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Dynamic and wave propagation investigation of FGM plates with porosities using a four variable plate theory

Riadh Bennai[1], Hocine Fourn[2], Hassen Ait Atmane*[1,2], Abdelouahed Tounsi[2,3] and Aicha Bessaim[2,4]

[1]Department of civil engineering, Faculty of civil engineering and architecture, University of Hassiba Benbouali of Chlef, Algeria
[2]Material and Hydrology Laboratory, Faculty of Technology, Department of Civil Engineering, University of Sidi Bel Abbes, Algeria
[3]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia
[4]Department of Civil Engineering, Université Mustapha Stambouli de Mascara, Mascara, Algeria

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Abstract. In this paper, an analytical analysis for the study of vibratory behavior and wave propagation of functionally graded plates (FGM) is presented based on a high order shear deformation theory. The manufacture of these plates' defects can appear in the form of porosity. This latter can question and modify the global behavior of such plates. A new shape of the distribution of porosity according to the thickness of the plate was used. The field of displacement of this theory is present of indeterminate integral variables. The modulus of elasticity and the mass density of these plates are assumed to vary according to the thickness of the plate. Equations of motion are derived by the principle of minimization of energies. Analytical solutions of free vibration and wave propagation are obtained for FGM plates simply supported by integrating the analytic dispersion relation. Illustrative examples are given also to show the effects of variation of various parameters such as (porosity parameter, material graduation, thickness-length ratio, porosity distribution) on vibration and wave propagation of FGM plates.

Keywords: functionally graded plate; higher-order plate theory; porosity; free vibration; wave propagation

1. Introduction

Since the beginning of the twentieth century, the use of composite materials in the form of plates and beams has grown considerably. Whether it is in the auto industry, construction, and morerecently in aeronautics. Composite materials have significant advantages over materials traditional. They bring many benefits functional: lightness, mechanical and chemical resistance, reduced maintenance, freedom of shape and service life extended. However, the disadvantage is the existence of concentrations of constraints at the interfaces between layers because of the abrupt change in mechanical properties from one layer to another. To overcome some difficulties a team of researchers Japanese (1980) proposed new materials called gradient property material (FGM). The materials to gradient properties (FGM) are composites of tip, whose microstructure is heterogeneous. Generally, these materials are made from isotropic components such as metals and ceramics (Barati and Shahverdi 2016). FGM find application in various fields such as aircraft, biomedical sectors and civil and industrial constructions (Bessaim *et al*. 2013, Besseghier *et al*. 2017, Bouafia *et al*. 2017). Therefore, the main question is an accurate description of material properties in the depth direction, to perform a satisfactory analysis of the mechanical behavior of FGM beams. Many studies on FGM structures have been studied in the literature (Bouderba *et al*. 2013, Tounsi *et al*. 2013, Ait Amar Meziane *et al*. 2014, Fekrar *et al*. 2014, Hamidi *et al.* 2015, Zemri *et al.* 2015, Taibi *et al*. 2015, Al-Basyouni *et al*. 2015, Attia *et al*. 2015, Meradjah *et al.* 2015, Bounouara *et al*. 2016, Bennoun *et al*. 2016, Bousahla *et al*. 2016, Hebali *et al*. 2016, Chikh *et al*. 2016, Laoufi *et al*. 2016, Beldjelili *et al*. 2016, Kolahchi *et al*. 2017a, Abdelaziz *et al*. 2017, Zidi *et al*. 2017, Abualnour *et al*. 2018, Attia *et al*. 2018, Bouhadra *et al*. 2018, Meksi *et al*. 2018).

When the application of the FGM increases, more accurate plate theories are required to predict the response of functionally graded (FG) plates. The first-order shear deformation theory (FSDT) accounts for the shear deformation effects by the way of linear variation of in-plane displacements through the thickness. Since the FSDT violates the conditions of zero transverse shear stresses on the top and bottom surfaces of the plate, a shear correction factor which depends on many parameters is required to compensate for the error due to a constant shear strain assumption through the thickness (Heireche *et al*. 2008, Bellifa *et al*. 2016). The higher-order shear deformation theories (HSDTs) account for the shear deformation effects, and satisfy the zero transverse shear stresses on the top and bottom surfaces of the plate, thus, a shear correction factor is not required (Ould Larbi *et al*. 2013). Hebali *et al*. (2014) proposed a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of FG plates. Bousahla *et al*. (2014) presented a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates.

∗Corresponding author, Assistant Professor
E-mail: aitatmane2000@yahoo.fr

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Zidi *et al* (2014) employed a four variable refined plate theory for bending analysis of FG plates under hygro-thermo-mechanical loading. Yaghoobi *et al*. (2014) presented an analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM. Bourada *et al*. (2015) discussed the bending and vibration responses of FG thick beams by proposing a novel simple shear and normal deformations theory. Mahi *et al*. (2015) developed a novel hyperbolic shear deformation model for static and dynamic analysis of isotropic, functionally graded, sandwich and laminated composite plates. Ait Atmane *et al*. (2015) used a variationally consistent shear deformation theory for dynamic behavior of thick FG beams with porosities. Attia *et al*. (2015) examined the dynamic response of FG plates with temperature-dependent properties by employing various four variable refined plate models. Larbi Chaht *et al*. (2015) studied the bending and buckling behaviors of FG size-dependent nanoscale beams including the thickness stretching effect. Kar and Panda (2015) examined nonlinear flexural vibration of shear deformable functionally graded spherical shell panel. Belkorissat *et al*. (2015) studied the dynamic properties of FG nanoscale plates using a novel nonlocal refined four variable theory. Bennai *et al*. (2015) proposed a novel higher-order shear and normal deformation theory for FG sandwich beams. Belabed *et al*. (2014) proposed hyperbolic function based higher-order shear deformation theory with five unknowns to investigate flexural and vibration characteristics of FGM plate. Ellali *et al*. (2018) presented the buckling of piezoelectric plates on Pasternak elastic foundation using higher-order shear deformation plate theories. The comparative study of the effect of various gradation laws (power-law, sigmoid or exponential function) on the mechanical behavior of FGM plates under transverse load was carried out by Chi and Chung (2006). Thai and Kim (2013) used the quasi-3D sinusoidal shear deformation theory with only five unknowns for bending behavior of simply supported FGM plates. Shahsavari *et al*. (2018) presented a novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation. Many studies on shear deformation theories have been developed in the literature to study FG structures and CNT-reinforced plates (Kolahchi and Bidgoli 2016, Arani and Kolahchi 2016, Kolahchi *et al*. 2016a, b, Bilouei *et al*. 2016, Madani *et al*. 2016, Draiche *et al*. 2016, Houari *et al*. 2016, Zamanian *et al*. 2017, Kolahchi and Cheraghbak 2017, Kolahchi 2017, Kolahchi *et al*. 2017b, c, Klouche *et al*. 2017, Mouffoki *et al*. 2017, Sekkal *et al*. 2017a, b, Chikh *et al*. 2017, Hajmohammad *et al*. 2017, Zarei *et al*. 2017, Shokravi 2017a, b, c, d, Bakhadda *et al*. 2018, Youcef *et al*. 2018). Recently, four variable plate theories with indeterminate integral terms are developed to make the theory more simple (Bellifa *et al*. 2017a, El-Haina *et al*. 2017, Khetir *et al*. 2017, Fahsi *et al*. 2017, Menasria *et al*. 2017, El-Haina *et al.* 2017, Hachemi *et al*. 2017, Zine *et al*. 2018, Yazid *et al*. 2018, Belabed *et al*. 2018, Benchohra *et al*. 2018).

In order to fully understand the different dynamic characteristics of functionally graduated structures, it is important to study wave propagation in this type of structures at large frequencies for their uses in different fields. Structural health monitoring or detection of damage is one such important application. As wave propagation deals with higher frequencies, diagnostic waves can be employed to predict the presence of even minute defects, which occur at initiation of damage and propagate them till the failure of the FGM structure. In many aircraft structures, the undesired vibration and noise transmit from the source to the other parts in form of wave propagation and this requires control or reduction, which is again an important application of wave propagation studies.

The study of wave propagation in FG plates has also received a lot of attention from various researchers. the behavior of wave dispersion in a FG plate with material properties varying in thickness direction has been studied by Chen *et al.* (2007). Han *et al.* (2001) used an analytical-numerical method for analyzing wave characteristics in FG cylinders. Han *et al*. (2002) also proposed a numerical method for studying the transient wave in FG plates excited by impact loads. Sun and Luo (2011a) also investigated wave propagation and dynamic response of functional gradient rectangular plates with complete tight supports under impulse loading. Considering the thermal effects and properties of temperature-dependent materials, Sun and Luo (2011b) investigated the propagation of a functionally infinite graded plate using the theory of higher order shear deformation plate.

In the manufacture of FGM parts, porosities may appear in these elements during the sintering process. This is due to the wide difference of the solidification temperature. In recent years, some studies about the porosity effect in the FG structures have been published in the literature; Wattanasakulpong *et al*. (2012) gives the discussion on porosities happening inside FGM samples fabricated by a multi-step sequential infiltration technique. Wattanasakulpong *et al*. (2014) also give a discussion of the porosities occurring inside the FGMs produced by the sequential infiltration technique. Şimşek and Aydın (2012) examined the forced vibration of FG microplates with porosity effects based on the modified couple stress theory. Jahwari and Naguib (2016) investigated FG viscoelastic porous plates with a higher order plate theory and a statistical based model of cellular distribution. Ait Yahia *et al*. (2015) investigated the wave propagationin FG plates with considering the porosity effect. Mouaici *et al*. (2016) proposed an analytical solution for the vibration of FGM plates with porosities. The analysis was based on the deformation theory of shear with taking into account the exact position of the neutral surface. Boukhari *et al*. (2016) introduced an efficient shear deformation theory for wave propagation of functionally graded material plates. Recently, Ait Atmane *et al*. (2016) is study the effect of stretching the thickness and porosity on the mechanical response of a FG beam resting on elastic foundations. Akbas SD (2017) studied the thermal effects on the vibratory behavior of FG beams with porosity. Benadouda *et al*. (2017) presented an efficient shear deformation theory for wave propagation in

FG material beams with porosities.

The objective of this work is to study the free vibration and wave propagation of a FGM plate by taking into account the effect of porosity using a high order shear deformation theory with four variables. A new shape of the distribution of porosity according to the thickness of the plate was used. The field of displacement of the theory is chosen according to an integral variation. The number of unknowns and motion equations of the theory are reduced and become simple to use. The equations governing wave propagation in the FGM plate are obtained using the Hamilton principle. The analytical dispersion relationships of FGM plates are obtained by solving a problem with eigenvalues. Will trace and analyze wave frequency curves and phase velocity in FGM plates, having porosity. We will also try to study the influence of the volume fraction index, number of wave, thickness ratio and porosity on the vibratory behavior and phase velocity of wave propagation in FGM plates.

2. Theory and formulation

2.1 Problem formulation

Consider a sandwich plate with porosity having total height (h), length (a), and width (b) referred to the Cartesian coordinates (x, y, z) as shown in Fig. 1. The top and bottom faces of the plate are at $z = \pm h/2$, and the horizontal edges of the plate are parallel to axes x and y. The plate is subjected to transverse load of intensity q(x) per unit length of the plate.

2.2. Material properties

A FG plate made from a mixture of two material phases, for example, a metal and a ceramic as shown in Fig. 1.

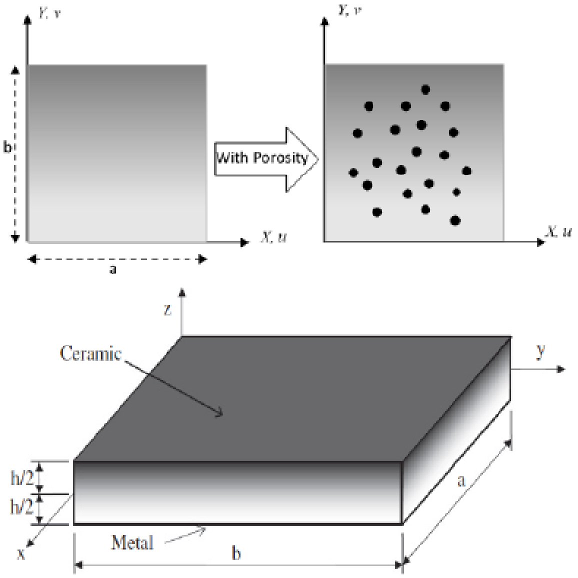


Fig. 1 The geometric configuration of FGM Plate with porosity

The material properties of FG plates are assumed to vary continuously through the thickness of the plate. In this investigation, the imperfect plate is assumed to have porosities spreading within the thickness due to defect during production. Consider an imperfect FGM with a porosity volume fraction, λ (λ <<1), distributed evenly among the metal and ceramic, the modified rule of mixture proposed by Ankit Gupta and Mohammad Talha (2017) is used as

$$P = P_c\left(V_c - \log\left(1+\frac{\lambda}{2}\right)\right) + P_m\left(V_m - \log\left(1+\frac{\lambda}{2}\right)\right) \quad (1)$$

λ is termed as porosity volume fraction (λ<1). λ= 0 indicates the non-porous functionally graded plate.

Now, the total volume fraction of the metal and ceramic is Vm+Vc =1, and the power law of volume fraction of the ceramic is described as

$$V_f = \left(\frac{1}{2}+\frac{z}{h}\right)^p \quad (2)$$

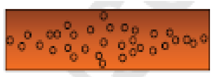
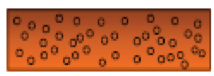
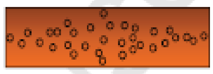
Where ‘p’ is the volume fraction index. The effective material property of porous FGM plate is given as

$$E(z) = [E_c - E_m]\left(\frac{2z+h}{2z}\right)^p - \xi[E_c + E_m]\left[1-\frac{2|z|}{h}\right] + E_m \quad (3a)$$

$$\rho(z) = [\rho_c - \rho_m]\left(\frac{2z+h}{2z}\right)^p - \xi[\rho_c + \rho_m]\left[1-\frac{2|z|}{h}\right] + \rho_m \quad (3b)$$

Where *P* denotes the effective material characteristic such as Young’s modulus *E* and mass density ρ subscripts *m* and *c* denote the metallic and ceramic components, respectively. ξ it is the factor of the distribution of the porosity according to the thickness of the plate (Table 1). It is noted that the positive real number p (0≤ p <∞) is the power law or volume fraction index, and z is the distance from the mid-plane of the FG plate. When p is set to zero (p = 0) the FG plate become a fully ceramic plate and fully metal plate for large value of p (p =∞). Since the influences of the variation of Poisson's ratio ν on the behavior of FG, plates are very small (Yang *et al*. 2005, Kitipornchai *et al*. 2006), it is supposed to be constant for convenience.

Table 1 Factor of the distribution of porosity ξ.

	ξ	geometric shape
Ankit Gupta *et al.* (2017)	$\log\left(1+\frac{\lambda}{2}\right)$	
Wattanasakulpong *et al.* (2013)	$\frac{\lambda}{2}$	
present	$1-e^{-\frac{\lambda}{2}}$	

2.3 Kinematics and strains

In this article, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. (Bouchafa *et al*. 2015) give the displacement field of the conventional HSDT

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_x(x,y,t) \tag{4a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\varphi_y(x,y,t) \tag{4b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{4c}$$

Where u_0, v_0, w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x=\int\theta(x,y)dx$ and $\varphi_y=\int\theta(x,y)dy$, the displacement field of the present model can be expressed in a simpler form as (El-Haina *et al*. 2017)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1\,f(z)\int\theta(x,y,t)\,dx \tag{5a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_2\,f(z)\int\theta(x,y,t)\,dy \tag{5b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{5c}$$

Where

$$f(z)=\frac{\cosh\left(\frac{\pi}{2}\right)}{\cosh\left(\frac{\pi}{2}\right)-1}-\frac{\cosh((\pi/h)*z)}{\cosh\left(\frac{\pi}{2}\right)-1} \tag{5e}$$

It can be seen that the displacement field in Eq. (5) introduces only four unknowns (u0, v0, w0 and θ).The nonzero strains associated with the displacement field in Eq. (5) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\ \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{6}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial x}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{Bmatrix},\ \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\frac{\partial}{\partial y}\int\theta\,dx+k_2\frac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix},\ \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix} \tag{7a}$$

$$g(z)=\frac{df(z)}{dz} \tag{7b}$$

And The integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\ \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y}$$
$$\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\ \int\theta\,dy=B'\frac{\partial\theta}{\partial y} \tag{8}$$

Where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A'=-\frac{1}{\kappa_1^2},\ B'=-\frac{1}{\kappa_2^2},\ k_1=\kappa_1^2,\ k_2=\kappa_2^2 \tag{9}$$

Where κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively. For elastic and isotropic FGMs, the constitutive relations can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\ C_{12}&C_{22}&0&0&0\\ 0&0&C_{66}&0&0\\ 0&0&0&C_{44}&0\\ 0&0&0&0&C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{10}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (1), stiffness coefficients, C_{ij}, can be given as

$$C_{11}=C_{22}=\frac{E(z)}{1-\nu^2},\ C_{12}=\frac{\nu\,E(z)}{1-\nu^2},\ C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)} \tag{11}$$

2.4 Equations of motion

Hamilton's principle is herein utilized to determine the equations of motion (Bellifa *et al.* 2017b)

$$\int_0^t(\delta U+\delta V-\delta K)\,dt=0 \tag{12}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]dV\\ &=\int_A\left[N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta k_x^b+M_y^b\delta k_y^b+M_{xy}^b\delta k_{xy}^b\right.\\ &\quad\left.+M_x^s\delta k_x^s+M_y^s\delta k_y^s+M_{xy}^s\delta k_{xy}^s+S_{yz}^s\delta\gamma_{yz}^s+S_{xz}^s\delta\gamma_{xz}^0\right]dA=0\end{aligned} \tag{13}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz \quad (i = x, y, xy)$$
$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{14}$$

The variation of the external work can be expressed as

$$\delta V = -\int_A q\delta w_0 dA - \int_A \begin{pmatrix} N_x^0 \dfrac{\partial w_0}{\partial x}\dfrac{\partial \delta w_0}{\partial x} + \\ 2N_{xy}^0 \dfrac{\partial w_0}{\partial x}\dfrac{\partial \delta w_0}{\partial y} + N_y^0 \dfrac{\partial w_0}{\partial y}\dfrac{\partial \delta w_0}{\partial y} \end{pmatrix} dA \tag{15}$$

Where q and (N_x^0, N_y^0, N_{xy}^0) are transverse and in-plane applied loads, respectively.

For the free vibration and wave propagation problems, the external work is zero. The variation of kinetic energy of the plate can be expressed as

$$\begin{aligned}
\delta K &= \int_V \left[\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w}\right]\rho(z)\,dV \\
&= \int_A \Bigg\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&- I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\
&+ J_1\left((k_1 A')\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right) + (k_2 B')\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right) \\
&+ I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) + K_2\left((k_1 A')^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + (k_2 B')^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\
&- J_2\left((k_1 A')\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + (k_2 B')\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right)\Bigg\} dA
\end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, K_i) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right)\rho(z)dz$$
$$\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(f, z\,f, f^2\right)\rho(z)dz \tag{17}$$

By substituting Eqs. (13), (15) and (16) into Eq. (12), the following can be derived

$$\begin{aligned}
&\delta u_0:\ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1 A' J_1\frac{\partial\ddot{\theta}}{\partial x} \\
&\delta v_0:\ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + k_2 B' J_1\frac{\partial\ddot{\theta}}{\partial y} \\
&\delta w_0:\ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - \\
&I_2\nabla^2\ddot{w}_0 + J_2\left(k_1 A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
&\delta\theta:\ -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial y} = \\
&-J_1\left(k_1 A'\frac{\partial\ddot{u}_0}{\partial x} + k_2 B'\frac{\partial\ddot{v}_0}{\partial y}\right) - K_2\left((k_1 A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + (k_2 B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) + \\
&J_2\left(k_1 A'\frac{\partial^2\ddot{w}_0}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)
\end{aligned} \tag{18}$$

Substituting Eq. (6) into Eq. (10) and the subsequent results into Eqs. (14), the stress resultants are obtained in terms of strains as following compact form:

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, S = A^s\gamma \tag{19}$$

Were

$$M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t \tag{20a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t,\quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t,\quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t \tag{20b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix},\quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}$$
$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{20c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix},\quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}$$
$$H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{20d}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}^t,\quad \gamma = \left\{\gamma_{xz}^0, \gamma_{yz}^0\right\}^t,\quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \tag{20e}$$

and stiffness components are given as

$$M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t \tag{21a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{21b}$$

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{20c}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{21c}$$

Introducing Eq. (19) into Eq. (18), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}
&A_{11}d_{11}u_0 + A_{66}\,d_{22}u_0 + \left(A_{12} + A_{66}\right)d_{12}v_0 - B_{11}d_{111}w_0 - \\
&\left(B_{12} + 2B_{66}\right)d_{122}w_0 + \left(B_{66}^s\left(k_1 A' + k_2\,B'\right)\right)d_{122}\theta + \\
&\left(B_{11}^s k_1 + B_{12}^s\,k_2\right)d_1\theta = I_0\ddot{u}_0 - I_1\,d_1\ddot{w}_0 + J_1\,A' k_1 d_1\ddot{\theta},
\end{aligned} \tag{22a}$$

$$\begin{aligned}
&A_{22}\,d_{22}v_0 + A_{66}\,d_{11}v_0 + \left(A_{12} + A_{66}\right)d_{12}u_0 - B_{22}\,d_{222}w_0 - \\
&\left(B_{12} + 2B_{66}\right)d_{112}w_0 + \left(B_{66}^s\left(k_1 A' + k_2\,B'\right)\right)d_{112}\theta + \\
&\left(B_{22}^s k_2 + B_{12}^s k_1\right)d_2\theta = I_0\ddot{v}_0 - I_1\,d_2\ddot{w}_0 + J_1\,B' k_2\,d_2\ddot{\theta},
\end{aligned} \tag{22b}$$

Table 2 Naturel frequencies of a porous FG plate for various thickness ratios, porosity parameters, power law indices and porosity distributions

a/h	p	λ=0			λ =0,1			λ =0,2		
		Nuttawit (2013)	Gupta (2017)	présent	Nuttawit (2013)	Gupta (2017)	présent	Nuttawit (2013)	Gupta (2017)	présent
5	0	78680,80	78680,80	78680,80	85629,23	81551,57	81550,31	96890,84	84841,06	84829,58
	0.5	53406,38	53406,38	53406,38	54182,36	53788,74	53788,58	55166,40	54179,50	54178,22
	1	46301,80	46301,80	46301,80	46166,29	46274,80	46274,82	45974,20	46227,97	46228,16
	5	36431,00	36431,00	36431,00	35427,32	35925,91	35926,13	34201,36	35368,65	35370,55
	10	34648,82	34648,82	34648,82	33532,33	34089,57	34089,81	32179,12	33474,25	33476,35
10	0	56646,44	56646,44	56646,44	61648,98	58995,93	58994,90	69756,81	61682,99	61673,63
	0.5	38361,90	38361,90	38361,90	38917,80	38825,08	38824,89	39627,60	39304,78	39303,19
	1	33289,58	33289,58	33289,58	33183,38	33441,90	33441,84	33034,31	33587,82	33587,36
	5	26537,07	26537,07	26537,07	25843,07	26364,23	26364,31	24993,52	26163,80	26164,50
	10	25211,47	25211,47	25211,47	24438,12	25001,00	25001,09	23501,75	24761,30	24762,13

$$\begin{aligned}&B_{11}d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0+\\&B_{22}d_{222}v_0-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0-\\&D_{22}d_{2222}w_0+\left(D_{11}^s k_1+D_{12}^s k_2\right)d_{11}\theta+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+\\&\left(D_{12}^s k_1+D_{22}^s k_2\right)d_{22}\theta=I_0\ddot{w}_0+I_1\left(d_1\ddot{u}_0+d_2\ddot{v}_0\right)-I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)\\&+J_2\left(k_1A'd_{11}\ddot{\theta}+k_2B'd_{22}\ddot{\theta}\right)\end{aligned}\tag{22c}$$

$$\begin{aligned}&-\left(B_{11}^s k_1+B_{12}^s k_2\right)d_1u_0-\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{122}u_0-\\&\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{112}v_0-\left(B_{12}^s k_1+B_{22}^s k_2\right)d_2v_0\\&+\left(D_{11}^s k_1+D_{12}^s k_2\right)d_{11}w_0+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)d_{1122}w_0+\\&\left(D_{12}^s k_1+D_{22}^s k_2\right)d_{22}w_0-H_{11}^s k_1^2\,\theta-H_{22}^s\,k_2^2\,\theta-2H_{12}^s k_1k_2\theta\\&-\left(\left(k_1A'+k_2B'\right)^2H_{66}^s\right)d_{1122}\theta+A_{44}^s\left(k_2B'\right)^2d_{22}\theta+A_{55}^s\left(k_1A'\right)^2d_{11}\theta=\\&-J_1\left(k_1A'd_1\ddot{u}_0+k_2B'd_2\ddot{v}_0\right)+J_2\left(k_1A'd_{11}\ddot{w}_0+k_2B'd_{22}\ddot{w}_0\right)-\\&K_2\left(\left(k_1A'\right)^2d_{11}\ddot{\theta}+\left(k_2B'\right)^2d_{22}\ddot{\theta}\right)\end{aligned}\tag{22d}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned}&d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\,d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\,d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m}\\&d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\,d_i=\frac{\partial}{\partial x_i},(i,j,l,m=1{,}2).\end{aligned}\tag{23}$$

2.5 Dispersion relations

We assume solutions for u_0, v_0, w_0 and θ_0 representing propagating waves in the x-y plane with the form

$$\begin{Bmatrix}u_0(x,y,t)\\v_0(x,y,t)\\w_0(x,y,t)\\\theta_0(x,y,t)\end{Bmatrix}=\begin{Bmatrix}U\exp\left[i(\kappa_1x+\kappa_2y-\omega t)\right]\\V\exp\left[i(\kappa_1x+\kappa_2y-\omega t)\right]\\W\exp\left[i(\kappa_1x+\kappa_2y-\omega t)\right]\\X\exp\left[i(\kappa_1x+\kappa_2y-\omega t)\right]\end{Bmatrix}\tag{24}$$

where U; V; W and X are the coefficients of the wave amplitude, κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency, $\sqrt{i}=-1$ the imaginary unit.

Substituting Eq. (24) into Eq. (23), the following problem is obtained

$$\left(\begin{bmatrix}S_{11}&S_{12}&S_{13}&S_{14}\\S_{12}&S_{22}&S_{23}&S_{24}\\S_{13}&S_{23}&S_{33}&S_{34}\\S_{14}&S_{24}&S_{34}&S_{44}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11}&0&m_{13}&m_{14}\\0&m_{22}&m_{23}&m_{24}\\m_{13}&m_{23}&m_{33}&m_{34}\\m_{14}&m_{24}&m_{34}&m_{44}\end{bmatrix}\right)\begin{Bmatrix}U\\V\\W\\X\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix}\tag{25}$$

Where

$$\begin{aligned}&S_{11}=-\left(A_{11}\kappa_1^{\,2}+A_{66}\kappa_2^{\,2}\right),\\&S_{12}=-\kappa_1\kappa_2\left(A_{12}+A_{66}\right),\\&S_{13}=\kappa_1\cdot i\cdot\left(B_{11}\kappa_1^{\,2}+B_{12}\kappa_2^{\,2}+2B_{66}\kappa_2^{\,2}\right),\\&S_{14}=i\cdot\kappa_1\left(k_1B_{11}^s+k_2B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\kappa_2^{\,2}\right),\\&S_{22}=-\left(A_{66}\kappa_1^{\,2}+A_{22}\kappa_2^{\,2}\right),\\&S_{23}=i\cdot\kappa_2\left(B_{22}\kappa_2^{\,2}+B_{12}\kappa_1^{\,2}+2B_{66}\kappa_1^{\,2}\right),\\&S_{24}=i\cdot\kappa_2\left(k_2B_{22}^s+k_1B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\kappa_1^{\,2}\right)\\&S_{33}=-\left(D_{11}\kappa_1^{\,4}+2(D_{12}+2D_{66})\kappa_1^{\,2}\beta^2+D_{22}\beta^4\right)\\&S_{34}=-k_1\left(D_{11}^s\kappa_1^{\,2}+D_{12}^s\kappa_2^{\,2}\right)+2(k_1A'+k_2B')D_{66}^s\kappa_1^{\,2}\kappa_2^{\,2}\\&-k_2\left(D_{22}^s\kappa_2^{\,2}+D_{12}^s\kappa_1^{\,2}\right),\\&S_{44}=-k_1\left(H_{11}^s k_1+H_{12}^s k_2\right)-(k_1A'+k_2B')^2H_{66}^s\kappa_1^{\,2}\kappa_2^{\,2}-\\&k_2\left(H_{12}^s k_1+H_{22}^s k_2\right)-\left(k_1A'\right)^2A_{55}^s\kappa_1^{\,2}-\left(k_2B'\right)^2A_{44}^s\kappa_2^{\,2}\\&m_{11}=-I_0,\quad m_{13}=i\cdot\kappa_1I_1,\quad m_{14}=-i\cdot J_1k_1A'\kappa_1,\\&m_{22}=-I_0,\ m_{23}=i\cdot\kappa_2\ I_1,\ m_{24}=-i\cdot k_2B'\ \kappa_2\ J_1,\\&m_{31}=-i\cdot\kappa_1I_1,\ m_{32}=-i\cdot\kappa_2\ I_1,\ m_{33}=-I_0-I_2(\kappa_1^{\,2}+\kappa_2^{\,2})\\&m_{34}=J_2\left(k_1A'\kappa_1^{\,2}+k_2B'\kappa_2^{\,2}\right),\ m_{41}=i\cdot J_1k_1A'\kappa_1\\&m_{42}=i\cdot k_2B'\ \kappa_2\ J_1,\ m_{43}=J_2\left(k_1A'\kappa_1^{\,2}+k_2B'\kappa_2^{\,2}\right),\\&m_{44}=-K_2\left(\left(k_1A'\right)^2\kappa_1^{\,2}+\left(k_2B'\right)^2\kappa_2^{\,2}\right)\end{aligned}\tag{26}$$

Table 3 The phase velocities of a porous FG plate for various thickness ratios, porosity parameters, power law indices and porosity distributions

a/h	p	λ=0			λ =0,1			λ =0,2		
		Nuttawit (2013)	Gupta (2017)	présent	Nuttawit (2013)	Gupta (2017)	présent	Nuttawit (2013)	Gupta (2017)	présent
5	0	7868,08	7868,08	7868,08	8562,92	8155,16	8155,03	9689,08	8484,11	8482,96
	0.5	5340,64	5340,64	5340,64	5418,24	5378,87	5378,86	5516,64	5417,95	5417,82
	1	4630,18	4630,18	4630,18	4616,63	4627,48	4627,48	4597,42	4622,80	4622,82
	5	3643,10	3643,10	3643,10	3542,73	3592,59	3592,61	3420,14	3536,86	3537,05
	10	3464,88	3464,88	3464,88	3353,23	3408,96	3408,98	3217,91	3347,43	3347,64
10	0	5664,64	5664,64	5664,64	6164,90	5899,59	5899,49	6975,68	6168,30	6167,36
	0.5	3836,19	3836,19	3836,19	3891,78	3882,51	3882,49	3962,76	3930,48	3930,32
	1	3328,96	3328,96	3328,96	3318,34	3344,19	3344,18	3303,43	3358,78	3358,74
	5	2653,71	2653,71	2653,71	2584,31	2636,42	2636,43	2499,35	2616,38	2616,45
	10	2521,15	2521,15	2521,15	2443,81	2500,10	2500,11	2350,17	2476,13	2476,21

The dispersion relations of wave propagation in the functionally graded beam are given by

$$\left| [K] - \omega^2 [M] \right| = 0 \tag{27}$$

The roots of Eq. (27) can be expressed as

$$\omega_1 = W_1(\kappa), \quad \omega_2 = W_2(\kappa), \quad \omega_3 = W_3(\kappa) \text{ and } \quad \omega_4 = W_4(\kappa) \tag{28}$$

They correspond to the wave modes M_1, M_2, M_3 and M_4 respectively. The wave modes M_1 and M_4 correspond to the flexural wave, the wave mode M_2 and M_3 corresponds to the extensional wave.

The phase velocity of wave propagation in the functionally graded plate can be expressed as

$$C_i = \frac{W_i(\kappa)}{\kappa}, \quad (i = 1,2,3,4) \tag{29}$$

3. Numerical results and discussions

In order to analyze the effect of porosity on the vibratory and behavior and phase velocity of the FGM plates, illustrative examples have been presented in this part. A functionally graduated plate is made from two Si3N4 / SUS304 materials; whose properties of these are presented in the following table

These properties change through the thickness of the plate according to the power law. The upper surface of FGM plate is rich in Si3N4 ceramic, while the lower surface of the FGM plate is rich in SUS304 metal. The thickness of the functionally graded plate is taken h=0.02 and 0.01 m. various numerical examples are presented and discussed to check the accuracy of present theory in investigating the wave propagation and free vibration of FG plates. The analysis based on the present model is carried out using MAPLE.

Tables 2 and 3 present the frequencies and phase velocities of an FGM plate for the three formulas of the porosity distribution factor. From the results presented in this two tables, we can observe the values of the frequencies and the velocity obtained by the present model are in good agreement with those of the Gupta (Ankit Gupta *et al*., 2017) model for the two cases λ = 0,1 and λ = 0, 2 regardless of the value of the ratio a / h.

The variation curves of the natural frequency (ω) and the phase velocity for the first four modes of the various functionally graded plates, as a function of the material power index (p) for different values of the porosity were respectively presented in Figs. 2 and 3.

From Figs. 2 and 3, it can be seen that the increase of the material index parameter induces the decrease of the natural frequency and the phase velocity in FG plates and this regardless of the wave number. However, the increase of the porosity factor leads to an increase of the frequency for the first two modes and a decrease for the modes 3 and 4. Consequently, the maximum frequency is obtained for a ceramic plate (p = 0) and a porosity factor λ = 0.3.

The natural frequency and the phase velocity of the wave propagation in the homogeneous plate is the maximum among those of all other FG plates. This is expected because the ceramic plate (p = 0) is the one with the highest rigidity. Therefore, it is clear that the heterogeneity of FGMs has a great influence on the phase velocity of the wave propagation and the natural frequency in the FG plate.

Fig. 4 show the frequency curves of the different FGM plates respectively obtained by using the proposed formula of the porosity distribution factor for different values of the latter as a function of the wave number kp. It can be seen from these curves that the frequency increases with the increase of kp for the same material power index. We can also observe that the frequency becomes maximum for the perfect plate (λ = 0).

Fig. 5 shows the influence of the phase velocity of a FGM plate as a function of wave number. The material power index is taken equal to p = 2. From this figure, the similarities in the evolutions of the vibration can be highlighted.

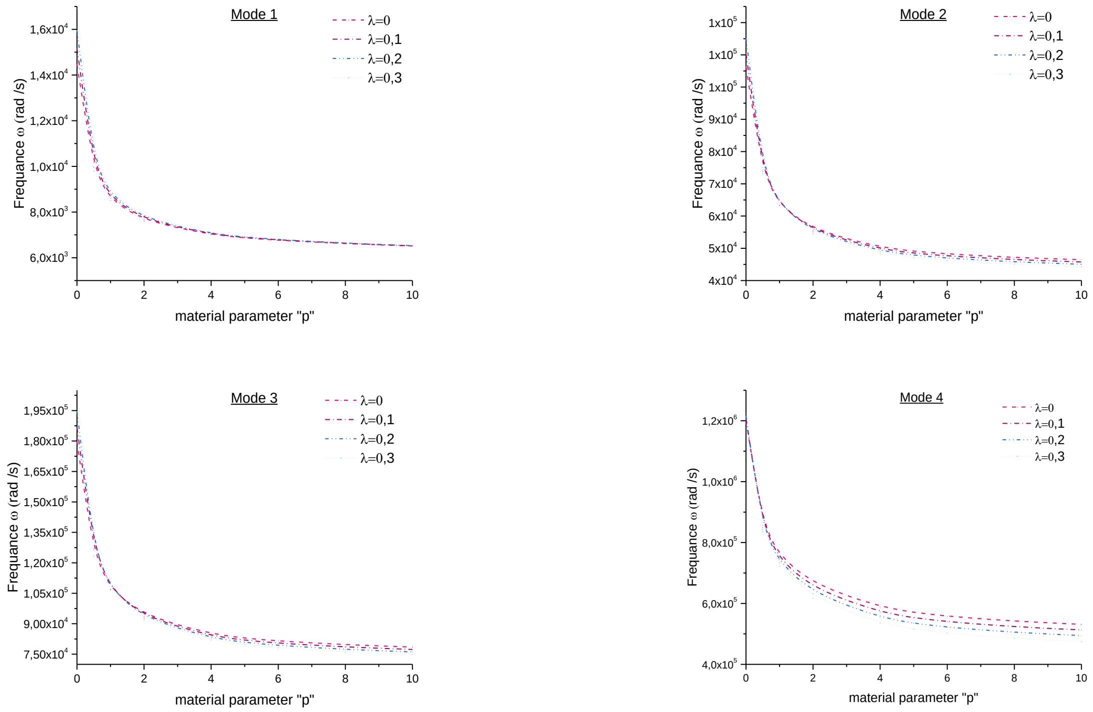


Fig. 2 Variation of the natural frequency of the FGM plates according to the material power index

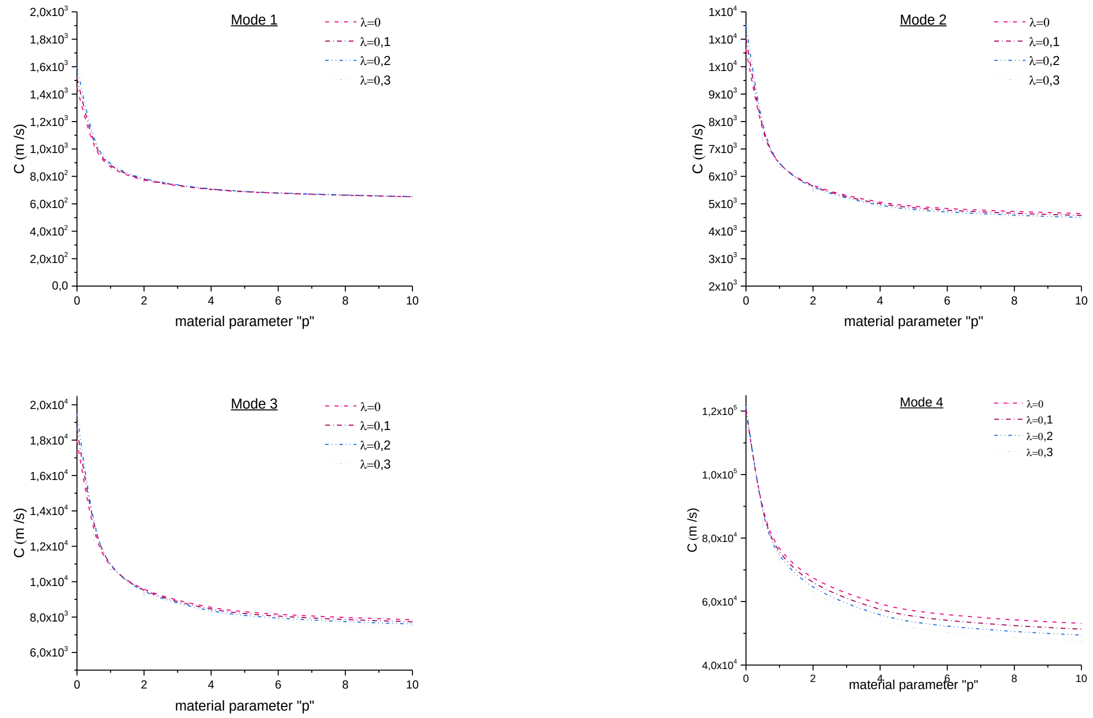


Fig. 3 Variation of the phase velocity of the FGM plates according to the material power index

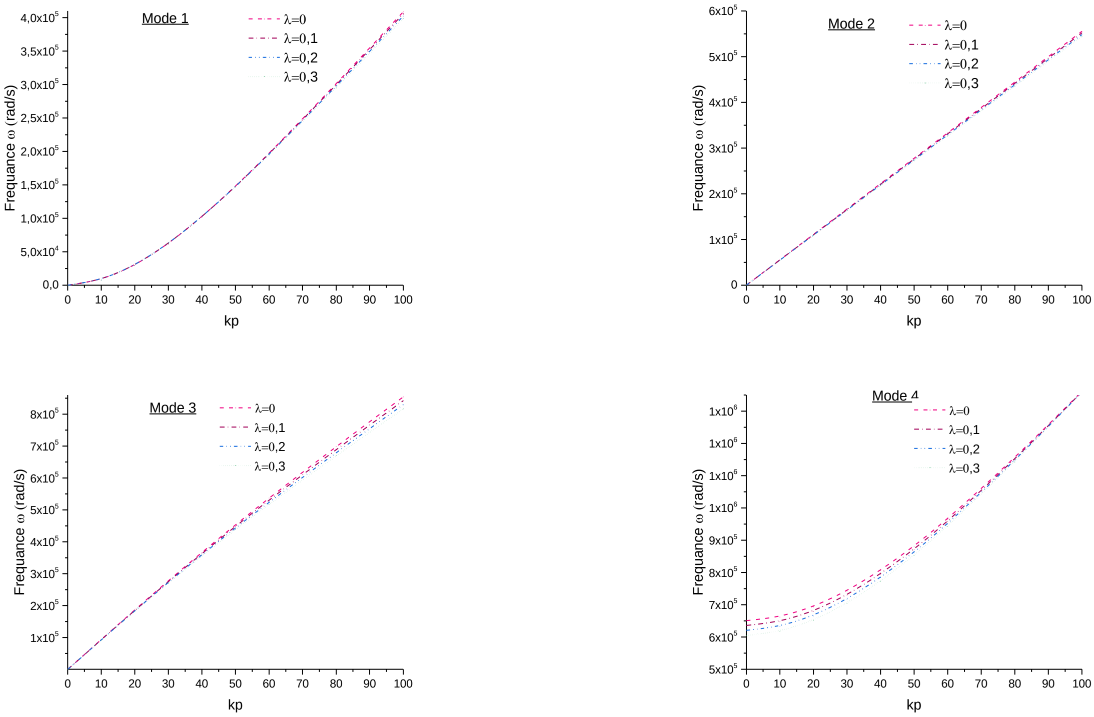


Fig. 4 The natural frequency curves of different functionally graded plates in terms of wave number

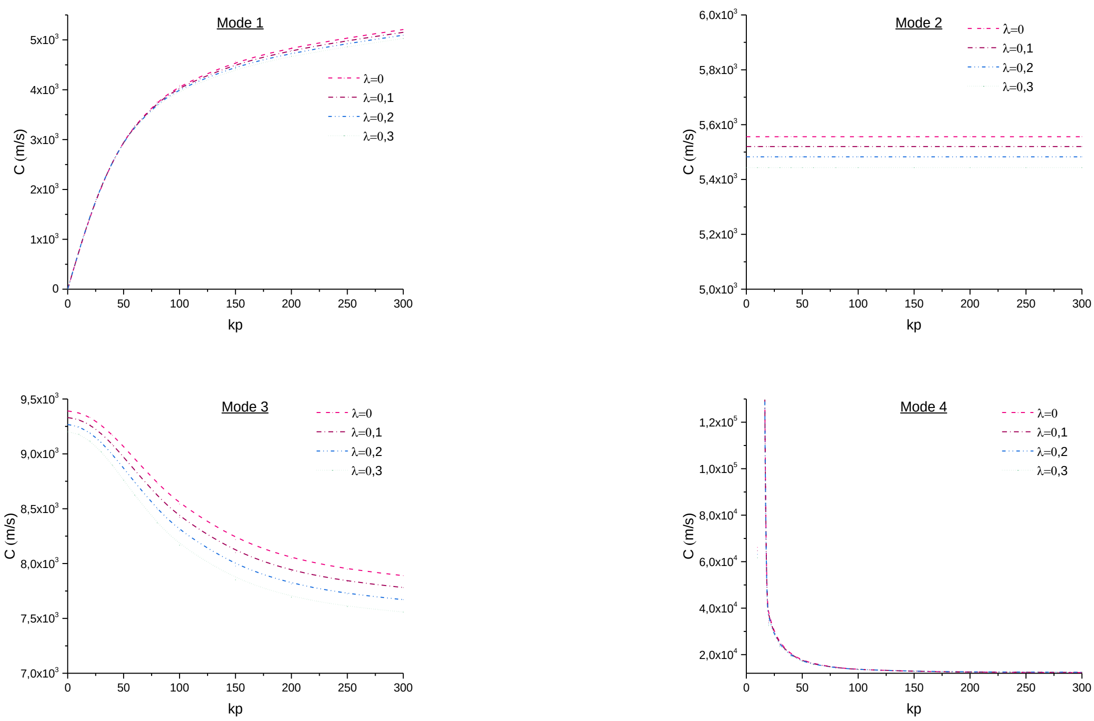
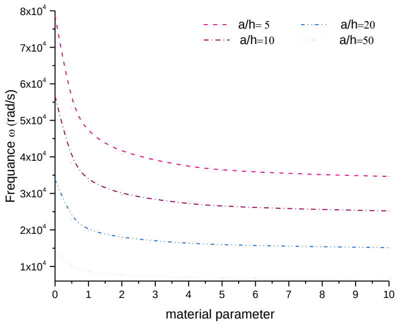
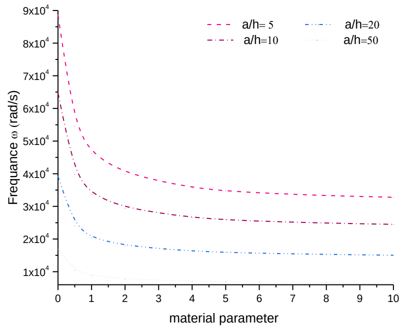


Fig. 5 The phase velocity curves of different functionally graded plates in terms of wave number

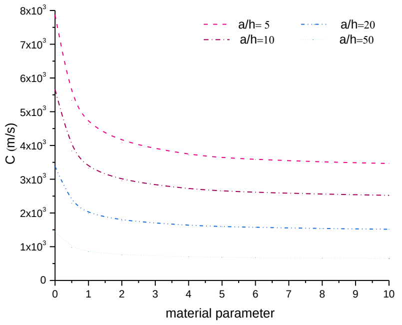


(a) λ =0

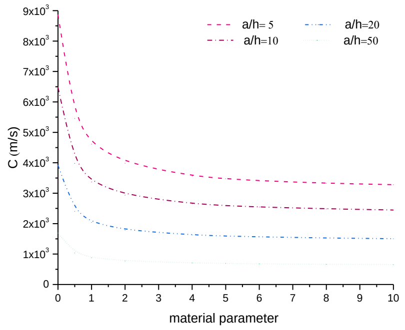


(b) λ =0,3

Fig. 6 Influence of thickness ratio on the natural frequency of the plate FGM

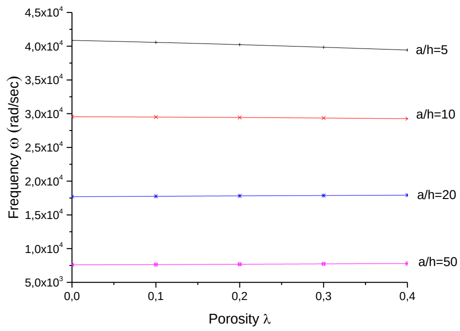


(a) λ =0



(b) λ =0,3

Fig. 7 Influence of thickness ratio on the phase velocity of the plate FGM



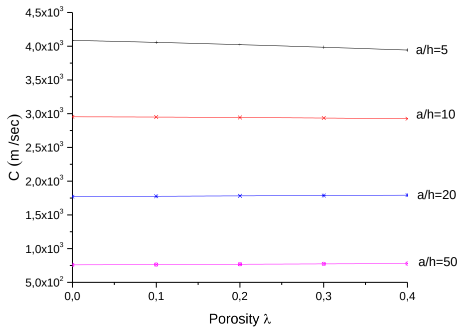


Fig. 8 Influence of the porosity on the natural frequency and phase velocity ofthe different plates

Table 4 The property of the materials used

Matériaux	E (GPa)	ρ (kg/m3)	ν
Si3N4	2370	2370	0.3
SUS304	201.04	8166	0.3

For the first mode, increasing in wave number leads to increase in the phase velocity parameter of the FG plate. For the second mode, the increase of wave number of the plate has no influence on the phase velocity. On contrast, for mode 3, the increase of the wave number of the plate results in a reduction of the phase velocity. It can be also seen that the phase velocity decreased with the increase of the porosity factor for all modes.

Figs. 6 and 7 show the influence of plate thickness ratio on natural frequency and phase velocity of wave propagation, respectively. Two values of the porosity parameter are considered (λ = 0 and λ = 0.3). The wave number value kp is taken equal to 10 and the depth of the beam is 0.02 m. It can be seen that the thickness ratio (a / h) has a considerable effect on the wave propagation frequency in the FGM plate (the latter decreases with the increase of this ratio).

Fig. 8 show the variation of the wave propagation frequency versus porosity factor using the formula proposed for FGM plates with different values of the thickness ratio (a / h).

4. Conclusions

In this work, the natural frequency and wave propagation of porous FG plate with a new model of the porosity distribution is investigated using a shear deformation theory with an integral displacement field. The properties of the material are assumed to vary in the direction of the height in the function of the modified mixing rule. The equations of motion are derived using the Hamilton principle. The analytical dispersion relation of a FG plate is obtained in solution to a problem of eigenvalue.
From the results obtained, it can be concluded that the effect of volume fraction distributions, thickness ratio and porosity volume index on vibratory behavior and wave propagation in FG plates is significant.

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