Wind and Structures, *Vol. 27, No. 6 (2018) 431-438*
DOI: https://doi.org/10.12989/was.2018.27.6.431

Vibration analysis of inhomogeneous nonlocal beams via a modified couple stress theory incorporating surface effects

Farzad Ebrahimi* and Hamed Safarpour

Mechanical Engineering department, faculty of engineering, Imam Khomeini International University, Qazvin, Iran

(Received August 8, 2017, Revised February 6, 2018, Accepted February 8, 2018)

Abstract. This paper presents a free vibration analysis of size-dependent functionally graded (FG) nanobeams with all surface effects considerations on the basis of modified couple stress theory. The material properties of FG nanobeam are assumed to vary according to power law distribution. Based on the Euler–Bernoulli beam theory, the modeled nanobeam and its equations of motion are derived using Hamilton's principle. An analytical method is used to discretize the model and the equation of motion. The model is validated by comparing the benchmark results with the obtained results. Results show that the vibration behavior of a nanobeam is significantly influenced by surface density, surface tension and surface elasticity. Also, it is shown that by increasing the beam size, influence of surface effect reduces to zero, and the natural frequency tends to its classical value.

Keywords: surface effects; vibration analysis; modified couple stress theory; functionally graded material; nanobeam

1. Introduction

In some applications the experiments show that size effects play an important role in mechanical properties (Chong *et al*. 2001). Thus, avoiding these effects may results incompletely hence wrong designs and incorrect solutions. The size effect is not considered in the classical continuum theories Thus; this theory does not fit the micro and nano scale devices. So we are looking for theories that consider the small scale effects.

The nonlocal elasticity theory assumes that the stress state at a reference point is a function of the strain at all neighbor points of the body. Hence, this theory could take into consideration the effects of small scales. Lots of studies have been performed to investigate the size-dependent response of structural systems based on Eringen's nonlocal elasticity theory (Ebrahimi and Salari 2015a, b, 2016, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Nasirzadeh 2015, Ebrahimi and Barati 2016 a, b, c, d, e, f, Ebrahimi and Hosseini 2016 a, b, c). One of the non-classical theories that consider the size effects is couple stress theory. Toupin (1962) investigated couple stress theory including higher order rotation gradients, which is in fact the asymmetric part of the deformation gradient. According to this theory, it includes four material constants (two classical and two additional) for isotropic elastic materials. As an example of this theory, Asghari *et al.* (2011) presented the size effects in Timoshenko beams based on the couple stress theory. It is mention that, it is difficult to determining the microstructure related length scale parameters. So, we're looking for the continuum theory that involves only one additional material length scale parameter. Modified couple stress theory is one of the best and most well-known continuum mechanics theories that include small scale effects with good accuracy in micro scale devices. Yang *et al.* (2002) presented a modified couple stress theory, in which the couple stress tensor is symmetric and only one internal material length scale parameter is involved, unlike classical couple stress theory mentioned above. Many researchers have used this theory to examine the dynamic and static behavior of microbeams and microplates (Reddy 2011, Shaat *et al.* 2014).

Due to providing many advantages and superior properties including high temperature resistance and high strength, functionally graded materials have the particular importance in modeling. Also, FGMs are a new class of composite materials, that broadly have been spread out into micro/nano scale devices and systems such as cantilever atomic force microscopes (AFMs cantilever) (Rahaeifard *et al*. 2009), mechanical systems at nano and micro scale (Lee *et al.* 2006) and the shape memory alloy fields (Witvrouw and Mehta 2005), to achieve desired performance and high sensitivity. Also, the effect of various boundary conditions on vibration and buckling characteristics of plates has been examined in recent works (Abdelaziz *et al*. 2017), also, current nanostructure can be used in smart systems (Sadeghi *et al*. 2017, Khanade *et al*. 2017).

Natarajan (2012) presents a brief overview of FG materials, based on Eringen's nonlocal elasticity and Reissner–Mindlin plate for application of nonlocal elasticity theory. Other papers presented the size-dependent vibration and static behavior of micro-beams made of functionally graded materials which are analytically investigated on the basis of the modified couple stress theory (Asghari *et al.* 2010). Nanomachines are of great importance in the study of nanotechnology. For example, in applications such as DNA nanomachines, programmable chemical synthesis and targeted drug delivery can be clearly observed the importance of nanomachines (Chen *et al.* 2012). Many

*Corresponding author, Professor
E-mail: febrahimy@gmail.com

Copyright © 2018 Techno-Press, Ltd.
http://www.techno-press.com/journals/was&subpage=7 ISSN: 1226-6116 (Print), 1598-6225 (Online)

researchers Earmark their studies to design nanostructures correctly. For example, design a nanotube which can be driven by fluid flow (Li *et al*. 2014). In addition, this material can be used in electrical devices such as those mentioned in Ref (Vafamehr *et al.* 2017)

Recently nanoscale simulations and experimental measurements showed that increasing the surface to volume ratio effects on mechanical properties. Hence for nanoscale structures and materials due to the high surface to volume ratio the surface effects play an important role in their Mechanical properties. In classical mechanics due to lower surface energy than volume energy, surface energy is not considered. The surface elasticity theory that considered surface energy on the mechanical properties was presented by Gurtin *et al*. (1975, 1998). They proposed the theory by modeling the surface by a two dimensional membrane adhering to the underlying bulk material without slipping. Also, the thermal effect on mechanical characteristics of FG structures have been investigated in recent researches (Bouderba *et al.* 2013). Wang and Feng (2007) investigated effects of both surface elasticity and residual surface tension on the natural frequency of microbeams. Yan and Jiang (2011) studied the influence of surface effects, including residual surface stress, surface elasticity and surface piezoelectricity, on the vibrational and buckling behaviors of piezoelectric nanobeams by using the Euler–Bernoulli beam theory. The natural frequency of nanotubes with consideration of surface effects has been presented using the nonlocal Timoshenko beam theory (Lei 2012). The boundary conditions and governing equations for the free vibration of nonlocal Timoshenko beams are derived via Hamiltonian's principle. Recently various refined and higher-order shear deformation theories have been introduced for analysis of FG structures by researchers (Abdelhak *et al.* 2016). Most recently various nonlocal zeroth-order, trigonometric and refined shear deformation theories have been introduced for the analysis of micro/nanostructures by researchers (Bouafia *et al*. 2017)

It is worth mentioning, none of the previous works have considered surface effects on the FG nanobeam using modified couple stress theory. The purpose of this paper is to propose a comprehensive analytical model for analysis the linear free vibration of FG-nanoscale Euler–Bernoulli beam using modified couple stress theory and considering the surface effects including surface density, surface elasticity and surface tension, also equation has developed by principle of minimum potential energy. The material for this work is functionally graded material and assuming that the bottom surface is aluminum and the top surface is silicon according to power law distribution. The results show, the vibration behavior of nanobeams is significantly influenced by surface density, surface tension and surface elasticity. Also it is shown that by increasing the beam size, influence of surface effects reduce to zero, and the natural frequency tends to its classical value.

2. Theory and formulation

2.1 Numerical simulation procedure

Fig. 1 shows an FG nanobeam of length L, thickness h, and width b. At the top and the bottom surfaces, the FG nanobeam is generally composed of two different materials. According to the power law distribution, bulk elastic modulus E (z), mass density ρ (z), surface elastic modulus E_S, and residual surface stresss τ_0(z) are assumed to be along the thickness direction. Volume fraction index n determines the variation profile of material properties across the FG nanobeam thickness and the superscripts +" and − " denote the top surface and the bottom surface, respectively. Assuming that the top surface (at z=−h/2) is metal and the bottom surface (at z=h/2) is functionally graded nanobeam ceramic, and for different values of n, the mechanical properties and surface elastic properties can be obtain by Eq (1).

$$\begin{aligned}
&\rho(z) = (\rho^+ - \rho^-)(\frac{z}{h} + \frac{1}{2})^k + \rho^-, \\
&E_S(z) = (E_S{}^+ - E_S{}^-)(\frac{z}{h} + \frac{1}{2})^k + E_S{}^-, \\
&\rho_S(z) = (\rho_S{}^+ - \rho_S{}^-)(\frac{z}{h} + \frac{1}{2})^k + \rho_S{}^-, \\
&\tau_0(z) = (\tau_0{}^+ - \tau_0{}^-)(\frac{z}{h} + \frac{1}{2})^k + \tau_0{}^-, \\
&\nu(z) = (\nu^+ - \nu^-)(\frac{z}{h} + \frac{1}{2})^k + \nu^-.
\end{aligned} \tag{1}$$

Also, the linear constitutive equations are in the following form

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{12} & E_{22} & E_{23} & 0 & 0 & 0 \\ E_{13} & E_{23} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} \tag{2}$$

where E_{ij} is represent elastic matrices. Because there is no slipping between upper and lower layers and the underlying material, the displacement in the whole of the beam is united. In an Euler–Bernoulli beam model, the displacements of an arbitrary point along the x- and z-axes are denoted by u_x(x, z, t) and u_z(x, z, t), respectively, So, we have

$$\begin{aligned}
&u_x(x,z,t) = -z\frac{\partial w(x,t)}{\partial x} \\
&u_z(x,z,t) = w(x,t)
\end{aligned} \tag{3}$$

where t is time, $u(x,t)$ and $w(x,t)$ are the axial and the transverse displacements of any point on the midplane. According to Euler–Bernoulli beam theory, the only nonzero strain is given by

$$\varepsilon_{xx} = -z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{4}$$

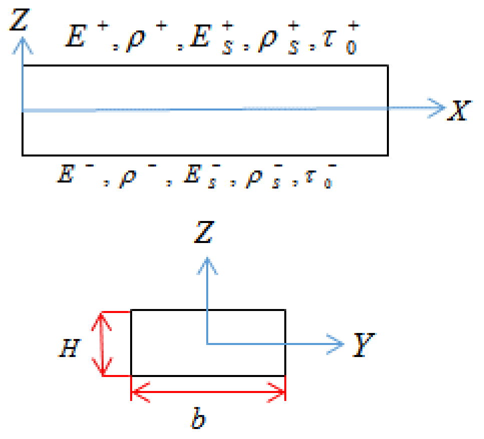

Fig. 1 Mesh grid of topographic model

Considering the relevant bulk stress–strain relation of functionally graded nanobeam and neglecting any residual stresses in the bulk due to surface stress also, assuming constant piezoelectric properties, the stress can be written as

$$\sigma_{xx} = E(z)\varepsilon_{xx} + \upsilon\sigma(zz) \tag{5}$$

The surface constitutive equation can be written as (Gurtin and Murdoch 1978)

$$\begin{aligned}(\tau_{xx})^{\pm} &= (\tau_0)^{\pm} + E_S\varepsilon_{xx}\\ (\tau_{zx})^{\pm} &= (\tau_0)^{\pm}\frac{\partial w}{\partial x}\end{aligned} \tag{6}$$

The stresses of the upper and lower surface layers must satisfy the following equilibrium relations

$$(\tau_{\beta i,\beta})^{\pm} - (\sigma_{iz})^{\pm} = \rho^{\pm}(\frac{\partial^2 u_i}{\partial t^2})^{\pm} \tag{7}$$

$\tau_{\beta i}{}^{+}$ and $\tau_{\beta i}{}^{-}$ are the upper and lower surface stresses of the FG nanobeam, respectively.$(\sigma_{iz})^{\pm}$ are the bulk stresses at z= $\pm h/2$. $(u_i)^{+}$ and $(u_i)^{-}$ are the displacements of surface layers along the i-direction at z= $\pm h/2$, respectively. In Eq. (7), $i = x, y, z$ and $\beta = x, y$ .The following equations can be obtained by substituting Eq. (6) into Eq. (7)

$$\begin{aligned}(\sigma_{zz})^{+} &= (\tau_0)^{+}(\frac{\partial^2 w}{\partial x^2})^{+} - (\rho_0)^{+}(\frac{\partial^2 w}{\partial t^2})^{+}\\ (\sigma_{zz})^{-} &= -(\tau_0)^{-}(\frac{\partial^2 w}{\partial x^2})^{-} + (\rho_0)^{-}(\frac{\partial^2 w}{\partial t^2})^{-}\end{aligned} \tag{8}$$

The bulk stress component σ_{zz}, is assumed to be zero in classical beam theory. However, the Eq. (8) must be true in the surface equilibrium equations of Gurtin–Murdoch model (Gurtin and Murdoch 1978). For satisfying the surface equilibrium equations Lu *et al.* (2006) assumed a linear distribution for the bulk stress component, σ_{zz} . Also, we assumed that the bulk stress component, σ_{zz} varies cubically through the beam thickness and its derivative about z besides the two surfaces are assumed to be zero, i.e., $\sigma_{zz,z}$ =0 at z = $\pm$h/2 (Lü *et al*. 2009). So, we have

$$\begin{aligned}\sigma_{zz} &= \mathrm{F}(z)\left[(\sigma_{zz})^{+} - (\sigma_{zz})^{-}\right]\\ &+ \frac{1}{2}\left[(\sigma_{zz})^{+} + (\sigma_{zz})^{-}\right]\end{aligned} \tag{9}$$

And we have $\mathrm{F}(z) = (\frac{2z}{h})(\frac{z}{h^2}) - \frac{3}{4}$. Substitution of Eq. (8) into Eq. (9) gives.

$$\begin{aligned}\sigma_{zz} &= \mathrm{F}(z)\left[(\rho_0^{+} + \rho_0^{-})\frac{\partial^2 w}{\partial t^2} - (\tau_0^{+} + \tau_0^{-})\frac{\partial^2 w}{\partial x^2}\right]\\ &+ \frac{1}{2}\left[-(\rho_0^{+} - \rho_0^{-})\frac{\partial^2 w}{\partial t^2} + (\tau_0^{+} - \tau_0^{-})\frac{\partial^2 w}{\partial x^2}\right]\end{aligned} \tag{10}$$

On the upper and lower surface of the functionally graded nanobeam the surface stresses is specified by the Laplace–Young equation (Chen *et al.* 2006). So, the distributed loading on the upper and lower surfaces are as follows

$$\begin{aligned}p^{+}(x) &= \tau_0{}^{+}b\frac{\partial^2 w}{\partial x^2}\\ p^{-}(x) &= \tau_0{}^{-}b\frac{\partial^2 w}{\partial x^2}\end{aligned} \tag{11}$$

Thus, the superposition of upper and lower surface stresses can be implemented by an effective transverse distributed loading into the functionally graded nanobeam along the longitudinal direction p(x) in the following form

$$p(x) = p^{+}(x) + p^{-}(x) = b(\tau_0{}^{+} + \tau_0{}^{-})\frac{\partial^2 w}{\partial x^2} \tag{12}$$

2.2 Modified couple stress theory

Yang *et al.* (200) has represented the modified couple stress theory in which the strain energy density is a function of both strain tensor and gradient of the rotation vector. These tensors are specified by two classical material constants for isotropic linear elastic materials and one independent material length scale parameter. The stored strain energy U is given by

$$U = \frac{1}{2}\iiint_V (\sigma_{ij}\varepsilon_{ij} + m_{ij}^{s}\chi_{ij}^{s})dV \tag{13}$$

In which

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i})\\ \chi_{ij}^{s} &= \frac{1}{2}(\varphi_{i,j} + \varphi_{j,i})\\ \varphi_i &= \frac{1}{2}\left[curl(u)\right]_i\\ m_{ij} &= 2\mu l^2\chi_{ij}^{s}\\ \mu(z) &= \frac{E(z)}{2(1+\nu(z))}\end{aligned} \tag{14}$$

in which ε_{ij} and χ_{ij}^{s} are the classical strain and symmetric rotation gradient tensors, respectively. Also φ_i is the infinitesimal rotation vector and l is material length scale parameter. For the equation of the motion, the Hamilton principle states that (Tauchert 1974)

$$\int_{t_2}^{t_1}(\delta U-\delta V-\delta T)dt=0 \tag{15}$$

where the strain energy, the potential of external loading δV and the kinetic energy δT are

$$\begin{aligned}&U=\frac{1}{2}\iiint_V(\sigma_{ij}\varepsilon_{ij}+m_{ij}^{s}\chi_{ij}^{s})dV\\&+\frac{1}{2}(\int_{s^-}\sigma_{ij}^{s^-}\varepsilon_{ij}ds^-+\int_{s^+}\sigma_{ij}^{s^+}\varepsilon_{ij}ds^+)\\&\Rightarrow\delta U=b\int\int_{-h/2}^{h/2}(\sigma_{xx}(z)\delta\varepsilon_{xx}\\&-m_{xy}\frac{\partial^2}{\partial x^2}\delta w)dzdx\\&+\int_{s^-}\sigma_{ij}^{s^-}\delta\varepsilon_{ij}ds^-\end{aligned}\tag{16}$$

$$\delta V=\int b\left\{(\tau_0^{+}+\tau_0^{-})\right\}\frac{\partial^2 w}{\partial x^2}dx \tag{17}$$

$$\begin{aligned}&\delta T=b\int\int_{-h}^{h}\rho(z)\begin{bmatrix}\left(-z\frac{\partial^2 w}{\partial t\partial x}\right)\left(-z\frac{\partial^2\delta w}{\partial t\partial x}\right)\\+\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t}\end{bmatrix}dzdx\\&+b(\int_{s^-}\rho_0^{-}\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t}+\int_{s^+}\rho_0^{+}\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t})\\&+b(\int_{s^-}\rho_0^{-}\left(-z\frac{\partial^2 w}{\partial t\partial x}\right)\left(-z\frac{\partial^2\delta w}{\partial t\partial x}\right)+\\&\int_{s^+}\rho_0^{+}\left(-z\frac{\partial^2 w}{\partial t\partial x}\right)\left(-z\frac{\partial^2\delta w}{\partial t\partial x}\right))=\\&-\left[I_0+b(\rho_0^{+}+\rho_0^{-})\right](\frac{\partial w}{\partial t})^2+\\&\left[I_2+\frac{bh^2}{4}(\rho_0^{+}+\rho_0^{-})\right]\left(\frac{\partial^4 w}{\partial t^2\partial x^2}\right)\end{aligned}\tag{18}$$

Where m_{xy} ,b, M and I are the deviatory symmetric part of the couple stress tensor, the beam width, moment resultant and mass moment inertia respectively which are given by

$$Y_{xy}=\int_{-h}^{h}m_{xy}dz \tag{19}$$

$$M=\int_{-h/2}^{h/2}z\sigma_{xx}(z)dz, \tag{20}$$

$$\left\{I_0,I_2\right\}=\int_{h}^{h}\rho(z)b\left\{1,z^2\right\}d(z) \tag{21}$$

By substituting Eqs. (16)-(18) into Eq. (15), we have

$$\begin{aligned}&\delta w:\frac{\partial^2}{\partial x^2}(M_{xx}+Y_{xy})+b(\tau_0^{+}+\tau_0^{-})\frac{\partial^2 w}{\partial x^2}\\&=\left[I_0+b(\rho_0^{+}+\rho_0^{-})\right]\frac{\partial^2 w}{\partial t^2}\\&-\left[I_2+\frac{bh^2}{4}(\rho_0^{+}+\rho_0^{-})\right]\frac{\partial^4 w}{\partial t^2\partial x^2}\end{aligned}\tag{22}$$

The local bending moment are given by

$$Y_{xy}=-bl^2\frac{\partial^2 w}{\partial x^2}\int_{-h/2}^{h/2}\mu(z)dz \tag{23}$$

$$\begin{aligned}M_{xx}=&\begin{bmatrix}-bC_0-C_{S0}+(\frac{3bZ_1^{\xi}}{2h}-\frac{2bZ_2^{\xi}}{h^3})\\(\tau_0^{+}+\tau_0^{-})-\frac{bh^2}{4}(E_S^{+}+E_S^{-})\end{bmatrix}(\frac{\partial^2 w}{\partial x^2})\\&+\left[(-\frac{3bZ_1^{\xi}}{2h}+\frac{2bZ_2^{\xi}}{h^3})(\rho_0^{+}+\rho_0^{-})\right](\frac{\partial^2 w}{\partial t^2})\end{aligned}\tag{24}$$

where parameters are used in Eq. (24) are given by

$$\begin{aligned}&\left\{Z_1^{\xi},Z_2^{\xi}\right\}=\int_{-h/2}^{h/2}\nu(z)\left\{z^2,z^4\right\}dz\\&C_0=\int_{-h/2}^{h/2}E(z)(z^2)dz\\&C_{S0}=2\int_{-h/2}^{h/2}E_S(z)\times z^2dz\end{aligned}\tag{25}$$

So governing equation of FG nanobeam with considering surface effects is given by

$$\begin{aligned}&\delta w:\left(EI\right)^{*}\frac{\partial^4 w}{\partial x^4}+\left(\rho I\right)^{*}\frac{\partial^4 w}{\partial x^2\partial t^2}\\&+C\frac{\partial^2 w}{\partial x^2}-\left(\rho A\right)^{*}\frac{\partial^2 w}{\partial t^2}=0\end{aligned}\tag{26}$$

The parameters are used in Eq. (26) are given by

$$\begin{aligned}&\left(EI\right)^{*}=\begin{bmatrix}-bC_0-C_{S0}+\\(\frac{3bZ_1^{\xi}}{2h}-\frac{2bZ_2^{\xi}}{h^3})(\tau_0^{+}+\tau_0^{-})\\-\frac{bh^2}{4}(E_S^{+}+E_S^{-})\\-bl^2\int_{-h/2}^{h/2}\mu(z)dz\end{bmatrix},\\&\left(\rho I\right)^{*}=\begin{bmatrix}(-\frac{3bZ_1^{\xi}}{2h}+\frac{2bZ_2^{\xi}}{h^3})(\rho_0^{+}+\rho_0^{-})\\+(I_2+\frac{bh^2}{4}(\rho_0^{+}+\rho_0^{-}))\end{bmatrix},\\&C=b(\tau_0^{+}+\tau_0^{-}),\quad\left(\rho A\right)^{*}=I_0+b(\rho_0^{+}+\rho_0^{-})\end{aligned}\tag{27}$$

3. Solution procedure

In this section on the basis of Navier procedure the governing equations of FG nanobeam are solved. In order to solve the governing equation of FG nanobeam and simply supported boundary condition, the Navier procedure is used by assuming the substitutions as follows

$$\{w(x,t)\} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{W_m \sin\left(\frac{n\pi}{L}x\right)e^{i\omega t}\right\} \quad (28)$$

where n is the axial wave numbers. By substituting Eq. (28) into Eq. (26) the motion equations are written as a matrix form as bellow

$$\left\{[k] - \omega^2 [M]\right\}\{d\} = 0 \quad (29)$$

where ω is natural frequency of FG nanobeam and $\{d\} = \{W_m\}$ is displacement amplitude vector. In Eq. (29) [M] is the mass matrix, also [k] is stiffness matrix. By setting the determinant of the coefficient matrix to zero, we can find the natural frequenciesω_n.

$$\left\{[k] - \omega^2 [M]\right\}\{d\} = 0 \quad (30)$$

4. Result and desiccation

Results are presented by two sections, the first one presents a validation of the proposed model with previous literatures. The second section shows influence of surface effect on each case. Also influence of each surface effects and length scale parameter on frequencies ratio will be discussed.

4.1 Validation

For result's verification of this work, Table 1 give a comparison of results for nondimensional frequency, Ψ, of simply supported nanobeam between the presented results with those obtained by Nateghi (2013). According to the Fig. 2, it is revealed that the proposed modeling can provide good accurate natural frequency results of the FG nanobeam as compared to the Nateghi (2013).

Table1 Comparison of results for natural frequencies (MHz) of simply supported nanobeam

Nondimensional scale parameter (*h/l*)	Present study	Nateghi (2013)
1	4.067040	4.067115
2	1.437997	1.430808
3	0.713966	0.713938
4	0.508455	0.508795
5	0.381126	0.381106
10	0.179607	0.179655

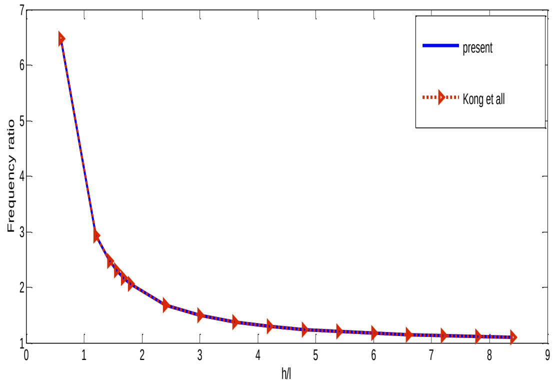

Fig. 1 Comparison of the natural frequency of FG microbeam with the results obtained by kong *et al.* (2008)

Table 2 Material and surface elastic properties of FGM constituents

Properties or surface elastic properties	Unit	Aluminum	Silicon
E	GPA	70	210
ρ	Kg/m^3	2700	2370
υ		0.3	0.24
E_S	N/m	5.1882	-10.6543
ρ_0	Kg/m^3	5.46*10^{-7}	3.17*10^{-7}
τ_0	N/m	0.9108	0.6048

4.2 Parametric result

The material for this paper is functionally graded material and it is assumed that the bottom surface is aluminum and the top surface is silicon. Volume fraction index (n) determines the variation profile of the material properties across the rotary FG nanobeam thickness. For the particular case (n=0) the material is not FG and certainly the material is the absolute metal. The material properties and surface elastic properties are given in Table 2. In addition, the geometry dimensions of nanobeam are assumed as: b (width) =0.2L nm and h (thickness) = 0.2L. Also the frequency ratio is given by following term

$$Frequency\,ratio = \frac{Frequency\ with\ considering\,all\ surface\ effects}{Frequency\ in\ each\ case} \quad (31)$$

4.2.1 Subtitle surface elasticity:

Fig. 2 shows the frequency ratio with respect to the length of a simply supported nanobeam for different values of material length scale parameter. It is observed that, by increasing the length the frequency ratio tends to decrease for different values of material length scale parameter. Also Fig. 2 shows, when the frequency ratio tend to decrease, this parameter converge to a constant value, and the natural frequency reaches its classical value. It is noted that, by increasing material length parameter, the frequency ratio tend to decrease. In addition, as it is shown in Fig. 2 the effect of this parameter is less than the other parameters, because among all of them this case has the biggest frequency ratio.

4.2.2 Surface density

Fig. 3 shows the frequency ratio with respect to the length of a simply supported nanobeam for different values of material length scale parameter. It is observed that, by increasing the length the frequency ratio tends to decrease for different values of material length scale parameter. Also, by increasing material length parameter, the frequency ratio tends to decrease.

4.2.3 Surface tension

Surface tension has opposite effects in compare with other, this parameter increases the amount of natural frequency versus the length of nanobeam. By increasing the beam length, the frequency ratio tends to increase. Also, when the frequency ratio tends to decrease, this parameter converges to a constant value, and the natural frequency reaches its classical value.

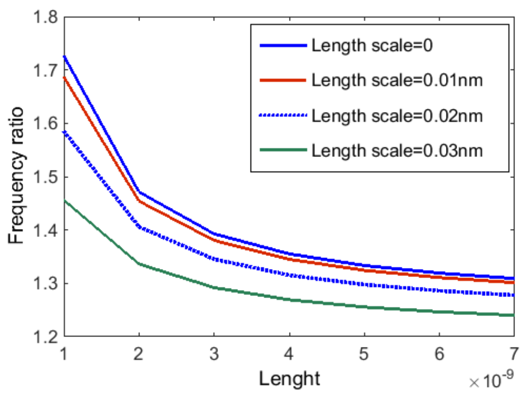

Fig. 2 Variation of natural fundamental frequency ratio versus length of nanobeam by considering Surface elasticity and N=1

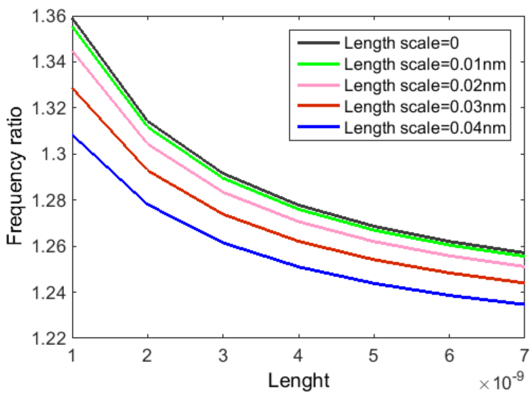

Fig. 3 Variation of natural fundamental frequency ratio versus length of nanobeam by considering Surface density and N=1

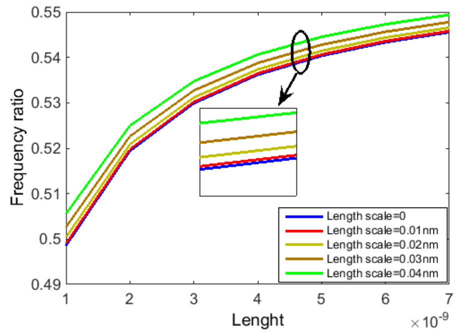

Fig. 4 Variation of natural fundamental frequency ratio versus length of nanobeam by considering surface tension and N=1density and N=1

4.2.4 Surface tension and elasticity

Fig. 5 shows the frequency ratio with respect to the length of a simply supported nanobeam for different values of material length scale parameter. It is observed that, by increasing the length the frequency ratio tends to increase for different values of material length scale parameter. Also, by increasing material length parameter, the frequency ratio tend to decrease.

4.2.5 Surface density and elasticity:

Considering two parameters among these three surface parameters will cause different behavior in natural frequencies. Assuming both surface density and elasticity which has opposite effects on natural frequency reduces the amount of frequency by increasing the nanobeam. According to Fig. 6 the frequency ratio will also decrease by increasing the size of beam.

4.2.6 Surface density and tension:

Fig. 7 shows the frequency ratio with respect to the length of a simply supported nanobeam for different values of material length scale parameter. It is observed that, by increasing the length the frequency ratio tends to increase for different values of material length scale parameter. Also, by increasing material length parameter, the frequency ratio tends to increase.

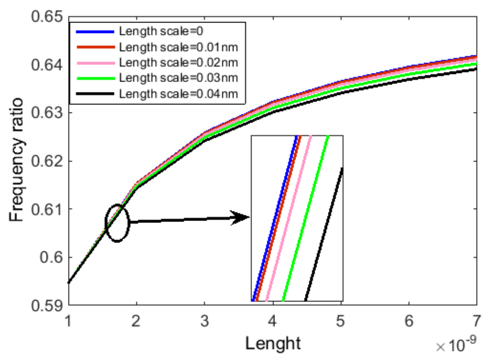

Fig. 5 Variation of natural fundamental frequency ratio versus length of nanobeam by considering both surface elasticity and tension and N=1

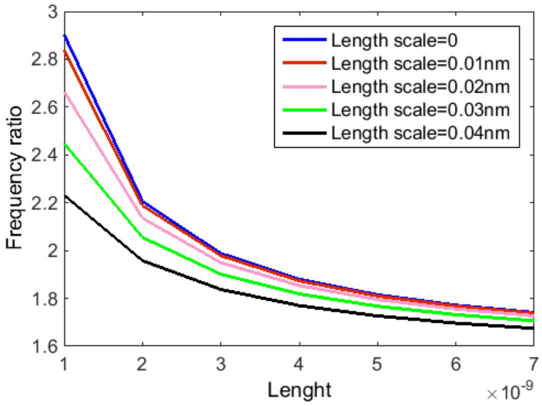

Fig. 6 Variation of natural fundamental frequency ratio versus length of nanobeam by considering both Surface elasticity and density and N=1

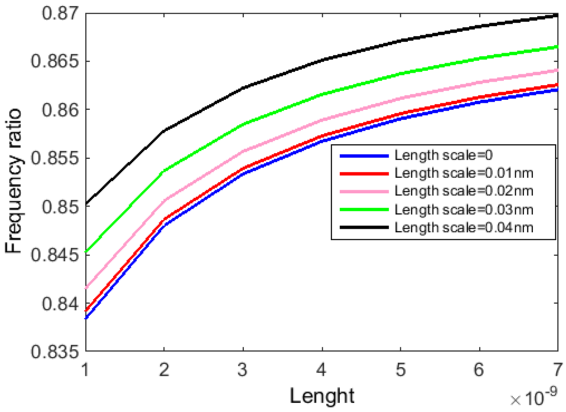

Fig. 7 Variation of natural fundamental frequency ratio versus length of nanobeam by considering both Surface density and elasticity and N=1

5. Conclusions

This paper presents the free vibration analysis of functionally graded simply-supported nanobeam modeled by considering all surface effect. Modified couple stress theory introduces the size-dependent effect. The frequency ratio of a nanobeam in each case is investigated with respect to the length of nanobeam for different material length scale parameter of FG nanobeam. The obtained results show that, the surface elasticity, surface density and surface tension play important roles on the natural frequency ratio.

References

Abdelhak, Z., Hadji, L., Khelifa, Z., Hassaine, D. and Adda Bedia, E.A. (2016), "Analysis of buckling response of functionally graded sandwich plates using a refined shear deformation theory", *Wind Struct*., **22**(3), 130-141.

Abdelaziz *et al*. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct*., 25(6), 693-704.

Asghari, M. *et al*. (2011), "Investigation of the size effects in Timoshenko beams based on the couple stress theory", *Arch. Appl. Mech*., **81**(7), 863-874.

Bouderba, B. *et al*. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct*., **14**(1), 85-104.

Chen, L. *et al*. (2012), "Engineering controllable bidirectional molecular motors based on myosin", *Nat. Nano*, 7(4), 252-256.

Chen, T., Chiu, M.S. and Weng, C.N. (2006), "Derivation of the generalized Young-Laplace equation of curved interfaces in nanoscaled solids", *J. Appl. Phys*., **100**(7), 074308.

Chong, A. *et al*. (2001), "Torsion and bending of micron-scaled structures", *J. Mater. Res*., **16**(4), 1052-1058.

Ebrahimi, F., Ghadiri, M., Salari, E., Hoseini, S.A.H. and Shaghaghi, G.R. (2015), "Application of the differential transformation method for nonlocal vibration analysis of functionally graded nanobeams", *J. Mech. Sci. Tech*., **29**, 1207-1215.

Ebrahimi, F. and Salari, E. (2015a), "Size-dependent thermo-electrical buckling analysis of functionally graded piezoelectric nanobeams", *Smart Mater. Struct*., **24**(12), 125007, 2015

Ebrahimi, F. and Salari, E. (2015b), "Nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment", *Acta Astronautica*, **113**, 29-50.

Ebrahimi, F. and Salari, E. (2015c), "Size-dependent free flexural vibrational behavior of functionally graded nanobeams using semi-analytical differential transform method", *Compos. B*, **79**, 156-169.

Ebrahimi, F. and Salari, E. (2015d), "A semi-analytical method for vibrational and buckling analysis of functionally graded nanobeams considering the physical neutral axis position", *Comput. Model. Eng. Sci*., **105**, 151-181.

Ebrahimi, F. and Salari, E. (2015e), "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments", *Compos. Struct*., **128**, 363-380.

Ebrahimi, F. and Salari, E. (2015f), "Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions", *Compos. B*, **78**, 272-290.

Ebrahimi, F. and Salari, E. (2016), "Effect of various thermal loadings on buckling and vibrational characteristics of nonlocal temperature-dependent functionally graded nanobeams", *Mech. Adv. Mater. Struct*., **23**(12), 1379-1397.

Ebrahimi, F., Salari, E. and Hosseini, S.A.H. (2015), "Thermomechanical vibration behavior of FG nanobeams subjected to linear and non-linear temperature distributions", *J. Therm. Stresses*, **38**(12), 1360-1386.

Ebrahimi, F., Salari, E. and Hosseini, S.A.H. (2016), "In-plane thermal loading effects on vibrational characteristics of functionally graded nanobeams", *Meccanica*, **51**(4), 951-977.

Ebrahimi, F. and Barati, M.R. (2016a), "Magneto-electro-elastic buckling analysis of nonlocal curved nanobeams", *Eur. Phys. J. Plus*, **131**(9), 346.

Ebrahimi, F. and Barati, M.R. (2016b), "Static stability analysis of smart magneto-electro-elastic heterogeneous nanoplates embedded in an elastic medium based on a four-variable refined plate theory", *Smart Mater. Struct*., **25**(10), 105014.

Ebrahimi, F. and Barati, M.R. (2016c), "Temperature distribution effects on buckling behavior of smart heterogeneous nanosize plates based on nonlocal four-variable refined plate theory", *Int. J. Smart Nano Mater*., 1-25.

Ebrahimi, F. and Barati, M.R. (2016d), "An exact solution for buckling analysis of embedded piezoelectro-magnetically actuated nanoscale beams", *Adv. Nano Res*., **4**(2), 65-84.

Ebrahimi, F. and Barati, M.R. (2016e), "Buckling analysis of smart size-dependent higher order magneto-electro-thermo-

elastic functionally graded nanosize beams", *J. Mech*., 1-11.
Ebrahimi, F. and Barati, M.R. (2016f), "A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams", *Arabian J. Sci. Eng*., **41**(5), 1679-1690.
Ebrahimi, F. and Hosseini, S.H.S. (2016a), "Double nanoplate-based NEMS under hydrostatic and electrostatic actuations", *Eur. Phys. J. Plus*, **131**(5), 1-19.
Ebrahimi, F. and Hosseini, S.H.S. (2016b), "Nonlinear electroelastic vibration analysis of NEMS consisting of double-viscoelastic nanoplates", *Appl. Phys. A*, **122**(10), 922.
Ebrahimi, F. and Hosseini, S.H.S. (2016c), "Thermal effects on nonlinear vibration behavior of viscoelastic nanosize plates", *J. Therm. Stresses*, **39**(5), 606-625.
Ebrahimi, F. and Nasirzadeh, P. (2015), "A nonlocal Timoshenko beam theory for vibration analysis of thick nanobeams using differential transform method", *J. Theor. Appl. Mech*., **53**(4), 1041-1052.
Eltaher, M.A., Emam, S.A. and Mahmoud, F.F. (2012), "Free vibration analysis of functionally graded size dependent nanobeams", *Appl. Math. Comput*., **218**(14), 7406-7420.
Gurtin, M.E. and Murdoch, A.I. (1975), "A continuum theory of elastic material surfaces", *Arch. Ration. Mech. An*., **57**(4), 291-323.
Gurtin, M.E. and Murdoch, A.I. (1978), "Surface stress in solids", *Int. J. Solids Struct*., **14**(6), 431-440.
Gurtin, M.E., Weissmüller, J. and Larche, F. (1998), "A general theory of curved deformable interfaces in solids at equilibrium", *Philos. Magazine A*, **78**(5), 1093-1109.
Khanade K., Sasangohar, F., Sadeghi, M., Sutherland, S. and Alexander, K. (2015), "Deriving information requirements for a smart nursing system for intensive care units", *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*.
Kong, S. *et al*. (2008), "The size-dependent natural frequency of Bernoulli–Euler micro-beams", *Int. J. Eng. Sci*., **46**(5), 427-437.
Lee, Z. *et al*. (2006), "Metallic NEMS components fabricated from nanocomposite Al–Mo films", *Nanotechnology*, **17**(12), 3063.
Lei, X.W. *et al.* (2012), "Surface effects on the vibrational frequency of double-walled carbon nanotubes using the nonlocal Timoshenko beam model", *Compos. Part B*: *Eng*., **43**(1), 64-69.
Li, J. *et al.* (2014), "Rotation motion of designed nano-turbine", *Sci. Rep*., 4.
Lu, P. *et al.* (2006), "Dynamic properties of flexural beams using a nonlocal elasticity model", *J. Appl. Phys*., 2006. **99**(7).
Lü, C., Lim, C.W. and Chen, W. (2009), "Size-dependent elastic behavior of FGM ultra-thin films based on generalized refined theory", *Int. J. Solids Struct*., **46**(5), 1176-1185.
Natarajan, S. *et al.* (2012), "Size-dependent free flexural vibration behavior of functionally graded nanoplates", *Comput. Mater. Sci*., **65**, 74-80.
Nateghi, A. and Salamat-talab, M. (2013), "Thermal effect on size dependent behavior of functionally graded microbeams based on modified couple stress theory", *Compos. Struct*., **96**, 97-110.
Rahaeifard, M., Kahrobaiyan, M. and Ahmadian, M. (2009), "Sensitivity analysis of atomic force microscope cantilever made of functionally graded materials", *Proceedings of the ASME 2009 international design engineering technical conferences and computers and information in engineering conference*, American Society of Mechanical Engineers.
Reddy, J. (2011), "Microstructure-dependent couple stress theories of functionally graded beams", *J. Mech. Phys. Solids*, **59**(11), 2382-2399.
Sadeghi M., Thomassie, R. and Sasangohar, F. (2017), "Objective assessment of functional information requirements for patient portals", *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*.
Shaat, M. *et al.* (2014), "Size-dependent bending analysis of Kirchhoff nano-plates based on a modified couple-stress theory including surface effects", *Int. J. Mech. Sci*., **79**, 31-37.
Tauchert, T.R. (1974), Energy principles in structural mechanics, McGraw-Hill Companies.
Toupin, R.A. (1962), "Elastic materials with couple-stresses", *Arch. Ration. Mech. An*., **11**(1), 385-414.
Vafamehr, A. *et al*. (2017), "A framework for expansion planning of data centers in electricity and data networks under uncertainty", *IEEE T. Smart Grid*.
Wang, G.F. and Feng, X.Q. (2007), "Effects of surface elasticity and residual surface tension on the natural frequency of microbeams", *Appl. Phys. Lett*., **90**(23), 231904.
Witvrouw, A. and Mehta, A. (2005), The use of functionally graded poly-SiGe layers for MEMS applications. in Materials science forum, Trans Tech Publ.
Yang, F. *et al*. (2002), "Couple stress based strain gradient theory for elasticity", *Int. J. Solids Struct*., **39**(10), 2731-2743.
Yan, Z. and Jiang, L. (2011), "The vibrational and buckling behaviors of piezoelectric nanobeams with surface effects", *Nanotechnology*, **22**(24), 245703.

CC