Theoretical and experimental modal responses of adhesive bonded T-joints

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Abstract. The modal frequency responses of adhesive bonded T-joint structure have been analyzed numerically and verified with own experimental data. For this purpose, the damped free frequencies of the bonded joint have been computed using a three-dimensional finite element model via ANSYS parametric design language (APDL) code. The practical relevance of the joint structure analysis has been established by comparing the simulation data with the in-house experimental values. Additionally, the influences of various geometrical and material parameters on the damped free frequency responses of the joint structure have been investigated and final inferences discussed in details. It is observed that the natural frequency values increase for the higher aspect ratios of the joint structure. Also, the joint made up of Glass fiber/epoxy with quasi-isotropic fiber orientation indicates more resistance towards free vibration.

Keywords: ANSYS APDL; fiber orientation; free vibration; glass/epoxy composite; T-joint, FEM

1. Introduction

Improved performance with reduced material and manufacturing of adhesively bonded joints permit to use these products widely in many high-end technical applications such as automotive, marine industries, wind turbines, space and aeronautics, etc. One of the techniques of joining composites is through the use of an adhesively bonded structural T-joint. Generally, T-joints are used in aerospace applications that include spar-wing skin and stiffener-wing skin interfaces. These joints are also used in the construction of fuselage bulkhead-to-skin and longer on-to-skin interfaces. For the complete use of bonding technology of these joint structures, it is essential to determine the mechanical properties and behavior of both adhesive and adhered connection of which properties depend on different parameters like geometrical shape, environmental effects, boundary and loading conditions (Mortensen and Thomsen 2002, Shenoi and Violette 1990,

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In the design of adhesive bonded composite joint structures, for the minimum vibration response, a detailed knowledge about the dynamic behavior of the joint structure is important. It is understood that the system damping capacity can be enhanced by adhesively bonded joints. Ghoneam et al. (2009) performed dynamic analysis of adhesively bonded composite joint structures. They investigated the dynamic behavior of the test specimens by considering the influences of adhesive bonded configuration and the corresponding end boundary restrictions. Also, the dynamic responses of an adhesively bonded double containment cantilever joint subjected to a transverse excitation force measured by Apalak et al. (2009). In their analysis, Fast Fourier Transform (FFT) method adopted to compute the damped free vibration responses to determine the first bending natural frequencies and corresponding loss factors. He (2010) studied the dynamic behavior of the single-lap adhesive joints by considering the effect of adhesive layer thickness. The results indicated that the damping of the single-lap adhesive joint increases with increase in thickness of the adhesive layer. Kaya et al. (2004) conducted 3D finite element analysis (FEA) to compute the effects of dynamic responses in the adhesively bonded joint structures subjected to dynamic loadings. Kim et al. (2006) performed both the vibration tests and FEA of the adhesive bonded L-shaped joint for combining bed and column of the micro-EDM (electrical discharge machining) machine to examine dynamic performance like damping characteristics. The structural responses (frequency, deflection and critical buckling load) of the composite and smart structures have also been analyzed largely by different group of researcher (Tounsi and his co-authors,

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2011a, b, 2014, 2016, 2017, 2018, 2019, Civalek 2017a, b) using different kind of kinematic model to reduce the total computational time without hampering the final accuracy. Subsequently, the frequency parameters are predicted by Kolahchi and his co-author (2016, 2018) including layered composite structure as in Tornabene *et al.* (2014). From the results, they recommended the optimal geometrical configuration and material parameters for the high precision micro-EDM machines.

The natural frequencies and the associated loss factors of coupled longitudinal and flexural frequencies are the important factors while analyzing adhesive joints. This has been investigated by Saito and Tani (1994). Finite element (FE) analysis is adopted commonly for the calculation of free vibration frequencies of the isotropic/orthotropic structures. Ko *et al.* (1995), Reddy (1979) and Lin *et al.* (1997) have used this method for evaluating vibration response of bonded composite plates. Oyadiji (2001) analysed the effect of various structural adhesives on the transverse free vibration in case of single-lap cantilevered beam joints. Their analysis showed that adhesive Young's modulus relates directly to the natural transverse frequencies of the single-lap cantilevered beam joints while Poisson's ratio remains the same.

Finding the effects of the adhesive material properties on the mode shapes of the adhesive bonded joint structure is important. Therefore, three-dimensional free vibration and stress analysis were carried out by Gunes et al. (2007) to evaluate the effect of the first ten natural frequencies. Cheng and Nicolas (1992) found out free vibration solutions analytically for the finite circular cylindrical shell having flat end subjected to different support boundary conditions. Free vibration analysis of the delaminated cantilever composite plate was also conducted by Tenek et al. (1993), who followed the finite element method (FEM) in conjunction with 3D-elasticity theory. The analysis to identify the effect by varying dimensions and position on the laminated composite plate's frequency observed by Ju et al. (1995). The dynamic responses of the composite plate for various fiber orientation and different boundary conditions are also analysed Grimes et al. (1994).

It is understood from the above review that, both numerical and analytical approaches have already been reported for the modeling of adhesive bonded joints and analysis of dynamic/frequency responses. The review indicates that the geometry, loading and boundary affect the frequency responses of the adhesive bonded joint structure considerably. However, it is also understood that none of the above literature has done extensive work on the free vibration frequency responses of the adhesive bonded Tjoint structure and its subsequent experimental verification. Hence, to bridge the gap in the literature, the current work aim is to analyse the frequency responses of adhesive bonded T-joints numerically using the commercial FEA. Further, the numerical result validity established by comparing with own experimental results. Lastly, the responses of natural frequency for different aspect ratio and varying thickness has been investigated including the different material properties and varying fiber orientation of the adhesively bonded T-joints.



Fig. 1 Schematic representation of adhesive bonded T-joint



Fig. 2 FE mesh and boundary condition of adhesive bonded T-joint

2. Numerical modeling

The schematic geometrical configuration of adhesive bonded T-joint made with GFRP/epoxy composite material is shown in Fig. 1 and Table 1 shows the dimensions of the sample joint. The meshed view of the adhesive bonded Tjoint along with the boundary conditions is shown in Fig. 2. The eight-nodded three-dimensional structural volume elements termed as SOLID 185 of FE package software ANSYS 17.2 (2017) and material properties as shown in Table 2 have been used for modeling of the adhesive bonded T-joint. Much better-quality finite elements have been used near the overlap regions to confirm convergence of the solution. The system has a built-in edge at the left horizontal end and free at the right and top end. To know about its dynamic behavior analysis has been done as shown in Fig. 2.

In case structure having constant stiffness, mass effects and no time-varying boundary conditions, i.e., displacements, forces, equation motion for an undamped system is represented by

$$[M]{\{\ddot{u}\}} + [k]{\{u\}} = \{0\}$$
(1)

Tuble I Dimensions of the utility's bonded I joint									
Length (L) in meter	Width (W) in meter	Height (h) in meter	Thickness (<i>t</i>) in meter						
0.16	0.18	0.16	0.0025						

Table 1 Dimensions of the adhesive bonded T-joint

Table 2 Mechanical	properties	of the	adhesive	bonded	T	joint
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	Ex (GPa)	<i>E</i> _y (GPa)	Ez(GPa)	ν	G _{xy} (GPa)	Gyz(GPa)	G _{xz} (GPa)	P (kg/m ³)
Glass Fiber	8.739	7.926	7.926	0.17	3.75	1.875	3.75	14.276



Fig. 3 (a) Glass fiber, (b) Epoxy and hardener and (c) Mold release spray

where, nodal $\{\ddot{u}\}$ and $\{u\}$ are acceleration and displacement vectors, [M] is mass matrix and [k] is stiffness matrix. Finite element method yields

$$\left(\left[K\right] - \lambda\left[M\right]\right)\left\{\overline{D}\right\} = \left\{0\right\}$$
(2)

where, $\lambda = \omega^2$

Solution of the above eigenvalue problem is given by

$$\det\left(\left[K\right] - \lambda\left[M\right]\right) = \{0\}$$
(3)

Each of eigenvalue λ_i related to an eigenvector $\{\overline{D}\}$,

which is defined as natural mode. Eigenvalues and eigenvectors are calculated by using the block-Lanczos eigenvalue extraction method, as models have large degree of freedoms (Cook 2007, Apalak *et al.* 2008).

3. Experimental design

3.1 Design and fabrication of laminated composite T-joint

The following raw materials were used for the preparation of laminated composite T-joint (Fig. 3).

- Woven glass fiber
- Epoxy
- Hardener
- Polyvinyl alcohol spray

The hand layup method has been adopted in the current study to fabricate the laminated composite T-joint. Initially, the epoxy and the hardener have been mixed by 10:1 ratio. Two rectangular boxes have been used as a fabrication platform. The mold release sheet spread with the silicone spray is stick on the rectangular boxes. Further, two layers are stacked together in L-shape on both the boxes and join them tightly. Now, two more layers have been added at the top and a flat plate with 40 Kg weight applied above them. The plate is further left for 72 hours for the proper curing. After adequate curing, the T-joint composite has been released from prepared on a required size, i.e., 0.16 m × 0.16 m × 0.18 m as shown in Fig. 4.



Fig. 4 Sample of adhesive bonded T-joint specimen

3.2 Experimental procedure for modal test

In this section, the experimental investigation on free vibration responses of the layered composite T-joint has been presented. The experimental vibration test set up has been presented in Fig. 5. The T-joint Glass-Epoxy composite (1) has been fixed on a clamped (2). The initial excitation has been given by the hammer (086C03) (3) and the acceleration signal has been captured by the accelerometer (352C03) (4) attached on the composite surface. The captured acceleration signal is further transferred to the eight-channel compact data acquisition (cDAQ-9178) (5), for the signal conditioning and conversion (analog to digital) process. Now, the signal is sent to the virtual instrument (VI) circuit (6) made in the LabVIEW platform. The VI program is mainly made for the conversion of the raw acceleration signal from the time domain into the frequency domain via fast furrier transformation (FFT). Now, the final acceleration amplitude-frequency graph can be seen from the output window (7) and the frequency responses for different mode can be recorded for the comparison purpose.

4. Results and discussion

4.1 Validation study

For the validation and comparison study, an adhesively bonded double containment cantilever joint has been analysed (Apalak *et al.* 2008). The necessary input parameters, i.e., geometry and material properties including the end boundaries etc. required for the comparison purpose are taken as same as the reference (Apalak *et al.* 2008). The first five natural frequencies are obtained and the comparison between the present and the reference plotted in Fig. 6. The comparison between the present FE and reference values are following very good agreement.



Fig. 5 Experimental set up to measure free vibration.1. Glass-epoxy composite T-joint 2. Fixture 3. Impact hammer 4. Accelerometer 5. cDAQ 6. Block diagram of the LABVIEW 7. Output window



Fig. 6 Natural frequencies with corresponding mode shapes of adhesive bonded double containment cantilever joint

4.2 Comparison of numerical with experimental results

In this section, the comparison between the own experimental frequencies and the corresponding simulation data (FEA) for the adhesive bonded laminated composite Tjoint. The comparison of experimental and FEA results has been plotted in Fig. 7. The Figure shows that the natural frequencies from both experiment and FEA are in good agreement, but experimental results are slightly less than those predicted using FEA. This is due to the reason that in the experimental case the support may not be perfectly rigid, as it is manually applied. However, in case of FE analysis, the support is more uniform and accurate, there is no chance of having a gap with the support.



Fig. 7 Comparison between numerical and experimental Result



Fig. 8 Mode shapes of the adhesive bonded laminated composite T-joint for free vibration

Further, the vibration responses of the T-joint structure for different modes are obtained from the current simulation results and plotted in Fig. 8. The first mode shows the displacement in the *x*-direction is comparatively less vibrant than that in *y*-direction. This could cause an impact on the dimensions of the system. It is identified that the adhesive layer has slopes in two different directions in the second mode. If no tearing and peering occur in the bonding and the bonding has sufficient strength, then identical results can be seen in other modes. In-plane bending modes resulted in the first three modes, whereas the fourth one is longitudinal. Tearing or peeling should be taken care of during the calculation of free vibration.

4.3 Parametric study

4.3.1 Natural frequencies of adhesive bonded laminated composite T-joint with varied aspect ratio

Natural frequencies of T-bonded layered structure have been evaluated by varying aspect ratio (L/h), i.e., 0.5 to 2. Here, the length of joint remains constant whereas the height of the joint varies. Fig. 9 shows the variation of five different modes of natural frequency w.r.t the aspect ratios. It is observed from the graph that the natural frequency values increase while the aspect ratio increases. This is due to the bulky nature of the joint structure i.e. the structural instability increases while the width to height ratio not maintained within the allowable limit (0.5 to 0.6). Also, it observed that the joint structural vibration is more predominate when aspect ratio > 1. Hence, L/h ratio should be maintained so that it will not be exceed unity for the Tbonded structure for the current analysis considering the necessary input parameter (Geometry, loading and boundary conditions).

4.3.2 Natural frequencies of adhesive bonded T-joint with varied thickness

The free vibrated frequency data of the adhesive bonded Tjoints have been carried out in this example for varying thickness of the joint structure. Here, the other dimensions, boundary and loading condition of joint remains same as that of experimental analysis. It is found that with an increase in thickness of the joint the natural frequencies increase, which has been depicted in Fig. 10.



Fig. 9 Variation of natural frequencies of adhesive bonded laminated composite T-joint with different L/h ratio



Fig. 10 Variation of natural frequencies of T-bonded layered structure with different thicknesses

Fibre	E _x (GPa)	E y (GPa)	E _z (GPa)	ν	G _{xy} (GPa)	G yz (GPa)	G_{xz} (GPa)	p (kg/m ³)	Order of Anisotropy $\binom{E_x}{E_z}$
Glass	8.74	7.93	7.93	0.17	3.75	1.86	3.75	14.28	1.10
Graphite	26.20	1.49	1.49	0.28	1.04	1.04	1.04	13.80	17.58
Carbon	181.00	10.30	10.30	0.28	7.71	7.17	7.71	13.80	17.57
Boron	207.00	21.00	21.00	0.30	7.00	7.00	7.00	25.20	9.86

Table 3 Material properties of T-joint with varying degree of anisotropy (Jones 2014)

Table 4 Natural Frequency of T-joint with varying orientation

Natural	Orientation								
Frequency	O 1	O ₂	O3	O_4	O5	O ₆			
Mode-1	19.693	19.571	19.419	19.418	19.238	27.114			
Mode-2	48.068	47.943	47.181	47.186	47.515	55.589			
Mode-3	57.168	56.774	56.346	56.335	55.825	77.606			
Mode-4	88.707	88.501	86.898	86.904	87.949	98.367			
Mode-5	164.40	163.40	162.14	162.14	160.67	224.74			
Mode-6	219.15	218.76	215.32	215.41	217.03	256.51			
Mode-7	229.01	227.64	226.81	226.73	224.31	306.85			
Mode-8	252.10	252.68	254.11	254.13	254.55	321.28			
Mode-9	328.72	327.49	322.85	322.83	324.34	396.94			
Mode-10	399.87	400.24	397.29	397.30	400.46	470.28			

This is due to the increase in structural joint stiffness. Moreover, the frequency of the joint structure does not follow a specific trend i.e., increasing or decreasing sharply for the higher mode while the thickness value increases more than 0.0015 m. Hence, the thickness for the T-structure utilized for the current analysis need to be maintained within 0.0015 m for the stable type of structure.

4.3.3 Natural frequencies of adhesive bonded T-joint with varied material properties (degree of anisotropy)

In this case, an attempt has been made to study the vibration behavior of adhesive bonded T-joint made with Glass fiber, Graphite fiber, Carbon fiber and Boron fiber materials (Table 3) with varying order of anisotropy. Order of anisotropy of a material is the ratio between the in-plane elastic modulus (i.e., E_x) and transverse modulus (i.e., E_z). The natural frequencies for five different modes of the joint structure has been plotted in the Fig. 11. The results show that the natural frquencies increases with increase in order of anisotropy.It is also observed that the joint structure with Glass/epoxy fiber laminate exhibits less vibration in compared to the other three types of fiber laminate. Whereas, One exception can be spotted in case of graphite fibre. Though, it hasorder of anisotropy more than boron and carbon fibres, the natural frequencies are less compared to them. This behavior may be attributed due to the simultaneous effects of both mass and the order of anisotropy of the joint structure. Hence, for the T-bonded joint structure (Geometry, loading and boundary conditions) made with Glass/epoxy fiber laminate would be suitable as it shows less vibration.



Fig. 11 Natural frequency (HZ) of T-bonded structure with different materials

4.3.4 Natural frequencies of adhesive bonded T-joint with varied fiber orientation

In this section, free vibration analysis is being evaluated for the T-joint where the laminated structure made with Glass/epoxy fiber having different lamination lay-up. The different lay-up schemes and natural frequency variations are presented in Table. 4. The different lamination schemes are: $O_1 = [0]_4$ (Unidirectional), $O_2 = [0/90]_s$, $O_3 = [\pm 45]_s$, O_4 $= [\pm 30]_2$, $O_5 = [0/\pm 45/90]$, O₆(Hybrid)=0(Glass)/90(Graphite)/90(Carbon)/0(Boron).

Table 4 indicates that natural frequency of the adhesive bonded T-joint with Quasi-isotropic lamination scheme has less vibration among six lamination schemes investigated. However, Hybrid lamination scheme shows predominately more vibration due to coupling of material anisotropy.

5. Conclusions

In this present study, both experimental and FE free vibration analyses of adhesive bonded laminated composite T-joint have been conducted. The modal test has been conducted using PXIe-1071, an industrial computer (National Instrument make). The vibration frequencies are recorded for the laminated composite T-joint under CFFF support through the accelerometer mounted on the joint. The FE analyses have been carried out using ANSYS (APDL) 17.2. The influences of various geometrical and material parameters on the dynamic responses of the joint structure has been investigated by varying aspect ratio (L/h), thickness and different lamination schemes. The salient observations from the computation are given in the following lines:

- It is observed that natural frequency increases with an increase in aspect ratio (*L/h*). This is due to the bulky nature of the joint structure.
- The responses indicate an increasing frequencies data for the higher thickness of the T-joint.
- It is found that the joint structure with Glass/epoxy fiber laminate exhibits less vibrational frequenies in comparison to other three types of fiber material adopted for the joint structure. This may be due to the simultaneous effects of both mass and degree of anisotropy of the joint structure.
- The study also indicates that the T-joint made with quasi-isotropic lamination scheme shows more resistance to free vibration among six lamination schemes.

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