Wind and Structures, *Vol. 28, No. 2 (2019) 89-97*
DOI: https://doi.org/10.12989/was.2019.28.2.089

Linearized analysis of the internal pressures for a two-compartment building with leakage

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(Received November 14, 2017, Revised November 27, 2018, Accepted December 13, 2018)

Abstract. The non-linear equations governing wind-induced internal pressures for a two-compartment building with background leakage are linearized based on some reasonable assumptions. The explicit admittance functions for both building compartments are derived, and the equivalent damping coefficients of the coupling internal pressure system are iteratively obtained. The RMS values of the internal pressure coefficients calculated from the non-linear equations and linearized equations are compared. Results indicate that the linearized equations generally have good calculation precision when the porosity ratio is less than 20%. Parameters are analyzed on the explicit admittance functions. Results show that the peaks of the internal pressure in the compartment without an external opening (Compartment 2) are higher than that in the compartment with an external opening (Compartment 1) at lower Helmholtz frequency. By contrast, the resonance peak of the internal pressure in compartment 2 is lower than that in compartment 1 at higher Helmholtz frequencies.

Keywords: internal pressure; governing equation; linearization; background leakage; admittance function

1. Introduction

Unfavorable opening may occur for doors or windows that have been left open accidentally or broken in severe windstorms. Internal pressures in the room fluctuate because of the entry of outside airflow, which can exacerbate the net load on the roof. Thus, the building might incur damages under the joint action of the external and internal pressures (Shanmugasundaram *et al*. 2000).

To describe the wind-induced fluctuating internal pressure for a building with a dominant opening, Holmes (1979) derived a second-order nonlinear differential equation according to the Helmholtz acoustic resonator principle. A number of studies have revised the internal pressure governing equation for single-cell buildings worldwide. The governing equations presented by Vickery and Bloxham (1992) are more suitable than the ones presented by Liu and Saathoff (1981); the former used the opening loss coefficient to describe the characteristic of damping. Roof flexibility, background leakage, and internal partitioning are the most important factors that affect wind-induced internal pressure. Researchers have theoretically studied the effects of roof flexibility and background leakage on internal pressure for single-cell buildings (Sharma and Richards 1997, Yu *et al.* 2008). The other influence factors including multiple openings, opening location, Helmholtz resonance effect and overshooting effect have recently been experimentally investigated (Pan *et al*. 2013, Wang and Li 2015, Tecle *et al*. 2013, Guha *et al.* 2013a). Besides, opening parameters such as inertial coefficient and loss coefficient that will significantly affect the simulated precision of internal pressure have also been studied by wind tunnel experiments (Xu *et al.* 2014, 2017). However, few focus on internal partitioning.

Saathoff and Liu (1983) pointed out that a series of governing equations can be set up for any room with multiple openings to obtain the time-varying internal pressure. Furthermore, the variation of internal pressure in each room for single-room, double-room and four-room buildings was numerically analyzed. The results show that the peaks of internal pressure and Helmholtz resonance frequencies increase with increasing opening area or decreasing internal volume when sudden breakage of windows and/or doors occur. Miguel *et al.* (2001) presented a second-order non-linear differential model for the internal pressures in enclosures with structures ranging from pores to large apertures. This model also accounts for both the flexibility of the enclosure envelope and deformation of the objects within the enclosure. Sharma (2003) first derived the linearized equations governing the internal pressure for a two-compartment building without background leakage. The effectiveness of the linearized equations was validated by a wind tunnel test. The results indicate that the mean internal pressure coefficients are nearly identical in the two compartments, and the root-mean-square (RMS) of the internal pressure coefficient in the compartment without an external opening is higher than that in the other compartment. Guha *et al.* (2013b) further theoretically obtained the admittance functions and phase functions of internal pressure for a two-compartment building without background leakage. Yu *et al.* (2012) investigated the effects of background leakage on internal pressure for a two-compartment building by using non-linear equations. The precision of the non-linear equations was validated by a wind tunnel test on a rigid model of a low-rise building.

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http://www.techno-press.com/journals/was&subpage=7
ISSN: 1226-6116 (Print), 1598-6225 (Online)

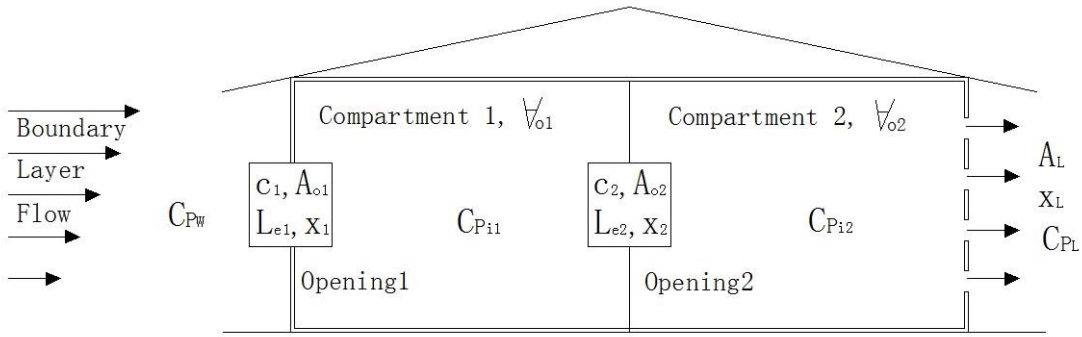


Fig. 1 Model of a two-compartment building with a windward opening and background leakage

Although the non-linear governing equations have high suitability, the explicit admittance functions of both compartments cannot be obtained because of complexity. Numerically solving these equations is also very difficult for engineers. Thus, these equations are not considered for practical applications.

This paper is involved in the equivalent linearized analysis method of non-linear internal pressure for a two-compartment building with background leakage. The non-linear governing equations are first briefly described in this paper. Thereafter a linearized form of the governing equations and corresponding explicit admittance functions are presented. The equivalent damping coefficients of the coupling internal pressure system are iteratively obtained. Finally, the RMS values of the internal pressure coefficients from the linearized governing equations are compared with that from the non-linear governing equations. The process validates the calculation precision of the linearized governing equations. Parameters analyses are also conducted on the explicit admittance functions.

2. Basic assumptions

There is a dominant opening on the windward wall and a dominant opening on the partition wall of the two-compartment building with background leakage. The openings areas are at least 200 times larger than that of single leakage holes. Besides, the following assumptions are made:

(i) The area of the single leakage hole is small, thus, the mass of a single air slug is also small. The role of wall thickness increases as the opening becomes smaller and the damping effect becomes obvious (Oh *et al.* 2007). Thus, the effect of inertial on background leakage can be neglected compared with the damping effect.

(ii) The building is rigid, thus, the effect of building flexibility on the fluctuating internal pressures can be neglected.

(iii) Internal pressures have nothing to do with flow separation and circulation, so they are relevant in their respective compartments. This assumption is the precondition for the theory of internal pressure responses, which has already been verified by experiments (Holmes 1979).

(iv) The adiabatic law is applied for the air in both compartments. The state equation governing the variation of the internal air pressure and density is defined as follows

$$P_i / \rho_i^{\gamma} = \mathrm{C} \tag{1}$$

where γ is the heat ratio of air and is generally equal to 1.4; C is a constant, and P_i and ρ_i are the internal air pressure and density, respectively. After differentiating and simplifying both sides of Eq. (1), the following equation is obtained

$$dP_i = \frac{\gamma P_a}{\rho_i} d\rho_i \tag{2}$$

where P_a is the atmospheric pressure.

(v) The mean external pressure coefficients on the leeward wall are close (Yu *et al.* 2008). Furthermore, the leeward fluctuating external pressures are negligible compared with the fluctuating internal pressures. Therefore, a single mean pressure coefficient can be used to consider the leeward external wind pressure.

(vi) The internal pressure fluctuations in both compartments obey the Gaussian distribution function in the linearization process, which has been validated by a wind tunnel test.

3. Analysis methods of internal pressure

3.1 Non-linear model

A simplified model of wind-induced internal pressure for a two-compartment building with a windward dominant opening and background leakage is shown in Fig. 1. C_{PW}, C_{Pi1}, C_{Pi2} and C_{PL} are the transit wind pressure coefficients of the windward wall opening, Compartment 1, Compartment 2, and leeward wall, respectively. x_1, x_2, and x_L are the air slug displacements at Opening 1, Opening 2, and lumped background leakage, respectively. L_{e1} and L_{e2} are the effective lengths for the air slug at

Openings 1 and 2, respectively. c_1 and c_2 are the flow coefficients at Openings 1 and 2, respectively. A_{o1} and A_{o2} are the areas for Openings 1 and 2, respectively. A_L is the area of the lumped leakage on the leeward side. $\forall_{01}$ and $\forall_{02}$ are the internal volumes of Compartments 1 and 2, respectively.

The non-linear equations governing the internal pressures for a two-compartment building with a windward dominant opening and background leakage are shown in Eqs. (3) and (4). Full details of derivation process can be found in Yu *et al*. (2012).

$$\begin{aligned}&\frac{\rho_a L_{e1}\forall_{01}}{\gamma P_a c_1 A_{o1}}\ddot{C}_{Pi1}+\frac{\rho_a L_{e1}\forall_{02}}{\gamma P_a c_1 A_{o1}}\ddot{C}_{Pi2}+\frac{C_{L1}\rho_a q_0 \forall_{01}^2}{2\gamma^2 P_a^2 c_1^2 A_{o1}^2}\cdot\\&\left|\dot{C}_{Pi1}+\frac{\gamma P_a A_L U_0}{q_0\forall_{01}}\sqrt{\frac{(C_{Pi2}-\bar{C}_{PL})}{C_L'}}+\frac{\forall_{02}}{\forall_{01}}\dot{C}_{Pi2}\right|\cdot\\&\left(\dot{C}_{Pi1}+\frac{\gamma P_a A_L U_0}{q_0\forall_{01}}\sqrt{\frac{(C_{Pi2}-\bar{C}_{PL})}{C_L'}}+\frac{\forall_{02}}{\forall_{01}}\dot{C}_{Pi2}\right)+\\&\frac{\rho_a L_{e1}A_L U_0}{2q_0 c_1 A_{o1}\sqrt{(C_{Pi2}-\bar{C}_{PL})C_L'}}\dot{C}_{Pi2}+C_{Pi1}=C_{PW}\end{aligned}\tag{3}$$

$$\begin{aligned}&\frac{\rho_a L_{e2}\forall_{02}}{\gamma P_a c_2 A_{o2}}\ddot{C}_{Pi2}+\frac{C_{L2}\rho_a q_0 \forall_{02}^2}{2\gamma^2 P_a^2 c_2^2 A_{o2}^2}\cdot\\&\left|\dot{C}_{Pi2}+\frac{\gamma P_a A_L U_0}{q_0\forall_{02}}\sqrt{\frac{(C_{Pi2}-\bar{C}_{PL})}{C_L'}}\right|\cdot\\&\left(\dot{C}_{Pi2}+\frac{\gamma P_a A_L U_0}{q_0\forall_{02}}\sqrt{\frac{(C_{Pi2}-\bar{C}_{PL})}{C_L'}}\right)+\\&\frac{\rho_a L_{e2}A_L U_0}{2q_0 c_2 A_{o2}\sqrt{(C_{Pi2}-\bar{C}_{PL})C_L'}}\dot{C}_{Pi2}+\\&C_{Pi2}-C_{Pi1}=0\end{aligned}\tag{4}$$

where C_{L1}, C_{L2} and C_L' are the loss coefficients at Openings 1, 2 and the lumped leeward opening, respectively. $\bar{C}_{PL}$ is the mean external pressure coefficient on the leeward wall, $q_0 = 0.5\rho_a U_0^2$ is the approaching wind pressure and is selected as the reference wind pressure for all wind pressure coefficients.

3.2 Equivalent linearized model

Although the non-linear internal pressure governing equations have wide application ranges and good generalities, the explicit admittance functions of internal pressure for both compartments cannot be obtained because of complexity. Thus, linearizing the non-linear governing equations is necessary.

3.2.1 Linearized governing equations

According to the unsteady form of the Bernoulli equation (Vickery 1986), the relationship between the airflow rate and the transient pressure drop for Opening 1 is provided by the following equation

$$q_0\left(C_{PW}-C_{Pi1}\right)=\frac{1}{2}C_{L1}\rho_a\dot{x}_1\left|\dot{x}_1\right|+\rho_a L_{e1}\ddot{x}_1 \tag{5}$$

Similarly, the relationship between the airflow rate and the transient pressure drop for Opening 2 is expressed as follows

$$q_0\left(C_{Pi1}-C_{Pi2}\right)=\frac{1}{2}C_{L2}\rho_a\dot{x}_2\left|\dot{x}_2\right|+\rho_a L_{e2}\ddot{x}_2 \tag{6}$$

The linearized equations of motion for air slugs entering the openings on the windward wall and partition wall (Eqs. 5 and 6) can be written as follows

$$\rho_a L_{e1}\ddot{x}_1+c_{j1}\dot{x}_1=q_0(C_{PW}-C_{Pi1}) \tag{7}$$

$$\rho_a L_{e2}\ddot{x}_2+c_{j2}\dot{x}_2=q_0(C_{Pi1}-C_{Pi2}) \tag{8}$$

where c_{j1} and c_{j2} are the equivalent damping coefficients for the two air slugs. Subscripts 1 and 2 refer to the number of the opening.

The relationship between the airflow rate and the transient pressure drop for the lumped leeward opening can be described by considering Assumption (i)

$$q_0\left(C_{Pi2}-C_{PL}\right)=\frac{1}{2}C_L'\rho_a\dot{x}_L{}^2 \tag{9}$$

According to Assumption (vi), the mean of Eq. (9) can be linearized because of the apparent quasi-steady behavior of this equation (Vickery 1986). The resulting equation is expressed as follows

$$\rho_a U_0\sqrt{\left|\bar{C}_{Pi2}-\bar{C}_{PL}\right|}\sqrt{C_L'}\,\dot{x}_L=q_0 C_{Pi2} \tag{10}$$

The law of conservation of mass $\rho_i\Sigma Q=\frac{d}{dt}(\rho_i\forall)$ and Assumption (ii) yields the following

$$\rho_{i1}(c_1A_{o1}\dot{x}_1-c_2A_{o2}\dot{x}_2)=\forall_{01}\frac{d\rho_{i1}}{dt} \tag{11}$$

$$\rho_{i2}(c_2A_{o2}\dot{x}_2-A_L\dot{x}_L)=\forall_{02}\frac{d\rho_{i2}}{dt} \tag{12}$$

Outside air modifies the air density in a building and condenses the air. The relationship between changes in air density and air pressure can be obtained by using the adiabatic law shown in Eq. (2). By substituting Eq. (2) into Eqs. (11) and (12) and simultaneously solving these equations, the following are obtained

$$\dot{x}_1 = \frac{c_2 A_{o2}}{c_1 A_{o1}} \dot{x}_2 + \frac{\forall_{01} q_0 \dot{C}_{Pi1}}{\gamma P_a c_1 A_{o1}} \tag{13}$$

$$\dot{x}_2 = \frac{A_L}{c_2 A_{o2}} \dot{x}_L + \frac{\forall_{02} q_0 \dot{C}_{Pi2}}{\gamma P_a c_2 A_{o2}} \tag{14}$$

By substituting $\dot{x}_L$ from Eq. (10) into Eq. (14), then Eqs. (13) and (14) become the following

$$\dot{x}_1 = \frac{A_L}{c_1 A_{o1}} \frac{q_0 C_{Pi2}}{\rho_a U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}} + \frac{\forall_{02} q_0 \dot{C}_{Pi2}}{\gamma P_a c_1 A_{o1}} + \frac{\forall_{01} q_0 \dot{C}_{Pi1}}{\gamma P_a c_1 A_{o1}} \tag{15}$$

$$\dot{x}_2 = \frac{A_L}{c_2 A_{o2}} \frac{q_0 C_{Pi2}}{\rho_a U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}} + \frac{\forall_{02} q_0 \dot{C}_{Pi2}}{\gamma P_a c_2 A_{o2}} \tag{16}$$

Differentiating both sides of Eqs. (15) and (16) yields the following equations

$$\ddot{x}_1 = \frac{A_L}{c_1 A_{o1}} \frac{q_0 \dot{C}_{Pi2}}{\rho_a U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}} + \frac{\forall_{02} q_0 \ddot{C}_{Pi2}}{\gamma P_a c_1 A_{o1}} + \frac{\forall_{01} q_0 \ddot{C}_{Pi1}}{\gamma P_a c_1 A_{o1}} \tag{17}$$

$$\ddot{x}_2 = \frac{A_L}{c_2 A_{o2}} \frac{q_0 \dot{C}_{Pi2}}{\rho_a U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}} + \frac{\forall_{02} q_0 \ddot{C}_{Pi2}}{\gamma P_a c_2 A_{o2}} \tag{18}$$

Substituting Eqs. (15) to (18) into Eqs. (7) and (8) yields the following

$$\ddot{C}_{Pi1} + \frac{\omega_{12}^2}{\omega_{22}^2} \ddot{C}_{Pi2} + \omega_{c1} \dot{C}_{Pi2} + \omega_{j1} \left(\omega_{c1} C_{Pi2} + \frac{\omega_{12}^2}{\omega_{22}^2} \dot{C}_{Pi2} + \dot{C}_{Pi1} \right) = \omega_{11}^2 C_{PW} - \omega_{11}^2 C_{Pi1} \tag{19}$$

$$\ddot{C}_{Pi2} + \omega_{c2} \dot{C}_{Pi2} + \omega_{j2} \left(\omega_{c2} C_{Pi2} + \dot{C}_{Pi2} \right) = \omega_{22}^2 C_{Pi1} - \omega_{22}^2 C_{Pi2} \tag{20}$$

where $\omega_{11} = \sqrt{\frac{\gamma P_a c_1 A_{o1}}{\rho_a L_{e1} \forall_{01}}}$, $\omega_{22} = \sqrt{\frac{\gamma P_a c_2 A_{o2}}{\rho_a L_{e2} \forall_{02}}}$, $\omega_{12} = \sqrt{\frac{\gamma P_a c_2 A_{o2}}{\rho_a L_{e2} \forall_{01}}}$, $\omega_{c1} = \frac{A_L \gamma P_a}{\rho_a \forall_{01} U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}}$, $\omega_{c2} = \frac{A_L \gamma P_a}{\rho_a \forall_{02} U_0 \sqrt{\left|\bar{C}_{Pi2} - \bar{C}_{PL}\right|} \sqrt{C_L^{'}}}$, $\omega_{j1} = \frac{c_{j1}}{\rho_a L_{e1}}$, and $\omega_{j2} = \frac{c_{j2}}{\rho_a L_{e2}}$.

Eqs. (19) and (20) are the linearized equations governing the internal pressures for a two-compartment building with a windward dominant opening and background leakage.

3.2.2 Admittance functions of internal pressures

Assuming that $C_{Pi1}(0) = \dot{C}_{Pi1}(0) = 0$ and $C_{Pi2}(0) = \dot{C}_{Pi2}(0) = 0$, then $L(\dot{C}_{Pi1}) = SC_{Pi1}(s)$, $L(\ddot{C}_{Pi1}) = S^2 C_{Pi1}(s)$, $L(\dot{C}_{Pi2}) = SC_{Pi2}(s)$, $L(\ddot{C}_{Pi2}) = S^2 C_{Pi2}(s)$, and $L(C_{PW}) = C_{PW}(s)$; L stands for the Laplace transform.

To obtain the explicit admittance functions, Eqs. (19) and (20) are Laplace transformed into frequency domains

$$\left(S^2 + \omega_{j1} S + \omega_{11}^2\right) C_{Pi1}(s) + \left(\frac{\omega_{12}^2}{\omega_{22}^2} S^2 + \omega_{c1} S + \omega_{j1} \omega_{c1} + \frac{\omega_{12}^2}{\omega_{22}^2} \omega_{j1} S \right) C_{Pi2}(s) = \omega_{11}^2 C_{PW}(s) \tag{21}$$

$$\omega_{22}^2 C_{Pi1}(s) - \left(S^2 + \omega_{c2} S + \omega_{j2} \omega_{c2} + \omega_{j2} S + \omega_{22}^2\right) C_{Pi2}(s) = 0 \tag{22}$$

By simultaneously solving Eqs. (21) and (22), the admittance functions can be obtained as follows

$$\left|\chi_{Pi1/Pw}\right|^2 = \left|\frac{C_{Pi1}}{C_{PW}}\right|^2 = \frac{\omega_{11}^4 \left[\left(-\omega^2 + \omega_{j2}\omega_{c2} + \omega_{22}^2\right)^2 + \left(\omega_{c2}\omega + \omega_{j2}\omega\right)^2 \right]}{\left(\omega^4 - \omega_{bb}^2 \omega^2 + \omega_{11}^2 \omega_{22}^2 + \omega_{22}^2 \omega_{j1} \omega_{c1} + \omega_{11}^2 \omega_{j2} \omega_{c2}\right)^2 + \left(\omega_{cc}^3 \omega - \omega_{aa} \omega^3\right)^2} \tag{23}$$

$$\left|\chi_{Pi2/Pw}\right|^2 = \left|\frac{C_{Pi2}}{C_{PW}}\right|^2 = \frac{\omega_{11}^4 \omega_{22}^4}{\left(\omega^4 - \omega_{bb}^2 \omega^2 + \omega_{11}^2 \omega_{22}^2 + \omega_{22}^2 \omega_{j1} \omega_{c1} + \omega_{11}^2 \omega_{j2} \omega_{c2}\right)^2 + \left(\omega_{cc}^3 \omega - \omega_{aa} \omega^3\right)^2} \tag{24}$$

$$\left|\chi_{Pi2/Pi1}\right|^2 = \left|\frac{C_{Pi2}}{C_{Pi1}}\right|^2 = \frac{\omega_{22}^4}{\left(-\omega^2 + \omega_{j2}\omega_{c2} + \omega_{22}^2\right)^2 + \left(\omega_{c2}\omega + \omega_{j2}\omega\right)^2} \tag{25}$$

where ω_{aa}, ω_{bb}^2, and ω_{cc}^3 can be defined as the following

$$\begin{aligned}
&\omega_{aa} = \omega_{j1} + \omega_{j2} + \omega_{c2} \\
&\omega_{bb}^2 = \omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \omega_{j1}\omega_{j2} + \omega_{j1}\omega_{c2} + \omega_{j2}\omega_{c2} \\
&\omega_{cc}^3 = \omega_{j1}\omega_{22}^2 + \omega_{j1}\omega_{12}^2 + \omega_{j2}\omega_{11}^2 + \omega_{11}^2\omega_{c2} + \omega_{22}^2\omega_{c1} + \omega_{j1}\omega_{j2}\omega_{c2}
\end{aligned} \tag{26}$$

When the area of the lumped leeward opening is zero ($A_L = 0$), Eqs. (23) and (24) can be simplified as the admittance functions proposed by Sharma (2003).

Background leakage does not affect the Helmholtz resonance frequencies. Furthermore, Sharma (2003) specified that damped natural frequencies are proximate to undamped natural frequencies for non-leaky buildings. Thus, the Helmholtz frequencies (ω_{oa} and ω_{ob}) can be obtained by setting these coefficients (ω_{c1}, ω_{c2}, ω_{j1}, and ω_{j2}) and the denominator of Eqs. (23) and (24) to zero.

$$\omega_{oa} = 2\pi f_{oa} = \sqrt{\frac{h_1 - \sqrt{h_1^2 - 4h_2}}{2}} \tag{27}$$

$$\omega_{ob} = 2\pi f_{ob} = \sqrt{\frac{h_1 + \sqrt{h_1^2 - 4h_2}}{2}} \tag{28}$$

where $h_1 = \omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2$ and $h_2 = \omega_{11}^2\omega_{22}^2$.

3.2.3 Equivalent damping coefficients

A comparison of Eqs. (5) and (7) yields the following

$$c_{j1} = \frac{1}{2}\rho_a C_{L1}\left|\dot{x}_1\right| \tag{29}$$

By substituting and linearizing $\dot{x}_1$ from Eq. (15) into Eq. (29), c_{j1} can be determined by following the work of Vickery and Bloxham (1992)

$$c_{j1} = \sqrt{8\pi}\frac{C_{L1}\forall_{01}\rho_a q_0 f_{oa}}{\gamma P_a c_1 A_{o1}} \left[\left(\frac{\omega_{c1}^2}{\omega_{oa}^2} + \frac{\forall_{02}^2}{\forall_{01}^2}\right)\tilde{C}_{Pi2}^2 + \tilde{C}_{Pi1}^2 + 2\rho_{12}\sqrt{\left(\frac{\omega_{c1}^2}{\omega_{oa}^2} + \frac{\forall_{02}^2}{\forall_{01}^2}\right)}\tilde{C}_{Pi1}\tilde{C}_{Pi2}\right]^{1/2} \tag{30}$$

Similarly, the damping coefficient c_{j2} is expressed as follows

$$c_{j2} = \sqrt{8\pi}\frac{C_{L2}\forall_{02}\rho_a q_0 f_{oa}}{\gamma P_a c_2 A_{o2}}\tilde{C}_{pi2}\left(1 + \frac{\omega_{c2}^2}{\omega_{oa}^2}\right)^{1/2} \tag{31}$$

where ρ_{12} is the correlation coefficient of the fluctuating internal pressures in both compartments. For the two-compartment building, $\rho_{12} = 1$.

The damping coefficients are coupled with the RMS values of the internal pressure coefficients. Thus, by assuming the initial values for $\tilde{C}_{Pi1}$ and $\tilde{C}_{Pi2}$, c_{j1} and c_{j2} can be calculated from Eqs. (30) and (31). Thereafter, the estimated values of ω_{j1} and ω_{j2} can be obtained, and the admittance functions in Eqs. (23) to (25) can be determined. Finally, the new RMS values of the internal pressure coefficients can be numerically integrated into the following equations

$$\tilde{C}_{Pi1} = \sqrt{\int_0^\infty \left|\chi_{Pi1/PW}\right|^2 S_{CpW}(f)df} \tag{32}$$

$$\tilde{C}_{Pi2} = \sqrt{\int_0^\infty \left|\chi_{Pi2/PW}\right|^2 S_{CpW}(f)df} \tag{33}$$

The convergent RMS values of the internal pressure coefficients and equivalent damping coefficients can be obtained by repeating the aforementioned iterative procedure.

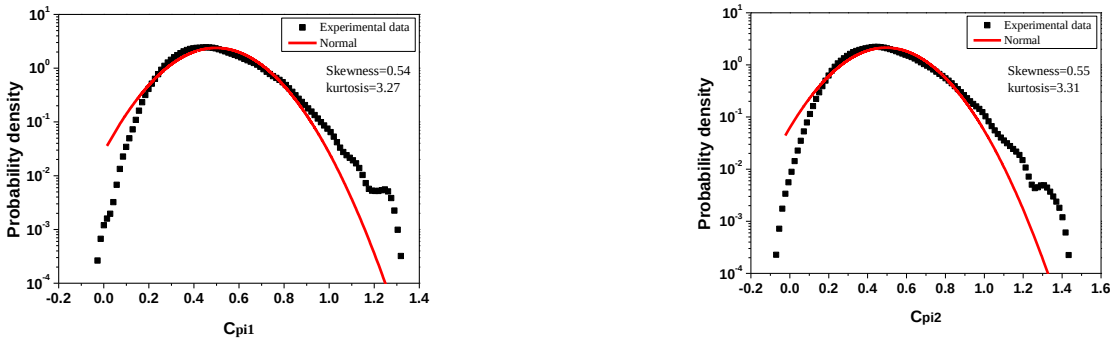
4. Verification of the basic assumptions

A 1:25 scale model of the Texas Tech University (TTU) test building is tested in the TJ-2 Boundary Layer Wind Tunnel Laboratory of the State Key Laboratory for Disaster Reduction in Civil Engineering at Tongji University. Details of the experiment have been described and the precision of the non-linear equations has also been validated (Yu *et al*. 2012).

Yu *et al*. (2012) validated Assumptions (iii) and (v) with wind tunnel test results. Assumption (vi) is validated in this section. The probability density curves of the fluctuating internal pressure coefficients for porosity ratios $A_L/A_{o1} = 0$ and $A_L/A_{o1} = 27.8\%$ are shown in Figs. 2(a) and 2(b) and Figs. 3(a) and 3(b), respectively. The fluctuating internal pressure coefficients for buildings with\without background leakage obey the Gaussian distributions function. The probability density distributions in both compartments are closer to the Gaussian distribution when background leakage is considered. However, background leakage is inherent in buildings. Thus, assuming that the fluctuating internal pressures in both compartments obey the Gaussian distribution is necessary. Therefore, Assumption (vi) is validated.

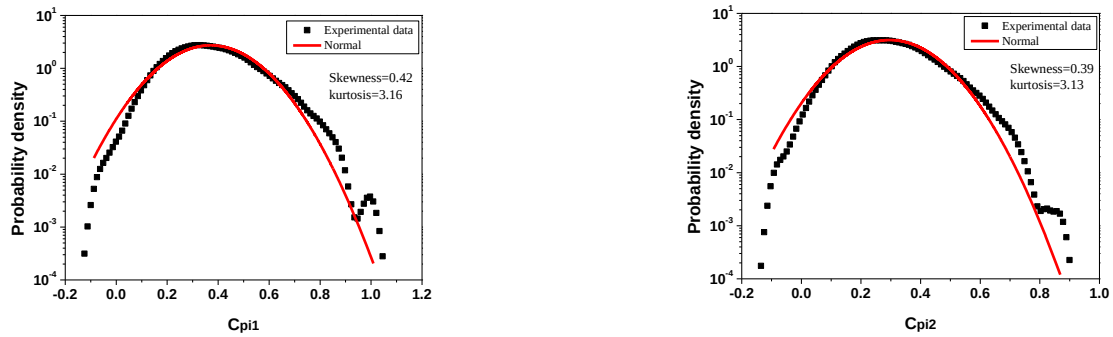
5. Numerical analysis

To describe the precision and characteristics of the linearized equations, the non-linear and linearized governing equations are numerically compared and analyzed in this section. The basic calculation parameters are as follows



(a) Compartment 1 (b) Compartment 2

Fig. 2 Probability density of fluctuating internal pressure coefficients when A_L/A_{o1} = 0



(a) Compartment 1 (b) Compartment 2

Fig. 3 Probability density of fluctuating internal pressure coefficients when A_L/A_{o1} = 27.8%

$\forall_{01} = 300m^3$, $\forall_{02} = 200m^3$, $A_{o1} = 2m^2$, $A_{o2} = 2m^2$, $U_0 = 30m/s$, $\rho_a = 1.22kg/m^3$, $\gamma = 1.4$, $c_1 = 0.6$, $c_2 = 0.6$, $C_{L1} = 1.2$, $C_{L2} = 1.2$, $C_L^{'} = 2.68$, $L_{e1} = 1.39\sqrt{A_{o1}/\pi}$, $L_{e2} = 1.29\sqrt{A_{o2}/\pi}$, $P_a = 101300(Pa)$, $\bar{C}_{PL} = -0.3$, $\bar{C}_{PW} = \bar{C}_{Pi2} = 1$, $q_0 = \frac{1}{2}\rho_a U_0^2 = 549(Pa)$.

The time series of wind speed can be simulated by the superposition of the harmonic based on the Davenport spectrum. The time series of the external pressure coefficients at the windward dominant opening can be obtained by using the quasi-steady state theory. The responses of the internal pressure can then be calculated by substituting the time series of the external pressure coefficients obtained in the last step into the governing equations.

5.1 Comparison between non-linear and linearized models

The RMS values of the internal pressure coefficients for both compartments are obtained from the linearized governing equations according to the iterative method presented in Section 3.2.3 when the same calculation parameters are adopted. Linear results are compared with those calculated from the non-linear equations to analyze the errors. Table 1 shows that the RMS values of the internal pressure coefficients decrease with increasing porosity ratio. When the porosity ratio isn't greater than 20%, the calculation error of the linearized model is within the allowable limits of engineering applications; otherwise, the error will become larger, especially for Compartment 2. A porosity ratio greater than 20% is uncommon for typical low-rise buildings; thus, the linearized governing equations are reasonably suited for practical applications.

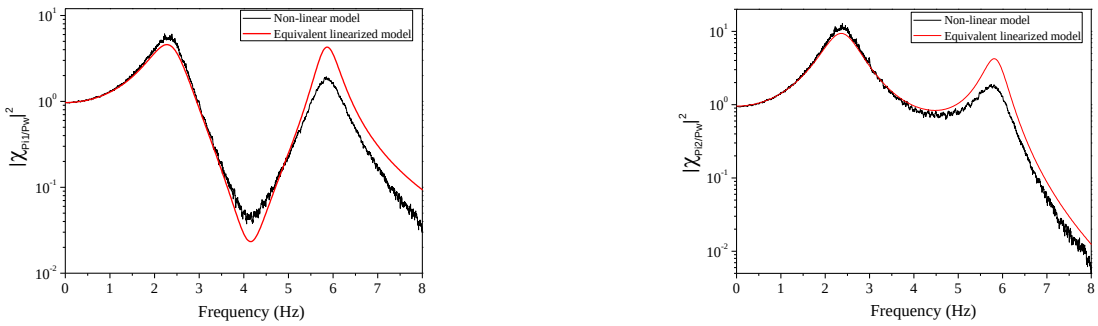
When porosity ratio A_L/A_{o1} =10%, the internal pressure admittance functions corresponding to both non-linear model and equivalent linearized model in Compartment 1 and Compartment 2 are shown in Figs. 4(a) and 4(b), respectively. It is shown that the equivalent linearized model has small calculation error at lower frequencies, while it has larger error at higher frequencies. Fortunately, the lower frequencies are dominant for internal pressure in a two compartment building, so the difference of RMS internal pressure coefficients between the two models is relatively small.

5.2 Parameter analysis through the linearized model

To show the characteristics of the admittance functions

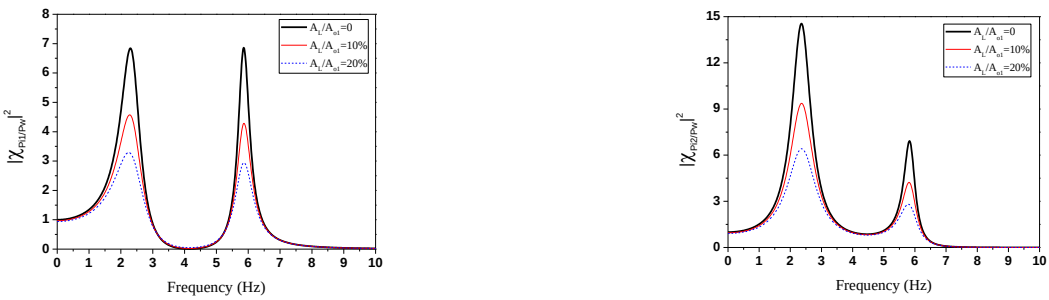
Table 1 Estimated RMS internal pressure coefficients for different background leakages

A_L/A_{o1}	C_{pi1}_rms			C_{pi2}_rms			C_{pw}_rms
	Non-linear model	Linear model	Error	Non-linear model	Linear model	Error	
0%	0.25	0.24	0.23	0.27	0.26	3.7%	0.23
10%	0.24	0.23	4.2%	0.25	0.24	4.0%	
20%	0.22	0.23	4.5%	0.22	0.23	4.5%	
30%	0.21	0.22	4.8%	0.19	0.22	15.8%	
40%	0.20	0.22	10.0%	0.17	0.21	23.5%	
50%	0.19	0.21	10.5%	0.15	0.20	33.3%	



(a) Compartment 1 (b) Compartment 2

Fig. 4 Internal pressure admittance functions of both models



(a) Compartment 1 (b) Compartment 2

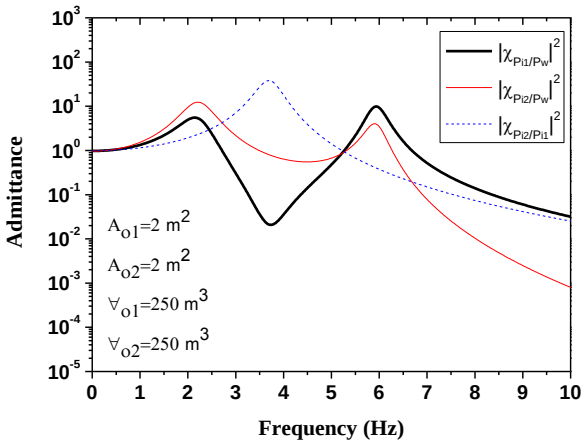
Fig. 5 Admittance functions of the internal pressure

of the internal pressure for both compartments, the effects of background leakage, opening area, and internal volume on admittance functions are analyzed through the linearized model.

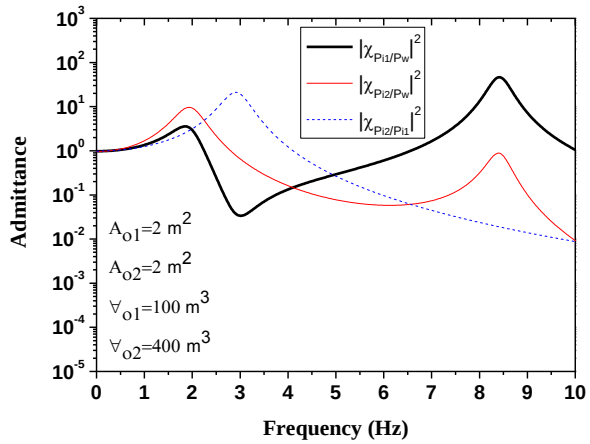
Figs. 5(a) and 5(b) show the admittance functions of the internal pressure in both compartments for a range of background leakages. Results indicate that the resonance peaks (2.4 and 5.9 Hz) decrease with increasing lumped leakage area. Furthermore, the peaks in Compartment 2 are obviously higher than those in Compartment 1.

The effects of the opening area and internal volume on the admittance functions of the internal pressure are also analyzed for a common two-compartment building with a background porosity ratio of 10%. Figs. 6(a) to 6(e) show that two resonance modes exist. The resonance peak for Compartment 2 is higher than that for Compartment 1 at lower Helmholtz frequency. By contrast, the resonance peak for Compartment 2 is lower than that for Compartment 1 at higher Helmholtz frequency. Moreover, the opening area and internal volume show remarkable effects on the admittance function of the internal pressure for Compartment 1.

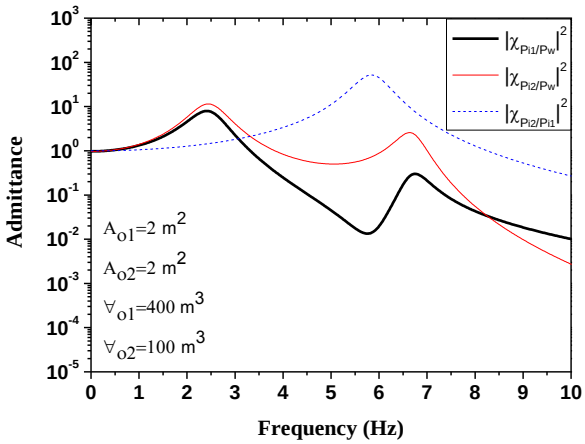
A Comparison of Fig. 6(a) with Figs. 6(b) and 6(c) shows that: (1) the higher Helmholtz frequencies of both compartments when $\forall_{01} < \forall_{02}$ are greater than those when $\forall_{01} = \forall_{02}$; (2) the resonance peak for Compartment 1 at higher Helmholtz frequency when $\forall_{01} < \forall_{02}$ is also greater than that when $\forall_{01} = \forall_{02}$; (3) the resonance peak



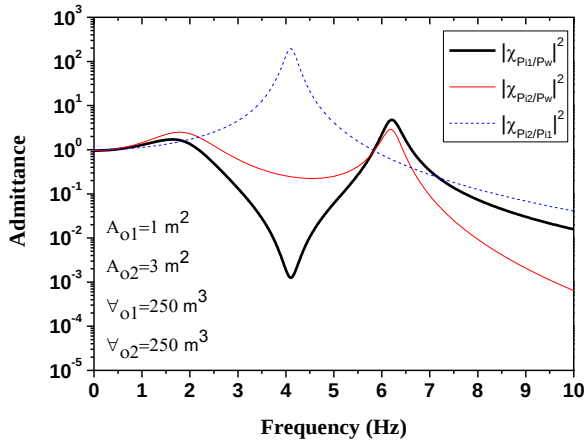
(a) $A_{o1} = A_{o2}$, $\forall_{01} = \forall_{02}$



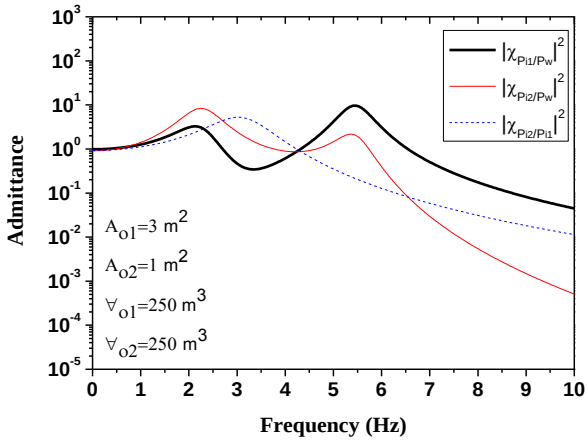
(b) $A_{o1} = A_{o2}$, $\forall_{01} < \forall_{02}$



(c) $A_{o1} = A_{o2}$, $\forall_{01} > \forall_{02}$



(d) $A_{o1} < A_{o2}$, $\forall_{01} = \forall_{02}$



(e) $A_{o1} > A_{o2}$, $\forall_{01} = \forall_{02}$

Fig. 6 Admittance functions in a two-compartment building

for Compartment 1 at higher Helmholtz frequency when $\forall_{01} > \forall_{02}$ is smaller than that when $\forall_{01} = \forall_{02}$.

A Comparison of Fig. 6a with Figs. 6d and 6e indicates that the resonance peaks, especially at lower Helmholtz frequencies, for both compartments when $A_{o1} < A_{o2}$ are smaller than those when $A_{o1} = A_{o2}$. The resonance peak for Compartment 2 at higher Helmholtz frequencies when $A_{o1} > A_{o2}$ is also smaller than that when $A_{o1} = A_{o2}$.

6. Conclusions

We linearized the non-linear equations governing wind-induced internal pressure for a two-compartment building with openings and background leakage. The corresponding linearized governing equations and explicit admittance functions of the internal pressure are presented. Moreover, we compared the RMS values calculated from the non-linear equations and linearized equations. The comparison results validate the precision of the linearized model. Our main conclusions are as follows:

(1) The linearized model has a favorable accuracy when the porosity ratio (A_L/A_{o1}) is less than 20%.

(2) The resonance peak of the internal pressure in the compartment without an external opening (Compartment 2) is higher than that in in the compartment with an external opening (Compartment 1) at lower Helmholtz frequencies. By contrast, the resonance peak of the internal pressure in Compartment 2 is lower than that in Compartment 1 at higher Helmholtz frequencies.

Acknowledgments

This research is fully supported by the National Natural Science Foundation of China (Grant No. 51778243), and the State Key Laboratory of Subtropical Building Science, South China University of Technology (Grant No. 2019ZB28). The financial supports are gratefully acknowledged.

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