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A consistent FEM-Vlasov model for hyperbolic cooling towers on layered soil under unsymmetrical wind load

Ali I. Karakas[a], Korhan Ozgan[*] and Ayse T. Daloglu[b]

Department of Civil Engineering, Karadeniz Technical University, Trabzon, Turkey

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Abstract. In this paper, the analysis of hyperbolic cooling tower on elastic subsoil exposed to unsymmetrical wind loading is presented. Modified Vlasov foundation model is used to determine the soil parameters as a function of vertical deformation profile within subsoil. The iterative parameter updating procedure involves the use of Open Application Programming Interface (OAPI) feature of SAP2000 to provide two way data flow during execution. A computing tool coded in MATLAB employing OAPI is used to perform the analysis of hyperbolic cooling tower with supporting columns over a hollow annular raft founded on elastic subsoil. The analysis of such complex soil-structure system is investigated under self-weight and unsymmetrical wind load. The response of the cooling tower on elastic subsoil is compared with that of a tower that its supporting raft foundation is treated as fixed at the base. The results show that the effect of subsoil on the behavior of cooling tower is considerable at the top and bottom of the wall as well as supporting columns and raft foundation. The application of a full-size cooling tower has demonstrated that the procedure is simple, fast and can easily be implemented in practice.

Keywords: hyperbolic cooling tower; modified Vlasov model; unsymmetrical wind load; open application programming interface; finite element analysis

1. Introduction

The hyperbolic cooling towers are essential components of many thermal power stations, oil refineries and other chemical plants. The cooling towers vary in size that can be up to 200 meters tall and 100 meters in diameter having a complex geometry with thin wall supported by variously oriented columns over an annular raft founded on soil stratum. The analysis of such complex soil-structure system under dead and variable wind and earthquake loading has attracted the attention of many researchers for more proper modeling. However, the flexibility of raft foundation and the compressibility of supporting soil stratum are usually neglected in most of the studies. The cooling tower failures in many parts of the world indicated that a fair estimation of the cooling tower-foundation-soil interaction is required for pure and robust design.

The studies on response analysis of fixed base or column supported cooling towers subjected to various static and dynamic loads have been performed by several researchers (Bosak and Flaga

∗Corresponding author, Associate Professor, E-mail: kozgan@ktu.edu.tr
[a] Ph.D. Student, E-mail: aliihsanka@yahoo.com
[b] Professor, E-mail: aysed@ktu.edu.tr

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1996, Karisiddappa *et al*. 1998, Christian 2002, Nasir *et al*. 2002, Esmaeil 2012, Murali 2012, Jia 2013, Prasahanth 2013, Tande 2013). Studies are also reported on the behavior of cooling towers incorporating the supporting soil medium in the analysis. Christian (2011a,b) studied the earthquake behavior of cooling towers supported by V-truss columns founded on Winkler vertical springs. Viladkar *et al*. (2006) worked on the numerical modeling of a column supported hyperbolic cooling tower and it's supporting annular raft-soil system under symmetrical wind loading. The soil medium was modeled using conventional linear elastic brick finite elements. Similarly Noorzaei (2006) investigated the behavior of cooling tower shell without supporting columns interacting with annular raft foundation and soil media using 20-noded solid isoparametric finite elements. The unsymmetrical wind loading and nonlinear hyperbolic soil model is considered in the analysis. Yang and Lu (1992) presented the static characteristics of soil-cooling tower interaction system by use of a coupling method combining the finite element method with the boundary element method.

However, a little attention has been paid on interactive analysis of cooling tower-column-annular raft-soil stratum system under unsymmetrical wind loading. Besides it is known that one parameter Winkler model and two parameter soil models have some discrepancies and limitations to reflect the soil structure interaction truly. Therefore, the present study focuses on a more proper estimation of the behavior of complex hyperboloid cooling tower on three parameter elastic foundation subjected to more realistic unsymmetrical wind loading using Open Application Programming Interface (OAPI) feature of SAP2000.

2. Sap2000 open application programming interface

The Open Application Programming Interface (OAPI) features of SAP2000 allow the user to access SAP2000 effectively by giving a chance to establish a direct bind to a broad range of supporting computer programming languages including MATLAB. A fast and robust coupling with SAP2000 provides two-way data flow during the execution of the analysis and design of the system as well as to facilitate pre and post processors.

In this study, OAPI features of SAP2000 is used interactively with a computing tool coded in MATLAB to perform the analysis of hyperboloid cooling towers on elastic subsoil using modified Vlasov foundation model as explained in the following sections.

3. Modified vlasov model

Subsoil reactions of a structure resting on a two-parameter elastic foundation may be given by

$$q_z = kw - 2t\nabla^2 w \tag{1}$$

depending on the displacement function w of the subsoil surface. k and $2t$ in above expression are subgrade reaction modulus and soil shear parameter respectively and may be defined as

$$k = \int_0^H \frac{E_s(1-\upsilon_s)}{(1+\upsilon_s)(1-2\upsilon_s)}\left(\frac{\partial\varphi(z)}{\partial z}\right)^2 dz \tag{2}$$

$$2t = \int_0^H G_s \phi(z)^2 \, dz \tag{3}$$

where H is subsoil depth, νs and G_s are Poisson's ratio and the shear modulus of subsoil respectively. The subgrade reaction, k, and soil shear parameter, $2t$, are considered to be constant in the classical two-parameter foundation models like Pasternak Model, Hetenyi Model and Vlasov Model etc. The drawback of these models lies in the difficulty of establishing the soil parameters, k and $2t$. Vallabhan *et al.* (1991) introduced another parameter, γ, to characterize the vertical deformation profile within the subsoil, and called the model as Modified Vlasov Model. The advantages of this model is the elimination of the necessity to determine the values of soil parameters, k and $2t$, arbitrarily because these values can be computed as a function of a new parameter, γ using an iterative procedure.

$\phi(z)$ in Eq. (3) is the mode shape function to describe the relationship between the vertical displacement of the subsoil and annular raft. The values of $\phi(z)$ are stipulated such that $\phi(0)=1$ and $\phi(H)=0$, Fig. 1.

Mode shape function $\phi(z)$ may be given depending on the subsoil surface vertical deformation parameter (γ) as below

$$\phi(z) = \frac{\sinh \gamma \left(1 - \frac{z}{H}\right)}{\sinh \gamma} \tag{4}$$

where γ is calculated using the equation shown below

$$\left(\frac{\gamma}{H}\right)^2 = \frac{(1-2\nu_s)}{2(1-\nu_s)} \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (\nabla w)^2 \, dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w^2 dxdy} \tag{5}$$

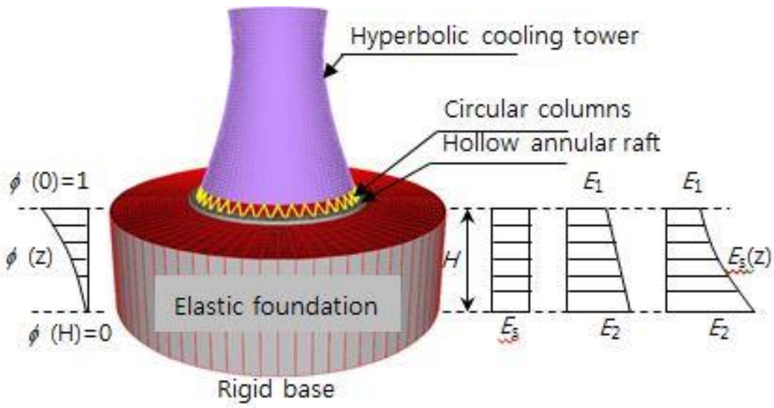

Fig. 1 Cooling tower on elastic foundation

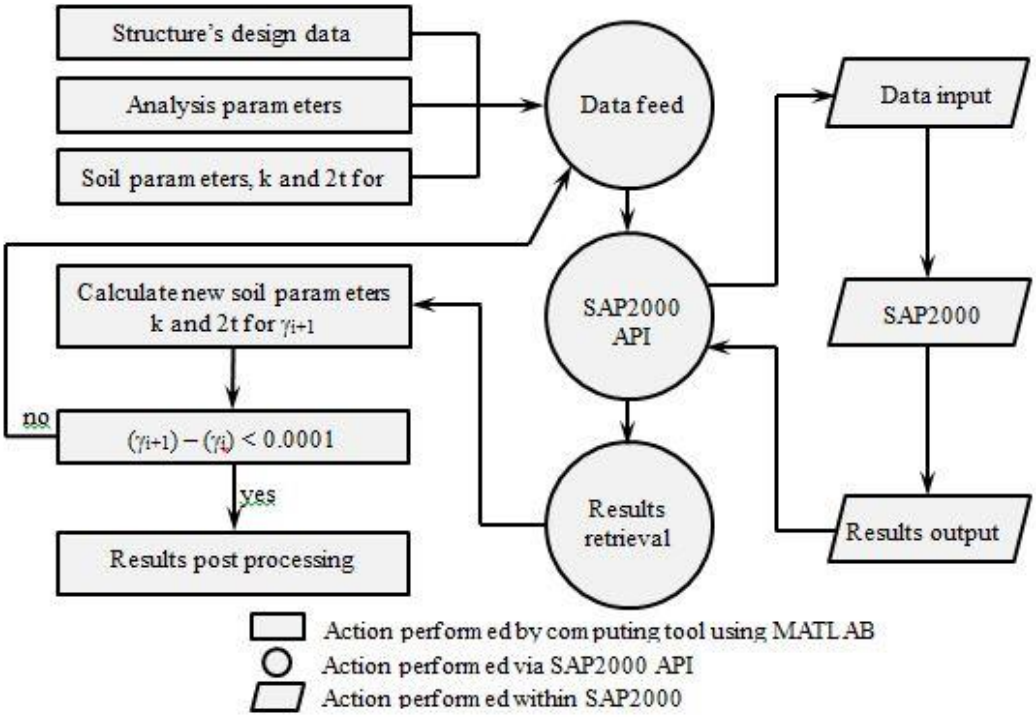

Fig. 2 Flowchart of the solution procedures

The important point here is that the modulus of subgrade reaction, *k*, and the soil shear parameter, 2*t*, are both dependent on the mode shape function $\phi(z)$ and the depth of the subsoil, *H*, as can be seen in Eqs. (2) and (3). Furthermore the value of the vertical deformation parameter within the subsoil,γ, varies with the displacement of the subsoil surface and the depth of the subsoil. So, the solution of this complex soil-structure interaction problem can be performed using an iterative technique.

For this purpose, initially a computing tool is developed using MATLAB to model hyperbolic cooling towers using SAP2000 software. As is known, the modulus of subgrade reaction, k, which is the only soil parameter used in Winkler Model is represented by elastic area springs in SAP2000. The interaction between the springs is ignored assuming each spring is acting independently.

A Shell-Layered/Nonlinear element with unit thickness is connected at the top of the springs to take the interaction between the springs into account. While Shell-Layered/Nonlinear element has only one degree of freedom (dof) at each node describing the displacement in z direction (*w*), plate element has three degrees of freedom at each node being the displacement in z direction (*w*) and two rotations (φ_x and φ_y). One of the main features of the SAP2000-OAPI is to provide data transfer and control of a structural model by different third-party applications simultaneously. A computing tool is developed in MATLAB and used to determine the soil parameters, *k* and 2*t* in terms of γ iteratively. Therefore, γ is initially set equal to one and subgrade reaction, *k*, and soil shear parameter, 2*t*, are calculated. Then, the structure-soil system is analyzed to find the surface displacements of the foundation which are the output of the structural model created by SAP2000.

A comparison between the new value of γ and previously calculated γ is then made. If the difference between the two successive γ values is within a prescribed tolerance, the analysis is terminated. Otherwise, another iteration is performed and the process is repeated until convergence is obtained. Solution procedure is given in Fig. 2.

4. Numerical verification

A circular hollow plate on elastic foundation given by Saygun and Çelik (2003) is analyzed for verification purposes using the proposed approach, Fig. 3. Saygun and Çelik (2003) used the full compatible ring sector finite element to evaluate the stiffness matrices of the plate and the soil. The material properties of the plate-soil system are as follows. Modulus of elasticity of the plate is 2.107 kN/m^2, Poisson's ratio of the plate is 0.16, modulus of elasticity of the subsoil is 80000 kN/m^2, Poisson's ratio of the subsoil is 0.25 and depth of the subsoil is 10 m. The analysis has been carried out with the same finite element mesh used by Saygun and Çelik (2003), and results are presented in Table 1 for comparison.

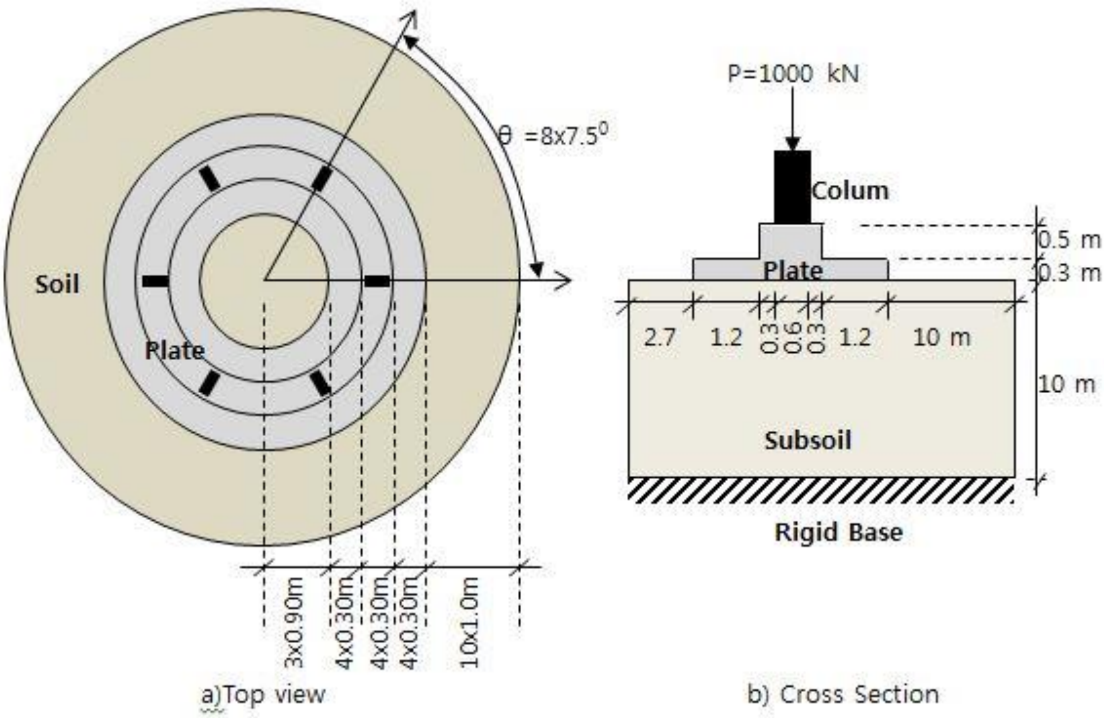

Fig. 3 A circular hollow plate on elastic foundation

Table 1 Soil parameters, central displacement and maximum moments for θ=0°

References	γ	k (kN/m³)	$2t$ (kN/m)	w (mm)	M_r (kNm)	M_θ (kNm)
Saygun and Çelik [16]	1.323	10081.85	86809.74	2.40	180	368
Present study	1.313	10068.92	87057.94	2.42	153	372

The soil parameters, the central displacement and the bending moments are very close to each other as seen in Table 1. So it can be said that the approach presented in this study is reliable and the model can be effectively and easily used for soil-structure interaction problems for any type of structure.

5. Hyperbolic cooling tower on elastic foundation

A case study is carried out for a hyperbolic cooling tower taken from literature (Noorzaei *et al.* 2006, Viladkar *et al.* 2006). The tower shell is discretized into 208 four-node thin shell elements in the circumferential direction and 104 elements in the meridional direction for the finite element analysis. Each shell element has variable thickness through the meridian. Also, two-node frame element activating all six degrees of freedom at both of its connected joints is used to model supporting columns. The raft foundation is modeled using 208 and 6 four-node thick-plate elements in circumferential and radial directions, respectively. C25 class of concrete for shell walls and C35 class for columns and ring raft are used, Table 2.

5.1 Geometry

The geometry of the hyperbolic wall is described by a hyperbolic equation as given below. Z coordinate in Eq. (6) is measured from the throat level. All dimensions in the R-Z plane are specified on the middle surface of the shell wall.

$$4R^2/d_T^2 - Z^2/b^2 = 1 \tag{6}$$

where b is a characteristic dimension of the shell that is evaluated for upper curve by

$$b = d_T Z_H / \sqrt{(d_H^2 - d_T^2)} \tag{7}$$

and for the lower curve by

$$b = d_T Z_U / \sqrt{(d_U^2 - d_T^2)} \tag{8}$$

The geometrical details and the values of b for both upper and lower curves of hyperbolic shell wall are presented in Table 3.

The shell wall is supported by 44 pairs of V-type columns having circular cross sections. And, they are placed equidistance and the adjacent top and bottom of the columns are connected. The shell has variable thickness and the transition is assumed to be linear as shown in Fig. 4.

Table 2 Material properties of concrete classes (ACI-318 2011)

Concrete Class	Elastic modulus(kPa)	Poisson's Ratio
C25	25170000	0.175
C35	29781000	0.175

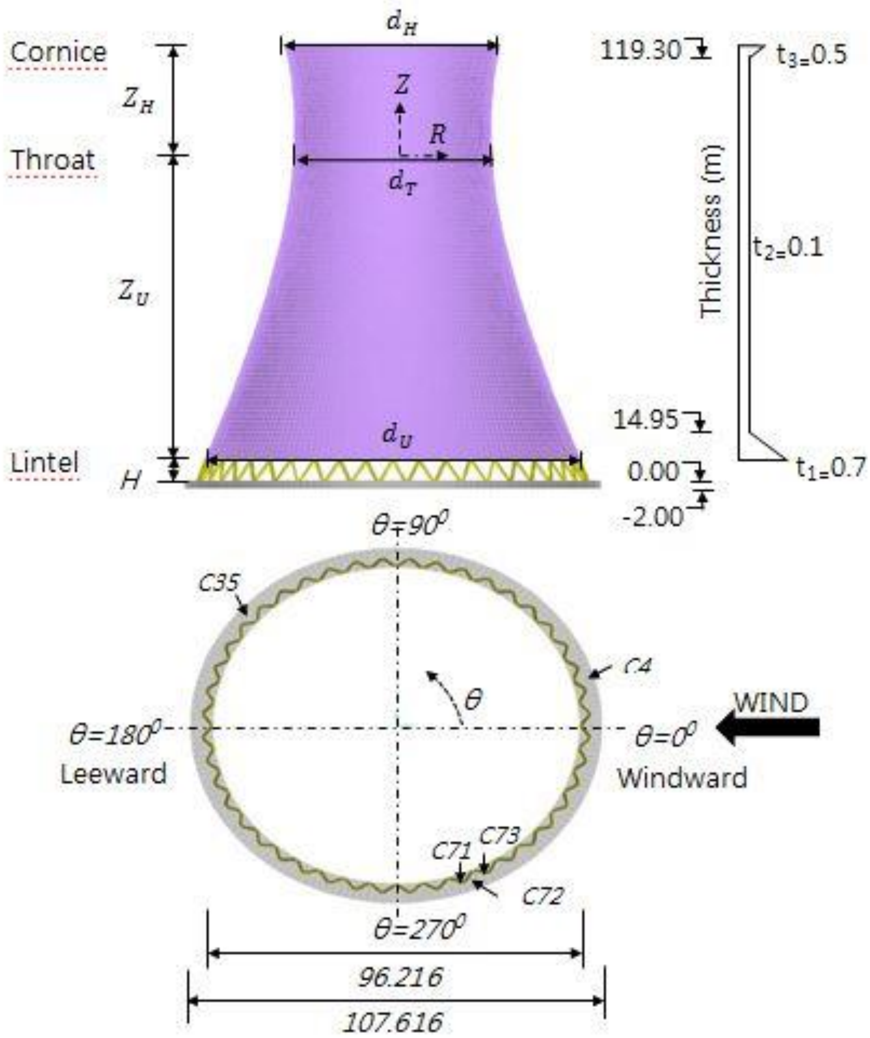

Fig. 4 Hyperbolic cooling tower

Table 3 Geometric details of hyperbolic cooling tower

Description	Symbol	Value (m)
Height above throat level	Z_H	24.090
Height below throat level	Z_U	91.260
Top diameter	d_H	55.070
Throat diameter	d_T	50.608
Shell base diameter	d_U	96.582
Characteristic dimension	b	56.143
Column diameter	-	0.7
Number of column pairs	-	44
Column height	H	6.95
Width of ring raft	-	5.7
Depth of ring raft	-	2

4.2 Loading

Cooling towers may be subjected to a variety of loading conditions such as dead load, wind load, seismic load, temperature variations, and settlement, etc. The behavior of the tower is investigated under the dead load and unsymmetrical wind pressure in this study. The dead load is computed as a body force by taking the unit weight of concrete as 24 kN/m^3. The wind load is treated as a quasi-static pressure load with a gusty wind effect. The external wind pressure acting at any point on the cooling tower shell is computed as (C 1977, Noorzaei *et al.* 2006)

$$p(z,\theta) = qH(z)C(\theta) \tag{9}$$

where $q = 0.0047\mathrm{v}_{10}^2$ kgf/m^2, $\mathrm{v}_{10} = 159.5$ km/h for a reference wind speed at 10 m above ground level, $H(\mathrm{z})$ is the vertical distribution of the design wind pressure profile at z above ground level given as

$$H(z) = \left[z/z_g\right]^{2\alpha} \tag{10}$$

in which α is the power law index and $\mathrm{z_g}$ is the gradient height. The values of $\alpha = 0.1$ and $\mathrm{z_g} = 10$ m are used in the analysis. $C(\theta)$ is the coefficient for circumferential distribution of external wind pressure as shown in Fig. 5. The circumferential distribution is unsymmetrical in nature and can be sufficiently represented by a Fourier sine-cosine series taking seven harmonics in the form of the following equation (Noorzaei *et al.* 2006).

$$C(\theta) = \sum_{n=0}^{7} a_n \cos(n\theta) + \sum_{n=1}^{7} b_n \sin(n\theta) \tag{11}$$

where θ is the horizontal angle measured from the windward meridian and the harmonic constants an and bn in the Fourier series expression in Eq. (11) are given in Table 4.

4.3 The analysis of the cooling tower on elastic foundation

Initially an iterative procedure is applied to determine the effective subsoil depth for a constant modulus of elasticity through the subsoil depth. Soil parameters are listed in Table 4 and variation of the settlement along the centerline of hollow annular circular plate is plotted in Fig. 6 for various subsoil depths. The gap between the curves closes as the subsoil depth increases as seen from Fig. 6. The difference in the settlement can be ignored for the higher values of subsoil depth after *H*=75 m. Therefore the subsoil depth is taken as 75 m for the case study here.

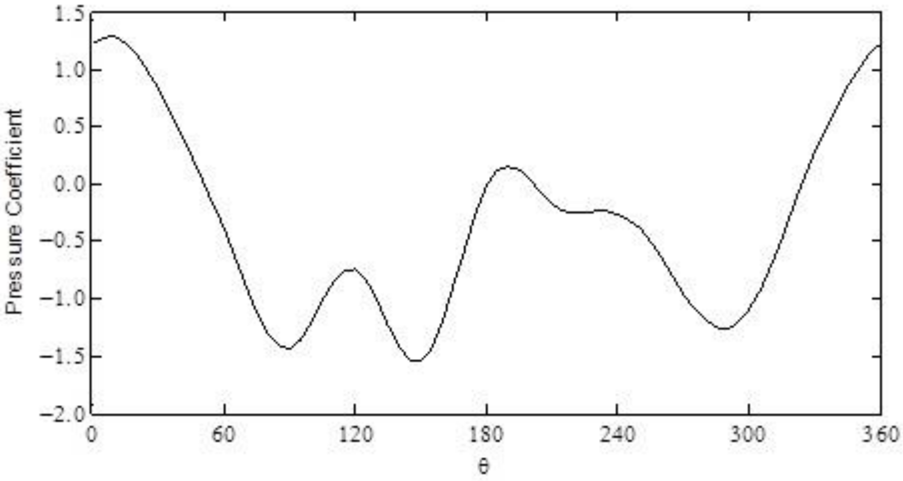

Fig. 5 Circumferential wind pressure coefficient C(θ) distribution

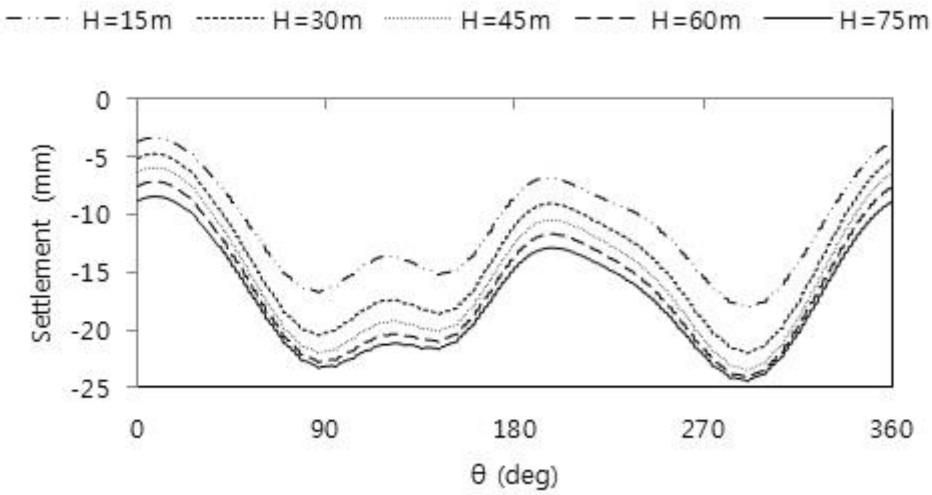

Fig. 6 Raft settlements for various subsoil depths

Table 4 Harmonic constants for circumferential wind distribution (Noorzaei *et al.* 2006)

n	a_n	b_n
0	-0.3562	-
1	0.5763	-0.1167
2	0.7537	0.4523
3	0.2865	-0.0578
4	0.0623	0.1074
5	-0.1995	-0.1336
6	0.1438	0.0285
7	-0.0385	0.0462

The cooling tower-raft foundation system has been assumed to be resting on a soil stratum made of sand. The settlements of the annular raft are calculated and plotted for constant and variable modulus of elasticity of the subsoil, Fig. 7. A constant value of modulus of elasticity for soil layer is considered first by taking E_1=E_2=47500 kN/m^2. Medium dense sand at the top and dense sand at the bottom is assumed next. Linear and quadratic variations of modulus of elasticity from top to bottom of soil stratum are considered by taking E_1=20000 kN/m^2 and E_2=75000 kN/m^2. The settlements of the raft in Fig. 1 show that the properties of the subsoil play an important role on the behavior of the annular raft foundation, and it is significant in the analysis of cooling tower-raft foundation system. The variation of the modulus of elasticity of the subsoil is assumed to increase linearly with depth for the case study here.

The response of cooling tower-column-raft system having fixed base at the bottom instead of soil layer has also been investigated to show the effect of elastic subsoil on the tower and supporting columns, and to compare the results.

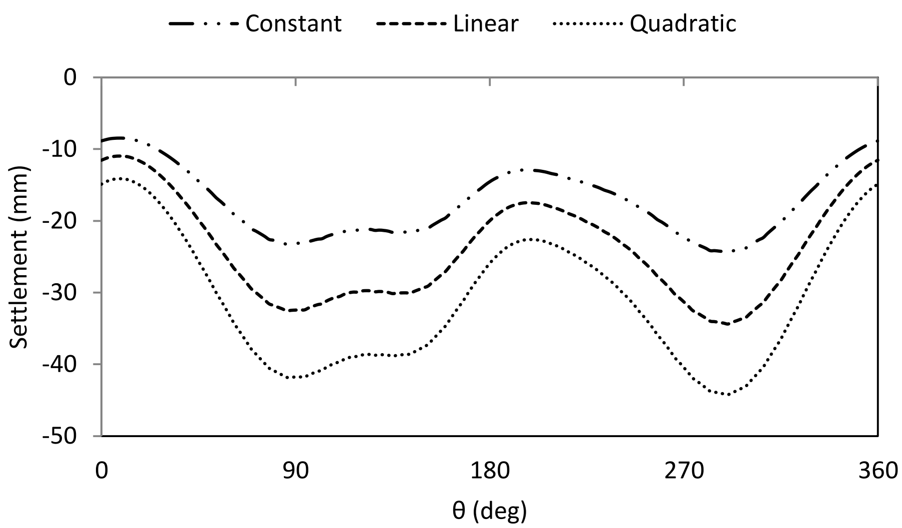

Fig. 7 Raft settlements for variable modulus of elasticity of subsoil with depth

The axial force, shear force, torsional and bending moments for the selected columns supporting the tower are depicted in Fig. 8. The maximum values of tension, compression, shear force, torsional and bending moments are observed in columns 4, 72, 71, 35, and 73, respectively, as locations are shown in Fig. 4 The positive value of axial force in Fig. 8(a) indicates that column 4 is in tension. Therefore the amount of reinforcement in the columns should be checked for maximum tensile forces. As far as boundary conditions are compared larger tensile forces and smaller compressive forces on columns are generated when soil-structure interaction is considered. Also Fig. 8(b)-8(d) display that the bending moments, torsions and shear forces for the structure resting on Vlasov foundation are greater than those obtained for the fixed supported case and the difference is considerably high.

The variation of the hoop and meridional membrane forces around the circumference of the cooling tower are plotted in Fig. 9 at cornice, throat and lintel. The remarkable increases are observed in the related internal forces when Vlasov foundation model is used except for the hoop force at throat and meridional force at throat and lintel. While the circumferential hoop force because of the wind load meridian is mostly tensile at cornice, the situation turns other way around through the lintel and becomes totally compressive after a certain height and at lintel.

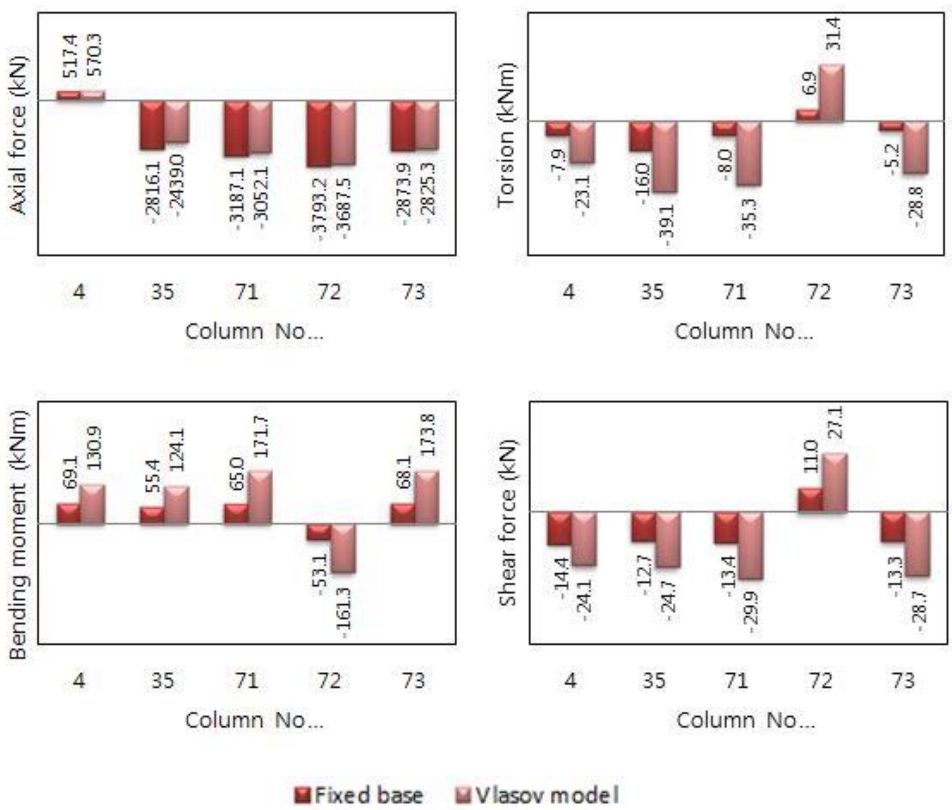

Fig. 8 Column forces (a) axial (b) torsion (c) bending moment (d) shear

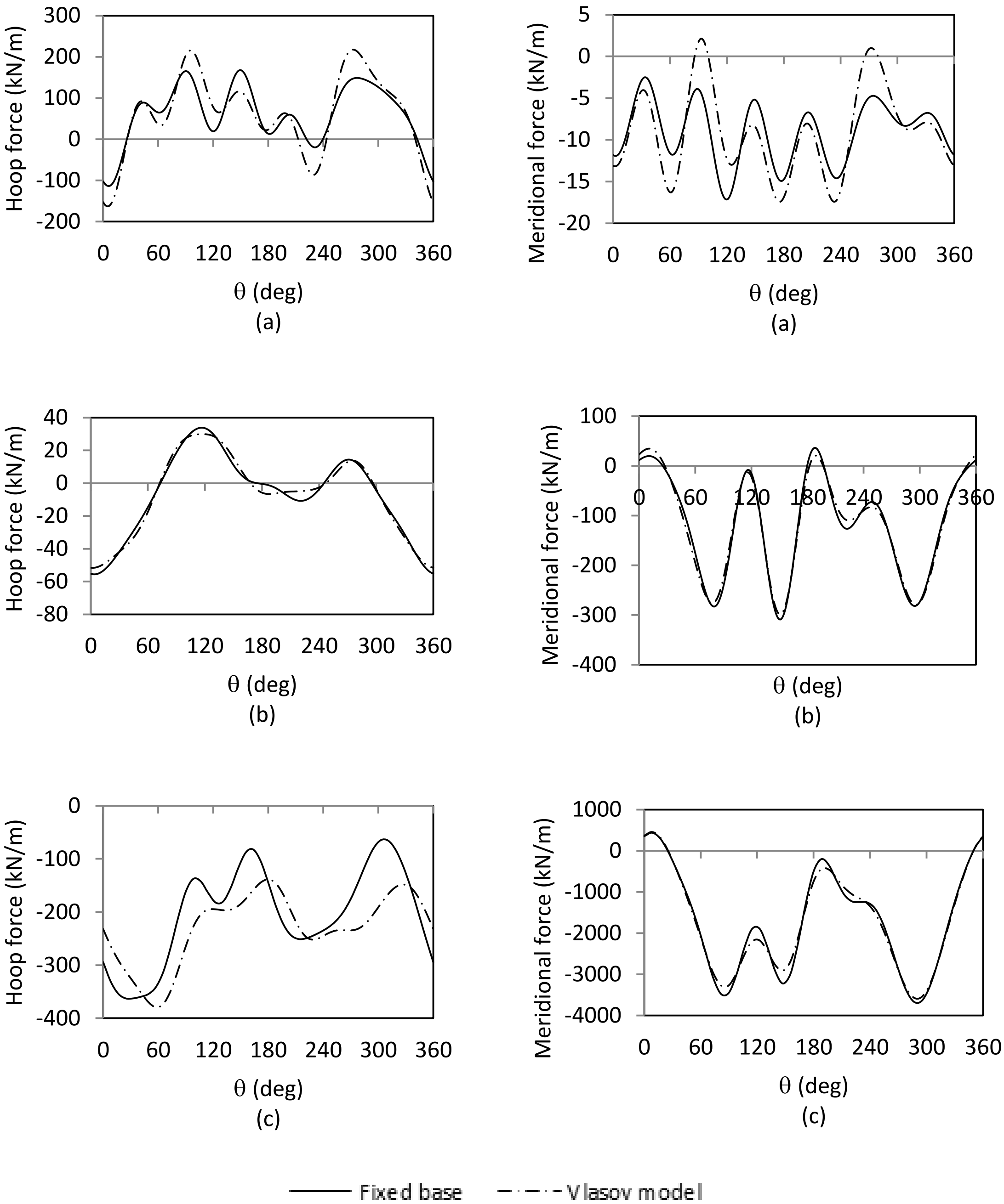

Fig. 9 Circumferential hoop and meridional forces at (a) cornice (b) throat (c) lintel

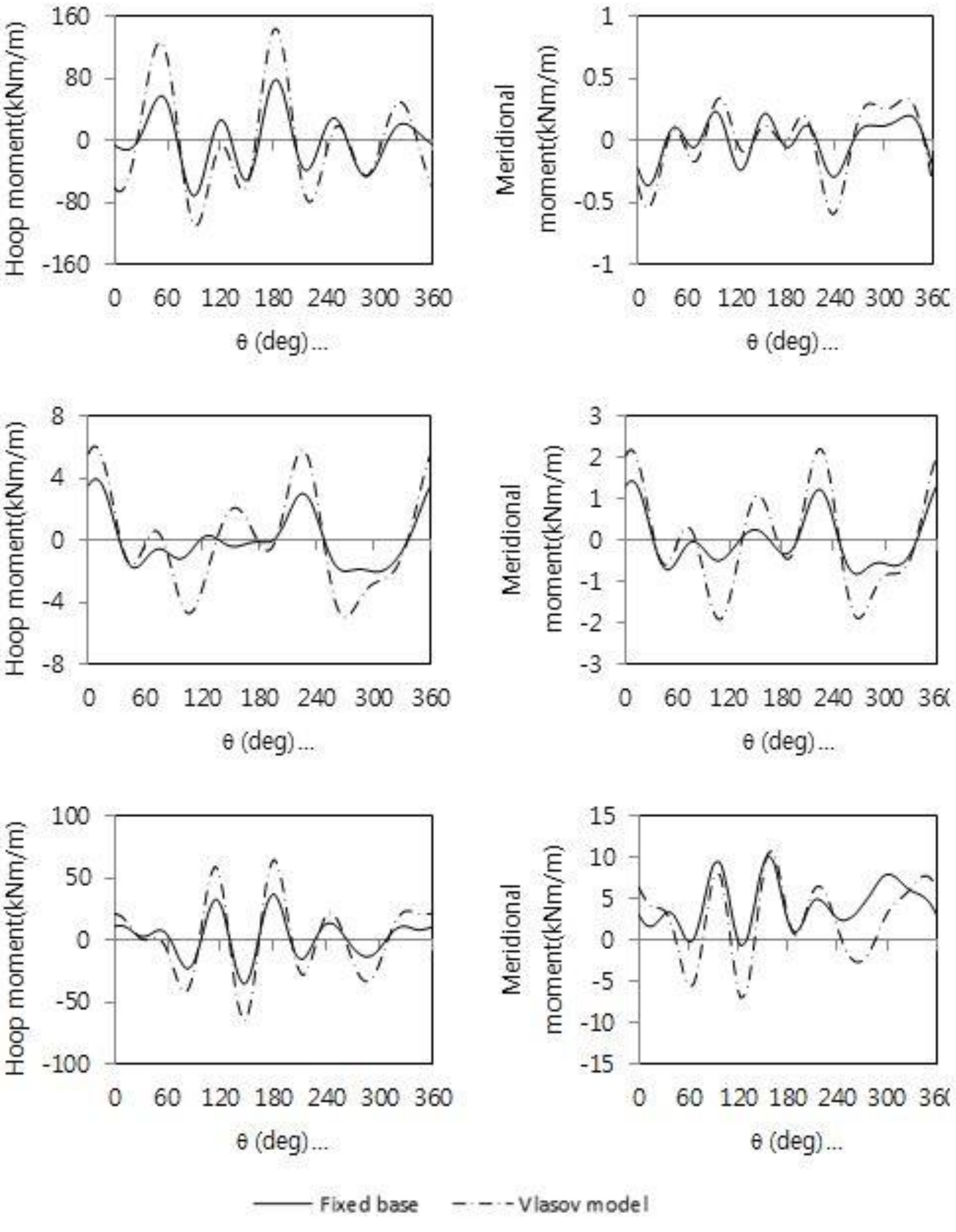

Fig. 10 Circumferential hoop and meridional moments at (a) cornice (b) throat (c) lintel

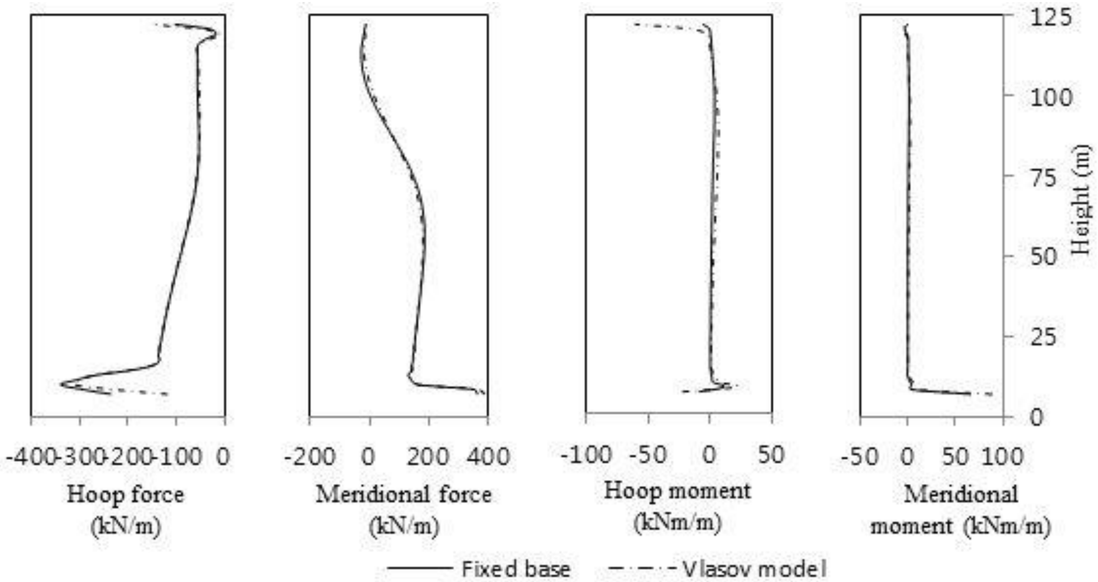

Fig. 11 Forces and moments along the height of the cooling tower at θ=0

The variation of the hoop and meridional moments around the circumference of the cooling tower are depicted in Fig. 10 at cornice, throat and lintel. Likewise there are a significant rise in the values of the hoop and meridional moments when soil-structure interaction is considered.

As can be seen from Fig. 11 the hoop and meridional forces and moments do not chance remarkably on the wall of the tower at θ=0^0 except at cornice and lintel. In other words, membrane forces and moments on the wall of the tower are not affected by the soil layer underneath.

The variation of radial and axial displacements along the height of the cooling tower with and without elastic foundation underneath are displayed in Fig. 12 at θ=0^0 meridian. Maximum radial displacement has occurred at the throat of the cooling tower having fixed support. The location of the maximum displacement moved down when the effect of subsoil taken into account. The maximum radial displacements are obtained as 4.37 cm at an elevation of 90.84 m for fixed supported tower and 9.65 cm at 78.85m for Vlasov model. Similarly, the axial displacements of both cases reflect influence of soil stratum considered in the analysis. The maximum axial displacements have been obtained as 1.15 cm and 4.48 cm, respectively.

The distribution of radial displacements over the cooling tower has also been demonstrated with elevation and plan views as given in Fig. 13. It can be easily seen that the circumferential distribution of radial displacements through the height of the cooling tower is similar to that of wind loading. The circumferential deformation of the cooling tower is shown in Fig. 13(e). The windward and leeward sides have deformed inwardly and lead to contraction of the cooling tower as seen in Figs. 13(a)-13(d). Additionally the elongation and contraction of the columns in Fig. 13(f) indicate the tension and compression regions of the tower.

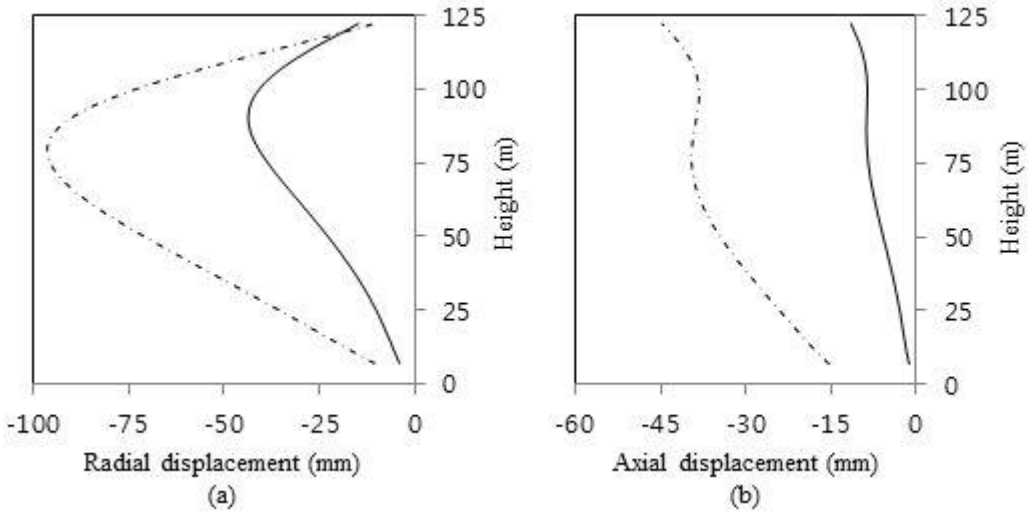

Fig. 12 (a) Radial and (b) Axial displacements along the height of the cooling tower at θ=0

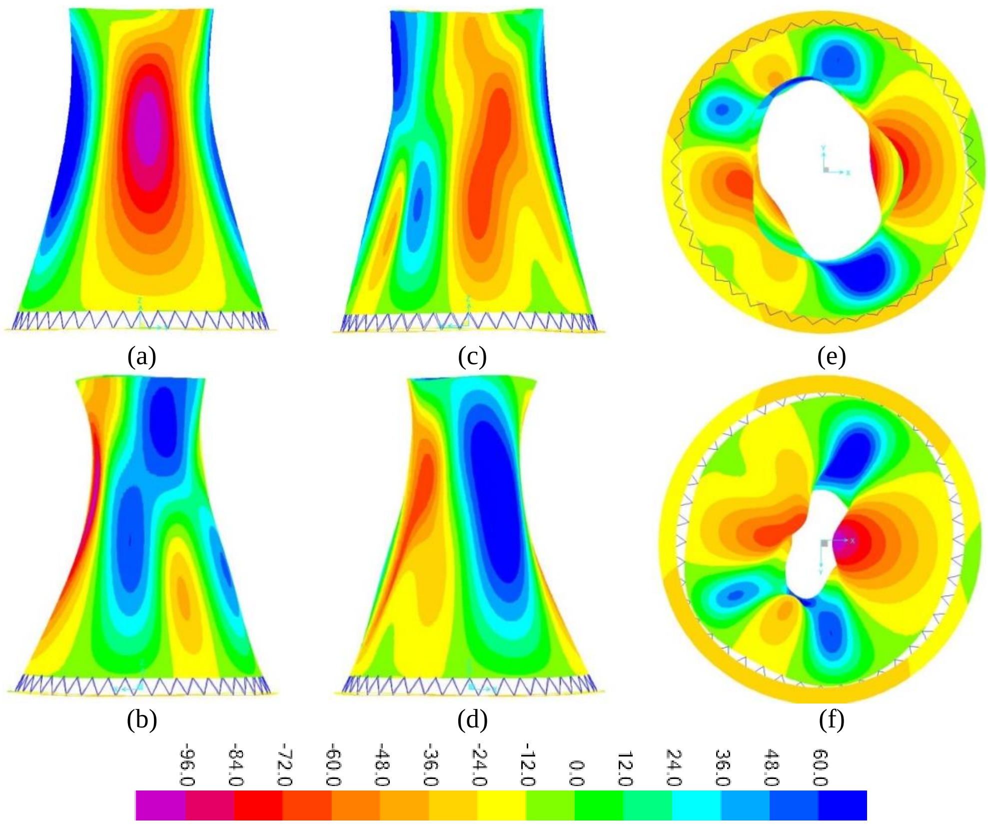

Fig. 13 Radial displacement (mm) views from (a) θ=0°, (b) θ=90°, (c) θ=180°, (d) 270°, (e) Top and (f) Bottom for Vlasov case

5. Conclusions

The Modified Vlasov foundation model is used for the analysis of hyperbolic cooling towers resting on elastic foundation subjected to dead and unsymmetrical wind loads. The behavior of the cooling tower is investigated under the effects of the subsoil depth, the variation of elasticity modulus of subsoil with depth, and the unsymmetrical wind load. A computer program is coded in MATLAB for the purpose to provide two way data flow between MATLAB and SAP2000 during execution using Open Application Programming Interface (OAPI). Soil parameters in modified Vlasov model are calculated as a function of vertical deformation profile within subsoil. An iterative parameter updating procedure is used for the analysis until the convergence is obtained. The program coded is verified by analyzing a circular hollow plate on elastic foundation taken from literature. A full-size hyperbolic cooling tower on elastic subsoil is studied and results are presented in graphical format. The following conclusions can be drawn from the study.

- Codes developed in any programming language are not for general purpose. Therefore, the codes require modifications for various types of structures and analysis methods. This leads to a serious loss of time and effort. However, the computing tool developed in the present study using MATLAB with OAPI feature is capable of carrying out static and dynamic analysis of any structure modeled via SAP2000 considering Vlasov elastic foundation.
- The complex realistic unsymmetrical wind pressure distribution is represented satisfactorily by the coded program as a distributed pressure load computed at 13728 points over the cooling tower using sufficient number of terms in Fourier sine-cosine series.
- The interactive behavior of cooling tower-column supports-raft-soil system leads to the redistribution of displacements, forces and moments.
- It is observed that the compressive forces in the supporting columns decrease while tensile forces increase when the interaction between the soil and the structure is considered. The values of the column bending moments, torsional moments and shear forces also increase because of the soil-structure interaction.
- Interestingly, the response of cooling tower wall is not affected much by the soil-structure interaction except at the column-shell wall connections and at cornice.

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