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The new odd-burr rayleigh distribution for wind speed characterization

Ibrahim Arik[1], Yeliz M. Kantar[*2] and Ilhan Usta[2]

[1]*Science and Art Faculty, Bilecik Seyh Edebali University, Dep. of Statistics, 11230, Bilecik, Turkey*
[2]*Faculty of Science, Department of Statistics, Eskisehir Technical University, 26470, Eskisehir, Turkey*

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Abstract. Statistical distributions are very useful in describing wind speed characteristics and in predicting wind power potential of a specified region. Although the Weibull distribution is the most popular one in wind energy literature, it does not seem to be able to perfectly fit all the investigated wind speed data in nature. Thus, many studies are still being conducted to find flexible distribution for modelling wind speed data. In this study, we propose a new Odd-Burr Rayleigh distribution for wind speed characterization. The Odd-Burr Rayleigh distribution with two shape parameters is flexible enough to model different shapes of wind speed data and thus it can be an alternative wind speed distribution for the assessment of wind energy potential. Therefore, suitability of the Odd-Burr Rayleigh distribution is investigated on real wind speed data taken from different regions in the South Africa. Numerical results of the conducted analysis confirm that the new Odd-Burr Rayleigh distribution is suitable for modelling most of the considered real wind speed cases and it also can be used for predicting wind power.

Keywords: wind speed; wind power; odd Burr-Rayleigh distribution; Weibull distribution

1. Introduction

The most popular and accepted source of renewable energy is wind energy, which produces inexhaustible and cost-effective energy. It is known that the wind energy potential of a specified region is evaluated based on the characteristic of wind speed, which is modelled by statistical distributions. In other words, types of wind speed, stability of wind data, finding a suitable distribution and estimation method for the chosen distribution are very important issues for the accurate estimation of the wind energy potential of a region (Souloukngа *et al*. 2018, Akdag and Guler 2009, Sedghi *et al*. 2015, Seshaiah and Sukkiramathi 2016). Although the Weibull distribution (WD) is an accepted-popular distribution in the wind energy literature (Safari and Gasore 2010, Philippopoulos *et al*. 2012, Ali *et al.* 2018), it does not seem to be able to perfectly fit all the investigated wind data in nature (Akpinar and Akpinar 2007, Kantar and Usta 2008, Zhou *et al*. 2010, Akdag *et al*. 2010, Usta and Kantar 2012, Soukissian 2013, Kantar and Usta 2015, Usta and Kantar 2016, Mohammadi *et al.* 2017, Kantar *et al.* 2018). Thus, alternative wind speed distributions have been introduced and tested for wind speed data. For example, WD and the Rayleigh distribution (RD), have been used to estimate the wind power potential of most of regions in the world (Safari and Gasore 2010, Ali *et al.* 2018). However, it is observed that WD and RD with respectively two and one parameters, are not enough flexibility for modelling most of the different wind speed types, such as low or high, skewed and/or kurtotic, multimodal wind speed (Kantar *et al.* 2018).Therefore, various statistical distributions (Zhou *et al*. 2010, Philippopoulos *et al.* 2012) are tested relative to WD and RD. Two-component Weibull mixtures are considered for modelling bimodal wind speed data (Akdag *et al.* 2010). Distribution derived from entropy principles have been introduced and evaluated for wind speed data although obtaining their estimates are very difficult in terms of computational problems (Akpinar and Akpinar 2007, Kantar and Usta 2008). The family of distributions such as Johnson SB and skewed generalized error distributions, are presented as wind speed distribution in (Usta and Kantar 2012, Soukissian 2013). On the other hand, upper-truncated Weibull distribution for wind speed data has been examined for the limited wind speed data within a specified range (Kantar and Usta 2015). Additionally, it is shown that while the Nakagami distribution can be an alternative distribution in estimating wind power (Kantar and Usta 2016), the good performances of the Kappa and Wakeby distributions are seen in Morgan *et al.* (2011). A good number of statistical distribution models such as the Birnbaum-Saunders distribution (Mohammadi *et al*. 2017) and extended generalized Lindley distribution (Kantar *et al.* 2018) are introduced and tested for modelling wind speed in the literature and they are seen to be other good alternative distributions for wind speed. More recently, while WD with multiple parameters is tested in (Chalamcharla and Doraiswamy 2016) for estimating the wind power, Hu *et al*. (2017) present a nonparametric kernel distribution to estimate the probability density function of wind speed. As a result of the mentioned studies above, studying flexible probability distribution of wind speed is critical to an accurate assessment of wind energy potential and characteristics for ascertain location.

In this study, we propose the new Odd-Burr Rayleigh distribution (*OBu-RD*) distribution with three parameters,

∗Corresponding author, Professor
E-mail: ymert@anadolu.edu.tr

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which is introduced in (Arik 2018, Arik and Kantar 2019), as an alternative to widely-used wind speed distributions. Thus, *OBu-RD* is applied for the first time to characterize wind speed data. Also, the *OBu-RD* is evaluated to show its capability in modelling real-wind speed data and in estimating wind power.

Considering this potential, *OBu-RD* is compared versus the well-known wind speed distributions such as, WD, RD, Lognormal distribution (LND) and Gamma distribution (GD) on real wind speed data taken from different regions of South Africa, based on different model selection criteria commonly used in the wind energy literature.

It is observed that *OBu-RD* with two shape and one scale parameters provides a more flexible distribution to model a variety of wind speed data and also it has explicit form of moment functions and distribution function to compute easily the wind power and probabilities in certain ranges.

The remainder of this paper is organized as follows: *OBu-RD*, its moments and estimation procedures for the parameters of *OBu-RD* are introduced in Section 2. Alternative wind speed distributions are briefly provided in Section 3. Next, the analysis and results to observe performance of *OBu-RD* are presented in Section 4. Finally, the study is concluded with a number of results in Section 5.

2. Odd Burr Rayleigh Distribution (*OBu-RD*)

Alizadeh *et al*. (2017) defined Odd Burr-G family based on T-X generator (Alzaatreh *et al*. 2013) with additional two shape parameters to obtain wider class of continuous distributions. This family is considered in (Altun *et al.* 2017) as Odd Burr-Lindley distribution and two real data sets on reliability are modelled with the newly obtained distribution. The cumulative density function (cdf) of this family is

$$F(x)=\int_0^{\frac{G(x)}{1-G(x)}}\frac{abt^{a-1}}{\left(1+t^a\right)^{b+1}}dt =1-\frac{\left[1-G(x)\right]^{ab}}{\left\{G(x)^a+\left[1-G(x)\right]^a\right\}^b},\quad x>0, \tag{1}$$

where $G(x)$ is the base distribution. By differentiating the equation (1), the corresponding probability density function (pdf) is obtained in the following form

$$f(x)=\frac{abg(x)G(x)^{a-1}\left[1-G(x)\right]^{ab-1}}{\left\{G(x)^a+\left[1-G(x)\right]^a\right\}^{b+1}} \tag{2}$$

where $a,b>0$ are additional shape parameters.

If $G(x)$ is taken as the classical Rayleigh, the cdf and pdf of arising distribution, called as Odd-Burr Rayleigh distribution (*OBu-RD*), are respectively given as follows

$$F(x)=1-\frac{\left[\exp\left(\frac{-x^2}{2c^2}\right)\right]^{ab}}{\left\{\left[1-\exp\left(\frac{-x^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x^2}{2c^2}\right)\right]^a\right\}^b} \tag{3}$$

$$f(x)=\frac{abx\left[\exp\left(\frac{-x^2}{2c^2}\right)\right]^{ab}\left[1-\exp\left(\frac{-x^2}{2c^2}\right)\right]^{a-1}}{c^2\left\{\left[1-\exp\left(\frac{-x^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x^2}{2c^2}\right)\right]^a\right\}^{b+1}} \tag{4}$$

where c is the scale parameter, a and b are the shape parameters. Therefore, *OBu-RD* with two shape parameters can be considered as a more flexible distribution to model various wind speed data compared to a shape parameterized WD. It can also be noted that *OBu-RD* includes the Rayleigh as a special case.

Mean wind power density (P_D) estimation based on a distributional model is calculated from the following formula

$$P_D=\frac{1}{2}\rho A\int_0^\infty v^3 f(v)\,dv=\frac{1}{2}\rho A\mu_3 \tag{5}$$

where $f(v)$ is the pdf and ρ is air density (kg/m^3), A is the wind turbine blade sweep area (m^2) and μ_3 is the third moment of a distribution. So that the third moment is very important to calculate the wind power density and r*th* moment of *OBu-RD* can be explicitly expressed as follow

$$\mu_r^{OBu-R}=E_{OBu-R}\left(X^r\right)=2^{r/2}c^r\Gamma\left(\frac{r}{2}+1\right) \times\sum_{k=0}^{\infty}n_{k+1}(k+1)\left[\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}}{(j-1)^{\frac{r}{2}+1}}\right] \tag{6}$$

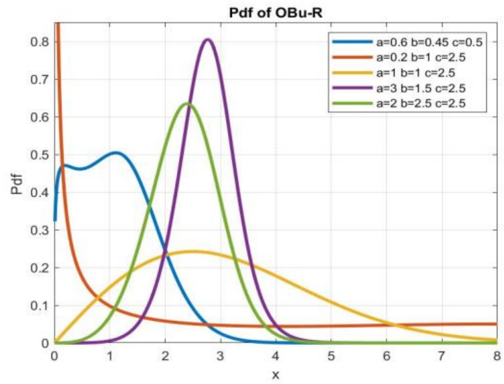

Fig. 1 Plots of the *OBu-RD* pdf for some parameter values

Also, as can be seen from the Fig. 1, the *OBu-RD* pdf is very flexible and it presents a wide range of shapes for the values of different parameters. Moreover, *OBu-RD* pdf can model bimodal data.

On the other hand, the other issue after determining wind speed distribution is parameter estimation method (Chang 2011). Maximum likelihood estimation (MLE) is one of the most common estimation method due to having some good statistical properties for large sample sizes (Kantar and Senoglu 2008, Gebizlioglu *et al.* 2012). Moreover, we have also analysed the performances of MLE for the *OBu-RD* versus other least squares estimation methods via simulation and it is observed that MLE is generally provides good performances. Estimation results are available upon request. For this reason, MLE method is used to estimate the parameters of *OBu-RD* in this study. MLE is obtained by maximizing the log-likelihood function given in (7).

$$\begin{aligned}\log L(a,b,c) &= n\log(ab)+\sum_{i=1}^{n}\log\left(\frac{x_i}{c^2}\right)-\frac{ab}{c^2}\sum_{i=1}^{n}x_i^2\\ &+(a-1)\sum_{i=1}^{n}\log\left\{1-\exp\left(\frac{-x^2}{2c^2}\right)\right\}\\ &-(b+1)\sum_{i=1}^{n}\log\left\{\left[1-\exp\left(\frac{-x^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x^2}{2c^2}\right)\right]^a\right\}\end{aligned} \tag{7}$$

In other word, the following Eqs. (8)-(10) are simultaneously solved with respect to the parameters a,b and c,

$$\begin{aligned}\frac{\partial \log L}{\partial a} &= \frac{n}{a}-\frac{b}{2c^2}\sum_{i=1}^{n}x_i^2-\sum_{i=1}^{n}\log\left\{1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right\}\\ &-(b+1)\sum_{i=1}^{n}\left[\frac{\left[1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a\log\left[1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right]}{\left[1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a}+\frac{\left[\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a\log\left[\exp\left(\frac{-x_i^2}{2c^2}\right)\right]}{\left[1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a}\right]=0\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial \log L}{\partial b} &= \frac{n}{b}-\frac{a}{2c^2}\sum_{i=1}^{n}x_i^2\\ &-\sum_{i=1}^{n}\log\left\{\left[1-\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a+\left[\exp\left(\frac{-x_i^2}{2c^2}\right)\right]^a\right\}=0\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial \log L}{\partial c} &= -\frac{2n}{c}+\frac{ab}{c^3}\sum_{i=1}^{n}x_i^2\\ &-(a-1)\sum_{i=1}^{n}\frac{x^2}{c^3\left[\exp\left(\frac{x^2}{2c^2}\right)-1\right]}\\ &-(b+1)\sum_{i=1}^{n}\left[\frac{ax^2\left[\exp\left(-\frac{x^2}{2c^2}\right)\right]^a}{c^3\left[1-\exp\left(-\frac{x^2}{2c^2}\right)\right]^a+\left[\exp\left(-\frac{x^2}{2c^2}\right)\right]^a}-\frac{ax^2\exp\left(-\frac{x^2}{2c^2}\right)\left[1-\exp\left(-\frac{x^2}{2c^2}\right)\right]^{a-1}}{c^3\left[1-\exp\left(-\frac{x^2}{2c^2}\right)\right]^a+\left[\exp\left(-\frac{x^2}{2c^2}\right)\right]^a}\right]\\ &=0\end{aligned} \tag{10}$$

3. Well-known distributions for wind speed and wind power

The well-known statistical distributions for modelling wind speed are briefly introduced in the following subsections.

3.1 Weibull Distribution (WD)

WD is the most popular distribution in wind energy literature. The pdf of WD is given as follows

$$f(v)=\frac{b}{c}\left(\frac{v}{c}\right)^{b-1}\exp\left(-\left(\frac{v}{c}\right)^b\right). \tag{11}$$

where v is the wind speed (m/s), c and b are respectively scale and shape parameters of WD. WD cdf is given as follows

$$F(v)=1-\exp\left(-\left(\frac{v}{c}\right)^b\right). \tag{12}$$

The r *th* moment and the power density based on 3*rd* moment for WD are respectively given with equation numbers (13) and (14).

$$\mu_r^{WD}=E(V^r)=c^r\Gamma\left(1+\frac{r}{b}\right),\quad v>0 \tag{13}$$

$$P_{WD}=\frac{1}{2}A\rho c^3\Gamma\left(1+\frac{3}{b}\right). \tag{14}$$

3.2 Rayleigh Distribution (RD)

RD is a special case of WD. Thus, RD is less flexible than WD. The formulas of pdf, cdf, *r*th moment and power density corresponding to RD are respectively listed as follows

$$f(v)=\frac{2v}{c^2}\exp\left(-\left(\frac{v}{c}\right)^2\right),\quad v>0 \tag{15}$$

$$F(v)=1-\exp\left(-\left(\frac{v}{c}\right)^2\right), \tag{16}$$

$$\mu_r^{RD}=E\left(V^r\right)=c^r\Gamma\left(1+\frac{r}{2}\right) \tag{17}$$

$$P_{RD}=\frac{1}{2}A\rho c^3\Gamma\left(1+\frac{3}{2}\right). \tag{18}$$

3.3 Gamma Distribution (GD)

GD is another alternative distribution for modelling wind speed. Its applicability to model low wind speeds is reported in (Soholi *et al.* 2016). The pdf of gamma variable is

$$f(v)=v^{b-1}\frac{\exp\left(-\frac{v}{c}\right)}{c^b\Gamma(b)},\quad b,c>0 \tag{19}$$

where c and b are scale and shape parameters of GD, respectively. Gamma cdf is

$$F(v)=\frac{\gamma\left(b,\frac{v}{c}\right)}{\Gamma(b)} \tag{20}$$

where $\gamma(.)$ is the incomplete Gamma function. The *r*th m oment and power density function based on GD are pr ovided as follows

$$\mu_r^{GD}=E\left(V^r\right)=c^r\frac{\Gamma(r+b)}{\Gamma(b)},\quad r>-b \tag{21}$$

$$P_{GD}=\frac{1}{2}A\rho c^3\frac{\Gamma(3+b)}{\Gamma(b)}. \tag{22}$$

3.4 Lognormal Distribution (LND)

LND can be used to represent wind speed data and its pdf is provided as follows

$$f(v)=\frac{\exp\left(-\left(\ln\left(\frac{v}{c}\right)^2/2b^2\right)\right)}{vb\sqrt{2\pi}},\quad c>0 \tag{23}$$

where c and b are respectively scale and shape parameters of LND. The cdf corresponding to LND is

$$F(v)=\Phi\left(\frac{\ln\left(\frac{v}{c}\right)}{b}\right), \tag{24}$$

where Φ is standard normal distribution, the *r*th moment and wind power formula based on LND are respectively presented as follows

$$\mu_r^{LND}=E\left(V^r\right)=\exp\left(r\mu+0.5r^2c^2\right) \tag{25}$$

$$P_{LND}=\frac{1}{2}\rho A\exp\left(3\mu+4.5c^2\right) \tag{26}$$

4. Analysis and results

In order to evaluate the suitability of *OBu-RD*, various analyses have been made by using MATLAB software based on yearly and seasonal wind speed data measured in various regions of Sound Africa. The performance of the proposed distribution is compared with the other well-known distributions by using five model selection criteria, the coefficient of determination (R^2), the Kolmogorov-Smirnov test (KS), root mean square error (RMSE), the Chi-square test value (CHI) and the Akaike information criterion (AIC). In addition to statistical criteria, the power density error (PDE) criterion is used to determine the capability of distribution in estimating wind power. The formulas of the mentioned criteria are given in Table 1.

Table 1 The formulas of criteria for model evaluation

Criteria	Formulas
R^2	$1-\sum_{i=1}^{N}(y_i-x_i)^2\Big/\sum_{i=1}^{N}(y_i-z_i)^2$
KS	$\max_{1\le i\le N}\left(F(v_i)-(i-1)/N,\, i/N-F(v_i)\right)$
RMSE	$\left(\sum_{i=1}^{N}(y_i-x_i)^2\Big/N\right)^{1/2}$
CHI	$\frac{\sum_{j=1}^{n}\left(y_j-x_j\right)}{n-2}$
AIC	$2k-2\ln(L)$
PDE	$\left\lvert\frac{P_{REF}-P_D}{P_{REF}}\right\rvert\times 100$

Table 2 Descriptive numerical measurements for the annual wind speed data

Station	**Data Period**	**Geographic characteristics**			**Height**	**Min**	**Max**	**Mean**	**Var.**	**S**	**K**	**n**
		Lat.	**Long.**	**Elav.(m)**	**(m)**	**(ms^{-1})**	**(ms^{-1})**	**(ms^{-1})**	**(m^2 s^{-2})**			
Station 1	01.01.2015-31.12.2015	28^0 36' S	16^0 39' E	17	62	0.436	21.796	5.957	13.120	0.806	3.101	8760
Station 2	01.01.2014-31.12.2014	31^0 43' S	18^0 25' E	40	62	0.247	19.421	7.108	10.756	0.366	2.454	8760
Station 3	01.01.2014-31.12.2014	31^0 15' S	25^0 01' E	1498	62	0.594	24.213	8.091	12.548	0.667	3.450	8760
Station 3	01.01.2015-31.12.2015	31^0 15' S	25^0 01' E	1498	62	0.653	25.157	7.825	11.220	0.652	3.518	8760
Station 3	01.01.2014-31.12.2015	31^0 15' S	25^0 01' E	1498	62	0.594	25.157	7.958	11.901	0.666	3.499	17520

Note: Latitude, longitude, elevation, minimum, maximum, variance, skewness, kurtosis, number of observation are respectively denoted by Lat., Long., Elev., Min, Max, Var., S, K and n

Table 3 Estimates of parameters of RD, WD, LND, GD and *OBuRD* and the results of criteria for the considered stations

	RD	WD	LND	GD	*OBu-RD*	RD	WD	LND	GD	*OBu-RD*
	Station 1-2015					Station 2 -2014				
c	6.97191	6.70814	1.57792	2.31502	8.40146	7.82766	8.03614	1.83531	1.72010	2.14059
b	-	1.72702	0.68153	2.57334	2.79566	-	2.32266	0.53616	4.13209	0.09918
a	-	-	-	-	0.86544	-	-	-	-	1.52011
KS	0.08636	0.03109	0.04834	0.02784*	0.02778*	0.03704	0.02717*	0.06539	0.04004	0.03687*
R^2	0.86954	0.95070	0.95142	0.96664*	0.95517*	0.96956	0.96994*	0.89683	0.95367	0.97348*
RMSE	0.01624	0.00965	0.01110	0.00817*	0.00922*	0.00758	0.00741*	0.01463	0.00963	0.00681*
CHI	0.00028	0.00010	0.00014	0.00007*	0.00010*	0.00006	0.00006*	0.00024	0.00010	0.00005*
AIC	45776.9	45442.1	45790.4	45352.8*	45417.7*	45324.1	45032.2*	46096.8	45293.9	45096.2*
	Station 3-2014					Station 3 -2015				
c	8.83196	9.13774	1.98650	1.63154	5.00572	8.51167	8.83129	1.95772	1.51030	5.02611
b	-	2.43405	0.47796	4.95885	0.50059	-	2.49019	0.46692	5.18107	0.54154
a	-	-	-	-	1.39732	-	-	-	-	1.42432
KS	0.07627	0.02384	0.04731	0.02205*	0.01593*	0.08278	0.02490	0.04494	0.01886*	0.01321*
R^2	0.94255	0.98609	0.95218	0.99159*	0.99350*	0.93595	0.98681	0.95867	0.99391*	0.99604*
RMSE	0.01100	0.00508	0.00936	0.00388*	0.00337*	0.01200	0.00515	0.00903	0.00346*	0.00276*
CHI	0.00013	0.00003	0.00010	0.00002*	0.00001*	0.00015	0.00003	0.00009	0.00001*	0.00001*
AIC	46905.1	46376.5	46732.5	46228.2*	46225.2*	46114.9	45462.2	45818.8	45315.0*	45311.0*

Note: "*" denotes the first two best distributions determined according to each criterion.

The smallest RMSE, KS, CHI, AIC, PDE and the highest R^2 values demonstrate that the regarded distribution provides a better fitting than the others. However, it should be emphasized that there is no consensus as to which criterion is best for the distributional model in the empirical study (Usta *et al*. 2018).

The hourly wind speed data, used in analysis, is measured at 62 m above ground level in three different regions of South Africa. The data is taken from http://wasadata.csir.co.za/wasa1/WASAData and the regions are illustrated in map in Fig. 2. The Alexander Bay (denoted by Station 1), Vredendal (denoted by Station 2) and Noupoort (denoted by Station 3) are located at the south part of the South Africa. Wind speed data of Station 3 in the last row of Table 2 is considered as long-term wind data. While different statistical characteristics of wind speed data are seen for different stations, annual wind speed data of Station 3 demonstrates similar statistical characteristics with its long-term wind data.

To demonstrate the performance of the proposed *OBu-RD*, comparisons are made with the RD, WD, LND and GD, which are frequently used statistical distributions in the wind literature. The results of the criteria and the estimated values of parameters for annual data are presented for Station 1, Station 2 and Station 3 in Table 3. The long-term measured data for Station 3 is separately analysed in Table 4. The seasonal analyses are given in the Tables 6-8.
According to the Table 3, for Station 1, it can be seen that OBu-RD has the best performance with respect to KS. In term of the other criteria, the proposed OBu-RD follows GD. For Station 2, it is observed form Table 3 that OBu-RD has the best performance according to R2, RMSE and CHI criteria and it has the second best performance for the AIC and KS criteria where WD is the best. Finally, while OBu-RD provides the best performance according to the all considered criteria for the years 2014 and 2015, GD has the second best performance for the Station 3. Table 4 compares the effectiveness of OBu-RD versus all four alternative distributions to model long-term wind speed data at Station 3, in terms of KS, R2, RMSE, CHI and AIC. It is observed from Table 4 that OBu-RD is able to perfectly model a long-term wind data measured at Station 3 as in wind data for 2014 and 2015.

Also, graphs of pdfs corresponding to OBu-RD as well as the RD, WD, LD and GD are provided to get more insight of the fit of these distributions to wind speed data. It can be seen from Figs. 3-6 that OBu-RD pdf exhibits a good fitting for the annual data for Stations 1-3 and for the long term data for Station 3.

Fig. 2 Locations of wind speed observations used in this study

Table 4 Estimates of parameters of RD, WD, LND, GD and *OBu-RD* and the results of criteria for the Station 3 (Long Term)

	RD	WD	LND	GD	*OBu-RD*
Station 3 (2014-2015)					
c	8.67329	8.98493	1.97211	1.57245	4.98716
b	-	2.45809	0.47268	5.06075	0.51279
a	-	-	-	-	1.41278
KS	0.07886	0.02295	0.04509	0.01898*	0.01300*
R^2	0.94136	0.98765	0.95761	0.99415*	0.99600*
RMSE	0.01128	0.00490	0.00896	0.00332*	0.00271*
CHI	0.00013	0.00003	0.00009	0.00001*	0.00001*
AIC	93041.9	91877.5	92568.2	91568.4*	91562.8*

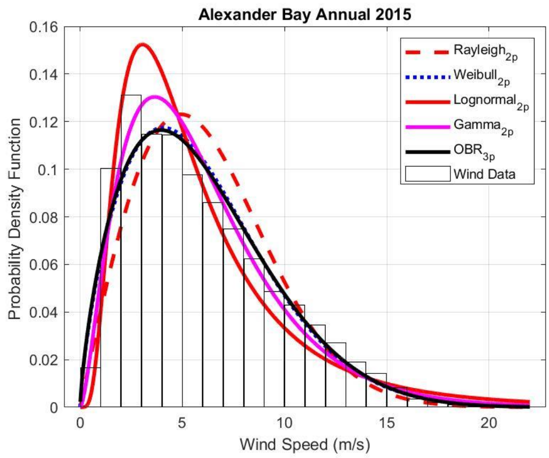

Fig. 3 The histogram and pdfs for yearly wind speed data measured in Station 1

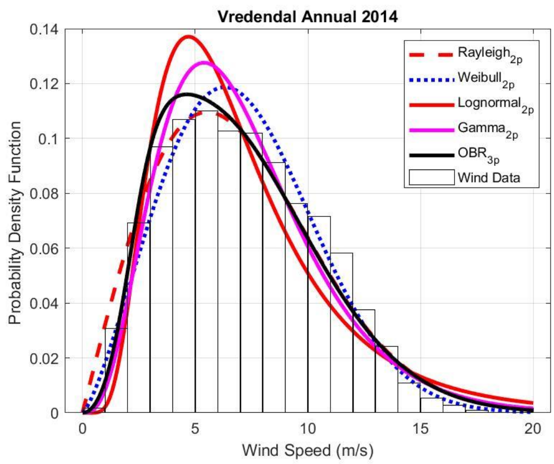

Fig. 4 The histogram and pdfs for yearly wind speed data measured in Station 2

Table 5 Descriptive numerical measurements for the seasonal wind speed data

Station	Min (ms^{-1})	Max (ms^{-1})	Mean (ms^{-1})	Var. (m^2 s^{-2})	S	K	*n*
			Autumn				
Station 1	0.467	18.013	5.254	11.180	0.962	3.404	2208
Station 2	0.247	19.421	6.813	10.228	0.385	2.425	2208
Station 3	0.850	19.526	7.721	11.839	0.623	3.072	2208
			Winter				
Station 1	0.456	14.835	4.946	8.578	0.765	2.899	2208
Station 2	0.475	17.593	6.765	8.502	0.518	3.095	2208
Station 3	0.594	24.213	9.430	16.678	0.434	3.122	2208
			Spring				
Station 1	0.436	19.858	6.681	14.386	0.662	2.811	2184
Station 2	1.016	17.157	7.306	9.953	0.178	2.205	2184
Station 3	1.291	19.568	7.941	11.021	0.492	2.842	2184
			Summer				
Station 1	0.470	21.796	6.979	15.358	0.572	2.666	2160
Station 2	0.507	17.620	7.558	13.974	0.267	2.128	2160
Station 3	1.083	20.550	7.250	7.908	0.631	3.666	2160

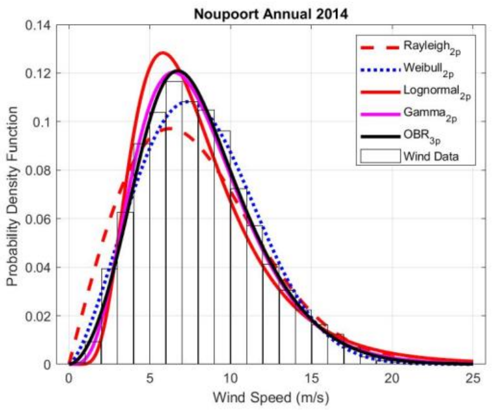

Fig. 5 The histogram and pdfs for yearly wind speed data measured in Station 3

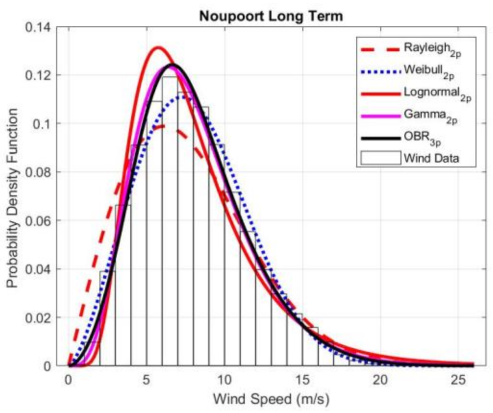

Fig. 6 The histogram and pdfs for long term wind speed data measured in Station 3

For the seasonal analysis, the descriptive numerical measurements for the seasonal wind data of three stations are given in the Table 5. It is seen from Table 5 that different mean, skewness and kurtosis values are observed at each station.

When the results of the seasonal wind speed data analysis are examined for Station 1, it can be seen from Table 6 that *OBu-RD* has comparable results for the considered criteria.

It can be seen from Fig. 7 that *OBu-RD* exhibits a good fitting for seasonal data for Station 1.

The seasonal analyses results are given in Table 7 for the Station 2. It can be seen from Table 7 that *OBu-RD* is the best for autumn, winter and summer for almost all considered criteria. *OBu-RD* has comparable results for spring.

Also, graphs of pdfs are provided for seasonal wind data measured in Station 2. It can be seen from Fig. 8 that *OBu-RD* shows a good fitting for seasonal data for Station 2.

When the results of the seasonal analysis of 2014 are examined for Station 3, it can be seen from Table 8 that *OBu-RD* provides better performances than the other considered distributions for almost all seasons in terms of all considered criteria.

Also looking at the pdfs corresponding to *OBu-RD* as well as RD, WD, LD and GD for seasonal wind data measured in Station 3, it can be seen from Fig. 9 that *OBu-RD* provides better fitting than the other considered distributions.

As a result, the results of analysis indicate that *OBu-RD* is suitable for most of the examined wind speed data cases compared to the considered other distributions commonly-used in the wind energy literature.

Table 6 Estimates of parameters of RD, WD, LND, GD and *OBuRD* and the results of criteria for the Station 1 (Seasonal)

	RD	WD	LND	GD	*OBu-RD*	RD	WD	LND	GD	*OBu-RD*
	Autumn					Winter				
c	6.22756	5.90953	1.44553	2.10531	9.87911	5.74771	5.57942	1.40363	1.81823	6.77436
b	-	1.66285	0.68137	2.49574	4.42186	-	1.77899	0.65979	2.72017	2.70293
a	-	-	-	-	0.83981	-	-	-	-	0.88922
KS	0.11324	0.04222	0.04095	0.03800*	0.03804*	0.08058	0.03666	0.04972	0.02642*	0.03359*
R^2	0.79275	0.91515	0.96417*	0.94690*	0.92076	0.79275	0.91515	0.96417*	0.94690*	0.92076
RMSE	0.02352	0.01480	0.01087*	0.01167*	0.01434	0.01926	0.01244	0.01379	0.01023*	0.01092*
CHI	0.00058	0.00024	0.00013*	0.00015*	0.00022	0.00040	0.00018	0.00022	0.00012*	0.00010*
AIC	11127.2	10988.6	10958.3*	10926.0*	10982.6	10604.1	10552.8	10631.1	10527.2*	10548.6*
	Spring					Summer				
c	7.68173	7.54257	1.71331	2.34784	0.98449	8.00321	7.87512	1.75451	2.48320	0.89068
b	-	1.85153	0.65106	2.84542	0.01772	-	1.86476	0.66141	2.81033	0.01217
a	-	-	-	-	1.85501	-	-	-	-	2.03662
KS	0.05982	0.03249*	0.06226	0.02872*	0.05960	0.04995	0.03084*	0.06319	0.03338*	0.04986
R^2	0.90993	0.94801*	0.92284	0.95769*	0.92633	0.91323	0.94534*	0.89491	0.94516*	0.92819
RMSE	0.01189	0.00875*	0.01285	0.00850*	0.01093	0.01134	0.00871*	0.01378	0.00921*	0.01046
CHI	0.00015	0.00009*	0.00018	0.00008*	0.00014	0.00013	0.00008*	0.00021	0.00009*	0.00013
AIC	11670.0	11649.9*	11810.1	11655.7	11641.7*	11718.0	11702.4*	11926.5	11735.5	11692.3*

Table 7 Estimates of parameters of RD, WD, LND, GD and *OBuRD* and the results of criteria for the Station 2 (Seasonal)

	RD	WD	LND	GD	*OBu-RD*	RD	WD	LND	GD	*OBu-RD*
	Autumn					Winter				
c	7.52618	7.70450	1.78829	1.70842	1.99035	7.36633	7.63364	1.80614	1.38223	5.72877
b	-	2.27773	0.54881	3.98803	0.09449	-	2.48015	0.48770	4.89425	1.09640
a	-	-	-	-	1.49073	-	-	-	-	1.28856
KS	0.03654	0.03078*	0.06209	0.03799	0.03025*	0.07885	0.01971*	0.06273	0.03470	0.01503*
R^2	0.96698*	0.96154	0.90148	0.95398	0.97267*	0.91914	0.98377*	0.89651	0.96363	0.98562*
RMSE	0.00802*	0.00864	0.01466	0.00985	0.00718*	0.01529	0.00636*	0.01629	0.00947	0.00587*
CHI	0.00007*	0.00008	0.00024	0.00011	0.00006*	0.00025	0.00005*	0.00030	0.00010	0.00004*
AIC	11286.4	11231.9*	11516.5	11296.6	11239.7*	11018.0	10864.7*	11074.0	10889.5	10856.0*
	Spring					Summer				
c	7.95770	8.24752	1.87590	1.59062	2.47577	8.43170	8.54983	1.87215	2.17151	1.63923
b	-	2.51381	0.50765	4.59315	0.12707	-	2.15345	0.59033	3.48061	0.04562
a	-	-	-	-	1.54273	-	-	-	-	1.66168
KS	0.05984*	0.04230*	0.07492	0.06028	0.06152	0.06047*	0.05626*	0.08814	0.07460	0.06050
R^2	0.88840*	0.91015*	0.79655	0.86728	0.88332	0.81458*	0.79320	0.73432	0.77522	0.84486*
RMSE	0.01373*	0.01285*	0.02021	0.01584	0.01398	0.01467*	0.01608	0.02067	0.01760	0.01385*
CHI	0.00020*	0.00019*	0.00046	0.00028	0.00024	0.00023*	0.00029	0.00048	0.00035	0.00023*
AIC	11268.1*	11109.0*	11433.6	11224.2	11195.8	11660.4	11644.7*	11943.5	11733.3	11618.8*

PDE is another criterion to choose the best distribution. It is natural to expect that the flexible distribution with the higher fitting is likely to provide less PDE. It can be seen from Table 9 that OBu-RD provides the smallest PDE among the considered distributions for Station 3 for years 2014, 2015 and long-term. For the other stations, OBu-RD is the second good performance in terms of PDE. In conclusion, the results of PDE analyses indicate OBu-RD can be an alternative distribution for the assessment of wind energy potential.

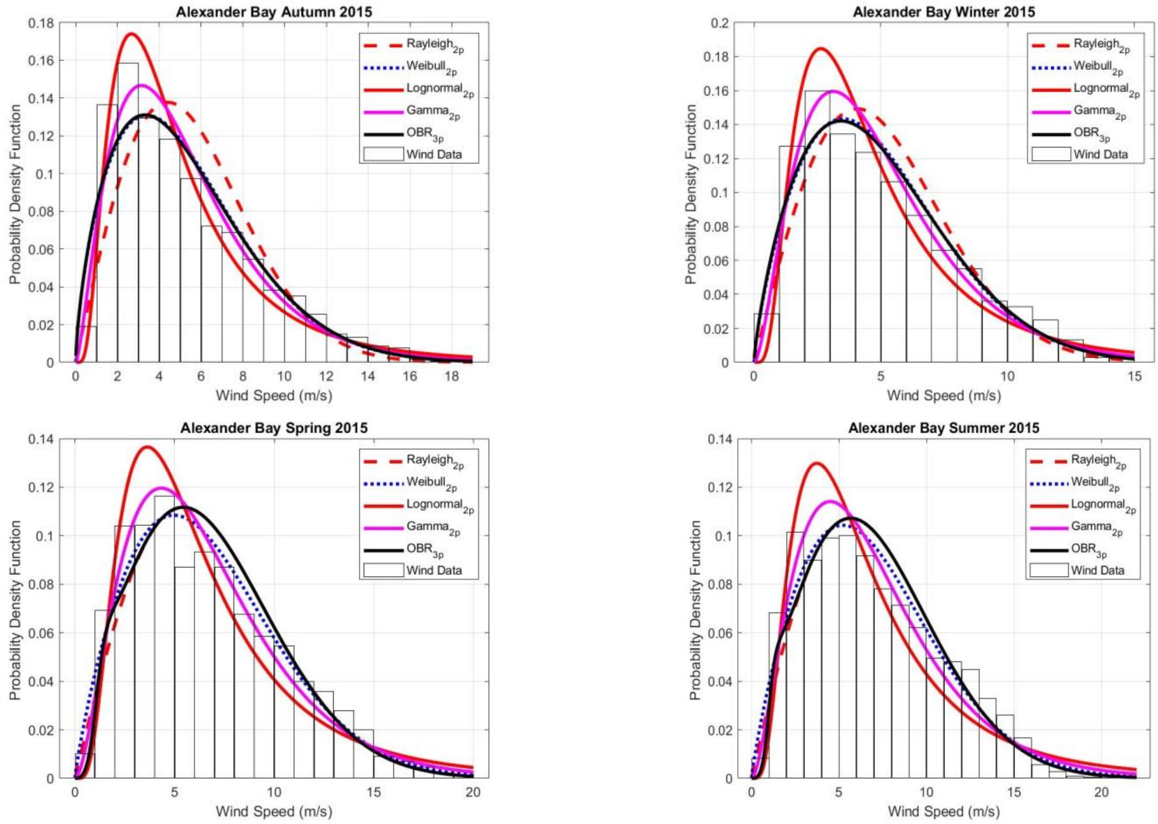

Fig. 7 Pdf graphs for seasonal wind speed data measured in Station 1

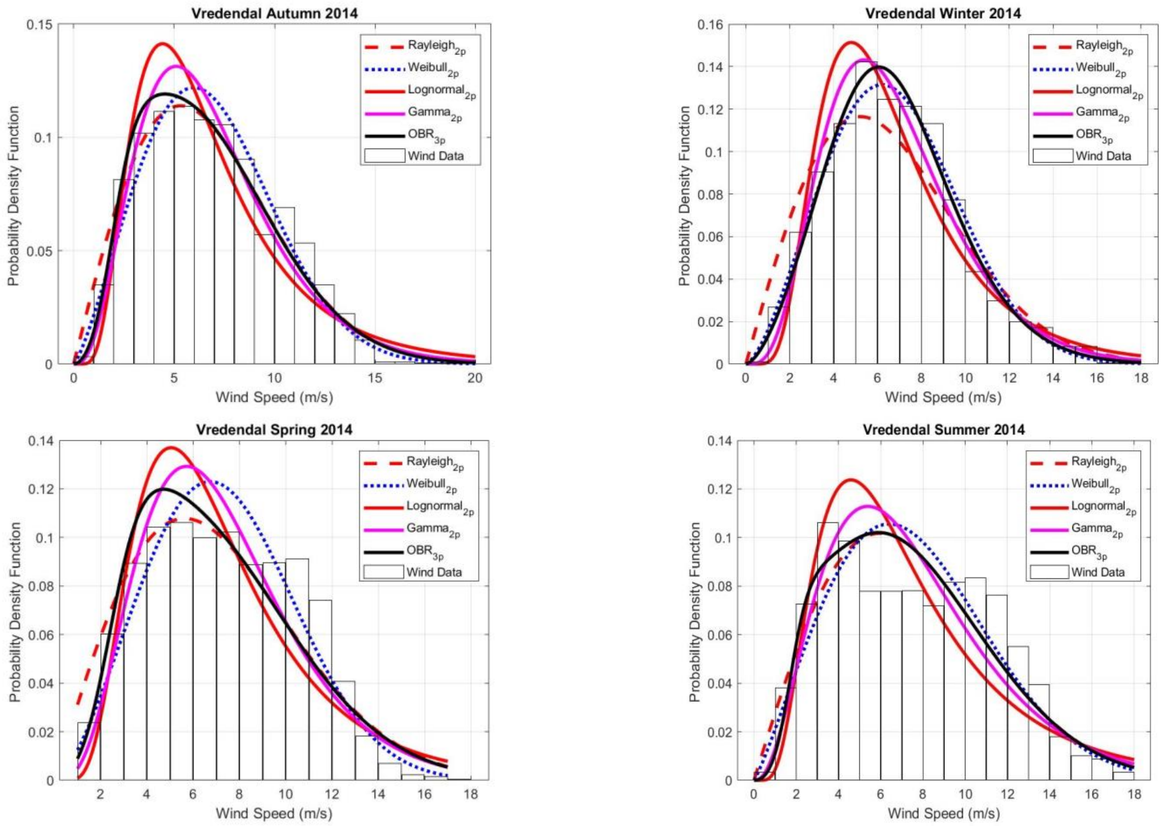

Fig. 8 Pdf graphs for seasonal wind speed data measured in Station 2

Table 8 Estimates of parameters of RD, WD, LND, GD and *OBuRD* and the results of criteria for the Station 3-2014 (Seasonal)

	RD	WD	LND	GD	*OBu-RD*	RD	WD	LND	GD	*OBu-RD*
			Autumn					Winter		
c	8,45277	8,72778	1,93593	1,61238	3,76508	10,27603	10,63366	2,13207	2,03631	10,76013
b	-	2.39869	0.48612	4.78869	0.27707	-	2.46504	0.50998	4.63097	2.25815
a	-	-	-	-	1.49906	-	-	-	-	1.23092
KS	0.07368	0.03045*	0.04912	0.03049	0.02981*	0.08637	0.01697*	0.08262	0.05013	0.01430*
R^2	0.93212	0.96082	0.94010	0.97418*	0.97263*	0.90910	0.98611*	0.83528	0.93563	0.98717*
RMSE	0.01217	0.00834	0.01102	0.00695*	0.00700*	0.01104	0.00408*	0.01448	0.00887	0.00390*
CHI	0.00016	0.00008	0.00013	0.00005*	0.00006*	0.00013	0.00002*	0.00023	0.00009	0.00002*
AIC	11659.9	11547.8	11632.8	11514.1*	11512.6*	12518.8	12376.0*	12710.6	12459.4	12374.3*
			Spring					Summer		
c	8.60651	8.95741	1.97510	1.49257	5.70596	7.77639	8.15164	1.90090	1.13292	5.50984
b	-	2.56974	0.46157	5.32024	0.72086	-	2.74539	0.41498	6.39971	0.79990
a	-	-	-	-	1.39140	-	-	-	-	1.53937
KS	0.09088	0.02436	0.04898	0.02421*	0.01476*	0.11970	0.02712	0.04811	0.02177*	0.01942*
R^2	0.90340	0.97846	0.93710	0.98527*	0.99016*	0.85961	0.97751	0.94399	0.97959*	0.98242*
RMSE	0.01431	0.00616	0.01126	0.00543*	0.00437*	0.02089	0.00808	0.01247	0.00743*	0.00688*
CHI	0.00022	0.00004	0.00014	0.00003*	0.00002*	0.00046	0.00007	0.00017	0.00006*	0.00006*
AIC	11519.5	11316.7	11451.1	11315.1*	11308.3*	10837.2	10513.3	10545.2	10448.5*	10444.4*

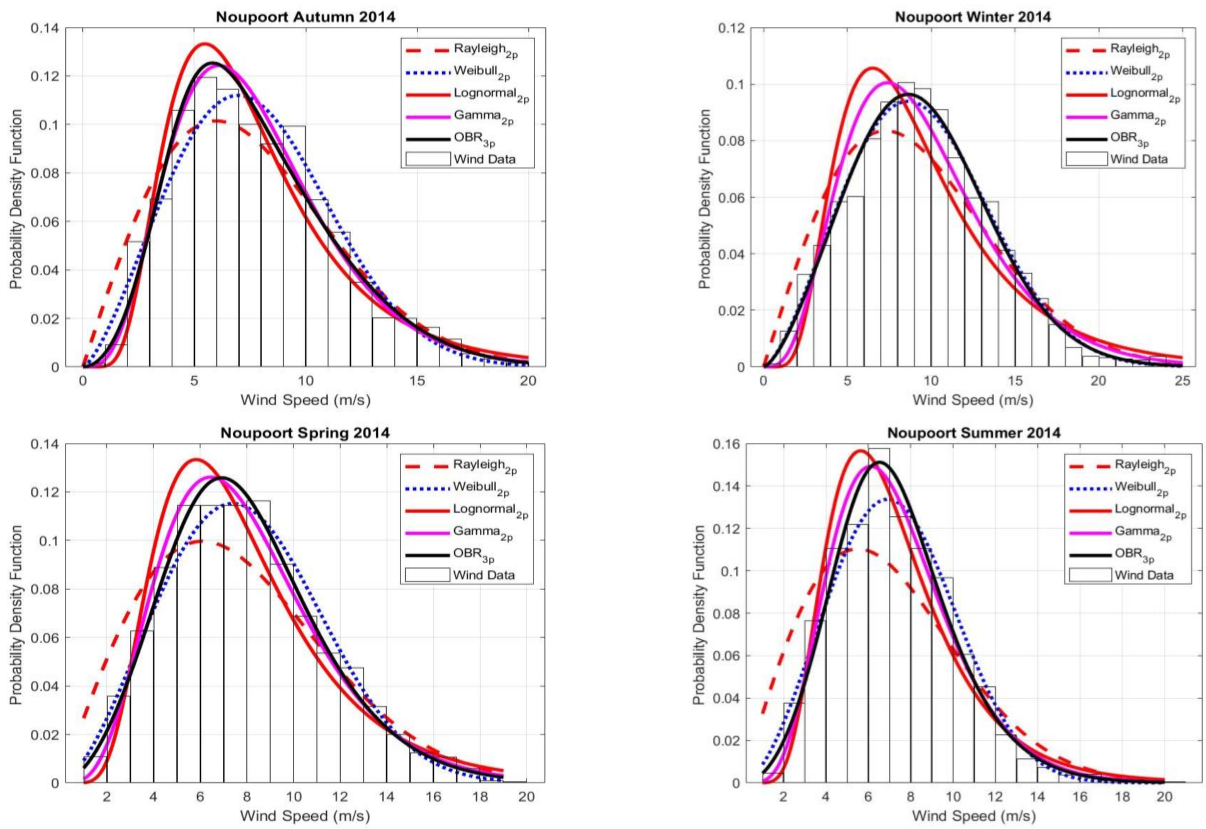

Fig. 9 Pdf graphs for seasonal wind speed data measured in Station 3

Table 9 Wind power density for the RD, WD, LND, GD and *OBu-RD* and corresponding wind power error values (%) and the results of criteria for the considered stations

	Station 1	Station 2	Station 3-2014	Station 3-2015	Station 3-Long Term
P_{REF}	297.2659	369.6861	531.2200	471.0267	501.1233
P_{RD}	275.9294	390.5151	560.9366	502.0957	531.2445
P_{WD}	294.2536	369.0095	524.2537	466.0182	495.0431
P_{LND}	563.2376	549.6713	663.3406	580.5675	621.1183
P_{GD}	319.5762	405.3603	546.9960	485.2525	515.7380
P_{OBuRD}	293.0337	387.4320	532.0130	472.8353	502.3307
PDE_{RD}	7.1776	5.6342	5.5940	6.5960	6.0107
PDE_{WD}	1.0133*	0.1830*	1.3114*	1.0633*	1.2133*
PDE_{LND}	89.4727	48.6859	24.8712	23.2558	23.9452
PDE_{GD}	7.5051	9.6499	2.9698	3.0202	2.9164
PDE_{OBuRD}	1.4237*	4.8002*	0.1493*	0.3840*	0.2409*

5. Conclusions

The main results obtained from the presented study can be listed as follows:

- *OBu-RD* with two shape parameters is proposed for the first time to model wind speed data.
- *OBu-RD* includes the Rayleigh as special case and it is very flexible.
- The performance of *OBu-RD* is evaluated with different model selection criteria and wind power error criterion. The results of analysis show that *OBu-RD* shows superiority over WD and also other well-known distributions for most of the considered seasonal and yearly wind speed measured in some regions of South Africa.
- The analysis for wind power density error criterion again point that *OBu-RD* can provide more accurate results than the considered alternative distributions in estimating the wind power.
- In conclusion, *OBu-RD* is an alternative distribution to be used in the assessment of wind energy potential.

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References

Akdag, S. and Guler, O. (2009), “Calculation of wind energy potential and economic analysis by using Weibull distribution-a case study from Turkey. Part 1: Determination of Weibull Parameters”, *Energy sources part B-economics planning and policy*, **4**, 1-8. https://doi.org/10.1080/15567240802532841.

Akdag, S.A., Bagiorgas, H.S. and Mihalakakou, G. (2010), “Use of two-component Weibull mixtures in the analysis of wind speed in the Eastern Mediterranean”, *Appl. Energy*, **87**(8), 2566-2573. https://doi.org/10.1016/j.apenergy.2010.02.033.

Akpinar, S. and Akpinar, E.K. (2007), “Wind energy analysis based on maximum entropy principle (MEP)-type distribution function”, *Energ. Convers. Manage.*, **48**(4), 1140-1149. https://doi.org/10.1016/j.enconman.2006.10.004.

Ali, S., Lee, S.M. and Jang, C.M. (2018), “Statistical analysis of wind characteristics using Weibull and Rayleigh distributions in Deokjeok-do Island – Incheon, South Korea”, *Renew. Energ.*, **123**, 652-663. https://doi.org/10.1016/j.renene.2018.02.087.

Alizadeh, M., Cordeiro, G.M., Nascimento, A.D.C., Lima, M.C.S. and Ortega, E.M.M. (2017), “Odd-Burr generalized family of distributions with some applications”, *J. Stat. Comput. Sim.*, **87**(2), 367-389. https://doi.org/10.1080/00949655.2016.1209200.

Altun, G., Alizadeh, M. Altun, E. and Ozel, G. (2017), “Odd Burr Lindley distribution with properties and applications”, *Hacettepe J. Math. Stat.*, **46**(2), 255-276.

Alzaatreh, A., Lee, C. and Famoye, F. (2013), “A new method for generating families of continuous distributions”, *Metron*, **71**, 63-79.

Arik, I. (2018), “New distribution families and their statistical properties for survival analysis”, Ph.D. Dissertation; Anadolu University, Eskisehir, Turkey.

Arik, I. and Kantar, Y.M. (2019), “New Odd Burr-Rayleigh distribution: theory and applications”, *Far East J. Theor. Stat.*, **55**(1), 53-82.

Chalamcharla, S.C.V. and Doraiswamy, I.D. (2016), “Mathematical modeling of wind power estimation using multiple parameter Weibull distribution”, *Wind Struct.*, **23**(4), 351-366. https://doi.org/10.12989/was.2016.23.4.351.

Chang T.P. (2011), “Estimation of wind energy potential using different probability density functions”, *Appl. Energ.*, **88**(5), 1848-1856. https://doi.org/10.1016/j.apenergy.2010.11.010.

Hu, B., Li, Y., Yang H. and Wang H. (2017), “Wind speed model based on kernel density estimation and its application in reliability assessment of generating systems”, *J. Modern Power Syst. Clean Energy*, **5**(2), 220-227.

Kantar, Y.M. and Usta, I. (2008), “Analysis of wind speed distributions: wind distribution function derived from minimum cross entropy principles as better alternative to Weibull function”, *Energ. Convers. Manage.*, **49**(5), 962-973. https://doi.org/10.1016/j.enconman.2007.10.008.

Kantar, Y.M. and Senoglu, B. (2008), “A comparative study for the location and scale parameters of the Weibull distribution with given shape parameter”, *Comput. Geosci.*, **34**(12), 1900-1909.

https://doi.org/10.1016/j.cageo.2008.04.004.

Kantar, Y.M. and Usta, I. (2015), "Analysis of the upper-truncated Weibull distribution for wind speed", *Energ. Convers. Manage.*, **96**, 81-88. https://doi.org/10.1016/j.enconman.2015.02.063.

Kantar, Y.M., Usta, I., Arik, I. and Yenilmez, I. (2018), "Wind speed analysis using the Extended Generalized Lindley Distribution", *Renew. Energ.*, **118**, 1024-1030. https://doi.org/10.1016/j.renene.2017.09.053.

Mohammadi, K., Alavi, O. and McGowan, J.G. (2017), "Use of Birnbaum-Saunders distribution for estimating wind speed and wind power probability distributions: A review", *Energ. Convers. Manage*, **143**, 109-122. https://doi.org/10.1016/j.enconman.2017.03.083.

Morgan, C.E., Lackner, M., Vogel, M.R. and Baise G.L. (2011), "Probability distributions for offshore wind speeds", *Energ. Convers. Manage.*, **52**(1), 15-26. https://doi.org/10.1016/j.enconman.2010.06.015.

Philippopoulos, K., Deligiorgi, D. and Karvounis, G. (2012), "Wind speed distribution modeling in the Greater Area of Chania", Greece, *Int. J. Green Energy*, **9**(2), 174-193. https://doi.org/10.1080/15435075.2011.622020.

Safari, B. and Gasore, J. (2010), "A statistical investigation of wind characteristics and wind energy potential based on the Weibull and Rayleigh models in Rwanda", *Renew Energ.***,** **35**(12), 2874-2880. https://doi.org/10.1016/j.renene.2010.04.032.

Sedghi, M., Hannani, S.K. and Boroushaki, M. (2015), "Estimation of weibull parameters for wind energy application in Iran's cities", *Wind Struct.*, **21**(2), 203-221. http://dx.doi.org/10.12989/was.2015.21.2.203

Seshaiah, C.V. and Sukkiramathi, K. (2016), "A mathematical model to estimate the wind power using three parameter Weibull distribution", *Wind Struct.*, **22**(4), 393-408. https://doi.org/10.12989/was.2016.22.4.393.

Soholi, V., Gupta, S. and Nema, R. (2016), "A comparative analysis of wind speed probability distributions for wind power assessment of four sites", *Turk. J. Elec. Eng. & Comp. Sci.*, **24**, 4724-4735.

Souloukngа, M.H., Doka, S.Y., Revanna, N., Djongyang, N. and Kofane, T.C. (2018), "Analysis of wind speed data and wind energy potential in Faya-Largeau, Chad, using Weibull distribution", *Renew. Energ.*, **121**, 1-8. https://doi.org/10.1016/j.renene.2018.01.002.

Soukissian, T. (2013), "Use of multi-parameter distributions for offshore wind speed modeling: The Johnson SB distribution", *Appl. Energ.*, **111**, 982-1000. https://doi.org/10.1016/j.apenergy.2013.06.050.

Usta, I. and Kantar, Y.M. (2012), "Analysis of some flexible families of distributions for estimation of wind speed distributions", *Appl. Energ.*, **89**(1), 355-367. https://doi.org/10.1016/j.apenergy.2011.07.045.

Usta, I. and Kantar Y.M. (2016), "Wind power potential estimation by using different statistical distributions", *DEU Muh. Fak. Fen ve Muh. Dergisi*, **18**(3), 362-380.

Usta, I., Arik, I., Kantar, Y.M. and Yenilmez, I. (2018), "A new estimation approach based on moments for estimating Weibull parameters in wind power applications", *Energ. Convers. Manage.*, **164**, 570-578. https://doi.org/10.1016/j.enconman.2018.03.033.

WASA (2018), Wind Atlas for South Africa; Department: Energy Republic of South Africa, South Africa, http://wasadata.csir.co.za/wasa1/WASAData.

Zhou, J., Erdem, E., Li G. and Shi J. (2010), "Comprehensive evaluation of wind speed distribution models: A case study for North Dakota sites", *Energ. Convers. Manage.*, **51**(7), 1449-1458. https://doi.org/10.1016/j.enconman.2010.01.020.

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