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Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory

Fouad Bourada*[1], Abdelmoumen Anis Bousahla[2,3], Mohamed Bourada[4], Abdelghani Azzaz[4], Amina Zinata[4] and Abdelouahed Tounsi[4,5]

[1]Département de Génie Civil, Institut de Technologie, Centre Universitaire de Ain Témouchent, Algérie
[2]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algérie
[3]Centre Universitaire Ahmed Zabana de Relizane, Algérie
[4]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algérie
[5]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia

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Abstract. This article present the free vibration analysis of simply supported perfect and imperfect (porous) FG beams using a high order trigonometric deformation theory. It is assumed that the material properties of the porous beam vary across the thickness. Unlike other theories, the number of unknown is only three. This theory has a parabolic shear deformation distribution across the thickness. So it is useless to use the shear correction factors. The Hamilton's principle will be used herein to determine the equations of motion. Since, the beams are simply supported the Navier's procedure will be retained. To show the precision of this model, several comparisons have been made between the present results and those of existing theories in the literature.

Keywords: porous FG beams; trigonometric deformation theory; free vibration; porosity

1. Introduction

In the monolayer, laminate or sandwich composite structures, the matrix and fibers are always stressed and can be damaged, and in addition the interfacial zone represents an area of accumulation and concentration of stresses can seriously influence the different types of composites previously mentioned. To avoid these problems, at the end of the 19 th century the Japanese research laboratories created the functionally graded materials which have a discrete variation across the thickness. Since its developments in the 1980s, FGMs are alternative materials widely employed in aerospace, nuclear reactor, energy sources, biomechanical, optical, civil, automotive, electronic, chemical, mechanical, and shipbuilding industries (Kar and Panda 2013, Zidi *et al.* 2014, Ait Amar Meziane *et al*. 2014, Al-Basyouni *et al*. 2015, Attia *et al*. 2015, Kar and Panda 2015a, b, c, d, Taibi *et al*. 2015, Belkorissat *et al*. 2015, Kar *et al*. 2016, Kar and Panda 2016a, b, c, d, e, Ahouel *et al*. 2016, Boukhari *et al*. 2016, Bounouara *et al.* 2016, Beldjelili *et al*. 2016, Kar *et al*. 2017, Kar and Panda 2017, Menasria *et al.* 2017, Mouffoki *et al*. 2017, El-Haina *et al*. 2017, Fahsi *et al*. 2017, Abdelaziz *et al*. 2017, Attia *et al*. 2018). Several researchers have used these materials in his research work such as for the vibration behavior analysis(Woo *et al.* 2006, Hu and Zhang 2011, Reddy 2011, Ruan and Wang 2014, Bellifa *et al.* 2016, Shahsavari *et al*. 2018), for the buckling analysis (Javaheri and Eslami 2002, Kiani *et al*. 2011, Ghannadpour *et al*. 2012, Ahmed 2014, Mohammadi and Saidi 2010, Kar *et al*. 2017) using the classical plate theory that neglects the transverse shear effect which gives imprecise results for thick plates and short beams. For this problem, a new theory has been developed by Reissner (1945) and Mindlin (1951) which introduces the transverse shear effect. Several works have been published for the studies of the free vibration of FG plates which are presented in (Chen 2005, Alijani *et al*. 2011, Fellah *et al*. 2013, Zhao *et al*. 2009, Hosseini Hashemi *et al*. 2010, Hosseini Hashemi *et al*. 2011, Efraim and Eisenberger 2007) using the first shear deformation theory (FSDT) which takes into account the transverse shear effect in uniform manner across the thickness of the plate which necessitates the introductions of a shear correction factor. In order to avoid introducing this factor each time, Reddy (1984) has developed a high order shear deformation theory (HSDT) that automatically satisfies the conditions of shear stresses nullity at the top and the bottom surfaces of the plate using the warping function. The use of this theory for the different behaviours of FG and nano structure can be found in (Chen *et al.* 2009, Jha *et al*. 2013, Akavci 2014, Mantari *et al.* 2014, Tounsi *et al.* 2016, Houari *et al.* 2016, Kolahchi *et al.* 2017a, b, c, 2016a, b, Madani *et al.* 2016, Bellifa *et al.* 2017a, b, Benadouda *et al.* 2017, Kolahchi and Cheraghbak 2017, Kolahchi 2017, Hajmohammad *et al.* 2017, Khetir *et al*. 2017, Klouche *et al*. 2017, Shokravi 2017 a, b, c, Xiang *et al*. 2013, Meftah *et al*. 2017, Xiang and Kang 2014, Mahi *et al.* 2015, Behravan Ra 2015, Kolahchi *et al.* 2015, Aldousari 2017, Hachemi *et al.*

*Corresponding author, Ph.D.
E-mail: bouradafouad@yahoo.fr

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2017, Bousahla *et al*. 2014, Bouderba *et al*. 2013, 2016, Fekrar *et al.* 2012, Bousahla *et al*. 2016, Draiche *et al*. 2016, Chikh *et al*. 2017, Besseghier *et al.* 2017, Bouafia *et al.* 2017, Benchohra *et al.* 2018, Yazid *et al*. 2018). Since, the use of functionally graded materials has attracted a lot of attention. Wattanasakulpong *et al.* (2012) have thought of introducing the porosity within the FGMs because of the great difference in the temperatures of solidification between the two materials during the production process of the FGM. Recently, several researchers have studied the effect of porosity in functionally graded materials. Ait Yahia *et al.* (2015) have used the various higher-order shear deformation plate theories for the studies of wave propagation in FG porous plate. Benferhat *et al.* (2016) have studied the effect of porosity on the bending and free vibration response of functionally graded plates resting on Winkler-Pasternak foundations. Hadji *et al.* (2015) presented a refined exponential shear deformation theory for free vibration of FG beam with porosities. Kolahchi and Bidgoli (2016) presented size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes. Arani and Kolahchi (2016) studied buckling response of embedded concrete columns armed with carbon nanotubes. Bilouei *et al.* (2016) discussed the buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP).Chen *et al*. (2016) have studied the free and forced vibrations of Timoshenko beams theory with non-uniform porosity distribution. The stability of a non-homogeneous porous plate has been published by Akbas (2017). The studies of the bending, the buckling and the vibration behaviours of functionally graded beams have been published by Fouda *et al.* (2017) using the finite element method. The mechanical response of a FG beams resting on elastic foundation under thickness stretching effect and porosities has been studied by Ait Atmane *et al*. (2015). A new expression of critical moment of lateral buckling for porous and non-porous beams under thermo mechanical loads has been provided by Ziane *et al.* (2017). Gupta and Talha (2017) proposed a new mathematical model to incorporate the effect of the porosity in the FG plate. Zamanian *et al*. (2017) investigated agglomeration effects on the buckling behavior of embedded concrete columns reinforced with SiO2 nano-particles. Zarei *et al.* (2017) examined seismic response of underwater fluid-conveying concrete pipes reinforced with SiO2 nanoparticles and fiber reinforced polymer (FRP) layer. Shokravi (2017d) presented vibration analysis of silica nanoparticles-reinforced concrete beams considering agglomeration effects. Mehar and Panda (2017) presented an experimental, numerical, and simulation study for elastic bending and stress analysis of carbon nanotube-reinforced composite plate. Mehar *et al.* (2017a) presented also a theoretical and experimental investigation of vibration characteristic of carbon nanotube reinforced polymer composite structure. Mehar *et al.* (2017b) provided nonlinear thermoelastic frequency analysis of functionally graded CNT-reinforced single/doubly curved shallow shell panels by FEM. Recently, the stretching effect is also included in structural analysis and the scientific literature can be consulted for this point (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Fekrar *et al.* 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Bourada *et al.* 2015, Hamidi *et al*. 2015, Abualnour *et al*. 2018).

In this paper, a new trigonometric high order shear deformation theory that takes into account the transverse shear effect will be presented for the free vibration analysis of imperfect (porous) FG beams. This theory contains only three unknowns. The equations of motion are determined from the Hamilton's principle. Using the Navier's method to determine the solutions of the free vibration of FG porous beams. A series of results will be presented and compared with those found in the literature.

2. Theoretical formulation

Consider a solid short porous beam of length L, thickness h and width b, made of functionally graded materials with the coordinate system as shown in Fig. 1 the beam examined occupies the following intervals

$$0 \le x \le L; \ -b/2 \le y \le b/2; \ -h/2 \le z \le h/2 \tag{1}$$

x, y, z are Cartesian coordinates.

2.1Effective materials properties of FG porous beams

During the manufacturing of the FG beams, the imperfection in the form of the pores occurs in the beam, this is due to the temperature of solidification between the materials constituting the FG beam (Zhu *et al*. 2001). For this concern Wattanasakulpong and Ungbhakorn (2014) have modified the mixing law by considering the porosity in the materials. The law mixing of material becomes

$$P = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right) \tag{2}$$

V_c *and* V_m are the volume fractions of ceramic and metal, respectively. The volume fraction of ceramic is given by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k, V_c + V_m = 1 \tag{3}$$

Where k is the material index, knowing that the material is totally ceramic where $(k = 0)$ and fully metal where $(k >>)$.Therefore the properties of an imperfect P-FGM beam can be given as follow

Fig. 1 Geometry of functionally graded beam

$$P = \left(P_c - P_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k + P_m - \left(P_c + P_m\right)\frac{\alpha}{2} \quad (4)$$

Based on the Eq. (4), the Young modulus $E(z)$ and material density $\rho(z)$ of the imperfect FG beam with porosity constant through the thickness can be written as follows

$$(FGM-I)\begin{cases} E(z) = \left(E_c - E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - \left(E_c + E_m\right)\frac{\alpha}{2} & (5a) \\ \rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k + \rho_m - \left(\rho_c + \rho_m\right)\frac{\alpha}{2} & (5b) \end{cases}$$

Where $(E_c \text{ and } E_m)$ are the Young modulus of ceramic and metal, $(\rho_c \text{ and } \rho_m)$ are material density and (α) is the parameter which takes into account the porosity.

A further distribution of porosity through the thickness was proposed by Wattanasakulpong and Ungbhakorn (2014) of such fate, the porosity is maximal at mid-plane of the cross section of the beam and tend to zero at the upper and lower surfaces of the beam because the material infiltration process in the zone is more difficult than infiltration in the top and the bottom surface area. The Eqs. (5(a)) and (5(b)) can be rewritten in the forms

$$(FGM-II)\begin{cases} E(z) = \left(E_c - E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - \left(E_c + E_m\right)\frac{\alpha}{2}\left(1-\frac{2|z|}{h}\right) & (6a) \\ \rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k + \rho_m - \left(\rho_c + \rho_m\right)\frac{\alpha}{2}\left(1-\frac{2|z|}{h}\right) & (6b) \end{cases}$$

Recently, a new mathematical expression is modeled by Gupta and Talha (2017), this expression is obtained with the help of the slight modification in the mixing law. The effective material properties are given as

$$(FGM-III)\begin{cases} E(z) = \left(E_c - E_m\right)\left(\frac{2z+h}{2h}\right)^k - \log\left(1+\frac{\alpha}{2}\right)\left(E_c + E_m\right)\left(1-\frac{2|z|}{h}\right) + E_m & (7a) \\ \rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{2z+h}{2h}\right)^k - \log\left(1+\frac{\alpha}{2}\right)\left(\rho_c + \rho_m\right)\left(1-\frac{2|z|}{h}\right) + \rho_m & (7b) \end{cases}$$

2.2 The basic assumptions

The basic assumptions considered in this paper are:

(i) The displacements are small with FG beam thickness and therefore, strains involved are infinitesimal.

(ii) The axial displacement u consist of extension, bending and shear components.

$$u = u_0 + u_b + u_s \quad (8)$$

The bending component u_b is assumed to be similar to the displacements given the classical beam theory (Euler Bernoulli Beam), therefore u_b can be expressed by

$$u_b = -z\frac{\partial w_b}{\partial x} \quad (9)$$

The shear component u_s give rise, in conjunction with w_s, to the parabolic variations of shear and strain γ_{xz} and hence to the shear stress τ_{xz} through the thickness of the beam h in such a way that shear stress τ_{xz} are zero at the top and bottom faces of the beam. Consequently, the expression for u_s and v_s can be given as (Benachour *et al.* 2011, Tounsi *et al.* 2013, Houari *et al.* 2013, Bennoun *et al.* 2016).

$$u_s = \left[z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right]\frac{\partial w_s}{\partial x} \quad (10)$$

(iii) The transverse displacement w includes two components of bending w_b and shear w_s.

$$w(x,z) = w_b(x) + w_s(x) \quad (11)$$

(iv) The transverse normal σ_z is negligible in comparison with in-plane stress σ_x.

2.3 Kinematics and constitutive equations:

Based on the assumptions of the preceding paragraph, the field of displacement of the present theory is given as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} + f(z)\frac{\partial w_s}{\partial x} \quad (12a)$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \quad (12b)$$

With

$$f(z) = z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \quad (13)$$

The strains associated with the displacement in Eqs. (12(a)) and (12(b))

$$\varepsilon_x = \varepsilon_x^0 + zk_x^b + f(z)k_x^s \quad (14a)$$

$$\gamma_{xz} = g(z)\gamma_{xz}^s \quad (14b)$$

Where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \gamma_{xz}^s = \frac{\partial\ w_s}{\partial x} \quad (15a)$$

$$f(z) = z - \frac{h}{\pi}\sin\frac{\pi z}{h},\quad g(z) = 1-\cos(\frac{\pi z}{h}) \quad (15b)$$

By assuming that the material elastic of the FG beam, the stresses in the beam can be written as

$$\sigma_x = \frac{E(z)}{1-\upsilon^2}\varepsilon_x \text{ and } \tau_{xz} = \frac{E(z)}{2(1+\upsilon)}\gamma_{xz} \quad (16)$$

Where (σ_x, τ_{yz}) and $(\varepsilon_x, \gamma_{xz})$ are the stresses and strains components.

The Hamilton's principle is utilised herin to derive the three equations of motion appropriate to the displacement filed. The principle can be expressed in analytical form as (Reddy 1984, Larbi Chaht *et al.* 2015, Zemri *et al*. 2015, Meradjah *et al*. 2015, Sekkal *et al.* 2017a, b, Zidi *et al*. 2017, Meksi *et al*. 2018, Youcef *et al.* 2018, Zine *et al*. 2018, Bakhadda *et al.* 2018, Belabed *et al.* 2018)

$$\int_0^t (\delta U - \delta K)\, dt = 0 \tag{17}$$

With

$$\delta U = \int_0^L \int_{-h/2}^{h/2} \left(\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}\right) dxdz \tag{18}$$

$$\delta K = \int_0^L \int_{-h/2}^{h/2} \rho\left[\ddot{u}_0\, \delta u_0 + (\ddot{w}_b + \ddot{w}_s)\delta(w_b + w_s)\right] dxdz \tag{19}$$

Where δU *and* δK are the variation of the strain and the kinetic energy, respectively.By substitution the Eqs. (18) and (19) into Eq. (17). The principle becomes in the following form

$$\begin{aligned}&\int_0^L \int_{-h/2}^{h/2} \left(\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}\right) dzdx - \\ &\int_0^L \int_{-h/2}^{h/2} \rho\left[\ddot{u}_0\, \delta u_0 + (\ddot{w}_b + \ddot{w}_s)\delta(w_b + w_s)\right] dzdx = 0\end{aligned} \tag{20}$$

Using Eq. (20) and integrating by parts, collecting the coefficients of $\delta u_0, \delta w_b$ and δw_s, the equations of motion for the perfect porous beam are obtained as follow

$$\delta u_0 : \frac{dN}{dx} = I_0\, \ddot{u}_0 - J_1 \frac{\partial \ddot{w}_s}{\partial x} \tag{21a}$$

$$\delta w_b : \frac{d^2 M_x}{dx^2} = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_2 \frac{d^2 \ddot{w}_b}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} + I_0(\ddot{w}_b + \ddot{w}_s) \tag{21b}$$

$$\delta w_s : \frac{d^2 Q_x}{dx^2} + \frac{dP_{xz}}{dx} = J_1 \frac{d\, \ddot{u}_0}{dx^2} - J_2 \frac{d^2 \ddot{w}_s}{dx^2} - K_2 \frac{\partial^2 \ddot{w}_s}{\partial x^2} + I_0(\ddot{w}_b + \ddot{w}_s) \tag{21c}$$

Where, the stresses result (N_x, M_x, Q_x and P_{xz}) are given as

$$\left(N_x, M_x, Q_x\right) = \int_{-h/2}^{h/2} \left(1, z, f(z)\right)\sigma_x dz,\; P_{xz} = \int_{-h/2}^{h/2} g(z)\tau_{xz} dz \tag{22}$$

And ($I_0, I_1, J_1, J_2, I_2, K_2$) are the masse inertia defined as

$$\left(I_0, I_1, J_1, J_2, I_2, K_2\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), zf(z), z^2, f^2(z)\right)\rho(z) dz \tag{23}$$

Using the Eqs. (12), (14), (15), (21), (22) and (23), the equations of motion can be expressed in term of displacements (u_0, w_b and w_s) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_b}{\partial x^3} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} = I_0\, \ddot{u}_0 - J_1 \frac{\partial \ddot{w}_s}{\partial x} \tag{24a}$$

$$\begin{aligned}&B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_b}{\partial x^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_2 \frac{\partial^2 \ddot{w}_b}{\partial x^2} \\ &- J_2 \frac{\partial^2 \ddot{w}_s}{\partial x^2} + I_0(\ddot{w}_b + \ddot{w}_s)\end{aligned} \tag{24b}$$

$$\begin{aligned}&B_{11}^s\frac{\partial^3 u_0}{\partial x^3} - D_{11}^s\frac{\partial^4 w_b}{\partial x^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} + A_{55}^s \frac{\partial^2 w_s}{\partial x^2} = J_1 \frac{\partial \ddot{u}_0}{\partial x} \\ &- J_2 \frac{\partial^2 \ddot{w}_b}{\partial x^2} - K_2 \frac{\partial^2 \ddot{w}_s}{\partial x^2} + I_0(\ddot{w}_b + \ddot{w}_s)\end{aligned} \tag{24c}$$

Where stiffness coefficients $(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s, A_{44}^s, A_{55}^s)$ are defined as

$$\{\, A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s \,\} = \int_{-h/2}^{h/2} \{\, 1, z, z^2, f(z), zf(z), f^2(z) \,\} \frac{E(z)}{(1-\nu^2)} dz \tag{25a}$$

$$\{\, A_{44}^s, A_{55}^s \,\} = \int_{-h/2}^{h/2} [g(z)]^2 \frac{E(z)}{2(1+\nu)} dz \tag{25b}$$

2.4 Exact solution for FGM beam:

The exact solutions of Eq. (24) for simply supported FG beam are derived by using the Navier's procedure. The followings representation for the displacements quantities that satisfy the above boundary conditions can be expressed as

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ W_{bm} \sin(\lambda x) e^{i\omega t} \\ W_{sm} \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \tag{26}$$

Where U_m , W_{bm} and W_{sm} are unknowns functions to be determined, ω is the frequency of the free vibration of the beam, $\sqrt{i} = -1$is the imaginary unite and $\lambda = m\pi / L$.

Substituting Eq. (26) into Eq. (24), the analytical solution for free vibration can be obtained in the form

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right) \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

In which

Table 1 Materials properties

Materials	$E(z)$ (GPa)	$\rho(kg/m^3)$	ν
Alumina (Al_2O_3)	380	3800	0.3
Aluminium (Al)	70.1	2700	

Table 2 Comparison of non-dimensional fundamental frequencies $(\bar{\omega})$ for homogeneous beams

h/L	ETB (Reddy 1999)	FSDBT (Koochaki 2011)	PSDBT (Koochaki 2011)	HSDBT (Ait Atmane *et al*. 2015)	Présent
0.01	2.985526	2.986137	2.9861380	2.9861344	2.9861350
0.0125	2.985232	2.985827	2.9858280	2.9858287	2.9858296
0.0142	2.984340	2.985556	2.9855680	2.9855821	2.9855833
0.0166	2.984865	2.985155	2.9851680	2.9851807	2.9851823
0.02	2.983701	2.984505	2.9845054	2.9845054	2.9845078
0.025	2.982588	2.983285	2.9832858	2.9832858	2.9832896
0.033	2.979668	2.980657	2.9806572	2.9807765	2.9807832
0.04	2.976570	2.978020	2.9780220	2.9780222	2.9780320
0.05	2.971688	2.973193	2.9731941	2.9731941	2.9732093
0.066	2.962858	2.962858	2.9628610	2.9633287	2.9633551
0.1	2.931568	2.934044	2.9340570	2.9340576	2.9341179

$$k_{11} = -A_{11}\lambda^2,\ k_{12} = B_{11}\lambda^3,\ k_{13} = B_{11}^s\lambda^3$$
$$k_{22} = -D_{11}\lambda^4,\ k_{23} = -D_{11}^s\lambda^4,\ k_{33} = -H_{11}^s\lambda^4 - A_{55}^s\lambda^2 \tag{28}$$
$$m_{11} = I_0,\ m_{13} = -J_1\lambda,\ m_{21} = -I_1\lambda$$
$$m_{22} = I_0 + (I_2\lambda^2),\ m_{23} = I_0 + (J_2\lambda^2),\ m_{33} = I_0 + (K_2\lambda^2)$$

3. Numerical results and discussions

In this part, the free vibration analysis of the simply supported perfect and imperfect functionally graded beam will be presented. The properties of the materials used in this work are summarized in the Table 1, these properties vary according to a power law through the thickness of the FG beam Eqs. (5)-(7).

To show the accuracy of the present model, several comparisons have been made between the present results and those given in the literature (Reddy 1999, Koochaki 2011, Sina *et al*. 2009, Ait Atmane *et al*. 2015).

For the simplicity, the non-dimensional fundamental frequency is defined as

$$\bar{\omega} = \frac{\omega L^2}{h}\sqrt{I_0 \Big/ \int_{-h/2}^{h/2} E(z)dz} \tag{29}$$

The Tables 2-4 present the comparisons of the non-dimensional fundamental frequencies of simply supported homogeneous $(k=0)$ functionally graded and FG porous beams, respectively.

Table 3 Non-dimensional fundamental frequencies $(\bar{\omega})$ for FG beams

L/h	k	Ait Atmane *et al.* (2015)	Sina *et al.* (2009)	Present
10	0	2.879551	2.879	2.879604
	0.3	2.774811	2.774	2.774963
30	0	2.922108	2.922	2.922114
	0.3	2.813328	2.813	2.813345
100	0	2.927100	2.927	2.927101
	0.3	2.817838	2.817	2.817840

It can be seen from the Table 2 that the results of the present theory are in good agreement with those obtained by the Euler Bernoulli theory (Reddy 1999), the first and third shear deformation theories presented in (Koochaki 2011) and the high shear deformation theory obtained by Ait Atmane *et al.* (2015) for the free vibration of homogeneous beams $(k=0)$.

From the Table 3, it should be noted that the present results are almost identical with those obtained by Sina *et al.* (2009) based on the first shear deformation beam theory and Ait Atmane *et al.* (2015) based on the high shear deformation beam theory for functionally graded beams with slenderness ratios $(L/h = 10{,}30\ \mathit{and}\ 100)$ and material index $(k = 0\ \mathit{and}\ 0.3)$.

The Table 4 present the six first non-dimensional frequencies of perfect $(\alpha = 0)$ and imperfect $(\alpha \neq 0)$ simply supported beam with $(L/h = 5)$ and $(\alpha = 0{,}0.1\ \mathit{and}\ 0.2)$. It can be seen that the increase of the porosity parameter leads

Table 4 Six first Non-dimensional frequencies $(\overline{\omega})$ of FG beams with (L/h=5)

k	α	Theories	$\overline{\omega_1}$	$\overline{\omega_2}$	$\overline{\omega_3}$	$\overline{\omega_4}$	$\overline{\omega_5}$	$\overline{\omega_6}$
0.5	0.0	Ait Atmane *et al.* (2015)	2.652071106	9.227581431	17.69478543	26.95936800	36.56997080	46.35346951
		Present	2.651404520	9.222989702	17.68927744	26.96711644	36.61350562	46.46069916
	0.1	Ait Atmane *et al.* (2015)	2.629935615	9.163948853	17.59574226	26.83563397	36.43006594	46.20291888
		Present	2.629095059	9.157756657	17.58597937	26.83628014	36.46414042	46.29905062
	0.2	Ait Atmane *et al.* (2015)	2.601215142	9.080767423	17.46517379	26.67110200	36.24244879	45.99940418
		Present	2.600145287	9.072433633	17.44962742	26.66195061	36.26329319	46.07982094
1.0	0.0	Ait Atmane *et al.* (2015)	2.581070224	8.998837152	17.29734773	26.41699106	35.91550139	45.61898733
		Present	2.579605128	8.987361174	17.27512611	26.40026499	35.93249080	45.70457565
	0.1	Ait Atmane *et al.* (2015)	2.527515127	8.838878481	17.03790125	26.08073189	35.52391636	45.18878939
		Present	2.525680432	8.823840712	17.00575143	26.04673934	35.51719093	45.24609360
	0.2	Ait Atmane *et al.* (2015)	2.450084220	8.604783550	16.65295765	25.57471517	34.92630325	44.52326210
		Present	2.447754961	8.584757926	16.60621160	25.51405359	34.88101624	44.53195045
2.0	0.0	Ait Atmane *et al.* (2015)	2.586406159	8.950808821	17.11088871	26.04796748	35.35579761	44.88145525
		Present	2.584168457	8.933596473	17.07699261	26.01796437	35.36484062	44.97218788
	0.1	Ait Atmane *et al.* (2015)	2.487791307	8.651090665	16.61507660	25.39454980	34.58585936	44.03039853
		Present	2.484992536	8.628516661	16.56633880	25.33899122	34.56054326	44.08187906
	0.2	Ait Atmane *et al.* (2015)	2.316541140	8.123771581	15.72853020	24.20495086	33.15759214	42.42204192
		Present	2.313031197	8.093515694	15.65577683	24.10268919	33.06068680	42.37831636
5.0	0.0	Ait Atmane *et al.* (2015)	2.792915979	9.395559685	17.55065355	26.27845939	35.25569424	44.39249113
		Present	2.790094096	9.375602079	17.51329197	26.24542352	35.26170671	44.47932333
	0.1	Ait Atmane *et al.* (2015)	2.694743432	9.069598562	16.96555175	25.44949237	34.21042237	43.15861543
		Present	2.690906635	9.041849564	16.91007416	25.38909982	34.18331811	43.21092956
	0.2	Ait Atmane *et al.* (2015)	2.450980911	8.307099364	15.65914662	23.66287326	32.02232974	40.64091433
		Present	2.445861574	8.268602520	15.57640692	23.55803999	31.93785016	40.63019776

Table 5 Six first Non-dimensional frequencies $(\overline{\omega})$ of FG beams with (L/h=5)

k	α	$\overline{\omega_1}$	$\overline{\omega_2}$	$\overline{\omega_3}$	$\overline{\omega_4}$	$\overline{\omega_5}$	$\overline{\omega_6}$
0.5	0.0	2.651404520	9.222989702	17.68927744	26.96711644	36.61350562	46.46069916
	0.1	2.667088064	9.253060483	17.70899929	26.95740885	36.56438287	46.36845834
	0.2	2.682673156	9.281584724	17.72448224	26.94035708	36.50481699	46.26284334
1.0	0.0	2.579605128	8.987361175	17.27512611	26.40026499	35.93249080	45.70457566
	0.1	2.589269446	8.996280576	17.25541147	26.33373509	35.81165400	45.52861876
	0.2	2.597958420	9.000615766	17.22589725	26.25191828	35.67032521	45.32741930
2.0	0.0	2.584168457	8.933596473	17.07699261	26.01796437	35.36484062	44.97218788
	0.1	2.585363734	8.904969901	16.97747800	25.82646471	35.07609009	44.58961471
	0.2	2.582516521	8.860779895	16.84776401	25.59022945	34.72921746	44.13687388
5.0	0.0	2.790094096	9.375602079	17.51329197	26.24542352	35.26170671	44.47932333
	0.1	2.801594643	9.318458500	17.28043907	25.78004975	34.54435672	43.51131978
	0.2	2.803080159	9.207152348	16.92776350	25.12350439	33.56558474	42.21534677

to decrease in the non-dimensional frequency and this is due to the decrease of the stiffness of the beam.

Table 5 shows the effect of the porosity and the material index on the first six non-dimensional natural frequencies for perfect and imperfect FG beams using the new porosity distribution (FGM-III), it can be seen that the 6th mode gives the highest non-dimensional frequencies.

In the Figs. 2-5, we study the influence of the different geometric and materials parameters on the dynamic response of the perfect and imperfect FG beams.

Fig. 2 shows the non-dimensional fundamental frequency as a function of the slenderness ratio for a non-porous (perfect) functionally graded beam using the proposed theory. The results are in good agreement with those obtained in the literature (Ait Atmane *et al*. 2015). According to the obtained results, it can be seen that the increase of the index power (k) makes the beam flexible. In addition, the fundamental frequency increases with the increase of the slenderness ratio (L/h). However, it is also observed that for the slender beam, the frequencies remains constant, this is due to the effect of shear which negligible in this case.

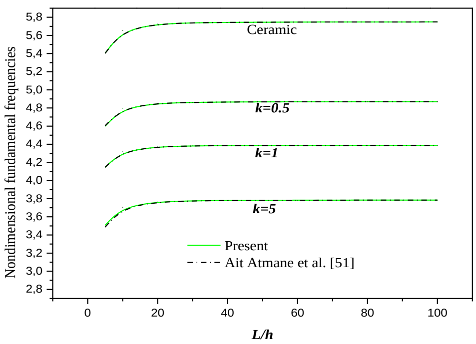

Fig. 2 Variation of the non-dimensional fundamentalfrequencies $(\hat{\omega} = (\omega L^2/h)(\sqrt{\rho_m/E_m})$ of the FG beams with the slenderness (L/h) for different values of the material index

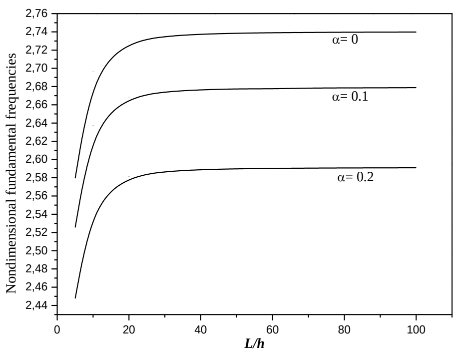

Fig. 3 Variation of the non-dimensional fundamental frequencies of the FG beams $(k=1)$ with the slenderness for different values of the porosity parameter (FGM-I)

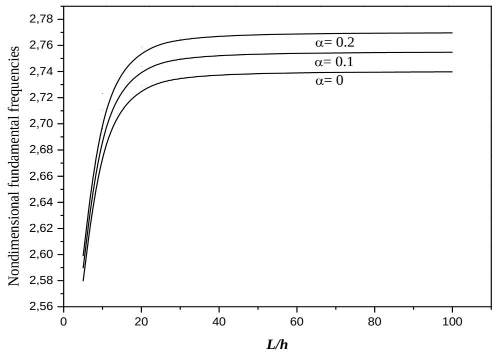

Fig. 4 Variation of the non-dimensional fundamental frequencies of the FG beams $(k=1)$ with the slenderness for different values of the porosity parameter (FGM- II)

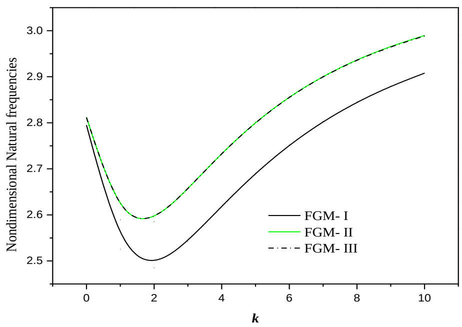

Fig. 5 The variation of the non-dimensional natural frequency with the material index (k) using the three solutions (FGM-I, FGM-II and FGM-III)

The Fig. 5 illustrate the variation of the non-dimensional fundamental frequency as a function of the material index (k) of the three types of the porosity distribution in the FGM (FGM-I, FGM-II and FGM-III) with $(L/h = 5 \;\; and \;\; \alpha)$. It can be seen from the obtained results that the frequencies increase with the increase of the material index (k) when the letter takes values greater than 2. It can be observed that the results obtained using the (FGM-II) gives a high frequencies compared with those obtained using (FGM-I). However, it is remarkable that the solution III gives values of non-dimensional frequencies identical to those determined using solution II (FGM-II).

Fig. 6 present the variation of the Young's modulus through the thickness of the beam for different values of material index (k). It can be seen that the increase of the material index reduce the value of the Young's modulus and consequently the beam tends to be entirely metallic. It can be also noted that the solutions II and III gives almost the same results.

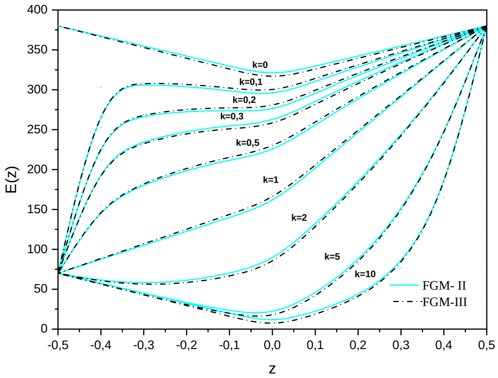

Fig. 6 The variation of the Young's modulus through the thickness for different values of the material index (k) with $\alpha = 0.3$

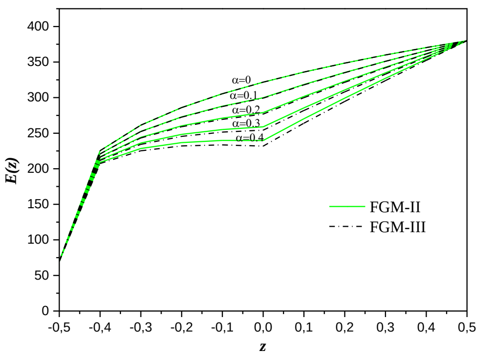

Fig. 7 The variation of the Young's modulus across the thickness for different values of porosity (α) with $k = 0.3$

The Fig. 7 shows the effect of the porosity on the Young's modulus. It should be noted that the porosity reduce the Young's modulus in particular in the central zone of the beam. Again, it can be noted that the results obtained using solutions II and III are almost identical.

4. Conclusions

In the present research, the high order trigonometric deformation beams theory was used for the free vibration analysis of perfect and imperfect (porous) functionally graded beams with different distribution of porosity across the thickness. The theory does not require the shear correction factor and ensures the nullity of the shear stresses at the top and the bottom surface of the beam. The equations of motion are solved using the Navier's procedure. The impact of several parameters influencing the fundamental frequency such as power law exponent, geometry ratios and the different types of porosity distribution are purposed and discussed in detail.

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