7	1.1551	1.7747	2.7171	3.4068	3.8936	4.6565	6.2951	6.6710	7.2728
	Present n = 9	1.1563	1.7775	2.7231	3.4158	3.9049	4.6719	6.3203	6.6985	7.3044
0.3	FSDT [(a)]	1.0608	1.5768	2.3188	2.8385	3.1962	3.7449	4.8862	5.1426	5.5497
	HSDT [(a)]	1.0612	1.5780	2.3227	2.8454	3.2059	3.7602	4.9191	5.1807	5.5972
	Present n = 3	1.0612	1.5780	2.3227	2.8454	3.2059	3.7602	4.9191	5.1807	5.5972
	Present n = 5	1.0632	1.5819	2.3294	2.8538	3.2153	3.7704	4.9282	5.1889	5.6033
	Present n = 7	1.0660	1.5874	2.3398	2.8681	3.2324	3.7919	4.9588	5.2214	5.6388
	Present n = 9	1.0681	1.5918	2.3482	2.8797	3.2464	3.8098	4.9852	5.2497	5.6703
0.4	FSDT [(a)]	0.9664	1.3970	1.9934	2.4004	2.6771	3.0969	3.9585	4.1505	4.4546
	HSDT[(a)]	0.9673	1.3996	2.0008	2.4130	2.6942	3.1230	4.0118	4.2116	4.5302
	Present n = 3	0.9673	1.3996	2.0008	2.4130	2.6942	3.1230	4.0117	4.2116	4.5300
	Present n = 5	0.9698	1.4037	2.0064	2.4188	2.6995	3.1266	4.0066	4.2034	4.5157
	Present n = 7	0.9734	1.4102	2.0175	2.4332	2.7162	3.1464	4.0318	4.2296	4.5431
	Present n = 9	0.9762	1.4155	2.0268	2.4454	2.7305	3.1639	4.0557	4.2548	4.5704

[(a)] Taken from Shufrin and Eisenberger (2005)

The last analysis represents the largest part that is generally devoted to the study of the free vibration behavior in the case of simply supported multilayered square laminated composite plates. Non-dimensional natural frequencies $\varpi = (\omega a^2 / h) / \sqrt{\rho / E_2}$ of various cross-ply and angle-ply laminated composite plates are obtained and compared with the other theories, considering the effects of thickness ratio (*a/h*), fiber angle (θ) and The number of layers (*N*) with the same thickness. The natural frequencies obtained by using the present theory of two-layer antisymmetric cross-ply (0°/90°) square plates are reported in Table 5. In order to verify with another type of laminates, the non-dimensional natural frequencies of four-layer symmetric cross-ply (0°/90°/90°/0°) square plates are demonstrated in Table 6. As well, to extended the present analytical method, the next example reported in Table 7 shows the non-dimensional natural frequencies of eight-layer antisymmetric cross-ply $(0°/90°)_4$ square plates for different values of thickness ratio (*a/h*) and for the different modes of vibration (*r*, *s*).All the results obtained in this section are compared with corresponding values of the first order shear deformation theory FSDT and higher order shear deformation theory HSDT which are included in the Reddy's (1985) reference, and the numerical results given by Senthilnathan *et al*. (1988) based on the simplified higher-order shear deformation plate theory SHSDT to predict the vibration responses of simply supported laminated composite plates and the results generated by the classical plate theory CPT. It should be noted that the "*nth*-HSDT" considered involves four independent variables as against five in case of FSDT and HSDT, hence the present theory does not require shear correction factor. It can be clearly seen that all types of thick laminates plates (*a/h* is taken from 2 to 5); the FSDT underestimates the natural frequencies when compared to the HSDT and SHSDT, but can generally, be considered acceptable for moderately thick laminates plates. It is also observed that the CPT overestimates the natural frequencies of laminated plates due to neglect of the transverse shear deformation effect for thin plates. It can be noticed in our case that the increase in the thickness ratio has a significant effect on the behavior of free vibration of the square laminated composite plates as he can considerably increase the non-dimensional natural frequencies. However, the analysis of Tables 5-7 reveals that the results obtained using present theory are in

Table 4 Comparison of non-dimensional natural frequencies $\overline{\varpi}$ of simply supported orthotropic square plate (*a/h* = 10)

r	*s*	Exact [(a)]	HSDT [(b)]	FSDT [(b)]	MFPT [(c)]	Present			
						n = 3	n = 5	n = 7	n = 9
1	1	0.0474	0.0474	0.0474	0.0474	0.0476	0.0477	0.0477	0.0478
1	2	0.1033	0.1033	0.1032	0.1032	0.1039	0.1041	0.1042	0.1043
2	1	0.1188	0.1189	0.1187	0.1187	0.1197	0.1199	0.1202	0.1204
2	2	0.1694	0.1698	0.1691	0.1691	0.1721	0.1725	0.1729	0.1732
1	3	0.1888	0.1888	0.1883	0.1883	0.1898	0.1901	0.1905	0.1908
3	1	0.2180	0.2184	0.2175	0.2175	0.2196	0.2202	0.2211	0.2218
2	3	0.2475	0.2477	0.2465	0.2469	0.2520	0.2525	0.2532	0.2538
3	2	0.2624	0.2629	0.2619	0.2614	0.2675	0.2682	0.2692	0.2700
1	4	0.2969	0.2969	0.2959	0.2949	0.2979	0.2985	0.2993	0.3000
4	1	0.3319	0.3330	0.3311	0.3299	0.3340	0.3350	0.3367	0.3382
3	3	0.3320	0.3326	0.3310	0.3297	0.3407	0.3416	0.343	0.3441
2	4	0.3476	0.3479	0.3463	0.3446	0.3533	0.3542	0.3554	0.3563
4	2	0.3707	0.3720	0.3696	0.3677	0.3774	0.3785	0.3805	0.3821

[(a)] Taken from Srinivas *et al*. (1970), [(b)] Taken from Reddy (1984b), [(c)] Taken from Zenkour (2001)

Table 5 Comparison of non-dimensional natural frequencies $\overline{\varpi}$ of simply supported two-layer antisymmetric cross-ply (0°/90°) square laminates

a/h	CPT [(b)]	FSDT [(a)]	HSDT [(a)]	SHSDT [(b)]	Present			
					n = 3	n = 5	n = 7	n = 9
2	8.606	5.191	5.699	5.717	5.7170	5.4833	5.429	5.4182
4	10.424	7.975	8.294	8.354	8.3546	8.2247	8.2023	8.2037
5	10.720	8.757	9.010	9.087	9.0871	8.9872	8.9717	8.9744
10	11.153	10.355	10.449	10.567	10.5680	10.5329	10.5287	10.5309
12.5	11.208	10.622	10.686	10.813	10.8135	10.7900	10.7871	10.7887
20	11.269	10.941	10.968	11.105	11.1051	11.0953	11.0942	11.095
25	11.283	11.020	11.037	11.176	11.1768	11.1704	11.1697	11.1702
50	11.302	11.127	11.132	11.275	11.2751	11.2734	11.2733	11.2734
100	11.306	11.155	11.156	11.300	11.3001	11.3000	11.3000	11.3000

[(a)] Taken from Reddy (1985), [(b)] Taken from Senthilnathan *et al*.(1988)

excellent agreement with SHSDT proposed by Senthilnathan *et al*. (1988) for various values of thickness ratio (*a/h*) when the parameter nth-order of the shape function $f(z)$ takes a value of 3.

To prove the credibility and precision of the present theory, another example is added for a range of a simply supported angle-ply laminated composite plates for different values of the thickness ratio (*a/h*). The results of non-dimensional natural frequencies are presented in Tables 8 and 9 for both laminates, two-layer (45°/-45°) and eight-layer $(45°/-45°)_4$ square antisymmetric angle-ply, respectively. All the individual layers have equal thickness. The obtained results are also compared with those proposed by Senthilnathan *et al*. (1988) using the SHSDT and CPT theories and those given by Reddy (1985) using the FSDT and HSDT theories. It is found that the present results are in good agreement with the results provided by Reddy's HSDT and Senthilnathan *et al*. (1988) using the SHSDT for all values of the thickness ratio.

In the last example, a comparison of non-dimensional natural frequencies with the thickness ratio *a/h*=10 is shown in Table 10 for a simply supported antisymmetric angle-ply square laminated composite plates for a different number of layers, with the lamination scheme (45°/-45°) and (30°/-30°). The results obtained from the present method are compared with the other higher order theories predicted by Reddy (1979), Bert and Chen (1978) and Maiti and Sinha (1996). It is also pointed out from the numerical results presented in Table 10 that the present nth-order theory in the particular case where (n = 3), predicts good values of natural frequencies as compared to that obtained by Bert and Chen (1978).

Table 6 Comparison of non-dimensional natural frequencies $\overline{\varpi}$ of simply supported four-layer symmetric cross-ply (0°/90°/90°/0°) square laminates

a/h	CPT [(b)]	FSDT [(a)]	HSDT [(a)]	SHSDT [(b)]	Present			
					n = 3	n = 5	n = 7	n = 9
2	15.902	5.492	5.576	6.0017	6.0017	5.8765	5.8876	5.9180
4	17.989	9.369	9.497	10.230	10.2032	10.2119	10.2816	10.3440
5	18.298	10.820	10.989	11.770	11.7710	11.8020	11.8776	11.9418
10	18.737	15.083	15.270	15.940	15.9405	15.9792	16.0317	16.0733
12.5	18.792	16.120	16.276	16.828	16.8288	16.8598	16.8998	16.9313
20	18.852	17.583	17.668	17.993	17.9938	18.0096	18.0289	18.0440
25	18.866	17.991	18.050	18.301	18.3010	18.3118	18.3249	18.3350
50	18.885	18.590	18.606	18.738	18.7381	18.7410	18.7445	18.7473
100	18.889	18.751	18.755	18.852	18.8526	18.8534	18.8543	18.8550

[(a)] Taken from Reddy (1985), [(b)] Taken from Senthilnathan *et al*.(1988)

Table 7 Comparison of non-dimensional natural frequencies $\overline{\varpi}$ of simply supported eight-layer antisymmetric cross-ply $(0°/90°)_4$ square laminates

a/h	CPT [(b)]	FSDT [(a)]	HSDT [(a)]	SHSDT [(b)]	Present			
					n = 3	n = 5	n = 7	n = 9
2	15.441	5.623	5.9263	5.9263	5.9263	5.8274	5.8519	5.8896
4	17.585	9.9843	10.080	10.080	10.0800	10.1091	10.1892	10.2576
5	17.903	11.565	11.618	11.618	11.6183	11.6668	11.7512	11.8204
10	18.351	15.681	15.673	15.673	15.6735	15.7203	15.7766	15.8201
12.5	18.407	16.541	16.530	16.530	16.5303	16.5670	16.6096	16.6423
20	18.469	17.657	17.649	17.649	17.6496	17.6679	17.6884	17.7039
25	18.483	17.949	17.944	17.944	17.9441	17.9565	17.9703	17.9807
50	18.502	18.363	18.362	18.361	18.3622	18.3656	18.3693	18.3721
100	18.507	18.472	18.471	18.470	18.4717	18.4725	18.4735	18.4742

[(a)] Taken from Reddy (1985), [(b)] Taken from Senthilnathan *et al*.(1988)

Table 8 Comparison of non-dimensional natural frequencies $\overline{\varpi}$ of simply supported two-layer antisymmetric angle-ply (45°/-45°) square laminates

a/h	CPT [(b)]	FSDT [(a)]	HSDT [(a)]	SHSDT [(b)]	Present			
					n = 3	n = 5	n = 7	n = 9
2	6.882	5.520	6.283	6.337	6.3368	5.9386	5.8326	5.8021
4	13.505	9.168	9.759	9.759	9.7593	9.5004	9.4444	9.4367
5	13.885	10.335	10.840	10.839	10.8398	10.6285	10.5855	10.5818
10	14.439	13.044	13.263	13.263	13.2631	13.1772	13.1621	13.1629
12.5	14.509	13.550	13.704	13.704	13.7040	13.6444	13.6342	13.6350
20	14.587	14.179	14.246	14.246	14.2463	14.2206	14.2164	14.2169
25	14.605	14.338	14.383	14.382	14.3828	14.3660	14.3632	14.3636
50	14.629	14.561	14.572	14.572	14.5723	14.5680	14.5673	14.5674
100	14.635	14.618	14.621	14.621	14.6212	14.6201	14.6200	14.6200

[(a)] Taken from Reddy (1985), [(b)] Taken from Senthilnathan *et al*.(1988)

Table 9 Comparison of non-dimensional natural frequencies $\bar{\omega}$ of simply supported eight-layer antisymmetric angle-ply $(45°/-45°)_4$ square laminates

a/h	CPT [b]	FSDT [a]	HSDT [a]	SHSDT [b]	Present			
					n = 3	n = 5	n = 7	n = 9
2	6.882	5.848	6.283	6.314	6.3140	6.0598	6.0417	6.0648
4	13.765	10.842	10.991	10.990	10.9905	10.9644	11.0486	11.1293
5	17.207	12.892	12.972	12.971	12.9720	12.9956	13.1007	13.1927
10	25.052	19.289	19.266	19.265	19.2660	19.3446	19.4480	19.5294
12.5	25.128	20.196	20.888	20.888	20.8885	20.9586	21.0443	21.1108
20	25.212	23.259	23.239	23.238	23.2388	23.2800	23.3269	23.3626
25	25.231	23.924	23.909	23.909	23.9091	23.9384	23.9711	23.9960
50	25.257	24.909	24.905	24.904	24.9046	24.9131	24.9224	24.9294
100	25.264	25.176	25.174	25.174	25.1745	25.1767	25.1791	25.1809

[a] Taken from Reddy (1985), [b] Taken from Senthilnathan *et al*.(1988)

Table 10 Comparison of non-dimensional natural frequencies $\bar{\omega}$ of simply supported multilayered antisymmetric angle-ply $(\theta°/-\theta°)_n$ square laminates, (a/h = 10)

Configuration	Theory	Number of layers		
		2	4	6
$(45°/-45°)_n$	Ref [a]	15.714	18.609	18.295
	Ref [b]	13.040	18.460	19.290
	Ref[c]	14.482	17.411	18.326
	Present n =3	13.2631	18.3221	19.0248
	Present n =5	13.1772	18.4757	19.1207
	Present n =7	13.1621	18.5895	19.2314
	Present n =9	13.1629	18.6681	19.3148
$(30°/-30°)_n$	Ref [a]	15.001	17.689	18.002
	Ref [b]	12.680	17.630	18.230
	Ref[c]	13.941	16.642	11.471
	Present n =3	12.9283	17.6618	18.3353
	Present n =5	12.8578	17.7972	18.4218
	Present n =7	12.8476	17.8981	18.5205
	Present n =9	12.8502	17.9679	18.5948

[a] Taken from Reddy (1979), [b] Taken from Bert and Chen (1978), [c] Taken from Maiti and Sinha (1996)

5. Conclusions

A simple refined nth-higher-order shear deformation theory is used to analyze the free vibration of isotropic, orthotropic and laminated composite plates. By effecting a modification in the kinematic of the displacement field, with the insertion of an undetermined integral term, the number of independent unknowns and governing equations of motion is reduced to four. The present theory satisfies the shear stress-free boundary conditions on the top and bottom surfaces of the plate and obviates the need of a transverse shear correction factor. Analytical solutions for simply supported thick to thin cross-ply and angle-ply laminated composite plates are solved using Navier’s solution technique. The results of natural frequencies obtained by present theory are compared with those obtained by other theories available in the literature. Through all several problems studied, It is clear that the proposed model is not only accurate but also provides an elegant and efficient approach for vibration behavior of isotropic, orthotropic and laminated composite plates. It can be concluded that the present "*nth*-HSDT" theory give a same numerical results of natural frequencies as compared to HSDT, SHSDT theories and even with the results investigated by Bert and Chen (1978). Other effects can be considered in future such as magnetic field (Bayones and Abd-Alla 2018). An improvement of the present formulation will be considered in the future work to consider the thickness stretching influence by employing quasi-3D shear deformation theories (Bessaim *et al*. 2013, Bousahla *et al*. 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Bourada *et al*. 2015, Hamidi *et al*. 2015, Larbi Chaht *et al*. 2015, Draiche *et al*. 2016, Bouafia *et al*. 2017, Bennoun *et al*. 2016, Benahmed *et al*. 2017, Sekkal *et al*. 2017b, Abualnour *et al*. 2018, Bouhadra *et al*. 2018, Younsi *et al*. 2018, Benchohra *et al*. 2018, Karami *et al*. 2018c,d, Boukhlif *et al*. 2019, Khiloun *et al*. 2019, Zarga *et al*. 2019, Zaoui *et al*. 2019, Boutaleb *et al*. 2019, Bendaho *et al*. 2019) and the wave propagation problem (Yahia *et al*. 2015, Boukhari *et al*. 2016, Benadouda *et al*. 2017, Ait Atmane *et al*. 2017, Karami *et al*. 2017, Fourn *et al*. 2018).

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