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A bimodal Weibull distribution - capacity factor for different heights at sulur

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Abstract. Due to developing environmental concern use of renewable energy source is very essential. The great demand for the energy supply coupled with inadequate energy sources creates an emergency to find a new solution for the energy shortage. The appropriate wind energy distribution is the fundamental requirement for the assessment of wind energy potential available at the particular site essential for the design of wind farms. Hence the proper specification of the wind speed distribution plays a vital role. In this paper the Bimodal Weibull distribution is used to estimate the Capacity factor at the proposed site. The shape and scale parameters estimated using Maximum likelihood method is used as the initial value for extrapolation. Application of this model will give an accurate result overwhelming the concept of overestimation or underestimation of Capacity factor.

Keywords: bimodal Weibull distribution ;two parameter Weibull distribution; v_{cut-in} ; $v_{cut-off}$; v_{rated} ; capacity factor; $v_{turbine}$

1. Introduction

Wind resource assessment is of paramount vital requirement for a wind farm project to be considered viable. If the wind regime does not suit for energy production a wind farm project will not definitely be feasible nevertheless of how much favorable other conditions are Patel (2006).

At a specific wind farm the available electricity generated by a wind power generator system depends on mean wind speed, standard deviation. Since variation on annual mean wind speed is hard to predict, wind speed variations during a year can be well characterized in terms of the probability density function (pdf). In literature, the pdf is defined as a mathematical function describing the relative likelihood for this RV to occur at a given point in the observation space. The wind speed probability distribution for a certain location is crucial in determining the performance of energy conversion systems.

Several authors have indicated that W-pdf should not be used in a generalized way, as it is unable to represent some wind regimes, such as those which describe wind speed frequency histograms which present bimodality. Alternatives for such regions include a bimodal probability distribution, proposed for La Ventosa, Mexico, by Jaramillo to overcome such situations. In this paper, the Bimodal Weibull distribution is used to model single-site hourly average wind speeds. A Bimodal Weibull distribution is even more useful because it is additionally able to represent heterogeneous wind regimes in which there is evidence of bimodality. The observed wind data describe wind speed frequency histogram which present bimodality.

For the proper assessment, the variability of the wind over time can be divided into three distinct time scales. Firstly, the large time scale variability describes the variations of the amount of wind from one year to another, or even over periods of decades or more. Secondly, the medium time scale covers periods up to a year. These seasonal variations of the wind are much more predictable. Finally, the short term time scale variability covers time scales of minutes to seconds, also well known by the term "turbulence" and which is of critical interest in the wind turbine design process. Therefore, accurate knowledge about the wind characteristics is needed for planning, design and operation of wind turbines .

The height of wind mast / towers available in Indian Wind Resource Assessment programme were of 20 m, 25 m, 50 m in olden days and the same are now raised to 80 m, 100 m and 120 m. Exact measurements of wind speed and turbulence design of boundary layer at turbine hub height is vitally important Park (1981). While structuring wind farm care should be taken to decrease turbine interference and wake effect created in order to increase energy yield for the wind farm as a long term process. Installation of apt wind turbines in more complex orographic areas will lead to increased electricity generation from wind farm. Tian Pau (2011).

Installation of these turbines need a little research study before being established. A commercial wind farm or an offshore installation of wind turbines need the wind resource to be determined in the area of proposed specific site. Hence several monitoring systems are available but may be cost expensive Burton *et al* (2007). A high

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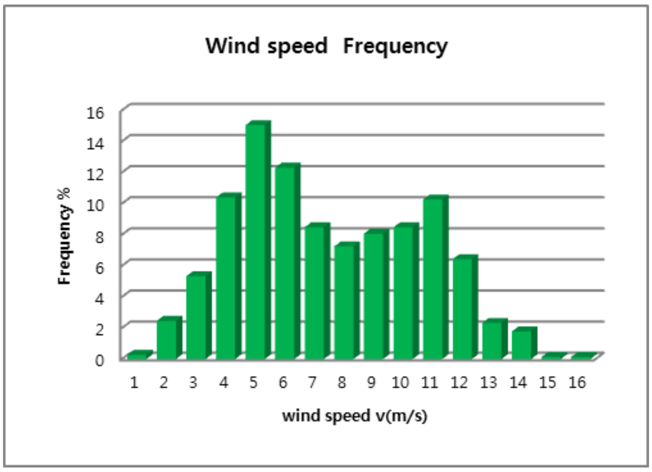

Fig. 1 Distribution of wind speed

percentage of the hardware cost is entirely spent on the tower designed to support the wind turbine. Our aim is to estimate the capacity factor at particular location which may help the manufacturers to design a turbine based on the capacity factor. This may reduce the production cost.

In this study we have discussed about the wind resources at Sulur site and we have estimated the Capacity factor using the mathematical model which was formulated in the previous paper by using Bimodal Weibull & Weibull factors and the two parameter Weibull factors .In the second section we have discussed about some specifications of the preferred site. In the third section we have defined Weibull distributions and tabulated the parametric factors of the Weibull distributions. In the fourth section we have discussed about the extrapolation of the wind at various heights. Fifth chapter consists of some characteristics of turbines, its capacity factor, expected power, etc.

2. Site specification

Wind characteristics has been studied based on a typically measured and observed two year data source at 10 meter height for every 10 minutes interval at Sulur site, Tamil Nadu, India. Average wind speed data at Sulur were observed during the time period 2012-2013. The wind speed data was collected for 24 months measured at Sulur at a mass height of 10 m. Observed wind speed data are usually available in time series format ,in which every data point resembles either an instantaneous sample wind speed or an mean wind speed over a particular time period. A sample data (giving hourly averages over a 24hr period) is given in Table 1.

Table 1 Wind speed data in 24 hour time-series format

Hour	Wind Speed (km/hr)	Hour	Wind Speed (km/hr)
1	EAST 14.8	*13*	ESE 11.1
2	ENE 7.4	*14*	ENE 7.4
3	ENE C	*15*	ESE 14.8
4	ENE 7.4	*16*	ENE 18.5
5	NE 7.4	*17*	ENE 11.1
6	NE 9.3	*18*	ENE 11.1
7	NE 9.3	*19*	ENE 11.1
8	NE 11.1	*20*	EAST 11.1
9	NE 13.0	*21*	EAST 7.4
10	NNE 9.3	*22*	EAST 7.4
11	NNE 9.3	*23*	ESE 11.1
12	ENE 9.3	*24*	ESE 20.4

3. Weibull distribution

The Weibull distribution is named after the familiar Swedish physicist Weibull. He applied it when studying a material in tension and fatigue in the 1930s which provides a better approximation to the probability rules of several natural phenomena.

3.1 Two parameter Weibull distribution

Let (Ω,F,P) be a probability space. A real valued random variable V: Ω →R is said to have a two parameter Weibull distribution if it has a probability density function (PDF) Sedghi *et al* (2015).

$$f(v:\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right] & ,v \geq 0 \\ 0 & ,v < 0 \end{cases} \quad (1)$$

where $\alpha > 0$ is the dimensionless shape parameter & $\beta > 0$ is the scale parameter.

And the cumulative distribution function is given by

$$F(v:\alpha,\beta)=\begin{cases}0 & ,v<0\\ 1-exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right] & ,v\geq 0\end{cases} \quad (2)$$

3.2 Bimodal Weibull & Weibull distribution

A random variable V which is distributed as V_i is said to have a two-component mixture Weibull and Weibull Distribution with mixing parameters ω_i (such that $\omega_1+\omega_2=1$) is supposed to have a Bimodal Weibull & Weibull distribution with the probability density function

$$ff(v;\alpha_1,\beta_1,\alpha_2,\beta_2,=\omega f(v;\alpha_1,\beta_1)+(1-\omega)f(v;\alpha_2,\beta_2) \quad (3)$$

$$=\omega\left\{\frac{\alpha_1}{\beta_1}\left(\frac{v}{\beta_1}\right)^{\alpha_1-1}\exp\left[-\left(\frac{v}{\beta_1}\right)^{\alpha_1}\right]\right\}+(1-\omega)\left\{\frac{\alpha_2}{\beta_2}\left(\frac{v}{\beta_2}\right)^{\alpha_2-1}\exp\left[-\left(\frac{v}{\beta_2}\right)^{\alpha_2}\right]\right\} \quad (4)$$

The cumulative distribution function (CDF) of the Bimodal Weibull and Weibull distribution with parameters $\alpha_1,\beta_1,\alpha_2,\beta_2>0$ is expressed by Carta *et al.* (2007).

$$FF(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega)=P(V\leq v)$$
$$=\omega F(v;\alpha_1,\beta_1)+(1-\omega)F(v;\alpha_2,\beta_2), \quad (5)$$

$$=\omega\left\{1-\exp\left[-\left(\frac{v}{\beta_1}\right)^{\alpha_1}\right]\right\}+(1-\omega)\left\{1-\exp\left[-\left(\frac{v}{\beta_2}\right)^{\alpha_2}\right]\right\},\quad v\geq 0 \quad (6)$$

The bimodal Weibull and Weibull distribution, is expressed as WW $(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega)$,where $v[m/s]$ is the wind speed, the scale parameters α_1 and α_2 ,dimensionless shape parameters $\beta_1 and\ \beta_2$ are established by the left and right Bimodal Weibull & Weibull distribution respectively and ω denotes the weight component of the left Bimodal Weibull & Weibull distribution $(0<\omega<1)$. The weight component ω can be calculated from the following formulas.

$$\bar{v}=\omega\,\bar{v}_1+(1-\omega)\,\bar{v}_2$$

$$\text{And } \sigma^2=\omega\,(\sigma_1{}^2-(\omega-1)(\bar{v}_1-\bar{v}_2)^2)-(\omega-1)\sigma_2{}^2 \quad (7)$$

$\bar{v}[m/s]$ is the wind speed, $\bar{v}_1$ and $\bar{v}_2$ represents the average wind speed of the left and right Bimodal Weibull & Weibull distribution peak respectively , $\sigma_1{}^2 and\ \sigma_2{}^2$ are the variance of the left and right Bimodal Weibull & Weibull distribution peak respectively (Jaramillo *et al.* 2004).

3.3 Estimated parameters

The estimation of parameters using Least square method, Maximum likelihood method Energy Pattern factor method and empirical method was detailed in the previous paper (Seshaiah and Indhumathy 2017). The estimated parameters are listed in Tables 2 and 3.

3.3.1 Energy pattern factor method

This is a new method suggested by Akdag and Ali (2009). It is related to the averaged data of wind speed. This method has simpler formulation, easier implementation and also requires less computation. The method is defined by the following equations.

$$E_{pf}=\frac{\overline{v^3}}{(\overline{v})^3}, \quad (8)$$

$$\alpha=1+\frac{3.69}{(E_{pf})^2} \quad (9)$$

$$\beta=\frac{\bar{v}}{\Gamma\left(1+\frac{1}{\alpha}\right)} \quad (10)$$

Eq. (8) is known as the energy pattern factor method which can be solved numerically or approximately by power density technique. Once α is determined, β can be estimated using Eq. (10).
Similarly parameters of Bimodal Weibull & Weibull distribution calculated using Maximum likelihood method gave a best fit. The parameters (Celik *et al.* 2004) are given in Table 2.

3.3.2 The maximum likelihood method

The Maximum likelihood method to find the value of the parameters $\alpha_1,\alpha_2,\beta_1,\beta_2$ which maximizes the function of log likelihood is given by Ahmed (2013).

$$l(\alpha_1,\beta_1,\alpha_2,\beta_2)=\prod_{i=1}^{n}f(v_i;\ \alpha_1,\beta_1,\alpha_2,\beta_2,\omega)$$

$$=\prod_{i=1}^{n}\omega\left\{\frac{\alpha_1}{\beta_1}\left(\frac{v_i}{\beta_1}\right)^{\alpha_1-1}\exp\left[-\left(\frac{v_i}{\beta_1}\right)^{\alpha_1}\right]\right\}+\prod_{i=1}^{n}(1-\omega)\left\{\frac{\alpha_2}{\beta_2}\left(\frac{v_i}{\beta_2}\right)^{\alpha_2-1}\exp\left[-\left(\frac{v_i}{\beta_2}\right)^{\alpha_2}\right]\right\} \quad (11)$$

$$=\omega^n\left(\frac{\alpha_1}{\beta_1}\right)^n\exp\left[-\frac{1}{\beta_1{}^{\alpha_1}}\sum_{i=1}^{n}v_i{}^{\alpha_1}\right]\frac{\prod_{i=1}^{n}v_i{}^{\alpha_1-1}}{\beta_1{}^{n(\alpha_1-1)}}+(1-\omega)^n\left(\frac{\alpha_2}{\beta_2}\right)^n\exp\left[-\frac{1}{\beta_2{}^{\alpha_2}}\sum_{i=1}^{n}v_i{}^{\alpha_2}\right]\frac{\prod_{i=1}^{n}v_i{}^{\alpha_2-1}}{\beta_2{}^{n(\alpha_2-1)}} \quad (12)$$

Take log on both the sides we get

$$\text{Log } l(\alpha_1,\beta_1,\alpha_2,\beta_2)=n\log\alpha_1-n\alpha_1\log\beta_1-\frac{1}{\beta_1{}^{\alpha_1}}\sum_{i=1}^{n}v_i{}^{\alpha_1}+\sum_{i=1}^{n}(\alpha_1-1)\log v_i+n\log\alpha_2-\frac{1}{\beta_2{}^{\alpha_2}}\sum_{i=1}^{n}v_i{}^{\alpha_2}+\sum_{i=1}^{n}(\alpha_2-1)\log v_i-n\,\alpha_2\log\beta_2 \quad (13)$$

Take partial derivatives w.r.t α_1, β_1, α_2, β_2 and equating to zero we obtain

$$\rightarrow \widehat{\beta_1} = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_1}}{n}\right)^{\frac{1}{\alpha_1}} \quad \text{or} \quad \widehat{\beta_1}^{\alpha_1} = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_1}}{n}\right) \qquad (14)$$

$$\widehat{\beta_2} = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_2}}{n}\right)^{\frac{1}{\alpha_2}} \quad \text{or} \quad \widehat{\beta_2}^{\alpha_2} = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_2}}{n}\right) \qquad (15)$$

$$\frac{\sum_{i=1}^{n} v_i^{\hat{\alpha}_1}\log(v_i)}{\sum_{i=1}^{n} v_i^{\hat{\alpha}_1}} - \frac{1}{\hat{\alpha}_1} = \frac{2\sum_{i=1}^{n}\log(v_i)}{n} \qquad (16)$$

$$\frac{\sum_{i=1}^{n} v_i^{\hat{\alpha}_2}\log(v_i)}{\sum_{i=1}^{n} v_i^{\hat{\alpha}_2}} - \frac{1}{\hat{\alpha}_2} = \frac{2\sum_{i=1}^{n}\log(v_i)}{n} \qquad (17)$$

To evaluate the performance of the Weibull distribution three statistical methods of accuracy were used. RMSE test , R^2 and $Chi - Square$ tests and the data are tabulated. (Tables 4 and 5).

Table 2 Estimation of two parameter Weibull parameters

METHODS	α	β
LS	2.584320	3.763621
MLE	2.431344	3.785243
EPF	2.384077	3.779660
EMPIRICAL	2.439150	3.778000

Table 3 Parameters of bimodal Weibull distributions

METHODS	α_1	α_2	β_1	β_2
LS	3.38192	6.29637	2.66000	5.41492
MLE	3.25216	6.38860	2.37929	5.26293
EPF	3.85618	6.40486	2.63343	5.26294
EMPIRICAL	3.51636	6.17992	2.64675	5.27318

Table 4 Goodness of fit for two parameter Weibull

TESTS	RMSE	CHI SQUARE	R^2
LS	0.044231	0.001789	0.999763
MLE	0.039334	0.001703	0.999812
EPF	0.043777	0.001691	0.999824
EMPIRICAL	0.039250	0.001715	0.999811

Table 5 Goodness of fit for bimodal Weibull

TESTS	RMSE	CHI SQUARE	R^2
LS	0.029648	0.001021	0.990943
MLE	0.029796	0.001144	0.990852
EPF	0.039656	0.001146	0.990938
EMPIRICAL	0.037753	0.001131	0.990879

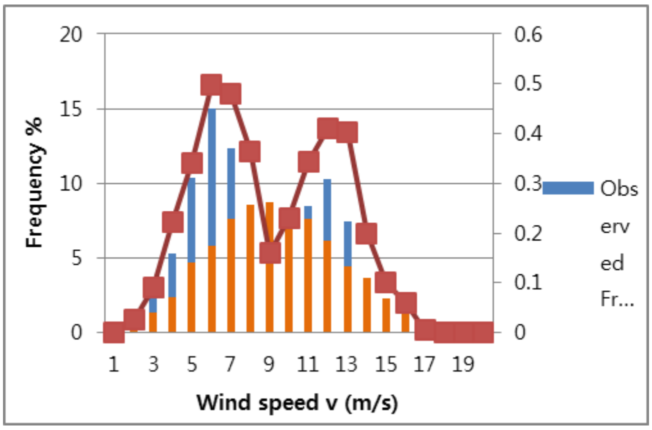

Fig 2 Comparison of Weibull PDF

Table 6 Initial parameters for evaluation

Distribution	TEST	α_1	α_2	β_1	β_2	ω
Two PARA	MLE	2.38407	-	3.77966	-	-
Bimodal	EPF	3.25216	6.38860	2.37929	5.26293	0.61548

The parameters of Two parameter Weibull distribution estimated using Energy Pattern Factor method gave a best fit when compared with other statistical methods and parameters of Maximum likelihood method gave a best fit (Ulgen *et al.* 2002).

The Bimodal Weibull & Weibull PDF and the two parameter Weibull PDF using these parameters are represented in Fig. 2.

The Bimodal Weibull & Weibull PDF exactly fits the Bimodal (two peak) frequency representation of the measured wind speed is clearly identified from the graphical representation and moreover this is proved by the goodness of fit test.

4. Wind speed extrapolation

The wind speed varies with altitude, however, wind blows relatively slow at lower altitude and wind speed then increases with altitude. Different relations are found in the literature to calculate wind speed at some height other than the anemometer level height (Justu *et al.* 1976).

Bimodal Weibull factors estimated using Maximum likelihood method are considered as the initial value (Cook 2001). The Weibull parameters and wind speed at desired height are calculated using the following expressions (Murthy *et al.* 2015).

$$\alpha_h = \frac{\alpha_{10}}{\left\{1 - 0.0881\ln\left(\frac{z_h}{z_{ref}}\right)\right\}} \qquad (18)$$

$$\beta_h = \beta_{10}\left(\frac{z_h}{z_{ref}}\right)^{\eta} \qquad (19)$$

$$\eta = 0.37 - 0.0881 \ln \beta_{10} \tag{20}$$

$$v_h = \beta_h \, \Gamma\left(1 + \frac{3}{\alpha_h}\right) \tag{21}$$

where α_{10} and β_{10} are the shape factor and scale factor at a height of 10 m and η is the power law coefficient , h is the desired height and z_{ref} is the reference height. The extrapolated parameters and mean wind speed at various heights are represented in Table 7 and 8.

The graphical representation of the calculated average wind speed at different heights using Bimodal Weibull & Two parameter Weibull distributions are shown in Fig. 3.

Table 7 Parameters at different height (Bimodal)

Height (*m*)	α_1	β_1	α_2	β_2	*mean ws* v (*m/s*)
10	3.609000	2.644	6.40486	5.242930	3.350200
20	3.843722	3.220003	6.821419	6.209182	4.080051
30	3.995739	3.613484	7.091201	6.854976	4.578628
40	4.111099	3.921491	7.295930	7.353511	4.968902
50	4.205272	4.178367	7.463058	7.765040	5.294389
60	4.285480	4.400692	7.605403	8.118323	5.576096
70	4.355722	4.597869	7.730060	8.429529	5.825939
80	4.418456	4.775799	7.841394	8.708736	6.051393
90	4.47531	4.938451	7.942293	8.962679	6.257488
100	4.527423	5.088637	8.034777	9.196108	6.447788

Table 8 Parameters at different height (Two parameter)

Height (*m*)	α	β	*mean ws* v (*m/s*)
10	2.431344	3.785243	3.350200
20	2.589473	4.511540	3.993023
30	2.691886	4.999391	4.424804
40	2.769603	5.377195	4.759187
50	2.833046	5.689797	5.035861
60	2.887081	5.958653	5.273817
70	2.934402	6.195856	5.483758
80	2.976666	6.408950	5.672360
90	3.014968	6.602987	5.844097
100	3.050075	6.781531	6.002121

5. Power distribution of wind turbine

5.1 Definition of $P_{turbine}$

For wind turbine machines that has power with maximum efficiency between rated and cut-off speed and an increasing power between cut-in and rated speed, wind speed of wind turbine with power coefficient C_p and efficiency coefficient $\eta \in (0,1)$ producing power P is given by Waleed *et al.* (2016).

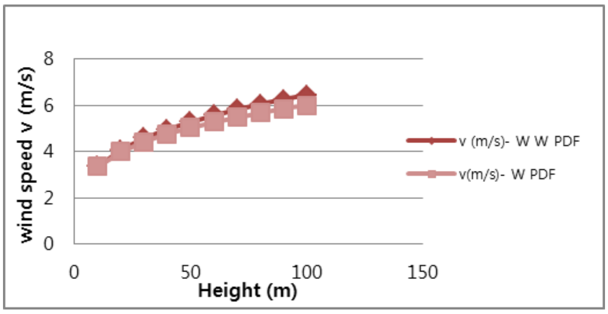

Fig. 3 Mean wind speed at various heights

$$P_{turbine} = \begin{cases} 0 & V_{turbine} < v_{cut-in} \\ \frac{1}{2}\rho C_p \eta A V^3 & v_{cut-in} \le V \le v_{rated} \\ \frac{1}{2}\rho C_p \eta A v_{rated}{}^3 & v_{rated} < V < v_{cut-off} \\ 0 & V \ge v_{cut-off} \end{cases} \tag{22}$$

The cdf of $P_{turbine}$ is defined by

$$FF_{Pturbine}(v) = \begin{cases} 0 & , -\infty < v < 0 \\ \left\{1-\omega\left(e^{-\left(\frac{v_{cut-in}}{\beta_2}\right)^{\alpha_2}} - e^{-\left(\frac{v_{cut-in}}{\beta_1}\right)^{\alpha_1}}\right) + \omega\left(e^{-\left(\frac{v_{cut-off}}{\beta_2}\right)^{\alpha_2}} - e^{-\left(\frac{v_{cut-ioff}}{\beta_1}\right)^{\alpha_1}}\right)\right\} & , 0 \le v \le \frac{1}{2}\rho C_{P\eta A v^3_{cut-in}} \\ 1-\omega e^{-\frac{1}{\beta_1^{\alpha_1}}\left(\frac{2v}{C_P\eta\rho A}\right)^{\frac{\alpha_1}{3}}} - (1-\omega) e^{-\frac{1}{\beta_2^{\alpha_2}}\left(\frac{2v}{C_P\eta\rho A}\right)^{\frac{\alpha_2}{3}}} + +\omega\left(e^{-\left(\frac{v_{cut-off}}{\beta_1}\right)^{\alpha_1}} - e^{-\left(\frac{v_{cut-ioff}}{\beta_2}\right)^{\alpha_2}}\right) + e^{-\left(\frac{v_{cut-in}}{\beta_2}\right)^{\alpha_2}} & , \frac{1}{2}\rho C_{P\eta A v^3_{cut-in}} < V < \frac{1}{2}\rho C_{P\eta A v^3_{rated}} \\ 1 & , \frac{1}{2}\rho C_{P\eta A v^3_{rated}} \le v \le \infty \end{cases} \tag{23}$$

The complete derivation of the above cumulative distribution function is detailed in the previous paper by (Seshaiah and Indhumathy 2016), where the wind speed experienced by the turbine is given by

$$V_{turbine} = \begin{cases} 0 & V_{turbine} < v_{cut-in} \\ V & v_{cut-in} \le V \le v_{rated} \\ v_{rated} & v_{rated} < V < v_{cut-off} \\ 0 & V \ge v_{cut-off} \end{cases} \tag{24}$$

Power curve defining cut-in,cut-off, rated wind speed is represented in Fig. 4. (Camilo *et al.* 2014).

There are three key points on this curve:

(1) cut-in speed below which the turbine will not produce power;

(2) rated speed at which the rated power of the turbine is produced; and

(3) cut-off speed beyond which the turbine is not allowed to deliver power.

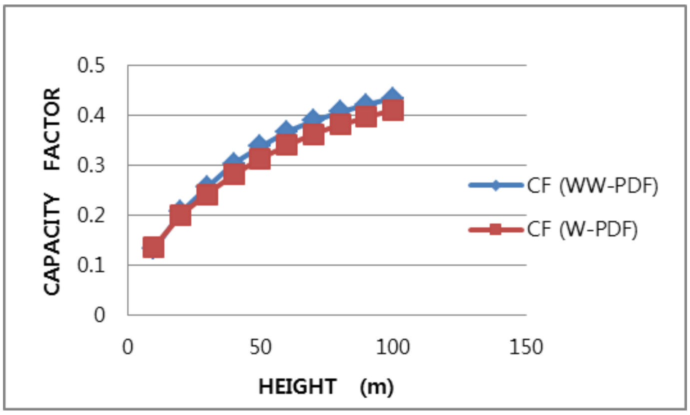

Fig. 5 Comparison of capacity factors

Table 9 CF using bimodal Weibull & Weibull parameters

Height (m)	α_1	β_1	α_2	β_2	mean ws v (m/s)	V_c	V_r	V_f	CF
10	3.609	2.644	6.40486	5.24293	3.3502	2.65	7.5	15	0.134499
20	3.843722	3.220003	6.821419	6.209182	4.080051	2.9	7.75	15.5	0.207615
30	3.995739	3.613484	7.091201	6.854976	4.578628	3.05	8	16	0.256895
40	4.111099	3.921491	7.29593	7.353511	4.968902	3.15	8.1	16.2	0.303882
50	4.205272	4.178367	7.463058	7.76504	5.294389	3.3	8.2	16.4	0.339071
60	4.28548	4.400692	7.605403	8.118323	5.576096	3.35	8.3	16.6	0.367401
70	4.355722	4.597869	7.73006	8.429529	5.825939	3.45	8.4	16.8	0.389235
80	4.418456	4.775799	7.841394	8.708736	6.051393	3.5	8.5	17	0.407342
90	4.47531	4.938451	7.942293	8.962679	6.257488	3.55	8.6	17.2	0.422207
100	4.527423	5.088637	8.034777	9.196108	6.447788	3.6	8.7	17.4	0.434587

5.2 Expected power of turbine

In particular , $p_{turbine}$ has mean

$$E[p_{turbine}] =$$

$$\left(\frac{1}{2}\rho C_p \eta A\right)\left\{\beta_1{}^3\omega\left(\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v_{rated}}{\beta_1}\right)^{\alpha_1}\right)-\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v_{cut-in}}{\beta_1}\right)^{\alpha_1}\right)\right)\right\}$$

$$+\left(\frac{1}{2}\rho C_p \eta A\right)\left\{\beta_2{}^3(1-\omega)\left(\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v_{rated}}{\beta_2}\right)^{\alpha_2}\right)-\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v_{cut-in}}{\beta_2}\right)^{\alpha_2}\right)\right)\right\} \quad (25)$$

$$+v_{rated}{}^3\omega\left\{e^{-\left(\frac{v_{rated}}{\beta_1}\right)^{\alpha_1}}-e^{-\left(\frac{v_{cut-off}}{\beta_1}\right)^{\alpha_1}}\right\}$$

$$+v_{rated}{}^3(1-\omega)\left\{e^{-\left(\frac{v_{rated}}{\beta_2}\right)^{\alpha_2}}-e^{-\left(\frac{v_{cut-off}}{\beta_2}\right)^{\alpha_2}}\right\}$$

5.3 Capacity factor

Capacity factor is the ratio of the actual energy produced by a turbine in a time period to the maximum possible capacity of production of the turbine when it operates full time as mentioned by the turbine manufacturer. (Rehman *et al.* 2004). We can define the capacity factor of a wind turbine to be the ratio.

$$CF = \frac{E[p_{turbine}]}{\frac{1}{2}\rho C_p \eta A v_{rated}{}^3} \quad (26)$$

An expression for CF using (26)

CF=

$$\left(\frac{\beta_1}{v_{rated}}\right)^3\left\{\omega\left(\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v_{rated}}{\beta_1}\right)^{\alpha_1}\right)-\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v_{cut-in}}{\beta_1}\right)^{\alpha_1}\right)\right)\right\}$$

$$+\left(\frac{\beta_2}{v_{rated}}\right)^3\left\{(1-\omega)\left(\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v_{rated}}{\beta_2}\right)^{\alpha_2}\right)-\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v_{cut-in}}{\beta_2}\right)^{\alpha_2}\right)\right)\right\} \quad (27)$$

$$+\omega\left\{e^{-\left(\frac{v_{rated}}{\beta_1}\right)^{\alpha_1}}-e^{-\left(\frac{v_{cut-off}}{\beta_1}\right)^{\alpha_1}}\right\}$$

$$+(1-\omega)\left\{e^{-\left(\frac{v_{rated}}{\beta_2}\right)^{\alpha_2}}-e^{-\left(\frac{v_{cut-off}}{\beta_2}\right)^{\alpha_2}}\right\}$$

From Figs. 1 and 2 it is clear that only Bimodal Weibull & Weibull distribution PDF fits the observed frequency of the wind speed. Comparison of estimated Capacitor factor also resembles the same.

The Capacity factor at various heights using the Bimodal Weibull & Weibull parameters and Two parameter Weibull at different v_{rated} , v_{cut-in} and $v_{cut-off}$ wind speed are given in Tables 9 and 10.

Table 10 CF using two parameter Weibull

Height (m)	α	β	mean ws $v\ (m/s)$	V_c	V_r	V_f	CF
10	2.431344	3.785243	3.3502	2.65	7.5	15	0.134315
20	2.589473	4.51154	3.993023	2.9	7.75	15.5	0.189797
30	2.691886	4.999391	4.424804	3.05	8	16	0.225204
40	2.769603	5.377195	4.759187	3.15	8.1	16.2	0.26086
50	2.833046	5.689797	5.035861	3.3	8.2	16.4	0.287334
60	2.887081	5.958653	5.273817	3.35	8.3	16.6	0.312061
70	2.934402	6.195856	5.483758	3.45	8.4	16.8	0.330868
80	2.976666	6.40895	5.67236	3.5	8.5	17	0.348089
90	3.014968	6.602987	5.844097	3.55	8.6	17.2	0.362608
100	3.050075	6.781531	6.002121	3.6	8.7	17.4	0.374906

6. Wind power

The computed P_{avg}, defined by $P_{avg} : \lfloor v_{cut-in}; v_{cut-off}) \rightarrow R$ is given as

$$P_{avg}(v) =$$

$$\left(\frac{1}{2}\rho C_p \eta A\right) \beta_1{}^3\omega\left(\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v}{\beta_1}\right)^{\alpha_1}\right)-\Gamma\left(1+\frac{3}{\alpha_1},\left(\frac{v_{cut-in}}{\beta_1}\right)^{\alpha_1}\right)\right)$$

$$+\beta_2{}^3(1-\omega)\left(\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v}{\beta_2}\right)^{\alpha_2}\right)-\Gamma\left(1+\frac{3}{\alpha_2},\left(\frac{v_{cut-in}}{\beta_2}\right)^{\alpha_2}\right)\right) \quad (28)$$

$$+v^3\omega\left\{e^{-\left(\frac{v}{\beta_1}\right)^{\alpha_1}}-e^{-\left(\frac{v_{cut-off}}{\beta_1}\right)^{\alpha_1}}\right\}$$

$$+v^3(1-\omega)\left\{e^{-\left(\frac{v}{\beta_2}\right)^{\alpha_2}}-e^{-\left(\frac{v_{cut-off}}{\beta_2}\right)^{\alpha_2}}\right\}$$

7. Conclusions

This study reveals that only Bimodal Weibull and Weibull distribution exactly fits the bimodality frequency distribution of the wind speed representation at Sulur site.

- This underestimation leads to loss in production of power at the particular location
- a great loss in electricity generation of the nation on the whole.

Major part of the investment is spend on the designing of wind turbines. Hence the estimation of expected capacity factor help the manufacturers to fix the v_{rated}, v_{cut-in} and $v_{cut-off}$ speed while designing the turbine for a particular site. The proposed mathematical model

- estimates the capacity factor accurately
- which may reduce the designing cost but at the same time increase the production.

can be applied to all locations experiencing bimodality frequency distribution.

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References

Ahmed, S.A. (2013), "Comparitive study of four methods of estimating Weibull parameters for Halabja, Iraq", *IJPS*, **8**(5), 186-192.

Akdag, S.A. and Ali, D. (2009), "A new method to estimate Weibull parameters for Wind energy applications", *Energ. Convers. Manage* , **50**, 1761-1766.

Burton, T., Sharpe, D., Jenkins, N. and Bossanyi, E. (2007), *Wind*

Camilo C., Cidras, J., Diaz-Dorado, E. and Obando-Montano, A.F. (2014), "An approach to determine the Weibull parameters for wind energy analysis: The case of Galicia (Spain)", *Energies*, **7**, 2676-2700.

Carta, J.A. and Ramirez, P. (2007), "Analysis of two-component mixture Weibull statistics for estimation of Wind speed distributions", *Renew. Energ.*, **32**,518-531.

Celik, A.N. (2004), "On the distribution parameters used in assessment of the suitability of wind speed probability density functions", *Energ. Convers. Manage.*, **45**(11), 1735-1747.

Chalamcharla, S.C.V. and Indhumathy, D. (2016), "Mathematical modeling of wind power estimation using multiple parameter Weibull distribution", *Wind Struct.*, **23**(4), 351-366.

Energy Handbook, Wiley I Edition, New York.

Jaramillo, O.A. and Borja, M.A. (2004), "Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case", *Renew Energ.*, **29**, 1613-1630.

Justu, C.G. and Mikhilai, A. (1976), "Height variation of wind speed and wind Distribution statistics", *Geophys. Res. Lett.*, **3**, 261- 264.

Murthy, K.S.R. and Rahi O.P. (2015), "Vertical extrapolation of Weibull parameters and Wind energy potential estimation of Himachal Pradesh, India", *India Conference, INDICON, Annual EEE*, India.

Park, J. (1981), *The wind power book*, Cheshire Books.

Patel, M.R. (2006), *Wind and solar power systems-design, analysis and operation*, second Ed., New York: CRCPress.

Rehman, S. and Ahmad, A. (2004), “Assesment of wind energy and capacity factor calculations for two locations of Saudi Arabia”, *Proceedings of the 2nd BSME-ASME international Conference On Thermal Engineering*, **2**, Dhaka, Bangladesh, BUET , 906-911.

Sedghi, M., Hannani, S.M. and Boroushaki, M. (2015), “Estimation of Weibull parameters for wind energy application in Iran”, *Wind Struct*., **21**(2), 203-221.

Seshaiah, C.V and Indhumathy, D. (2017), “Analysis of wind speed at Sulur - a bimodal Weibull & Weibull distribution”, *IJLEMR*, **2**(10), 29-37.

Tian Pau, C. (2011), “Estimation of wind energy potential using different probability density functions”, *Appl. Energ.*, **88**(5), 1848-1856.

Ulgen K and Hepbasli A. (2002), “Determination of Weibull parameters for wind energy analysis of Izmir”, *Turkey. Int. J. Energ. Res.*, **26**, 495-506.

Waleed, I.J.A.L.R.F. and Hamam, M. (2016), “Analysis of wind power density in Ninava Governorate/IRAQ”, *Int. J. Latest Res. Eng. Technol.*, **2**(1), 56-65.

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