Wind and Structures, *Vol. 27, No. 6 (2018) 369-380*
DOI: https://doi.org/10.12989/was.2018.27.6.369

Investigating nonlinear thermal stability response of functionally graded plates using a new and simple HSDT

Ismail Bensaid[*1], Ahmed Bekhadda[1], Bachir Kerboua[2] and Cheikh Abdelmadjid[1]

[1]*IS2M Laboratory, Faculty of Technology, Department of Mechanical engineering, University Abou Beckr Belkaid (UABT), Tlemcen, Algeria*
[2]*Department of Mechanical engineering, University Abou Beckr Belkaid (UABT), Tlemcen, Algeria*

(Received January 9, 2018, Revised June 14, 2018, Accepted June 16, 2018)

Abstract. In this research work, nonlinear thermal buckling behavior of functionally graded (FG) plates is explored based a new higher-order shear deformation theory (HSDT). The present model has just four unknowns, by using a new supposition of the displacement field which enforces undetermined integral variables. A shear correction factor is, thus, not necessary. A power law distribution is employed to express the disparity of volume fraction of material distributions. Three kinds of thermal loading, namely, uniform, linear, and nonlinear and temperature rises over *z*-axis direction are examined. The non-linear governing equations are resolved for plates subjected to simply supported boundary conditions at the edges. The results are approved with those existing in the literature. Impacts of various parameters such as aspect and thickness ratios, gradient index, type of thermal load rising, on the non-dimensional thermal buckling load are all examined.

Keywords: thermal stability; functionally graded materials; refined plate theory; thermal load

1. Introduction

Normally Actually, with the progress of new industries conception and modern production technologies, many structures and machines may be subjected to several thermal environments, resultant in various types of thermal loads (Noda *et al*. 2003). For these reasons a new kind of composite materials known as functionally graded materials or sandwich (FGMs) which are designed to withstand high temperature gradients and waves have been incorporated successfully in various engineering applications (Houari *et al*. 2013, Tounsi et *al*. 2013, Bouderba et *al*. 2013, Bessaim *et al*. 2013, Zidi et *al*. 2014, Ait Amar et *al*. 2014, Belabed *et al*. 2014, Bousahla et *al*. 2014, Hamidi et *al*. 2015, AitYahia *et al*. 2015, Boukhari et *al*. 2016, Saidi et *al*. 2016). Further, FGMs have also attracted growing interest and are considered to be the most promising materials for applications in nanoengineering, which were mainly focused on the study of their mechanical behavior by using both classical and higher order shear deformation models (Ebrahimi et *al*. 2015, Ebrahimi and Salari 2015a, 2016, Ebrahimi and Farazmandnia 2017, Ebrahimi and Barati 2016a, b, c, Ebrahimi and Hosseini 2016, Ebrahimi and Jafari 2016, Bellifa et *al*. 2017, Bouafia et *al*. 2017). Also, many other researchers investigated the mechanical and thermal stabilities of homogenous and non-homogenous nanostructures (Ebrahimi and Salari 2015b, 2016, Ebrahimi and Barati 2016a, h, i, Bellifa et *al*. 2017, Besseghier et *al*. 2017).

On the other hand, the thermo-mechanical effect on FG structures at both macro and nano scales is studied by many researchers (Ebrahimi et *al*. 2015, 2016, Ebrahimi and Salari 2015a, c, d, 2016, Ebrahimi and Farazmandnia 2016, Ebrahimi and Barati 2016a, b, c, Ebrahimi and Hosseini 2016). Beldjelili *et al*. (2016) investigated a refined trigonometric shear deformation theory for Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations. Abdelaziz et *al*. (2017) used an efficient hyperbolic shear deformation theory to investigate the bending, buckling and free vibration of FGM sandwich plates with various boundary conditions.

However in such cases, the in-plane compressible forces can be produced due to the temperature growing in plates which make the structures to be buckled prior to get to a yield stress, and which can lead to undesired phenomenon. Consequently, studying and understanding the thermal stability response of FG plates plays a vital role in practical application to guarantee a proficient and reliable design. So, in the last years, thermal buckling of functionally graded plates (FGMP) has gained outstanding interest by several researchers to guarantee the integrity of structures (Song and Li 2007, Gossard *et al*. 1952). For the first time, Javaheri and Eslami (2002) obtained the equilibrium and stability equations to study the thermal behavior of a rectangular FGP under thermal loads via a higher order theory. Najafizadeh and Eslami (2002) employed the classical plate theory (CPT) to investigate the buckling analysis of clamped and simply supported circular FGM plate. Shen (2002) employed the Reddy's higher order shear deformation plate theory to explore the nonlinear bending analysis for a simply supported functionally graded plate exposed to a crossways uniform or sinusoidal load in thermal environments. Na and Kim (2004) researched the

∗Corresponding author, Ph.D.
E-mail: bensaidismail@yahoo.fr

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http://www.techno-press.com/journals/was&subpage=7
ISSN: 1226-6116 (Print), 1598-6225 (Online)

thermomechanical buckling response of an FG Plate composed of ceramic, FGM, and metal layers by using the three dimensional model. The impacts of various parameters on thermal buckling behaviors of FGP were explored. Shariat and Eslami (2007) contributed to the buckling analysis of thick functionally graded rectangular plates subjected to different types of mechanical and thermal loads. The analytical solution for the mechanical and thermal stability of a simply supported rectangular plate has been obtained by using third order shear deformation plate theory. Shariat and Eslami (2005) provided a study on the thermal buckling analysis of rectangular FGPs considering the geometrical imperfections and using the classical plate theory. They have considered in their study three types of thermal loading as uniform temperature rise, nonlinear temperature rise along the thickness and axial temperature rise. Matsunaga (2009) formulated a higher order deformation model to investigate the thermal stability of FGPs. A new technique called the power series expansion of displacement components; by set of fundamental equations of rectangular FGPs was provided. Zenkour and Mashat (2010) employed a sinusoidal shear deformation plate theory to study the thermal buckling response of FG plates. The thermo-elastic buckling of FGP based on first order shear plate theory was investigated by Bouazza *et al*. (2010); investigated the effects of changing various parameters such as, material constitution and volume fraction of constituent materials on the critical temperature difference of FGP with simply supported edges are also investigated. Bourada *et al*. (2012) formulated a new shear deformation model to explore the thermal buckling response of sandwich FGM plates by a new refined theory. In another work Bachir Bouiadjra *et al*. (2012) contributed to the thermal buckling bifurcation of FG plates based on the refined plate theory, in which three types of thermal loads were considered. R Bachir Bouiadjra *et al.* (2013) investigated the nonlinear thermal buckling behavior of FGM plates by employing a robust sinusoidal shear deformation theory, in which four unknowns were taken into account. Kettaf *et al*. (2013) investigated the thermal buckling of FG sandwich plates via a new hyperbolic shear deformation model. The effects of elastic foundation the thermal buckling analysis of FG plates have been investigated by Tebboune *et al*. (2015) based on an efficient and simple trigonometric shear deformation theory. Attia *et al*. 2015 studied the free vibration of functionally graded plates taking into account temperature-dependent properties and employing various four variable refined plate theories with polynomial and non-polynomial shear deformation theories. Bouderba *et al*. (2016) contributed to the thermal stability of FG sandwich plates via a simple shear deformation theory and various boundary conditions. Chikh *et al*. (2016) developed a new analytical model to study the thermomechanical post-buckling of symmetric S-FGM plates lying on Pasternak elastic foundations via a hyperbolic shear deformation model. El-Hassar *et al*. (2016) studied the thermal stability analysis of solar FG plates on elastic foundation using an efficient hyperbolic shear deformation theory. Bousahla et *al*. (2016) investigated the effect of gradation in the coefficient of thermal expansion on thermal stability of functionally plates. Kar, Panda *et al*. (2016) researched the thermal buckling behavior of shear deformable functionally graded single/doubly curved shell panel with TD and TID properties. Loc V, Nguyen-Xuan *et al*. (2016) developed an isogeometric approach to study the nonlinear bending and post-buckling analysis of functionally graded plates under thermal environment. As seen above, some works have been developed on the thermal stability of FG plates based on refined theory, in which only four variables are involved, the main assumption on which this theory is based is that the in-plane and transverse displacements consist of bending and shear components, make it simple to use. However, recently a new plate theory model has been developed by Tounsi and his co-workers (Bourada *et al*. 2016, Hibali *et al*. 2016, Merdaci, Tounsi *et al*, 2016, Meksi *et al*. 2017, Bessegheir *et al*. 2017, Abualnour *et al*. 2018) to study the mechanical behavior of FG structures, this theory uses a new displacement field which enforces undetermined integral variables by providing a reduction in the number of variables and equations of motion, and therefore in the computation time and convergence. Various contributions were made recently, dealing on thermal bending and stability of both functionally graded and sandwich plates (Benbakhti *et al*. 2016, Elmossouess *et al*. 2017, Khetir *et al*. 2017, Sekkal *et al*. 2017, Menasria, Tounsi *et al*. 2017, Chikh, Tounsi *et al*. 2017, Fahsi et *al*. 2017, El-Haina *et al*. 2017), they showed the efficiency of this model by comparing the obtained results with the existing ones in the literature, in which a good agreement was shown.

The main aim of the present work is to extend the newly developed refined plate model cited in the previous works, to analyze the nonlinear thermal buckling of functionally graded rectangular plates. By using the undetermined integral term in the plate kinematics a reduction in the number of variables and equations of motion will be achieved. It is supposed that material characteristics of the FG plate change continuously through the plate thickness according to power-law form of volume fractions of the constituents. The derived governing equations are solved analytically for simply-supported boundary conditions and subjected to different types of temperature rise, and supposed as uniform, linear and non-linear distribution across the thickness. The obtained results are checked and compared with the results of previous works existing in the literature and the good agreement between them validated the presented model. The impacts of different variables, such as aspect ratios and width-to-thickness ratios, power-law index, types of loading on the nondimensional critical buckling temperature are all explored.

2. Mathematical development

The In this work, we consider a rectangular plate made of FGMs of thickness h, length b, and width a made by mixing two distinct materials (metal and ceramic) is studied here. The coordinates x, y are along the in-plane directions and z is along the thickness direction, also the FG plate is subjected to three types of in-plane thermal loads as shown in (Fig. 1).

The properties of FGM are supposed graded in the thickness direction only (z-axis direction). Power-law model is commonly used to describe these variations of materials properties. However, the material properties of the FGM plate are assumed as follows (Bouazza *et al*. 2015, Bensaid *et al*. 2017).

$$P(z)=P_b+(P_t-P_b)V_t \tag{1}$$

in which P denotes the effective material properties like Young-modulus and P_t and P_b denote the corresponding properties of the top and bottom faces of the plate, respectively. Also V_t in Eq. (1) denotes the volume fraction of the top face constituent and it is assumed to be given by (Abdelhak *et al*. 2016, Bensaid and Kerboua 2017)

$$V_t=\left(\frac{z}{h}+\frac{1}{2}\right)^k; \tag{2}$$

where ***k*** represents a non-negative variable parameter (power-law exponent) which presents the material distribution profile across the thickness of the FG plate and z is the distance from the mid-plane of the FG beam.

2.1 Kinematics and strains

There are several types of plate and beam theories for modeling of shear deformation effect (Ebrahimi and Shafiei 2016, Ebrahimi and Barati 2016d, e, f, g, Hebali *et al*. 2014, Mantari and Soares 2014, Mahi *et al*. 2015, Bourada *et al*. 2015, Bellifa *et al*. 2016, Bennoun et *al*. 2016, Draiche et *al*. 2016, Houari et *al*. 2016, Zidi *et al*. 2017). In this research paper, the usual HSDT is reformulated by taking into consideration some simplifying suppositions so that the number of unknowns is reduced. The displacement field of the conventional HSDT is defined by (Bakora and Tounsi 2016)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_x(x,y,t) \tag{3a}$$

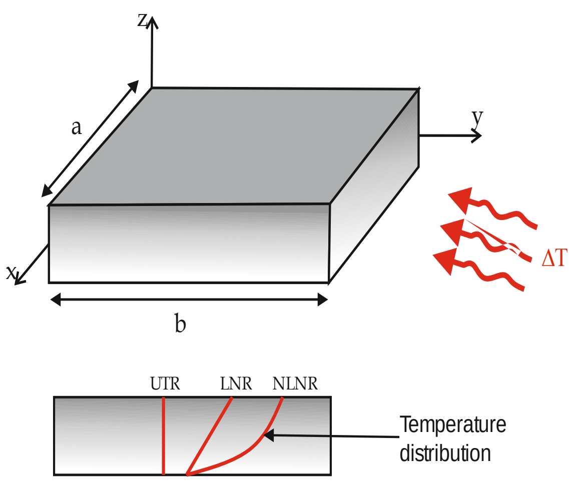

Fig. 1 Geometry and coordinates of FG plates subjected to in plane thermal loads

$$u(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_y(x,y,t) \tag{3b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{3c}$$

In which u_0; v_0; w_0, φ_x, φ_y represent the five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By making the new rotation angle supposition as, $\varphi_x=\int\theta(x,y)dx$, and $\varphi_y=\int\theta(x,y)dy$, the displacement field of the present model can be expressed in a simpler form as (Bourada *et al*. 2016, Hebali, Tounsi *et al*. 2016, Merdaci, Tounsi *et al*. 2016), as follow

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1 f(z)\int\theta(x,y,t)dx \tag{4a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_2 f(z)\int\theta(x,y,t)dy \tag{4b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{4c}$$

In this investigation, the shape function is defined by (Abualnour, Houari et *al*. 2018)

$$f(z)=\sin\left(\frac{\pi z}{h}\right) \tag{5}$$

One can see that the Kinematic in Eq. (4) gives only four unknowns (u_0, v_0, w_0 and θ). The non-linear von Karman strain–displacement equations are written as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{6}$$

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_b}{\partial x}+\frac{\partial w_s}{\partial x}\right)^2\\ \frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_b}{\partial y}+\frac{\partial w_s}{\partial y}\right)^2\\ \frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}+\left(\frac{\partial w_b}{\partial x}+\frac{\partial w_s}{\partial x}\right)\left(\frac{\partial w_b}{\partial y}+\frac{\partial w_s}{\partial y}\right)\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_b}{\partial x^2}\\ -\frac{\partial^2 w_b}{\partial y^2}\\ -2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix}, \tag{7a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\frac{\partial}{\partial y}\int \theta dx + k_2\frac{\partial}{\partial y}\int \theta dy \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1\int \theta dy \\ k_2\int \theta dx \end{Bmatrix}, \quad (7b)$$

and

$$g(z) = \frac{df(z)}{dz} \quad (8)$$

The integrals just defined in the above equations shall be resolved by a Navier type method and can
be written as follows

$$\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \ , \quad \int \theta dx = A'\frac{\partial\theta}{\partial x}\int \theta dy = B'\frac{\partial\theta}{\partial y}\ , \quad (9)$$

In which the coefficients A' and B' are expressed according to the type of solution used, in this case
via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2},\ B' = -\frac{1}{\beta^2},\ k_1 = \alpha^2,\ k_2 = \beta^2 \quad (10)$$

where α and β are fixed in expression (31).

2.2 Constitutive equations

The plate is subjected to a thermal load $T(x,y,z)$. The linear constitutive relations are written as follow

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{xy} \end{Bmatrix} \ and \ \begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (11)$$

where ($\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{yx}$) and ($\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{yx}$) are the stress and strain components, respectively, and $T(x, y, z)$ is the in plane temperature rise across-the-thickness. Using the material properties defined in Eq. (2), stiffness coefficients, $\boldsymbol{Q_{ij}}$, can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad (12a)$$

$$Q_{12} = \frac{\nu E(z)}{1-\nu^2}, \quad (12b)$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \quad (12c)$$

2.3 Stability equations

The total potential energy of the FG plate may be take the following form

$$U = \frac{1}{2}\iiint \begin{bmatrix} \sigma_x(\varepsilon_x - \alpha T) + \sigma_y(\varepsilon_y - \alpha T) \\ + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz} \end{bmatrix} dz\,dydx, \quad (13)$$

The principle of virtual work for the present problem may be expressed as follows

$$\begin{bmatrix} N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b \\ +M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s \\ +S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta k_{xz}^s \end{bmatrix} dxdy = 0 \quad (14)$$

where

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz, (i = x, y, xy)\ and\ \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g(\tau_{xz}, \tau_{yz})dz \quad (15)$$

Using Eq. (11) in Eq. (14), the stress resultants of the FG plate can be related to the total strains by

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ A & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, \ S = A^s\gamma, \quad (16)$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \ B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \ D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \quad (17)$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \ H^s = \begin{bmatrix} H_{11}^s & H_{11}^s & 0 \\ H_{11}^s & H_{11}^s & 0 \\ 0 & 0 & H_{11}^s \end{bmatrix}, \quad (18)$$

$$S = \left\{S_{yz}^s, S_{xz}^s\right\}^t, \ \gamma = \left\{\gamma_{yz}, \gamma_{xz}\right\}, \ A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \quad (19)$$

where $\boldsymbol{A_{ij}}$, $\boldsymbol{D_{ij}}$, etc., are the plate stiffness, are given as follows

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_x^{sT} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-\nu}\alpha(z)T \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \quad (20)$$

$$\begin{Bmatrix} A_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(1, z^2, f(z), zf(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \frac{1-\nu}{2} \end{Bmatrix} dz, \tag{21}$$

and

$$\left(A_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{22}$$

$$A_{44}^s = A_{55}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{2(1+\nu)} \left[g(z)\right]^2 dz, \tag{23}$$

By which, the stress and moment resultants, $N_x^T = N_y^T$, $M_x^{bT} = M_y^{bT}$, and $M_x^{sT} = M_y^{sT}$ due to thermal loading are given by

In an effort to determine the stability equations and study the thermal buckling behavior of the FG plate, the adjacent equilibrium criterion is used.

Suppose that the equilibrium state of the FG plate considering thermal loads is defined in expressions of the displacement parts ($u_0^0, v_0^0, w_b^0, w_s^0$). The displacement components of a neighboring stable state differ by ($u_0^1, v_0^1, w_b^1, w_s^1$) with respect to the equilibrium position. Thus, the whole displacements of a neighboring state are

$$\begin{aligned} u_0 &= u_0^0 + u_0^1, \quad v_0 = v_0^0 + v_0^1, \\ w_b &= w_b^0 + w_b^1, \quad w_s = w_s^0 + w_s^1, \end{aligned} \tag{24}$$

By substitution of Eqs. (6) and (24) into Eq. (14) and integrating by parts and then equating the coefficients of ($u_0^1, v_0^1, w_0^1, \theta^1$) to zero, singly, the general governing stability equations are obtained for the new shear deformation plate theories as

$$\begin{aligned} &\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0 \\ &\frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0 \\ &\frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} + \overline{N} = 0 \\ &-k_1 M_x^{s1} - k_2 M_y^{s1} - \left(k_1 A' + k_2 B'\right)\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} \\ &+k_1 A' \frac{\partial S_{xz}^{s1}}{\partial x} + k_2 B' \frac{\partial S_{yz}^{s1}}{\partial y} + \frac{\partial S_x^{s1}}{\partial y} = 0, \end{aligned} \tag{25}$$

in which

$$\overline{N} = \left[N_x^0 \frac{\partial^2 \left(w_b^1 + w_s^1\right)}{\partial x^2} + N_y^0 \frac{\partial^2 \left(w_b^1 + w_s^1\right)}{\partial y^2} \right], \tag{26}$$

where the expressions N_x^0 and N_y^0 are the pre-buckling force resultants obtained as

$$N_x^0 = N_y^0 = -\int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{\alpha(z)E(z)T}{1-\nu} dz, \tag{27}$$

However, the general stability equations in expressions of the displacement components may be achieved by substituting Eq. (16) into Eq. (25). Resulting equations are four stability equations based on the present refined shear deformation theory for FG plates in contact with two parameters elastic foundation

$$\begin{aligned} &A_{1} \frac{\partial^2 u_0^1}{\partial x^2} + A_{12}\frac{\partial^2 v_0^1}{\partial x \partial y} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial y^2} + \frac{\partial^2 v_0^1}{\partial x \partial y}\right) \\ &-B_{11}\frac{\partial^3 w_0^1}{\partial x^3} - B_{12}\frac{\partial^3 w_0^1}{\partial x \partial y^2} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x \partial y^2} \\ &+B_{11}^s A' k_1 \frac{\partial^3 \theta^1}{\partial x^3} B_{12}^s B' k_2 \frac{\partial^3 \theta^1}{\partial x \partial y^2} \\ &+B_{66}^s \left(A'k_1 + B'k_2\right)\frac{\partial^3 \theta^1}{\partial x \partial y^2} = 0, \end{aligned} \tag{28a}$$

$$\begin{aligned} &A_{12} \frac{\partial^2 u_0^1}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0^1}{\partial y^2} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial x \partial y} + \frac{\partial^2 v_0^1}{\partial x^2}\right) \\ &-B_{12}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} - B_{22}\frac{\partial^3 w_0^1}{\partial y^3} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} \\ &+B_{12}^s A' k_1 \frac{\partial^3 \theta^1}{\partial x^2 \partial y} + B_{22}^s B' k_2 \frac{\partial^3 \theta^1}{\partial y^3} \\ &+B_{66}^s \left(A'k_1 + B'k_2\right)\frac{\partial^3 \theta^1}{\partial x^2 \partial y} = 0, \end{aligned} \tag{28b}$$

$$\begin{aligned} &B_{11}\frac{\partial^3 u_0^1}{\partial x^3} + B_{12}\left(\frac{\partial^3 u_0^1}{\partial x \partial y} + \frac{\partial^3 v_0^1}{\partial x^2 \partial y}\right) \\ &+B_{22}\frac{\partial^3 v_0^1}{\partial y^3} + 2B_{66}\left(\frac{\partial^3 u_0^1}{\partial x \partial y^2} + \frac{\partial^3 v_0^1}{\partial x \partial y^2}\right) - D_{11}\frac{\partial^4 w_0^1}{\partial y^3} \\ &-2D_{12}\frac{\partial^4 w_0^1}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w_0^1}{\partial y^4} - 4D_{66}\frac{\partial^4 w_0^1}{\partial x^2 \partial y^2} \\ &+D_{11}^s A'k_1 \frac{\partial^4 \theta^1}{\partial x^4} + D_{12}^s\left(A'k_1 + B'k_2\right)\frac{\partial^4 \theta^1}{\partial x^2 \partial y^2} \\ &+D_{22}^s B'k_2 \frac{\partial^4 \theta^1}{\partial y^4} + 2D_{22}^s\left(A'k_1 + B'k_2\right)\frac{\partial^4 \theta^1}{\partial x^2 \partial y^2} \\ &+N_x^0 \frac{\partial^2 w_0^1}{\partial x^2} + N_y^0 \frac{\partial^2 w_0^1}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0^1}{\partial x \partial y} = 0, \end{aligned} \tag{28c}$$

$$\begin{aligned} &-B_{11}^s A'k_1 \frac{\partial^3 u_0^1}{\partial x^3} - B_{12}^s\left(A'k_1 \frac{\partial^3 v_0^1}{\partial x^2 \partial y} + B'k_2 \frac{\partial^3 u_0^1}{\partial x \partial y^2}\right) \\ &-B_{22}^s B'k_2 \frac{\partial^3 v_0^1}{\partial y^3} - D_{66}\left(\left(A'k_1 + B'k_2\right)\frac{\partial^3 u_0^1}{\partial x \partial y^2}\right. \\ &\left.+\left(A'k_1 + B'k_2\right)\frac{\partial^3 v_0^1}{\partial x^2 \partial y}\right) + D_{11}^s A'k_1 \frac{\partial^4 w_0^1}{\partial x^4} \\ &+D_{12}^s\left(A'k_1 + B'k_2\right)\frac{\partial^4 w_0^1}{\partial x^2 \partial y^2} + D_{22}^s B'k_2 \frac{\partial^4 w_0^1}{\partial y^4} + 2D_{66}^s \\ &\left(A'k_1 + B'k_2\right)\frac{\partial^4 w_0^1}{\partial x^2 \partial y^2} - H_{11}^s\left(A'k_1\right)^2 \frac{\partial^4 \theta^1}{\partial x^4} \\ &-2H_{12}^s A'k_1 B'k_2 \frac{\partial^4 \theta^1}{\partial x^2 \partial y^2} - H_{22}^s\left(B'k_2\right)^2 \frac{\partial^4 \theta^1}{\partial y^4} - H_{66}^s \\ &\left(A'k_1 + B'k_2\right)^2 \frac{\partial^4 \theta^1}{\partial x^2 \partial y^2} + A_{55}^s\left(A'k_1\right)^2 \frac{\partial^2 \theta^1}{\partial x^2} + A_{44}^s\left(B'k_2\right)^2 \frac{\partial^2 \theta^1}{\partial y^2} = 0, \end{aligned} \tag{28d}$$

3. Closed-form solution

Usually rectangular plates are classified in accordance with the type of support used. We are here concerned with the exact solution of Eq. (28) for a simply supported FG plate. The following boundary conditions are imposed for the present efficient sinusoidal shear deformation theory at the side edges

$$v_0^1 = w_0^1 = \theta^1 = \frac{\partial \theta^1}{\partial y} = N_x^1 = M_x^{b1} = M_x^{s1} = 0 \quad at \quad x = 0, a, \tag{29a}$$

$$u_0^1 = w_0^1 = \theta^1 = \frac{\partial \theta^1}{\partial x} = N_y^1 = M_y^{b1} = M_y^{s1} = 0 \quad at \quad y = 0, b, \tag{29b}$$

The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_0^1 \\ \theta^1 \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha x)\sin(\beta y) \\ V_{mn}\sin(\alpha x)\cos(\beta y) \\ W_{mn}\sin(\alpha x)\sin(\beta y) \\ X_{mn}\sin(\alpha x)\sin(\beta y) \end{Bmatrix}, \tag{30}$$

where U_{mn} , V_{mn} , W_{mn} , X_{mn} are arbitrary parameters to be determined. α and β are defined as

$$\alpha = m\pi / a \quad and \quad \beta = n\pi / b \tag{31}$$

Substituting Eq. (30) into Eq. (28), the closed-form solution of buckling load can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} + N_x^0\alpha^2 + N_y^0\beta^2 & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{32}$$

in which

$$a_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right),\; a_{12} = -\alpha\beta\left(A_{12} + A_{66}\right)$$

$$a_{13} = \alpha\left(B_{11}\alpha^2 + \left(B_{12} + 2B_{66}\right)\beta^2\right),$$

$$a_{14} = -\alpha\left(B_{11}^s A'k_1\alpha^2 + B_{12}^s B'k_2\beta^2 + B_{66}^s\left(A'k_1 + B'k_2\right)\beta^2\right)$$

$$a_{22} = -\alpha^2 A_{66} - \beta^2 A_{22},\; a_{23} = \beta\left(B_{22}\beta^2 + \left(B_{12} + 2B_{66}\right)\alpha^2\right)$$

$$a_{24} = -\beta\left(B_2^s B'k_2\beta^2 + \alpha^2\left(B_{12}^s A'k_1 + B_{66}^s\left(A'k_1 + B'k_2\right)\right)\right)$$

$$a_{33} = -\alpha^2\left(D_{11}\alpha^2 + \left(2D_{12} + 4D_{66}\right)\beta^2\right) - D_{22}\beta^4,$$

$$a_{34} = D_{11}^s A'k_1\alpha^4 + D_{12}^s\left(A'k_1 + B'k_2\right)\beta^2\alpha^2 + D_{22}^s B'k_2\beta^4 + 2D_{66}^s\left(A'k_1 + B'k_2\right)\beta^2\alpha^2$$

$$a_{44} = -\left(H_{11}^s\alpha^2 k_1 + 2k_1\beta^2 H_{66}^s + 2H_{66}^s\alpha^2 k_2 + H_{12}^s\alpha^2 k_2 + k_1\beta^2 H_{12}^s + k_2\beta^2 H_{22}^s + A_{44}^s k_1 + A_{55}^s k_2\right),$$

By using the condensation technique to eliminate the axial displacements $\boldsymbol{U_{mn}}$ and $\boldsymbol{V_{mn}}$, Eq. (31) can be rewritten as

$$\begin{bmatrix} \overline{a_{33}} + N_x^0\alpha^2 + N_y^0\beta^2 & \overline{a_{34}} \\ \overline{a_{43}} & \overline{a_{44}} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{33}$$

where

$$\begin{aligned} \bar{a}_{33} &= a_{33} - \frac{a_{13}\left(a_{13}a_{22} - a_{12}a_{23}\right) - a_{23}\left(a_{11}a_{23} - a_{12}a_{13}\right)}{a_{11}a_{22} - a_{12}^2} \\ \bar{a}_{34} &= a_{34} - \frac{a_{14}\left(a_{13}a_{22} - a_{12}a_{23}\right) - a_{24}\left(a_{11}a_{23} - a_{12}a_{13}\right)}{a_{11}a_{22} - a_{12}^2} \\ \bar{a}_{43} &= a_{34} - \frac{a_{13}\left(a_{14}a_{22} - a_{12}a_{24}\right) - a_{23}\left(a_{11}a_{24} - a_{12}a_{14}\right)}{a_{11}a_{22} - a_{12}^2} \\ \bar{a}_{44} &= a_{44} - \frac{a_{14}\left(a_{14}a_{22} - a_{12}a_{24}\right) - a_{24}\left(a_{11}a_{24} - a_{12}a_{14}\right)}{a_{11}a_{22} - a_{12}^2} \end{aligned} \tag{34}$$

The system of homogeneous Eq. (33) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients ($\boldsymbol{W_{mn}}$, $\boldsymbol{X_{mn}}$) must equal zero

$$\begin{bmatrix} \bar{a}_{33} + N_x^0\alpha^2 + N_y^0\beta^2 & \bar{a}_{34} \\ \bar{a}_{43} & \bar{a}_{44} \end{bmatrix} = 0 \tag{35}$$

The obtained equation may be solved for the buckling load for non trivial solution. This gives the following relation for buckling load

$$N_x^0\alpha^2 + N_y^2\beta^2 = \frac{\bar{a}_{34}\bar{a}_{43} - \bar{a}_{33}\bar{a}_{44}}{\bar{a}_{44}}, \tag{36}$$

In this paper, to check the effect of the considered kind of temperature disparity within the thickness on stability buckling response of FG sandwich plate, three types of thermal loading within the plate thickness are taken (Fig. 1).

3.1 Uniform temperature rise (UTR)

It is assumed that the initial uniform temperature of the FG sandwich plate is $\boldsymbol{T_i}$, and the temperature is uniformly elevated to a final value $\boldsymbol{T_f}$ such that the plate buckles. Thus, the temperature change is

$$T(z) = T_f - T_i = \Delta T \tag{37}$$

In this case, by manipulating the Eqs. (34), (27), and (35) the following equation for thermal buckling load is deduced

$$\Delta T(m,n) = \frac{1}{\alpha^2 + \beta^2} \frac{1}{\int_{-h/2}^{h/2} \frac{E(z)\alpha(z)}{1-\nu} dz}, \tag{38}$$

3.2 Linear temperature distribution through the thickness (LTD)

Here, For FG plates, the temperature change is not uniform. The temperature is assumed to be varied linearly through the thickness as follows

$$T(z)=\Delta T\left(\frac{z}{h}+\frac{1}{2}\right)+T_m\,,\qquad \Delta T=T_c-T_m \tag{39}$$

Identically to the UTR procedure, the following expression for thermal buckling load is derived

$$\Delta T(m,n)=\frac{1}{\displaystyle\int_{-h/2}^{h/2}\frac{E(z)\alpha(z)\left(\frac{z}{h}+\frac{1}{2}\right)}{1-\nu}dz}\times \left(\frac{1}{\alpha^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44}-\bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}}-\int_{-h/2}^{h/2}\frac{E(z)\alpha(z)T_m}{1-\nu}dz\right), \tag{40}$$

3.3 Buckling of FG plates subjected to graded temperature change across the thickness

We suppose that the temperature of the top surface is $\boldsymbol{T_M}$ and the temperature varies from $\boldsymbol{T_M}$, according to the power law variation through-the-thickness, to the bottom surface temperature $\boldsymbol{T_M}$ in which the plate buckles. In this instance, the temperature through-the-thickness is written as

$$T(z)=\Delta T\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma}+T_m\,, \tag{41}$$

Similar to the previous loading case, the critical buckling temperature change $\Delta\boldsymbol{T_{cr}}$ becomes by using Eqs. (27) and (34)

$$\Delta T(m,n)=\frac{1}{\displaystyle\int_{-h/2}^{h/2}\frac{E(z)\alpha(z)\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma}}{1-\nu}dz}\times \left(\frac{1}{\alpha^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44}-\bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}}-\int_{-h/2}^{h/2}\frac{E(z)\alpha(z)T_m}{1-\nu}dz\right), \tag{42}$$

4. Numerical results and discussion

In this section of the present investigation, some numerical results are analyzed for inspecting the accuracy of the present novel formulation in predicting the thermal buckling responses of thick FG plates subjected to uniform, linear and nonlinear thermal loading throughout the thickness. The obtained results are validated with those existing in the literature.

The functionally graded plate utilized in this investigation is made of a mixture of aluminum and alumina. The Young modulus, coefficient of thermal expansion and thermal conductivity for aluminum are $\boldsymbol{E_m}$ = 70 GPa, $\boldsymbol{\alpha_m}$ = 23.10^{-6}/C and for alumina are $\boldsymbol{E_c}$ = 380 GPa, $\boldsymbol{\alpha_c}$ = 7.410^{-6}/C, respectively. For the linear and nonlinear

Table 1 Critical thermal buckling of FG square plate under uniform temperature rise for different values of gradient index ***k*** and side-to-thickness ratio ***a/h***

k	Theory	***a/h***=5	***a/h***=10	***a/h***=20
0	Present	5.5855	1.61882	0.42154
	TPT[(a)]	5.58556	1.61882	0.42154
	HPT[(a)]	5.58344	1.61868	0.42154
	SPT[(a)]	5.58069	1.61862	0.42153
1	Present	2.6724	0.75845	0.19626
	TPT[(a)]	2.67241	0.75845	0.19627
	HPT[(a)]	2.67153	0.75840	0.19627
	SPT[(a)]	2.67039	0.75837	0.19626
5	Present	2.2713	0.67894	0.17850
	TPT[(a)]	2.27131	0.67895	0.17851
	HPT[(a)]	2.27501	0.67931	0.17854
	SPT[(a)]	2.35948	0.68678	0.17905
10	Present	2.2755	0.69254	0.18313
	TPT[(a)]	2.27551	0.69254	0.18313
	HPT[(a)]	2.27678	0.69269	0.18314
	SPT[(a)]	2.36822	0.70108	0.18373

(a) Zenkour and Sobhy (2011)

temperature rises through the thickness, the temperature rises 5C in the metal-rich surface of the plate (i.e., $\boldsymbol{T_m}$ = 5C).

For checking the correctness of the present novel formulation, the results have been recovered for FG plates subjected to uniform, linear and nonlinear thermal loading across the thickness according to the newly refined theory. The obtained results of buckling analysis for the plate under uniform temperature rise are tabulated in Tables 1, 2 and 3. These tables demonstrate the comparisons of the critical buckling temperature change ($t_{cr}=10^{-3}T_{cr}$,) obtained by the present theory with those given by Zenkour and Sobhy (2011) based on the higher plate theory (HPT), third order plate theory (TPT) and first order shear deformation plate theory, without considering the elastic foundation. One can see that the results of the present theory are in good agreement with TPT, HPT and FPT for both thin and thick FG plates. It is also to be noted that the number of unknowns in present model is only four, while the unknown functions in the existing HPT, TPT and FPT is five.

From the Tables 1 and 2, we can see that an increment in the gradient index (***k***) from 0 to 10 and in the dimension aspect ratio *a*/*h* lead to a decrement in critical buckling temperatures $\boldsymbol{t_{cr}}$. Furthermore, the critical buckling temperatures for homogeneous plates are considerably superior to those for the FG plates particularly for the reasonably thicker plates. The linear temperature rise gives large values compared to the uniform temperature distribution.

Table 2 Critical thermal buckling of FG square plate under linear temperature rise for different values of gradient index ***k*** and side-to-thickness ratio ***a/h***

k	Theory	***a/h***=5	***a/h***=10	***a/h***=20
0	Present	11.17111	3.23764	0.84308
	TPT[(a)]	11.16112	3.22764	0.83309
	HPT[(a)]	11.15688	3.22736	0.83307
	SPT[(a)]	11.15138	3.22725	0.83306
1	Present	5.01201	1.42244	0.36809
	TPT[(a)]	5.00264	1.41307	0.35872
	HPT[(a)]	5.00099	1.41297	0.35871
	SPT[(a)]	4.99885	1.41292	0.35871
5	Present	3.90958	1.16866	0.30726
	TPT[(a)]	3.90098	1.16006	0.29866
	HPT[(a)]	3.90735	1.16069	0.29871
	SPT[(a)]	4.05274	1.17354	0.29959
10	Present	4.03236	1.22723	0.31982
	TPT[(a)]	4.02350	1.21837	0.31566
	HPT[(a)]	4.02576	1.21864	0.31568
	SPT[(a)]	4.18778	1.23350	0.31672

(a) Zenkour and Sobhy (2011)

Table 3 Critical thermal buckling of FG square plate under nonlinear temperature rise for different values of gradient index ***k*** and side-to-thickness ratio ***a/h***

k	Theory	***a/h***=5	***a/h***=10	***a/h***=20
0	Present	22.3422	6.47528	1.67617
	TPT[(a)]	22.32223	6.45528	1.66618
	HPT[(a)]	22.31376	6.45473	1.66614
	SPT[(a)]	22.30276	6.45450	1.66614
1	Present	10.02693	2.84572	0.71640
	TPT[(a)]	10.00817	2.82696	0.71764
	HPT[(a)]	10.00488	2.82676	0.71763
	SPT[(a)]	10.00060	2.82667	0.71763
5	Present	6.79150	2.03014	0.52376
	TPT[(a)]	6.77655	2.01520	0.51882
	HPT[(a)]	6.78763	2.01628	0.51889
	SPT[(a)]	7.04019	2.03861	0.52043
10	Present	6.94087	2.11242	0.55860
	TPT[(a)]	6.92562	2.09717	0.54335
	HPT[(a)]	6.92950	2.09763	0.54338
	SPT[(a)]	7.20839	2.12321	0.54516

(a) Zenkour and Sobhy (2011)

The effects of non-linear temperature distribution on the thermal stability temperatures of FG plates are presented in Table 3. It may be observed that the critical thermal buckling temperature t_{cr} is sensitive with increasing of both material gradient index and the side-to- thickness ratios. This is due to the rise in flexibility of the FG plates, while the fraction of metal segment increases when power law index increases. In addition, it is seen that the present type of thermal loading gives large values compared to the previous ones.

4.1 Parametric investigations

In this part, the impacts of different parameters of plate and the type of thermal loads on the critical buckling load of the present FG plate will be highlighted, the obtained results are illustrated in Figs. 2 to 5.

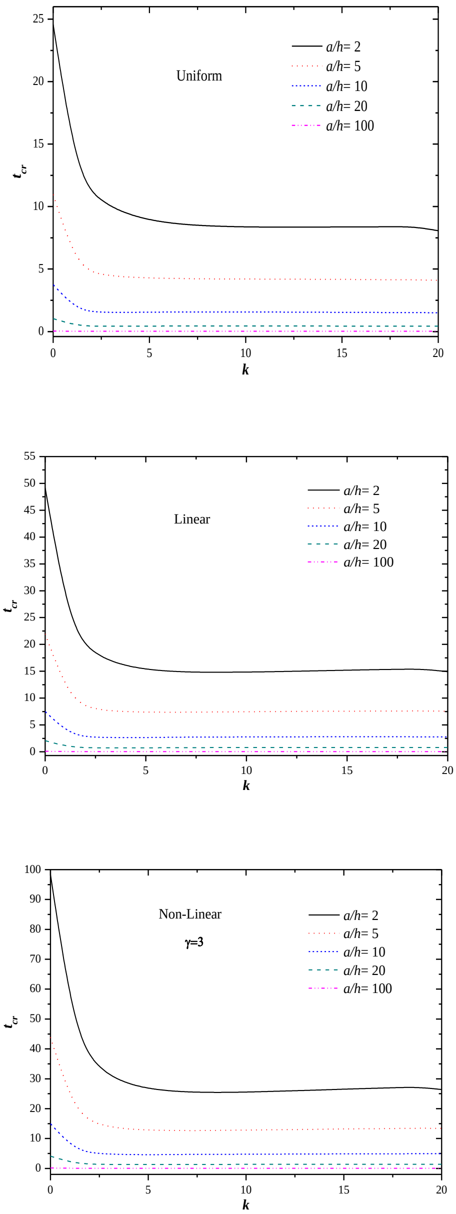

Fig. 2 Critical buckling temperature t_{cr} of FG plate (***a***/***b***=2) versus the power law index ***k*** for different values of the side-to thickness ratio ***a***/***h***

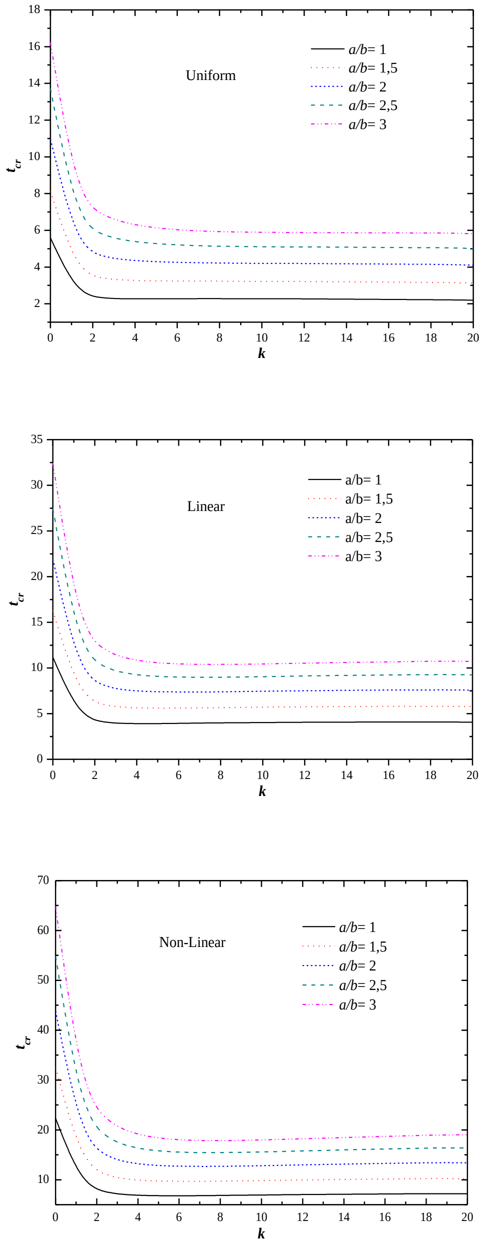

Fig. 3 Critical buckling temperature of FG plate (***a***/***h***=5) against the power law index ***k*** for various values of the aspect ratio ***a***/***b***

Figs. 2 and 3 examine the impact of material graduation index (*k*) on dimensionless thermal buckling loads of FG plates with simply boundary conditions, respectively, for various values of side to-thickness ratio *a/h* , aspect ratio *a/b* and a range of thermal loading types. It can be observed that from these figures that t_{cr} is quite sensitive to the variation the previously cited parameters, and must be well chosen in the design phase. However, regardless of the loading kind and the gradient index *k*, the critical buckling temperature t_{cr} decreases as the side to-thickness ratio ***a***/***h*** rises and it is diminished with the reduction of the aspect ratio ***a***/***b***. The high critical buckling temperature value for the FG plate is obtained for the ceramic phase ***k*** reach to zero. The reason is that the ceramic plate is stiffer than the other. Further, when the FG plate becomes either thin or large there is a noticeable difference in the peak values of t_{cr} between the thermal loading types.

Fig. 4 illustrates the change in critical temperatures t_{cr} of FG square plates over to various thermal loading cases with respect to the thickness ratio *a/h*. It can be seen that with rising *a/h*, the critical temperature t_{cr} reduces regularly. Note that in the case of the uniform temperature rise, the critical temperatures t_{cr} of the Graded plate get the less significant values than that of the plate over linear temperature rise and this latter is smaller than that of the plate under non linear temperature rise. Also, one can see that an increment in the nonlinearity parameter γ leads to an increases t_{cr} .

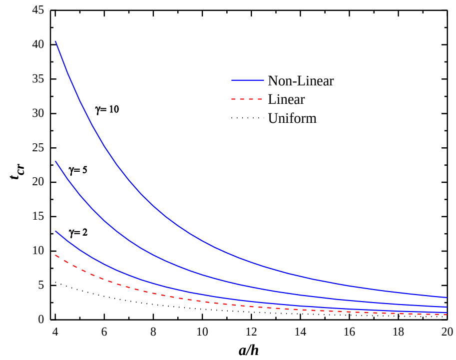

Fig. 4 Maximum critical buckling temperature t_{cr} caused by uniform, linear and non-linear temperature rise across the thickness versus the side-to-thickness ratio ***a***/***h*** and for distinct values of the non-linearity parameter γ (***k***=5 and ***a***/***b***=2)

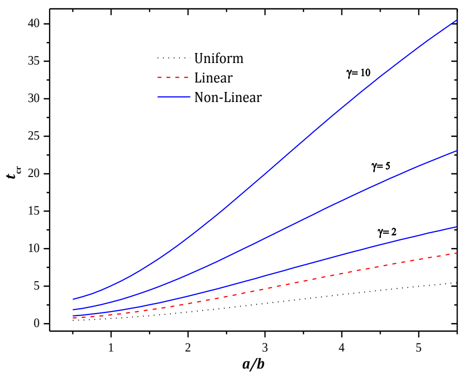

Fig. 5 Maximum critical buckling temperature t_{cr} caused by uniform, linear and non-linear temperature rise across the thickness versus the aspect ratio ***a***/***b*** and for different values of the non-linearity parameter γ. (***k***=5 and ***a***/***h***=10)

Fig. 5 exhibits the influences of the aspect ratio ***b/a*** on the extreme value of critical stability temperature t_{cr} of FG plate subjected to different types of thermal loads. It can be shown that, the critical temperature t_{cr} decreases progressively with rising the plate aspect ratio ***b/a*** wherever the thermal loading type. It is also remarked from Fig. 5 that the t_{cr} increases with increasing of the nonlinearity parameter γ.

5. Conclusions

In this paper, the nonlinear thermal buckling characteristics of functionally graded plates subjected to uniform, linear and non-linear temperature rises through the thickness path has been examined on the basis novel higher-order shear deformation theory. By making further simplifying suppositions to the existing HSDT, with the incorporation of an undetermined integral term, the quantity of unknowns and governing equations of the current theory is diminished to four as the other refined plate theories and hence, make this model simple and efficient to exploit. Material properties are graded in the thickness direction by a power-law distribution scheme. To confirm the exactness of the current theory, the obtained results by the present investigation have been matched with existing ones in the literature; good concordances have been observed. Influences of various parameters have been performed such as aspect ratio, plate thickness, thermal loading type and also material gradient index on the critical buckling temperature of functionally graded rectangular plate. Finally it can be concluded that, the current new model can enhance the numerical computational rate by reason of their diminished degrees of freedom.

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