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A joint probability distribution model of directional extreme wind speeds based on the t-Copula function

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Abstract. The probabilistic information of directional extreme wind speeds is important for precisely estimating the design wind loads on structures. A new joint probability distribution model of directional extreme wind speeds is established based on observed wind-speed data using multivariate extreme value theory with the t-Copula function in the present study. At first, the theoretical deficiencies of the Gaussian-Copula and Gumbel-Copula models proposed by previous researchers for the joint probability distribution of directional extreme wind speeds are analysed. Then, the t-Copula model is adopted to solve this deficiency. Next, these three types of Copula models are discussed and evaluated with Spearman's rho, the parametric bootstrap test and the selection criteria based on the empirical Copula. Finally, the extreme wind speeds for a given return period are predicted by the t-Copula model with observed wind-speed records from several areas and the influence of dependence among directional extreme wind speeds on the predicted results is discussed.

Keywords: directional extreme wind speeds; t-Copula; joint probability distribution; directionality; dependence

1. Introduction

The importance of considering wind directionality effect in estimating probabilistic wind load effects on structures has been well recognized (Zhang and Chen 2015). Cook (1983) analyzed the wind directionality effect for the first time and indicated the risk should be distributed uniformly by direction. Cook and Miller (1999) improved this method and showed the geographic variation of the correlation of extreme wind speeds between adjacent sectors. The pioneering work of Cook (1983) and Cook and Miller (1999) proposed a solution of wind directionality effect, while the definite expression of the joint probability distribution of directional extreme wind speeds has not been obtained. The estimation of a joint probability distribution model of directional extreme wind speeds can be decomposed into two aspects based on the Sklar theorem (Sklar 1959). The first aspect is the estimation of the one-dimensional marginal cumulative distribution function in each sector and the second one is the estimation of the Copula model to consider the correlation of extreme wind speeds among different sectors (Simiu *et al*. 1985, Kanda and Itoi 2001, Itoi and Kanda 2002, Zhang and

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Chen 2015, 2016).

The first aspect involves fitting the one-dimensional probability distribution based on the pioneering theoretical work of Fisher and Tippett (1928). Von Mises (1936) represented the three asymptotic distributions of Fisher and Tippett (1928) as the Generalised Extreme Value Distribution. Gumbel (1958) defined clear formulations for these asymptotic distributions as Fisher-Tippett Type I, Type II, Type III distributions. Cook and Harris (2004), Harris (1996, 2006, 2009) and Naess (2001) considered extreme wind speed distribution as belonging to the domain of the Fisher-Tippett Type I distribution because the parent distribution of wind speeds is generally regarded as the Weibull distribution. While, Simiu *et al.* (1996, 2001), Holmes and Moriarty (1999) and Kasperski (2007) considered that the Fisher-Tippett Type III distribution is appropriate for extreme wind speeds which are limited in magnitude for geophysical reasons because the Type III distribution curves in a way that approaches a limited value for long return periods, while the extreme value predicted with Fisher-Tippett Type I is unbounded as the return period increases. Lagomarsino *et al*. (1992) indicated that applying the asymptotic treatment to samples composed by a finite number of independent data, errors increase on increasing the return period of the estimates concerned. Cook and Harris (2004, 2008) developed the FT1 penultimate distribution that accounts for asymptotic convergence and avoids the associated errors; this method derives from classical asymptotic analysis, taking into account finite values for the annual rate of independent events (Torrielli *et al*. 2013). The above distribution of analysis of extreme wind speeds were tested by Torrielli *et al.* (2013) and Harris (2014) based on simulation methods for the macro-meteorological wind speed proposed by Torrielli *et al.* (2011), Torrielli *et al.* (2014) and Harris (2014). The FT1 penultimate distribution devised by Cook and Harris performs surprisingly well (Harris 2014) and leads to a very good data fitting (Torrielli *et al*. 2013). To solve the second problem, researchers analysed the correlation of extreme wind speeds among different sectors with Copula theory, including the two-dimensional Gumbel-Copula model (Simiu et al. 1985), Partially Nested Gumbel-Copula model (Kanda and Itoi, 2001), Fully Nested Gumbel-Copula model (Itoi and Kanda 2002, Zhang and Chen 2016) and Gaussian-Copula model (Zhang and Chen 2015, 2016).

For several researchers and practitioners, it appears to be sufficient to try to fit any stochastic model with the most convenient Copula family and confirm that the fitting procedure is not so “bad” (Jaworski *et al*. 2010). Gumbel-Copula is one of the Extreme-Value Copulas (Genest and Rivest 1989, Jaworski *et al.* 2010). It appears to be reasonable to build the joint probability distribution model of directional extreme wind speeds with the Gumbel-Copula model. However, the Gumbel-Copula model has only a single parameter, so it is not able to fully reflect the complex correlation among 16 (or even more than 16) different sectors (Haraguchi and Kanda 1999). The Partially Nested Gumbel-Copula and Fully Nested Gumbel-Copula models have multiple parameters, but it is difficult to build the suitable nested structure. In order to avoid producing a negative probability density function, the parameters in the nested function should be decline from the deepest nested structure to the shallowest one gradually (Whelan 2004, Embrechts *et al*. 2001). This makes the problem more complicated. Zhang and Chen (2016) and Itoi and Kanda (2002) sorted the nested sequence in the order of wind speed dominance, but it does not always satisfy the requirement. It is difficult to determine the sequence of the fully nested structure or divide the directional wind speeds into several groups to build the partially nested structure. Zhang and Chen (2015, 2016) considered the statistical dependence among directional extreme wind speeds with the Gaussian-Copula model, but the Gaussian-Copula model has no upper tail (long return period) dependence (Sibuya 1959, Falk *et al*. 2010). The dependence among directional extreme wind speeds for long return periods cannot be reflected correctly when the joint probability distribution model is established with the

Gaussian-Copula model, while the extreme wind speed for long return periods is mainly considered in estimating probabilistic wind load effects of structures. Jaworski *et al*. (2010) indicated that a significant estimation error can be induced when one tries to fit with Copulas that do not exhibit any peculiar behaviour in the tails. This may cause over estimating the wind load effects on structures.

The t-Copula model can reflect the dependence among multi-dimensional random variables conveniently since it has multiple parameters, and it usually does not require the nested structure (Jaworski *et al*. 2010). Although the t-Copula is not an Extreme-Value Copula theoretically, it can capture the dependence among multi-dimensional extremes in the upper tail (Embrechts *et al.* 2002, Nikoloulopoulos *et al*. 2009). Therefore, the t-Copula is proposed to establish the joint probability distribution model of directional extreme wind speeds in the present study.

2. Basic method of the proposed approach

Let $F\left(v_1,v_2,\cdots,v_N\right)$ be the joint probability distribution function of multi-dimensional variables $\left(v_1,v_2,\cdots,v_N\right)$, with one-dimensional margins $F\left(v_1\right),\cdots,F\left(v_N\right)$. According to the Sklar theorem (Sklar 1959), when $F\left(v_1\right),\cdots,F\left(v_N\right)$ are all continuous, there exists a unique Copula $C\left(p_1,p_2,\cdots,p_N\right)$ such that for all $\left(v_1,v_2,\cdots,v_N\right)$

$$F\left(v_1,v_2,\cdots,v_N\right)=C\left(F_1\left(v_1\right),F_2\left(v_2\right),\cdots,F_N\left(v_N\right)\right) \tag{1}$$

where N denotes the size of multivariate random variables, which is equal to the number of sectors (16 in the present case).

Based on the Sklar theorem, modelling the joint distribution of directional extreme wind speed can be divided into two aspects:

(1) The estimation of one-dimensional marginal cumulative distribution functions $F\left(v_1\right),\cdots,F\left(v_N\right)$ in each sector.

(2) The estimation of Copula model $C\left(p_1,p_2,\cdots,p_N\right)$ to consider the correlation of extreme wind speeds among different sectors.

This section introduces the process for building the joint distribution model of directional extreme wind speeds using the Sklar theorem. At first, the one-dimensional marginal distribution function of extreme wind speeds in each sector is analysed by the extreme analysis theory of Harris (1996, 1999, 2006, 2009) based on the monthly maximum wind speeds (see section 2.1). Then, the process of establishing the t-Copula model to consider the dependence among directional extreme wind speeds is introduced (see section 2.2). Finally, the parameter estimation method for the t-Copula model is discussed (see section 2.3).

2.1 The fitting process of one-dimensional marginal probability distribution functions

The extreme analysis theory of Harris (1996, 1999, 2004, 2006, 2009) eliminates as much of the error as possible and further considers the properties of extreme values among the fitting methods of one-dimensional extreme distributions (Zhang 2014). However, the Independent Storm method proposed by Harris (1996, 1999, 2004, 2006, 2009) is not appropriate for analysing wind speeds with the Sklar theorem since the extreme samples extracted with this method do not have the same lengths in different sectors, so it is difficult to build the Copula model. Therefore, the block maxima method, in which extracted extreme samples in different sectors are of the same length, is adopted in the present study instead of the Independent Storm method. The left-censoring (Harris 2004, 2009), improved reduced variate (Harris 1996, 1999) and weighted least-square (Harris 1996, 1999) methods are adopted to fit the one-dimensional marginal distributions based on the FT1 penultimate distribution (Cook and Harris, 2004, 2008, Harris, 2009).

Until recently, the total observation times of wind-speed records have generally been several decades or even a few years. For the block maxima, the small sample size of annual maximum wind speed will lead to large statistical error (Zhang and Chen 2015). The daily maximum wind speeds are also not appropriate for estimating extreme wind speed since their temporal correlation will result in the inapplicability of the extreme theory. The monthly maximum wind speeds not only eliminate the temporal correlation but also reduce the statistical error due to the increased sample size (Zhang and Chen 2015). Therefore, the monthly maximum wind speeds are used to fit one-dimensional marginal distributions in the present study. The unit of measurement of the return period is accordingly adjusted from year to month, e.g., a 50-year return period is expressed as a 600-month return period.

The fitting process of the one-dimensional marginal probability distribution with the monthly maximum wind speeds in each sector (e.g., the n^{th} sector) is introduced as follows:

(1) Statistically analysing the observed wind-speed data in each sector, extracting the monthly maximum wind speeds in the n^{th} sector, and then sorting them from smallest to largest, $v_{1n} \le \cdots \le v_{mn} \le \cdots v_{Mn}$, where M denotes the number of observed sample points, i.e., it denotes the number of monthly maximum wind speeds in the present case. $m\left(m = 1, \cdots, M\right)$ is the index of the monthly maximum wind speeds;

(2) Calculating the mean value, the mean square value and the variance of each reduced variate (Harris 1996, 1999)

$$\bar{y}_m = \frac{\Gamma(M+1)}{\Gamma(M-m+1)\Gamma(m)} \int_0^1 -\ln(-\ln(p)) \cdot p^{m-1} \cdot (1-\mathrm{p})^{M-m} dp \tag{2}$$

$$\overline{y_m^2} = \frac{\Gamma(M+1)}{\Gamma(M-m+1)\Gamma(m)} \int_0^1 (-\ln(-\ln(p)))^2 \cdot p^{m-1} \cdot (1-\mathrm{p})^{M-m} dp \tag{3}$$

$$\sigma_m^2 = \overline{\mathrm{y}_m^2} - \bar{y}_m^2 \tag{4}$$

where p denotes the non-exceedance probability and $y = -\ln\left(-\ln\left(p\right)\right)$ is the reduced variate for extreme analysis. The expression for the Γ function is $\Gamma\left(m\right) = \int_0^{+\infty} e^{-y} y^{m-1} dy \quad \left(m > 0\right)$;

(3) Calculating the weights (Harris 1996, 1999)

$$w_m = \frac{1/\sigma_m^2}{\sum_{m=1}^{M} \left(1/\sigma_m^2\right)} \tag{5}$$

(4) According to the left-censoring, only if $\bar{y}_m > 0$, the m^{th} monthly maximum wind speed v_{mn}

and its $\bar{y}_m$ and w_m are used to fit the FT1 penultimate distribution (Harris 2004, 2009);

(5) Fitting the FT1 penultimate distribution by the weighted least-square method (Cook and Harris 2004, Harris 2009)

$$P(v) = \exp(-\exp(-(\alpha v^{\tau} - \Pi))) \tag{6}$$

a) Calculating the fitting parameters α and Π

$$\alpha = \frac{\sum_{m=1}^{M} w_m \bar{y}_m v_{mn}^{\tau} - (\sum_{m=1}^{M} w_m \bar{y}_m)\cdot(\sum_{m=1}^{M} w_m v_{mn}^{\tau})}{\sum_{m=1}^{M} w_m v_{mn}^{2\tau} - (\sum_{m=1}^{M} w_m v_{mn}^{\tau})^2} \tag{7}$$

$$\Pi = \alpha \sum_{m=1}^{M} w_m v_{mn}^{\tau} - \sum_{m=1}^{M} w_m \bar{y}_m \tag{8}$$

where α , Π and τ are fitting parameters of the FT1 penultimate distribution. The initial value of τ is set as 1;

b) Calculating the error

$$S^2 = \sum_{m=1}^{n} w_m (\bar{y}_m - \alpha v_{mn}^{\tau} + \Pi)^2 \tag{9}$$

Steps (a) and (b) are repeated using the optimization algorithm to find the optimal parameter τ to minimize the error S^2. Then, by substituting α , Π and τ into Eq. (6), the one-dimensional marginal probability distribution for the n^{th} sector is obtained.

2.2 Establishing the t-Copula model

The expression for the N-dimensional t-Copula function is (Fang *et al.* 2002)

$$C_t\left(p_1, p_2, \cdots, p_N\right) = t_N\left(\left[\rho\right], k; t_1^{-1}\left(k; p_1\right), t_1^{-1}\left(k; p_2\right), \cdots, t_1^{-1}\left(k; p_N\right)\right) \tag{10}$$

Its probability density function is

$$\mathrm{c}_t\left(p_1, p_2, \cdots, p_N\right) = \left|\left[\rho\right]\right|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{k+N}{2}\right)\left[\Gamma\left(\frac{k}{2}\right)\right]^{N-1}}{\left[\Gamma\left(\frac{k+1}{2}\right)\right]^{N}} \frac{\prod_{n=1}^{N}\left(1+\frac{\left(t_1^{-1}\left(k; p_n\right)\right)^2}{k}\right)^{\frac{k+1}{2}}}{\left(1+\frac{1}{k}\vec{\xi}'\left[\rho\right]^{-1}\vec{\xi}\right)^{\frac{k+N}{2}}} \tag{11}$$

where p_n is the one-dimensional marginal probability of the n^{th} sector, n=1,2,$\cdots$,N, and

$t_N\left([\rho],k;x_1,x_2,\cdots,x_N\right)$ is the N-dimensional multivariate t-distribution function of the N-dimensional variables $\left(x_1,x_2,\cdots,x_N\right)$ (Härdle and Simar 2007):

$$t_N\left([\rho],\mathrm{k};x_1,x_2,...,x_N\right) = \int_{-\infty}^{x_N}\cdots\int_{-\infty}^{x_1} \frac{\Gamma\left(\frac{k+N}{2}\right)}{\Gamma(k/2)k^{N/2}\pi^{N/2}\left|[\rho]\right|^{\frac{1}{2}}\left(1+\frac{1}{k}(x_1,x_2,...,x_\mathrm{N})\cdot[\rho]^{-1}\cdot(x_1,x_2,...,x_N)'\right)^{\frac{k+N}{2}}}dx_1\cdots dx_N \tag{12}$$

where *k* is the number of degrees of freedom of t_N and $[\rho]$ is the dispersion matrix of t_N, both of which are fitted parameters. $[\rho]$ is an $N\times N$ positive-definite symmetric matrix with ones on the primary diagonal. $\left|[\rho]\right|$ is the determinant of $[\rho]$. $\vec{\xi}'$ is the transpose of $\vec{\xi}=\left(t_1^{-1}(k;p_1),t_1^{-1}(k;p_2),\cdots,t_1^{-1}(k;p_N)\right)'$. $t_1^{-1}(k;p)$ is the inverse function of the one-dimensional standard t-distribution $t_1(k;x)$ (t_1 is a function of *x* with *k* degrees of freedom, and the subscript 1 means that the expression for t_1 is one dimensional). The expression for $t_1(k;x)$ is

$$t_1(k;x)=\int_{-\infty}^{v}\frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{k\pi}\,\Gamma(k/2)}\left(1+x^2/k\right)^{-(k+1)/2}dx \tag{13}$$

where Γ is the gamma function.

Although the formula of the Copula model is complex, many mathematical software packages, such as MATLAB, have the built-in functions to estimate its parameters conveniently.

The joint probability distribution function $F\left(v_1,v_2,\cdots,v_N\right)$ of directional extreme wind speeds $v_1,v_2,\cdots,v_N$ can be obtained with Eq. (1), Eqs. (6) and (10)

$$\begin{aligned}F\left(v_1,v_2,\cdots,v_N\right)&=C_t\left(P_1(v_1),P_2(v_2),\cdots,P_N(v_N)\right)\\&=t_N\left([\rho],k;t_1^{-1}\left(k;P_1(v_1)\right),t_1^{-1}\left(k;P_2(v_2)\right),\cdots,t_1^{-1}\left(k;P_N(v_N)\right)\right)\end{aligned} \tag{14}$$

The relationship between the return period *R* and the joint probability distribution function $F\left(v_1,v_2,\cdots,v_N\right)$ is

$$1-\frac{1}{R}=F\left(v_1,v_2,\cdots,v_N\right) \tag{15}$$

Combining Eqs. (14) and (15), the directional extreme wind speeds $v_1,v_2,\cdots,v_N$ for return period *R* can be obtained from

$$1-\frac{1}{R}=t_N\left([\rho],k;t_1^{-1}\left(k;P_1(v_1)\right),t_1^{-1}\left(k;P_2(v_2)\right),\cdots,t_1^{-1}\left(k;P_N(v_N)\right)\right) \tag{16}$$

When the extreme wind speed of each sector is estimated for a given return period, in order to distribute the risk uniformly by direction (Cook 1983) and to avoid the infinitely many solutions of Eq. (16), it is assumed that the non-exceedance probability $P_n(v_n)$ of each sector is the same, equal to p. The Eq. (16) become

$$1-\frac{1}{R}=t_N\left([\rho],k;t_1^{-1}(k;p),t_1^{-1}(k;p),\cdots,t_1^{-1}(k;p)\right) \tag{17}$$

There is only a single unknown variable p when the return period R is given and the parameters $[\rho]$ and k are fitted. p can be obtained conveniently with Eq. (17) by a numerical method for a given return period. After p is obtained, the extreme wind speed in each sector can be solved with Eq. (6).

When the all-directional extreme wind speed is estimated for a given return period, instead of assuming that the non-exceedance probability $P_n(v_n)$ of each sector is the same, it is assumed that the extreme wind speed v_n of each sector is the same, and equal to v_{all}. The Eq. (16) become

$$1-\frac{1}{R}=t_N\left([\rho],k;t_1^{-1}\left(k;P_1(v_{all})\right),t_1^{-1}\left(k;P_2(v_{all})\right),\cdots,t_1^{-1}\left(k;P_N(v_{all})\right)\right) \tag{18}$$

There is only a single unknown variable v_{all} after the return period R is given, the parameters $[\rho]$ and k are fitted and the one-dimensional marginal probability distribution $P_n(\cdot)$ for each sector is obtained. v_{all} can be obtained conveniently with Eq. (18) by a numerical method for the given return period.

2.3 Estimation method of t-Copula model

If the values of $[\rho]$ and k of $c_t(p_1,p_2,\cdots,p_N)$ are directly estimated by the Maximum-Likelihood Estimation method, the calculation for the case of N=16 is too large to the present microcomputer. The method proposed by Lindskog *et al*. (2003) is adopted in the present study. First, Kendall's tau (an important measure of dependence) of the observed samples is estimated and $[\rho]$ is obtained from Kendall's tau based on the theoretical relationship between them. Then, $[\rho]$ is set to the known value and the parameter k is estimated by the Maximum-Likelihood Estimation method. The steps of the estimation are as follows:

(1) Calculating Kendall's tau (Embrechts *et al.* 2001) as follows

$$\tau_{0,n_1n_2}=P\left(\left(v_{m_1,n_1}-v_{m_2,n_1}\right)\left(v_{m_1,n_2}-v_{m_2,n_2}\right)>0\right)-P\left(\left(v_{m_1,n_1}-v_{m_2,n_1}\right)\left(v_{m_1,n_2}-v_{m_2,n_2}\right)<0\right) \tag{19}$$

where $P\left(\left(v_{m_1n_1}-v_{m_2n_1}\right)\left(v_{m_1n_2}-v_{m_2n_2}\right)>0\right)$ is the probability of concordance between the wind speeds v_{n_1} and v_{n_2}, and $P\left(\left(v_{m_1n_1}-v_{m_2n_1}\right)\left(v_{m_1n_2}-v_{m_2n_2}\right)<0\right)$ is the probability of discordance between them;

Calculating the correlation parameter, $\rho_{n_1n_2}$, using the inverse expression of Kendall's tau of the t-Copula (Embrechts *et al*. 2001, Lindskog *et al*. 2003)

$$\rho_{n_1 n_2} \approx \sin\left(\frac{\pi\tau_{0,n_1 n_2}}{2}\right) \tag{20}$$

(2) Estimating the parameter k (the number of degrees of freedom):
Setting $[\rho]$ to the known value and estimating k by the Maximum-Likelihood Estimation method

$$\frac{\partial}{\partial k}\prod_{m=1}^{M} c_t(p_{m1}, p_{m2}, \cdots, p_{mN}) = 0 \tag{21}$$

where the expression for $c_t\left(p_{m1}, p_{m2}, \cdots, p_{mN}\right)$ is equal to Eq. (11).

3. Testing and selection of Copula models

Since only the Fully Nested Gumbel-Copula model and Gaussian-Copula model were expanded to 16-dimensional functions for the current 16-wind-sector system (Zhang and Chen 2016), the proposed t-Copula model is compared with them here. At first, the t-Copula model of extreme wind speed is established based on the monthly maximum wind speeds. The dependence of the t-Copula model is compared with that of the observed wind speeds by Spearman's rho (an important measure of dependence) to test the accuracy of the present model (see section 3.1). In addition, the three Copula models are tested with the parametric bootstrap test based on the empirical Copula (see section 3.2). Finally, the fitting effects of the Copula models that passed the goodness-of-fit test are evaluated according to the selection criteria for Copula based on the empirical Copula (see section 3.3).

The directional hourly mean wind-speed data used in this study was processed based on the data set 6405 from Automated Surface Observation System (ASOS, NOAA) in DES MOINES IA USA, BISMARCK ND USA, ABERDEEN SD USA, LINCOLN NE USA, MINNEAPOLIS MN USA, SPRINGFIELD IL USA, and TOPEKA KS USA, dated from January 1st, 2000 to September 30th, 2016. All of the stations are inland stations and mainly affected by monsoon. The wind-speed data from these fixed weather stations are selected as observed samples to test and select the Copula models. The raw data contain two-minute mean wind speeds in unit of knot and a wind direction with a resolution of 1°. The observation height is 33 ft (10 m) throughout the observation time. The hourly mean wind speed and direction are then calculated with a vector-averaging algorithm (Zhang and Chen 2015). After post-processing, the hourly mean wind speed is in unit of m/s and the resolution of the wind direction is 1°. Next, these directional wind speeds are divided into 16 directional sectors, each of which sweeps an area from α_n-11.25° to α_n+11.25°, where α_n=0°,22.5°,⋯,337.5°, representing the directions *N*,*NNE*,⋯,*NW*,*NNW*.

The t-Copula model is established by the method proposed in section 3.2. Table 1 shows the dispersion matrix $[\rho]$, which is fitted with the monthly maximum wind-speed data from data set 6405 at DES MOINES IA USA; the parameter k (the number of degrees of freedom) of this t-Copula model is 25.00.

Table 1 The dispersion matrix $[\rho]$ of the t-Copula model

	N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	NNW
N	1.00															
NNE	0.41	1.00														
NE	0.27	0.62	1.00													
ENE	0.18	0.35	0.56	1.00												
E	0.13	0.22	0.25	0.56	1.00											
ESE	0.08	0.21	0.17	0.26	0.60	1.00							symmetry			
SE	0.14	0.08	0.09	0.24	0.34	0.42	1.00									
SSE	0.01	0.00	0.06	0.24	0.32	0.24	0.57	1.00								
S	0.09	0.07	0.14	0.19	0.22	0.19	0.34	0.57	1.00							
SSW	0.23	0.09	0.11	0.20	0.13	0.13	0.31	0.23	0.58	1.00						
SW	0.25	0.17	0.14	0.20	0.22	0.17	0.36	0.26	0.36	0.55	1.00					
WSW	0.17	0.06	0.08	0.20	0.23	0.11	0.21	0.22	0.20	0.23	0.52	1.00				
W	0.20	0.17	0.21	0.25	0.29	0.16	0.25	0.19	0.16	0.20	0.43	0.61	1.00			
WNW	0.32	0.07	0.04	0.09	0.31	0.18	0.29	0.15	0.12	0.14	0.35	0.44	0.57	1.00		
NW	0.43	0.06	0.05	0.08	0.19	0.00	0.13	0.11	0.09	0.15	0.36	0.38	0.44	0.76	1.00	
NNW	0.57	0.15	0.13	0.10	0.15	0.04	0.00	-0.01	0.10	0.18	0.29	0.34	0.29	0.43	0.69	1.00

3.1 Comparison of the dependence structures by Spearman's rho

Kendall's tau and Spearman's rho are two important measures of dependence (Embrechts *et al.* 2001). The t-Copula model established in the Section 3.3 has the same Kendall's tau as the observed samples, so Kendall's tau cannot be used to test the dependence structure of the proposed model. Therefore, Spearman's rho of the observed monthly maximum wind speeds and that of the fitted t-Copula model are compared to test the dependence structure of the fitted model.
Setting $(v_{m_1n_1}, v_{m_1n_2})$, $(v_{m_2n_1}, v_{m_2n_2})$ and $(v_{m_3n_1}, v_{m_3n_2})$ as three independent and identically distributed samples of two-dimensional continuous random variables, Spearman's rho is defined as (Embrechts *et al.* 2001)

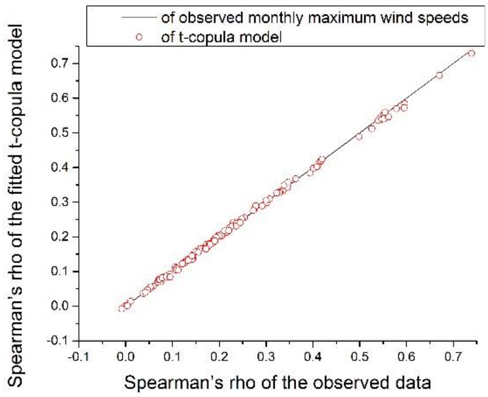
$$\begin{aligned}&\rho_s\left(v_{n1}, v_{n2}\right)\\&=3\Big(\mathrm{P}\Big(\big(v_{m1,n1}-v_{m2,n1}\big)\big(v_{m1,n2}-v_{m3,n2}\big)>0\Big)-\mathrm{P}\Big(\big(v_{m1,n1}-v_{m2,n1}\big)\big(v_{m1,n2}-v_{m3,n2}\big)<0\Big)\Big)\end{aligned} \tag{22}$$

Spearman's rho of the t-Copula model between sector n_1 and sector n_2 is (Embrechts *et al.* 2001)

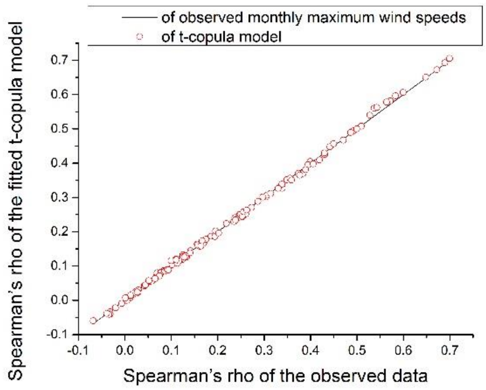
$$\rho_{s,n_1n_2}=12\int_0^1\int_0^1 C_t\left(p_{n1}, p_{n2}\right)dp_{n2}dp_{n1}-3 \tag{23}$$

where the expression of $C_t(p_{n_1}, p_{n_2})$ is given by Eq. (10) for the case of N=2.

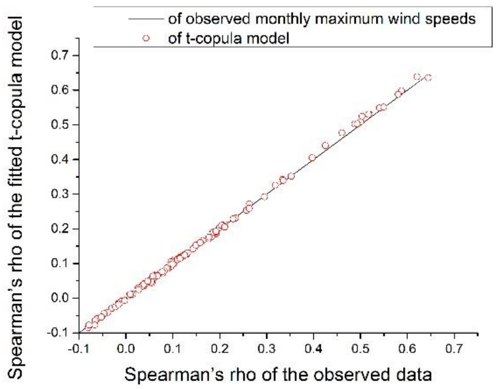
Fig. 1 compares Spearman's rho of the fitted t-Copula model and that of the observed monthly maximum wind-speed samples. The results for four areas are given in Fig. 1; the others are similar to them and are not listed here. The values of Spearman's rho between any two sectors are shown. The differences between them are small. It is shown that the dependence among samples from different areas can be reflected correctly by the fitted t-Copula model.



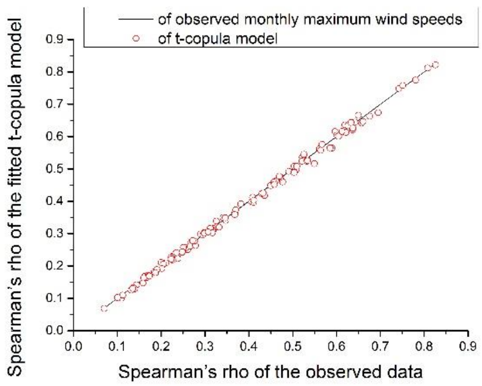
DES MOINES IA USA



TOPEKA KS USA



MINNEAPOLIS MN USA



SPRINGFIELD IL USA

Fig. 1 Spearman's rho of the observed monthly maximum wind speeds and that of the fitted t-Copula model

3.2 Parametric bootstrap test based on the empirical Copula

Genest *et al*. (2009) and Genest and Rémillard (2008) summarized the goodness-of-fit test methods of Copula models completely and compared these methods with one another. The parametric bootstrap test based on the empirical Copula has a less intricate form and is easier to understand. It also performs well in application, so it is selected to test the t-Copula, Gaussian-Copula and Fully Nested Gumbel-Copula models in the present study. The Cramer–von Mises statistic S is selected as the inspected value.

The expression for the empirical Copula is (Deheuvels 1979)

$$C_0(\vec{p}) = \frac{1}{M}\sum_{m=1}^{M} I(p_{m1} \le p_1, ..., p_{mN} \le p_N) \tag{24}$$

where $\vec{p}=\left[p_1,\cdots,p_N\right]$, $p_1,\cdots,p_N\in\left[0,1\right]$, and p_{mn} is the empirical estimate of the one-dimensional marginal non-exceedance probability for the m^{th} observed monthly maximum wind speed in the n^{th} wind sector. $I(\cdot)$ is the indicator function, i.e., if $p_{m1}\le p_1,\ldots,p_{mN}\le p_N$, then $I(\cdot)=1$; otherwise, $I(\cdot)=0$.

In goodness-of-fit testing, the greater the sample size, the better the test effect (Genest *et al*. 2009). However, in this case, the number of observed samples is limited. On one hand, the monthly maximum wind-speed samples are usually not enough to establish the 16-dimensional empirical Copula function to test the 16-dimensional distribution directly; on the other hand, when the correlation between variates is weak, the probability of rejecting the original hypothesis is very low by the bootstrap test (Genest *et al*. 2009). Considering the above factors, when the t-Copula and Gaussian-Copula models are tested, only the two-dimensional marginal cumulative distributions of the adjacent wind directions (for which the correlation of wind speeds is stronger) are examined.

When the Fully Nested Gumbel-Copula model is tested, the model should be examined according to the nested sequence.

The steps of the parametric bootstrap test based on the empirical Copula for each group of two-dimensional samples are as follows:

(1) Setting j=0 and calculating the empirical estimates of the one-dimensional marginal non-exceedance probabilities, $[p_j]_{M\times 2}$, with the observed monthly maximum wind speeds in both sectors, $[V]_{M\times 2}$;

(2) Calculating the empirical joint cumulative distribution function, $C_{0,j}(\vec{p})$, with $[p_j]_{M\times 2}$ by Eq. (24);

(3) Fitting the Copula model (t-Copula model, Gaussian-Copula model or Fully Nested Gumbel-Copula model), $C_{\theta,j}(\vec{p})$, with $[p_j]_{M\times 2}$. One subscript of $C_{\theta,j}(\vec{p})$ is set as θ to distinguish the fitted model $C_{\theta,j}(\vec{p})$ from the empirical Copula, $C_{0,j}(\vec{p})$;

(4) Calculating the Cramer–von Mises statistics $S_j=\sum_{m=1}^{M}\left(C_{0,j}\left(\vec{p}_m\right)-C_{\theta,j}\left(\vec{p}_m\right)\right)^2$, where $\vec{p}_m$ is the m^{th} row of $[p_j]_{M\times 2}$;

(5) If $j<J$ (J is often set to several hundred), setting $j=j+1$ and generating M two-dimensional random numbers, $[p_j]_{M\times 2}$, with $C_{\theta,0}(\vec{p})$. Returning to step 2 with the generated samples $[p_j]_{M\times 2}$. If $j=J$, going to step 6;

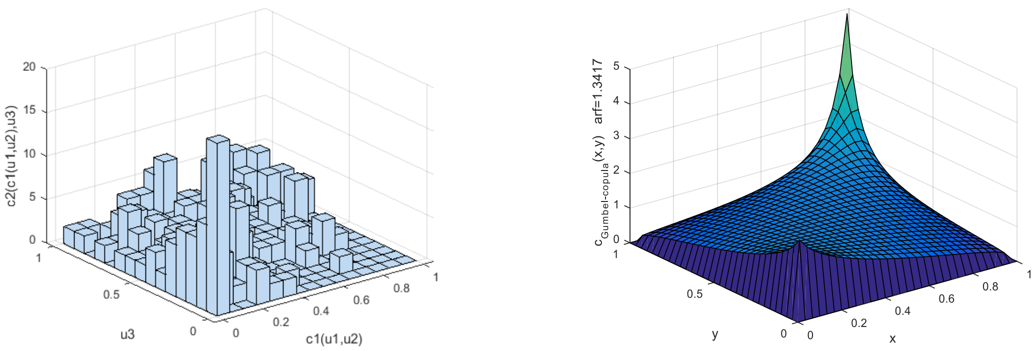
(6) Calculating the P-value approximately with $P_{value}\approx\frac{1}{J}\sum_{j=1}^{J}I\left(S_j>S_0\right)$. The P-value can be considered the probability that $S_j>S_0$;

(7) If $P_{value}\le P_0$, the original hypothesis is rejected, otherwise it is accepted. P_0 is the significance level and is usually set as 0.05.

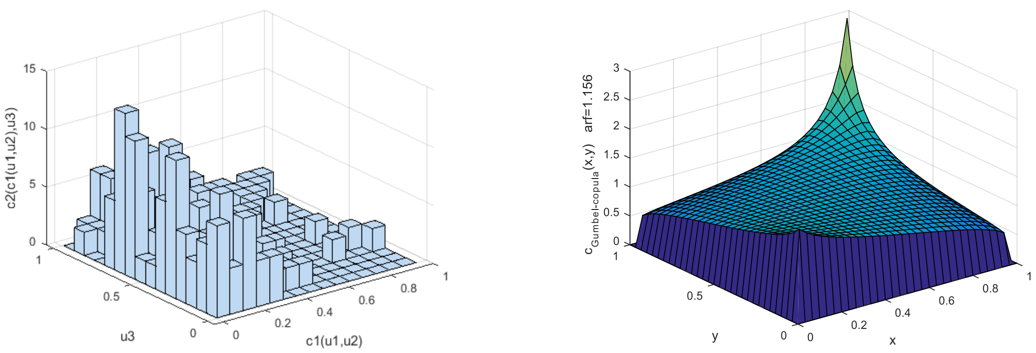
Testing is performed on the Copula models based on the wind-speed data at 7 representative areas in the USA. When the t-Copula and Gaussian-Copula models of the wind speeds of each region are tested, all of the two-dimensional marginal cumulative distributions of the adjacent wind directions pass the goodness-of-fit test. This indicates that the t-Copula and Gaussian-Copula models can be used to establish the joint probability distribution model of directional extreme wind speeds. However, the Fully Nested Gumbel-Copula models for wind speeds at all of the tested areas are rejected at the second-deepest nested structure, which indicates that the Fully Nested Gumbel-

Copula model is not appropriate for building the joint probability distribution model of directional extreme wind speeds.

The data from DES MOINES IA USA and TOPEKA KS USA are taken as examples to discuss the reason for this phenomenon. Fig. 2 (left) shows the frequency histogram of the variables in the second-deepest nested structure of the Fully Nested Gumbel-Copula model. Fig. 2 (right) shows the probability density plot of the fitted Gumbel-Copula model in the second-deepest nested structure. Obviously, the trends of the two figures are different. Fig. 2 (left) has obviously lower tail, while the upper tail of the probability density plot of the fitted Gumbel-Copula model is very tall in Fig. 2 (right). Fig. 2 (left) and Fig. 2 (right) do not match well, thus the Fully Nested Gumbel-Copula model is rejected at the second-deepest nested structure.



DES MOINES IA USA



TOPEKA KS USA

Fig. 2 The frequency histogram of the variables in the second-deepest nested structure (left)
The probability density plot of the fitted Gumbel-Copula model in the second-deepest nested structure (right)

3.3 Selection criteria based on the empirical Copula

For the evaluation of general statistical models, the Akaike information criterion (AIC) and Bayesian information criterion (BIC) have been widely used, however, the AIC and BIC are applicable for cases in which the number of samples is much larger than the number of parameters (Burnham and Anderson 2003). The dependence among extreme wind speeds in the 16 wind sectors should be considered and many parameters should be fitted in the present study, which may cause the AIC or BIC to give an unreasonable judgement. An improved Akaike information criterion, AIC_c, can correct the result from the AIC with small sample sizes (Burnham and Anderson 2003), but its correction term of multi-dimensional distribution is too complicated.

Trivedi and Zimmer (2007) summarized several methods for assessing Copula models. The selection criterion based on the empirical Copula is suitable for comparison between different kinds of Copula models, and it is easy to use due to its less intricate form (Durrleman *et al.* 2000, Trivedi and Zimmer 2007). Therefore, this criterion is proposed to select Copula models.

Durrleman *et al.* (2000) proposed an assessment method for Copula models based on the empirical Copula, in which the discrete L^2 norm between the empirical Copula and the fitted Copula model is used as the selection criterion of the fitted Copula model. The smaller the L^2 norm, the better the fitting effect. The expression for the L^2 norm between the empirical Copula and the fitted Copula model is

$$\left\| C_0\left([p]_{M\times N}\right) - C_\theta\left([p]_{M\times N}\right) \right\|_{L^2} = \left(\sum_{m=1}^{M} \left(C_0\left(\vec{p}_m\right) - C_\theta\left(\vec{p}_m\right) \right)^2 \right)^{1/2} \tag{25}$$

where $[p]_{M\times N}$ is the one-dimensional marginal non-exceedance probability for each sample, and $\vec{p}_m$ is the m^{th} row of $[p]_{M\times N}$. $C_0([p]_{M\times N})$ is the value of the empirical Copula (Eq. (24)) for each sample. $C_\theta([p]_{M\times N})$ is the value of the fitted Copula model for each sample. $C_0(\vec{p}_m)$ is the value of the empirical Copula for the m^{th} sample, and $C_\theta(\vec{p}_m)$ is the value of the fitted Copula model for the m^{th} sample.

Because the Fully nested Gumbel-Copula model does not pass the goodness-of-fit test, its further analysis is not needed. The L^2 norm between the empirical Copula and the fitted Copula model is calculated when the fitted Copula model is the t-Copula model or Gaussian-Copula model.

Table 2 The L^2 norm between the empirical Copula and the fitted Copula model (t-Copula model or Gaussian-Copula model)

areas	t-Copula	Gaussian-Copula
DSM	0.1379	0.1562
BIS	0.0945	0.0994
ABR	0.1078	0.1104
LNK	0.0785	0.0800
MSP	0.0904	0.0995
SPI	0.3007	0.3154
TOP	0.1012	0.1163

Table 2 shows the L^2 norms based on the wind-speed data at the 7 representative areas in the USA. All of the results show that the L^2 norm between the empirical Copula and the fitted t-Copula model is smaller than that between the empirical Copula and the fitted Gaussian-Copula model. This indicates that the t-Copula model generally fits better with the observed data than the Gaussian-Copula model.

4. Application and discussion

In this section, the influence of using different block maxima on the estimated parameters of the Copula model is first discussed based on the wind-speed data from DES MOINES IA USA (see section 4.1). After that, the extreme wind speeds for a given return period are estimated by the t-Copula model and the independent case (in which the dependence among directional wind speeds are ignored), for the 7 representative areas. At last, the influence of dependence among directional extreme wind speeds on the predicted results is discussed (see section 4.2).

4.1 The influence of using different block maxima on the estimated parameters of the Copula model

Zhang and Chen (2015) established the Gaussian-Copula model for annual maxima with monthly maxima and thought that the covariance matrix of monthly maximum variables was equal to that of the annual maximum variables. In this regard, we have a different view.

The underlying Gaussian variable is defined as follows (Zhang and Chen 2015)

$$y_n = \Phi^{-1}\left(p_n\right) \tag{26}$$

where Φ^{-1} is the inverse function of the one-dimensional standard normal distribution, and p_n is the one-dimensional marginal probability of the n^{th} sector, for n=1,2,⋯,N. When p_n is obtained from monthly maxima or annual maxima, y_n is named the monthly maximum variable or annual maximum variable, respectively.

The influence of using different block maxima on the estimated parameters of the Copula model is discussed here based on the wind-speed data from DES MOINES IA USA. Because the number of annual maxima is small, the covariance matrix [ρ_{Gau}] of the Gaussian-Copula model is estimated with the quarterly and monthly maximum variables. The results are shown in Fig. 3, where ρ_{Gau,n_1n_2} is the parameter in the n_1^{th} row and n_2^{th} column of [ρ_{Gau}]; the x-axis is set as ρ_{Gau,n_1n_2} of the quarterly maximum variables and the y-axis as ρ_{Gau,n_1n_2} of the monthly maximum variables.

The values of ρ_{Gau,n_1n_2} of the quarterly maximum variables are obviously different from those of the monthly maximum variables, as shown in Fig. 3. The covariance matrix of the monthly maximum variables is not equal to that of the quarterly maximum variables. It can be reasonably deduced that establishing the Gaussian-Copula model for annual maxima with the covariance matrix of the monthly maximum variables is not appropriate.

4.2 Estimation of extreme wind speeds and discussion

The cumulative distributions of extreme wind speeds are fitted for the 7 representative areas in the USA. The directional factors of the basic wind speeds, namely the ratio between the directional extreme wind speeds and the all-directional extreme wind speeds, are showed in Table 3 for a 50-year return period.

In Fig. 4, the top-left, top-right, bottom-left and bottom-right figures are the results from DES MOINES IA, TOPEKA KS, MINNEAPOLIS MN and SPRINGFIELD IL, respectively. The results of the other 3 areas are similar to the results shown in these figures, so they are not shown here. The cumulative distribution of extreme wind speeds is fitted by the t-Copula model. The fitting effect for each sector is shown in Fig. 4(a). The fitted results match well with the observed samples on the whole. Fig. 4(b) shows the directional extreme wind speeds for a 50-year return period predicted by the t-Copula model and the independent case. In Fig. 4(b), the results of the t-Copula model and the independent case are almost the same. Fig. 4(c) shows the all-directional extreme wind speeds predicted by the t-Copula model and the independent case for various return periods. In Fig. 4(c), the results for low return periods are obviously overestimated by the independent case. The overestimation decreases gradually with the increase of the return period, which means that the extreme wind speeds in different sectors become independent gradually with the increase of the return period.

In the independent case, the one-dimensional marginal probability distributions of each sector are independent of each other. The all-directional extreme wind speed of the independent case, V_{ind}, for a given return period R is obtained from

$$1-\frac{1}{R}=\prod_{n=1}^{N}P_n\left(v_{ind}\right) \tag{27}$$

where $P_n(\cdot)$ is the one-dimensional marginal probability distribution in the n^{th} sector, i.e., Eq. (6).

Table 3 The directional factors of the basic wind speeds for a 50-year return period for the 7 representative areas

	DSM	BIS	ABR	LNK	MSP	SPI	TOP
N	0.684	0.760	0.999	1.040	0.829	0.753	1.009
NNE	0.899	0.734	0.960	0.925	0.873	0.751	1.068
NE	0.721	0.846	1.098	0.898	0.913	0.910	0.734
ENE	0.793	0.913	0.835	0.738	1.172	0.975	0.842
E	0.711	0.817	0.805	0.998	0.851	0.701	0.962
ESE	0.618	0.822	0.834	0.834	0.864	0.776	0.722
SE	0.655	1.062	0.985	0.806	0.932	0.786	0.943
SSE	0.949	1.052	1.197	0.865	1.003	0.895	0.928
S	0.841	0.923	0.796	1.165	0.937	0.869	0.923
SSW	0.834	0.723	0.928	0.926	1.003	0.908	1.136
SW	0.990	0.757	0.807	0.904	1.051	1.061	0.935
WSW	1.221	1.171	0.954	0.828	1.088	1.157	1.053
W	1.073	1.074	0.917	0.906	0.877	0.940	0.883
WNW	1.021	1.026	0.903	0.806	0.977	0.823	0.829
NW	0.890	1.010	0.865	0.845	1.010	0.786	1.018
NNW	0.819	0.914	0.934	0.934	0.858	0.837	0.937

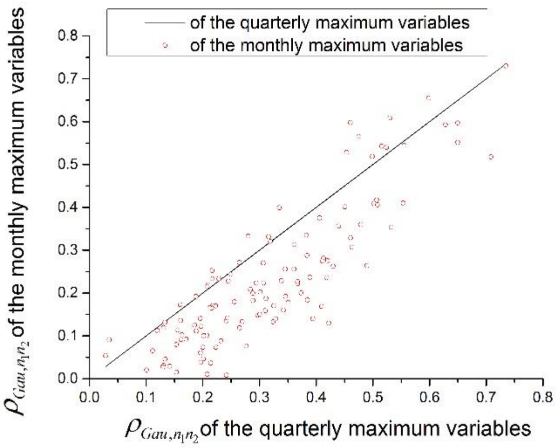
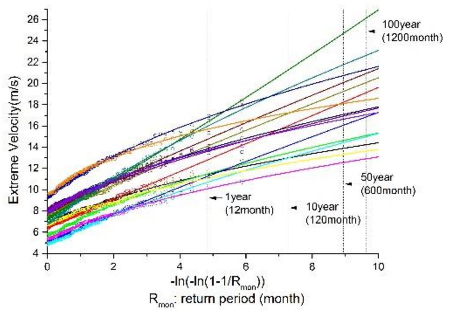
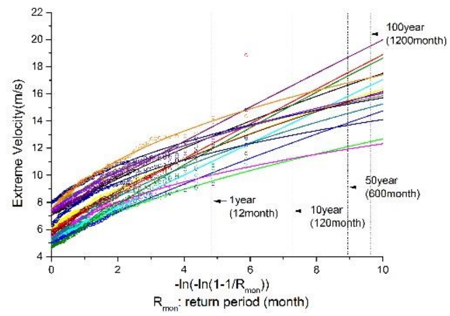


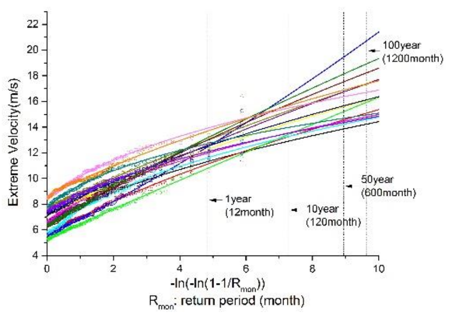
Fig. 3 The ρ_{Gau,n_1n_2} of the quarterly maximum variables and that of the monthly maximum variables (DES MOINES IA USA)



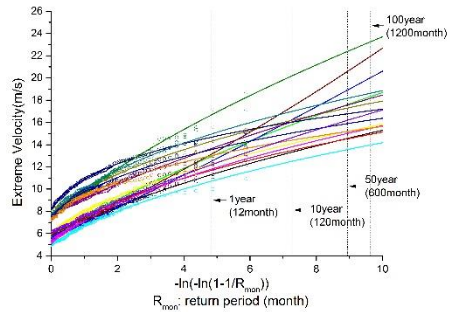
DES MOINES IA USA



TOPEKA KS USA



MINNEAPOLIS MN USA



SPRINGFIELD IL USA

(a) The fitted cumulative distribution of extreme wind speeds in each sector

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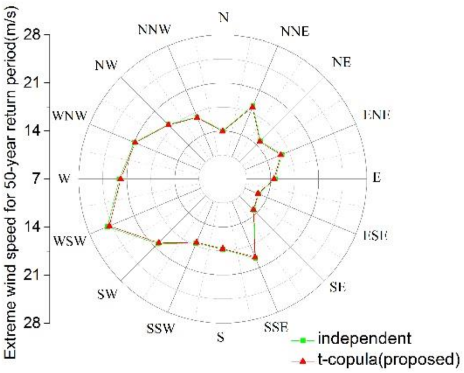
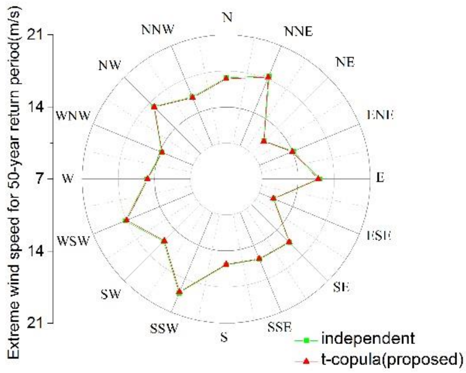
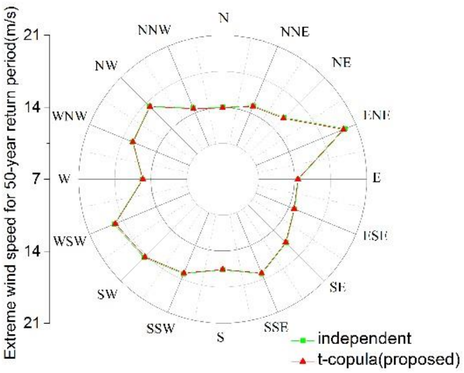


DES MOINES IA USA



TOPEKA KS USA



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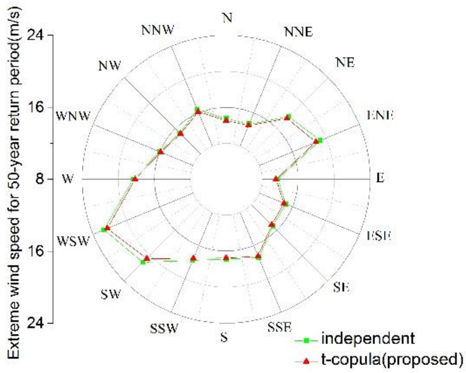
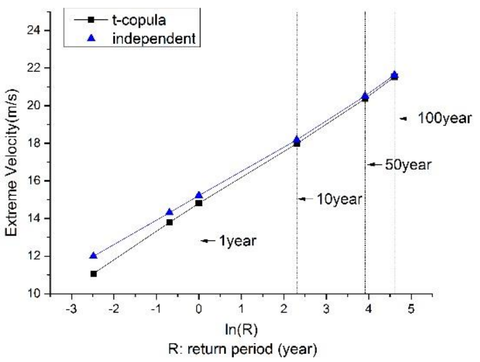
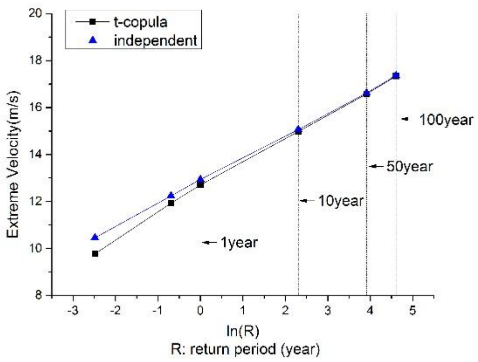


SPRINGFIELD IL USA

(b) The predicted directional extreme wind speeds for 50-year return period



DES MOINES IA USA



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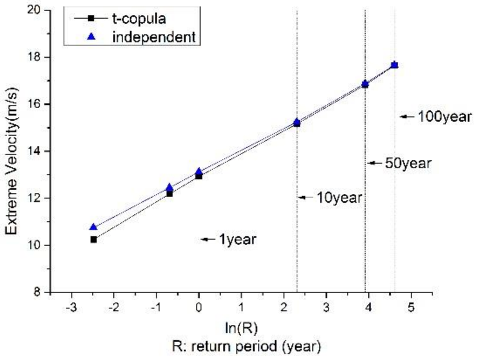

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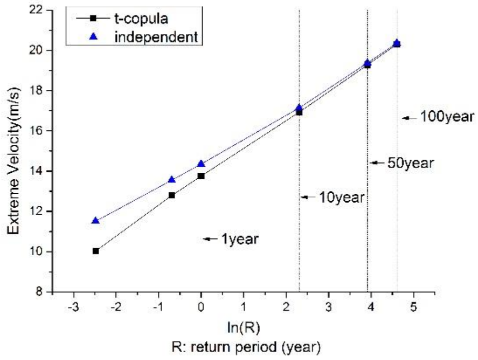

SPRINGFIELD IL USA

(c) The predicted all-directional extreme wind speeds for various return periods

Fig. 4 The predicted extreme wind speeds

Table 4 The directional dependence factors of the basic wind pressure for the 7 representative areas

	1 年	10 年	50 年	100 年
DSM	1.056	1.023	1.015	1.010
BIS	1.019	1.006	1.003	1.002
ABR	1.033	1.013	1.008	1.004
LNK	1.031	1.008	1.004	1.003
MSP	1.031	1.011	1.007	1.002
SPI	1.089	1.026	1.010	1.006
TOP	1.036	1.011	1.005	1.002
Maximum	1.089	1.026	1.015	1.010

The directional-dependence factor of the basic wind pressure is defined as

$$\gamma_{w_0\_cor} = (1/2\rho V_{ind}^2)/(1/2\rho V_t^2) \tag{28}$$

where V_t is the predicted all-directional extreme wind speeds of the proposed t-Copula model.

Table 4 shows the directional-dependence factors of the basic wind pressure for the 7 representative areas.

It is shown that the basic wind pressure is overestimated by up to 8.9% for a 1-year return period when the dependence is ignored; the overestimation decreases to less than 2.6% for a 10-year return period. The dependence factors are not larger than 1.5% for 50- and 100-year return periods, which are negligible in civil engineering. One can speculate that the directional extreme wind speeds become independent gradually when the target area is mainly affected by monsoon. In such a case, although the Gaussian-Copula model cannot correctly reflect the dependence among directional extreme wind speeds for long return periods in theory, it can produce similar results to the t-Copula model.

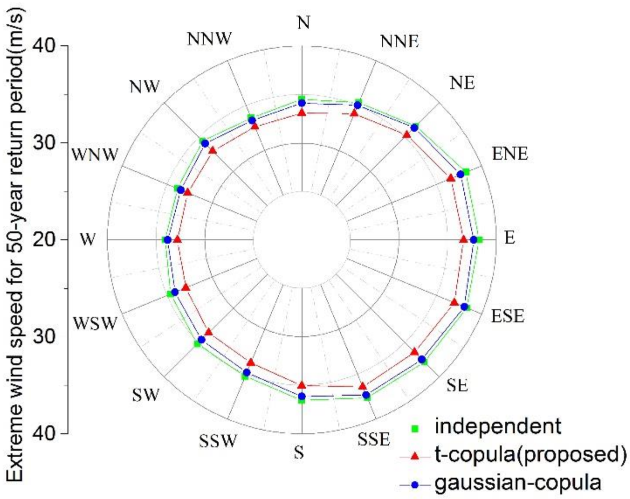

Fig. 5 The predicted directional extreme wind speeds for 50-year return period in Hong Kong China

The extreme wind speeds for long return periods are mainly caused by typhoons in typhoon-prone areas. A typhoon can cause high wind speeds in multiple directions simultaneously, so the directional extreme wind speeds for a long return period are very likely to be correlated with one another in typhoon-prone areas. Therefore, the result must be overestimated by the independent case. Fig. 5 shows the predicted directional extreme wind speeds for a 50-year return period in Hong Kong, China (a typhoon-prone area) of the t-Copula model, the Gaussian-Copula model and the independent case. These wind-speed data are obtained from weather stations and Monte Carlo typhoon simulation. The typhoon wind-field model in Meng *et al.* (1995) is adopted in this typhoon Monte Carlo simulation. The key typhoon parameters and probability models are referenced from Xiao *et al*. (2011).

In Fig. 5, the results of the Gaussian-Copula model are close to those of the independent case. The results are overestimated by the Gaussian-Copula model since the Gaussian-Copula model cannot correctly reflect the dependence among directional extreme wind speeds for long return periods. However, the t-Copula model can consider the dependence since its theory is more stringent for the upper tail (long return period). Its results should be more reasonable.

5. Conclusions

A new joint probability distribution model of directional extreme wind speeds is established by the multivariate extreme value theory with the t-Copula model. It is compared with other methods based on extreme wind speed data and several conclusions are obtained:

- The t-Copula model is more appropriate for building the joint probability distribution model of directional extreme wind speeds than the Gaussian-Copula model in theory due to its ability to capture the upper tail dependence among extreme values. The t-Copula model can reflect the dependence among multi-dimensional random variables conveniently because it has multiple parameters and usually does not require the construction of the nested structure similar to the nested Gumbel-Copula model.
- The comparison of Spearman's rho of the fitted t-Copula model with that of observed monthly maximum wind speeds shows that the dependence among samples in different sectors can be reflected correctly by the fitted t-Copula model.
- The goodness-of-fit test results show that the fitted Fully Nested Gumbel-Copula model cannot match well with the observed monthly maximum wind speeds, while the t-Copula model and Gaussian-Copula model are applicable for establishing the joint probability distribution model of directional extreme wind speeds.
- The L^2 norm between the empirical Copula and the t-Copula model is smaller than that between the empirical Copula and the Gaussian-Copula model, which indicates that the t-Copula model is generally more appropriate than the Gaussian-Copula model for fitting the joint probability distribution model of directional extreme wind speeds.
- The fitted covariance matrix of quarterly maximum variables and that of monthly maximum variables are different, and it is not appropriate to establish the Gaussian-Copula model for annual maxima with the covariance matrix of monthly maximum variables.
- The overestimation of predicted extreme wind speed of the independent case decreases gradually with the increase of the return period in a monsoon area. It can be speculated that the extreme wind speeds in different sectors become independent gradually in a monsoon area. Therefore, both the Gaussian-Copula model and the independent case are appropriate for predicting the extreme wind speeds for long return period in such an area. However, neither of them is appropriate in a typhoon-prone area.

Acknowledgments

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