

## A Bayesian network based framework to evaluate reliability in wind turbines

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**Abstract.** The growing complexity of modern technological systems requires more flexible and powerful reliability analysis tools. Existing tools encounter a number of limitations including lack of modeling power to address components interactions for complex systems and lack of flexibility in handling component failure distribution. We propose a reliability modeling framework based on the Bayesian network (BN). It can combine historical data with expert judgment to treat data scarcity. The proposed methodology is applied to wind turbines reliability analysis. The observed result shows that a BN based reliability modeling is a powerful potential solution to modeling and analyzing various kinds of system components behaviors and interactions. Moreover, BN provides performing several inference approaches such as smoothing, filtering, what-if analysis, and sensitivity analysis for considering system.

**Keywords:** wind turbine; reliability; risk; Bayesian network

### 1. Introduction

Technological changes and components interactions increase the risk of failure and the vulnerability in a complex technological system. Wind turbines are examples of complex systems in which technical, human, organizational and environmental factors interact with each other and most major power grid blackouts that have occurred in the past were initiated by a single event (or multiple related events) may lead to catastrophic failure or even entire system collapse.

On one hand, wind turbines have an important role in providing electricity due to the increase in capacities and the number of grid-connected wind turbines (Guo *et al.* 2009). Furthermore, wind power penetration in power systems increases at a significant rate and it is estimated to be the fastest growing renewable energy resource (Global Wind Energy Council (GWEC), 2012). On the other hand, the profitability of wind farms are significantly affected by system availability and consequently its reliability determining operation and maintenance costs (O&M costs).

A low reliable wind turbine encounters high turbine failure rate leading to a high cost of energy

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(CoE) due to high O&M costs, as well as lost revenue from electricity sales. O&M costs typically represent around 10% of the total COE for onshore wind farms and 35% for offshore sites, of which around 70% is due to unplanned maintenance (Tavner *et al.* 2008, Walford 2006). The baseline O&M costs for large wind turbines are estimate around \$7/kW per year (McMillan and Ault 2007) which can be controlled through comprehensive understanding of system reliability. Long-term cost analysis, including O&M costs minimization efforts via reliability and therefore availability improvement is an effective means of optimizing wind turbines economic performance.

The main objective of reliability analysis is to provide input to a decision problem. An example can be to examine the environmental conditions effect on a component's time to failure, and use this as input to an O&M cost reduction program. The model developed to analyze reliability should cover the following attributes to provide an efficient support system for decision makers:

- dealing with uncertain or random fluctuations of data,
- mathematically soundness as well as easy to understand for the decision maker,
- ability to combine historical data and or expert judgment,
- required quantities can be calculated efficiently

All of aforementioned requirements result in shifting focus from traditional methods such as fault trees (FTs) to more flexible modeling methods such as Bayesian network (BN). BN originated in the field of artificial intelligence, has gained popularity for inference from uncertain knowledge over the last decade. The history of BNs in reliability refers to Barlow (1988) and Almond (1992). BNs have been compared to reliability block diagrams (Torres-Toledano and Sucar 1998, Solano-Soto and Sucar 2001) and fault-trees (Portinale and Bobbio 1999, Bobbio *et al.* 2001) in terms of modeling and analysis features and the results indicate significant advantages of BNs over traditional methods (Langseth and Portinale 2007, Lam and Yang 2015, Caspeele and Taerwe 2013). BNs can provide a very valuable framework for modeling and analyzing reliability in complex systems illustrating dependencies among components reliabilities. We discuss some of these advantages in detail by way of a real-world case study.

The reliability of wind turbines as with most complex systems is affected by various factors and their interactions during field operation and it is required to adopt an appropriate approach and modeling tool adapting complex systems attributes. In particular, we showed how a series/parallel diagram representing the structure of the wind turbine can be converted into the equivalent BN. The proposed BN model can compute predictive or diagnostic measures concerning the state of the general system, or the state of its components.

In this paper we introduce a reliability BN model of a wind turbine with environmental variables as well as series and parallel connections of technical parts. In particular, components in parallel have to be modeled through an AND node (since the overall parallel subsystem fails if all the components fail), while series components have to be modeled through an OR node (since the overall series subsystem fails if at least one of the components fails). In the proposed approach, we generalized the OR and AND nodes into the nodes representing the state of series or parallel modules.

This paper is organized as follows: Section 2 describes wind turbine structure and how it can be structured in terms of series and parallel modules. Section 3 introduces BN model and its features. Section 4 presents the BN model for the case study of modeling wind turbine reliability. Section 5 provides several kinds of inference procedures such as filtering, predicting, and what-if-analysis and presents the quantitative-qualitative results of implementing BN model for reliability analysis of wind turbine and discusses the results. Finally, Section 6 gives a conclusion on model and its

applications.

## 2. Wind turbine: Structure and series and paralel modlues

Rotor, nacelle, tower, gearbox, generator, pitch system, yaw system, and the foundation of the system are eight major components of a wind turbine. Kinetic energy extracted from the wind is converted to mechanical energy through the rotor and rotating blades. Generator converts mechanical energy to electricity. The enclosure at the top of the tower is called nacelle which is the main structure and contains major components like the turbine generator, gearbox, drive train, electronic control system and grid interface (Ribrant 2006 and Saeed 2008). The drive train is composed of different mechanical components such as bearings and shaft and its main mission is to interconnect the hub, the gearbox and the generator (Ribrant 2006 and Sterzing 2004). Wind turbine gearbox transforms low speed revolution to high speed revolution. It is connected to an electric generator through high speed shaft (Ribrant 2006) which transforms the rotational mechanical energy to electrical energy. The generator output is then connected to the electrical grid station for supply of electrical energy. The main mission of pitch and yaw systems is to adjust blades and stop turbine performance during inappropriate climate and storm. The whole structure is supported by a tower fastened to wind turbine foundation and raises nacelle to a height where maximum wind speeds can be extracted (Sterzing 2004 and Saeed 2008).

Wind turbine components illustrate series or parallel modules/subsystems. In a series module, if one component of the series fails, the wind turbine performance is interrupted. On the other hand, in a parallel module, the component, establish several parallel ways to maintain the system function; therefore if one component fails, the other elements can still be exploited to system mission. The aforementioned structure of the wind turbine, become more visible if they are modeled as the series/parallel diagram. Climate condition and wind loading may affect system reliability (Zhang and Li 2007, Cheng and Li 2009). We refer to the mission of pitch and yaw systems to identify series and parallel reliability structure for a wind turbine. Since the main function of pitch and yaw systems is to stop turbine during inappropriate climate and storm. So, pitch and yaw systems are in a serial structure with other components of a wind turbine at inappropriate climate condition (Arabian-Hoseynabadi Tavner and Oraee 2010) (Fig. 1).

In a normal climate condition, the pitch and yaw systems are functioned to optimize the system performance and adjust nacelle position. So, pitch and yaw systems are in a serial-parallel structure with other components of a wind turbine in normal climate condition (Arabian-Hoseynabadi *et al.* 2010) (Fig. 2). In other words, all other components of the wind turbine except pitch and yaw systems are in a serial structure. This series sub module is combined with pitch and yaw systems in a parallel structure.

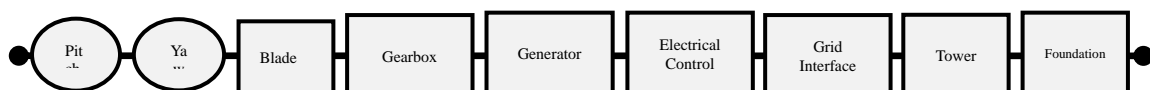


Fig. 1 Wind turbine serial structure in bad climate condition

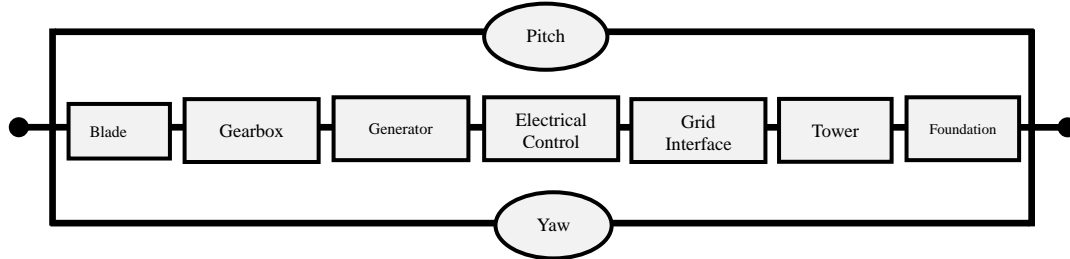


Fig. 2 Wind turbine parallel structure in normal climate condition

As shown in Figs. 1 and 2, the basic components are structured in series or parallel modules, in such a way to represent which configuration of components have to be working in order to determine the system functioning.

### 3. BN overview

BN is a directed acyclic graph (DAG) consisting of a set of nodes (variables) connected through a set of directed arcs (links). It is an instance of causal network that graphically encodes a multivariate statistical distribution function of a set of conditional independence statements into a compact concise formalism representing a joint probability distribution (Pearl 1988). BN is used for representing uncertain knowledge in probabilistic systems and have been applied to vast variety of real-world problems.

BN consists of the conditional probabilities between the parent and child variables captured through the directed links using rules of d-separation (Pearl 1988). Conditional probability table (CPT) of each child variable is defined in order to link condition states of the child to the parent variables. The set of conditional probabilities allows calculation of the joint probability distribution using as follows (Pearl 1988)

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{Parent } x_i) \quad (1)$$

The prior probability distribution of any can be calculated by marginalizing other variables out of the joint probability function in Eq. (1).

BN computation is based on the prior probabilities and CPTs. For instance, a basic BN consisting of two nodes  $X_1$  and  $X_2$  (each has two states) is illustrated in Fig. 3. The priori probabilities of node  $X_1$  as root node are defined as in Table 1. A CPT is associated to node  $X_2$  (Table 2) which defines the conditional probability distributions over the states of  $X_2$  given the states of  $X_1$ .



Fig. 3 Basic BN example

Table 1 Prior probabilities of the node  $X_1$ 

$X_1$	
$X_1=0$	$P(X_1=0)$
$X_1=1$	$P(X_1=1)$

Table 2 CPT of the node  $X_2$  given the node  $X_1$ 

$X_2 \backslash X_1$	$X_1=0$	$X_1=1$
$X_2=0$	$P(X_2=0   X_1=0)$	$P(X_2=0   X_1=1)$
$X_2=1$	$P(X_2=1   X_1=0)$	$P(X_2=1   X_1=1)$

BNs enable updating the model when new information regarding condition state of any variable becomes available, e.g., through observation.

We argue that BN is a suitable approach to both qualitative and quantitative modelling of risk and reliability in complex systems. Furthermore, BN provides not only a forward (or predictive) analysis but also a backward (diagnostic) analysis, in which the posterior probability of variables can be estimated. Although in most cases variables are defined discretely, continuous variables have been used in some studies. This paper is not limited to the use of discrete variables representing the nodes of the BN.

#### 4. BN model for wind turbine reliability

The BN for wind turbine reliability should capture the behavior of a series/parallel module composed by nine components of wind turbine and climate condition. The general BN for wind turbine reliability modeling consists of:

- the Time To Failure (TTF) variables of all aforementioned components and affecting parameters of each component TTF as nodes of BN model,
- each component reliability and the whole system reliability,
- climate condition of wind turbine environment,
- arcs illustrate interactions among nodes.

In BN model, the component's TTF is assumed as a random variable with a known probability density functions Weibull and exponential distribution for mechanical and electrical components respectively. The parameters of TTF probability density function are extracted from historical data.

In the BN model, continuous nodes are used to represent the TTF of the system's basic components. Nodes representing wind turbine mechanical components' TTF follow Weibull probability density function (Eq. (2)) with various scale and shape parameters. Weibull probability density function is a general failure function which allows modeling of different phases of the lifecycle of systems' components.

$$f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \quad (2)$$

Where  $\alpha$  and  $\beta$  are the scale and shape parameter of Weibull probability density function, respectively. The parameters of failure functions are modeled as discrete nodes in BN. The prior values of aforementioned parameters as the parent nodes of components TTF are extracted from historical data and experts' knowledge. Furthermore, they may vary for various components by time transition and system evolution. For wind turbine case, the value of parameter  $\beta$  is estimated considering prior knowledge about system and the scale parameter is extracted via applying a Maximum Likelihood Estimation (MLE) method (Eq. (3))

$$\hat{\alpha} = \left[ \sum_{i=1}^N \frac{t_i^{\beta}}{r} \right]^{\frac{1}{\beta}} \quad (3)$$

Where  $t_i$  is time to failure of component  $i$  and  $r$  is the number of failed or suspend components. Tables 3 and 4 show the CPTs of TTF and its parameters prior values for a wind turbine gearbox as an instance of mechanical components.

The TTF of nodes representing basic electrical components of wind turbine such as generator is modeled via exponential probability density function. Table 5 shows the CPT of wind turbine generator as an instance of electrical components.

In particular, each component reliability is represented in form of a binary variable expressing the working (value1) or the failure state (value 0). Then, the reliability of the component is estimated as a measure considering component working without failure in the time interval  $[0, t]$

$$\text{Reliability}(t) = 1 - \int_0^t f(\tau_i) d\tau_i \quad (4)$$

Where  $\tau_i$  is TTF of component ( $i$ ) and  $\int_0^t f(\tau_i) d\tau_i$  is its cumulative density function in the time interval  $[0, t]$ . So to determine the CPTs of components reliability a binary variable is defined which is true ("On") when the TTF of underlying component is greater than a given time (e.g., 30000 hours), otherwise its status is false ("Fail").

Table 3 Priori probabilities of the shape and scale parameters of gearbox TTF

Parameter	Initial value
shape parameter of gearbox TTF( $\alpha$ )	1
scale parameter of gearbox TTF( $\beta$ )	42000

Table 4 CPT of the node gearbox TTF

TTF Distribution	Weibull probability density function
( $\alpha$ )	1
( $\beta$ )	42000

Table 5 CPT of the node generator TTF

TTF Distribution	exponential probability density function
Rate	0.0000116

According to wind turbine structure, the BN model must contain both *series* and *parallel* modules. *Series module* models wind turbine behavior while all nine main components show a serial configuration, as shown in Fig. 1. *Parallel module* models wind turbine reliability while wind turbine operates as a parallel module in normal condition climate, as shown in Fig. 2. One the advantages of BN models is their capability of capturing these two kinds of modules in a single model. In other words, both modules are identical in BN as a graphical model and can be distinguished according to the expressions of conditional probabilities of reliability estimation. BN model also considers the dependencies among system components. The CPT of wind turbine reliability based on components serial/ parallel configuration is determined through IF, AND and OR logical gates based on and climate condition and components dependencies. The logic of reliability modeling for a series module is based on the fact that the wind turbine fails when each input component fails. So, the only state in which the series module works /conditional probabilities are true is

$$P(WT = 1 | PS = 1, YS = 1, Be = 1, Gx = 1, Gr = 1, EC = 1, GI = 1, Tr = 1, Fn = 1) = 1 \quad (5)$$

Where WT, PS, YS, Be, Gx, Gr, EC, GI, Tr, Fn respectively represent wind turbine, pitch system, yaw system, blades, gearbox, generator, electrical control system, tower, and foundation. Eq. (5) yields

$$\begin{aligned} P(WT = 1) &= P(PS = 1)P(YS = 1)P(Be = 1)P(Gx = 1)P(Gr = 1)P(EC = 1)P(GI = 1)P(Tr = 1)P(Fn = 1) \\ &= \mathcal{R}_{PS}(t)\mathcal{R}_{YS}(t)\mathcal{R}_{Be}(t)\mathcal{R}_{Gx}(t)\mathcal{R}_{Gr}(t)\mathcal{R}_{EC}(t)\mathcal{R}_{GI}(t)\mathcal{R}_{Tr}(t)\mathcal{R}_{Fn}(t) \end{aligned} \quad (6)$$

where  $\mathcal{R}_x(t)$  represents the reliability of each component of a wind turbine. In normal climate condition, the main function of pitch and yaw systems are optimizing the performance of wind turbine and adjusting nacelle position respectively. So, pitch and yaw systems are in parallel structure with other components of a wind turbine and the whole system shows a serial- parallel configuration. In other words, all other mentioned components of the wind turbine except pitch and yaw systems compose a series module. So, the overall reliability of other components (OC) except pitch and yaw systems is calculated such a series module as shown above and the failure probability of wind turbine as a combination of series-parallel module is expressed as follows

$$P(WT = 0) = P(PS = 0)P(OC = 0)P(YS = 0) = (1 - \mathcal{R}_{PS}(t))(1 - \mathcal{R}_{OC}(t))(1 - \mathcal{R}_{YS}(t)) \quad (7)$$

The CPT of wind turbine reliability is determined using “if” gate based on the climate node status. If the climate node shows inappropriate condition, all components operate as a series modules and the wind turbine reliability is “On” when all components are in “On” state. In normal condition Fig. 4 illustrates the BN model developed for wind turbine reliability. The commercially available software ‘AgenaRisk’ is used for the development and performing calculations related to this BN (AgenaRisk 2014).

After determining model CPTs using existing historical data, expert judgments and series-parallel modules rules, the model can be applied to perform inference about components and wind turbine reliabilities.

## 5. Results and discussion

As mentioned earlier, the main objective of reliability modeling is to calculate quantities meaningful for decision makers contributing to system targets such as O&M cost minimization. Therefore a comprehensive inference and understanding of the model is required.

The constructed BN model for wind turbine was verified through simple examinations using arbitrary extreme conditions of components. For example, when all components are failed, the wind turbine should be expected to be in the same state. In this section, we compute several predictive and diagnostic inferences using the BN model depicted in Fig. 4 and representing the behavior of the wind turbine.

### 5.1 Sensitivity analysis

A sensitivity analysis is carried on the BN model to assess the impact of the reliability of each component on the wind turbine condition. Using the BN model shown in Fig.4 as the basic framework, we assign a probability of '1' for being in the 'failed' state for each component, in turn, keeping the remaining components in their existing state and the resulting changes in the wind turbine condition are calculated.

The sensitivity of wind turbine condition on each of its components is computed by normalizing the change calculated individually in the wind turbine over the total change and is illustrated in Fig. 5.

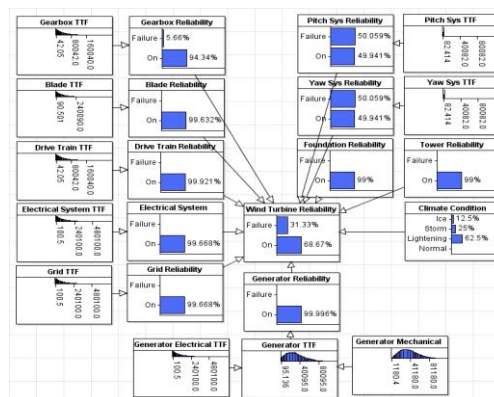


Fig. 4 BN model for wind turbine reliability

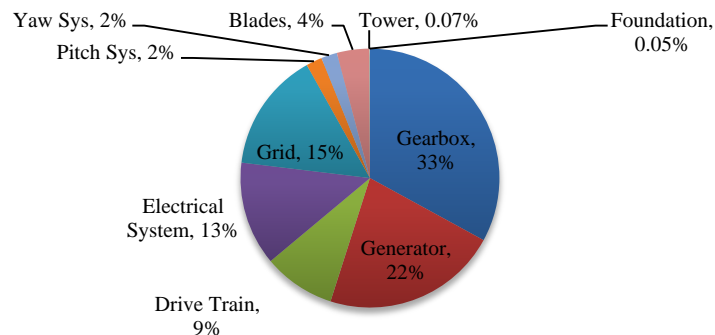


Fig. 5 Sensitivity of a given wind turbine reliability on its components



## 5.2 Filtering

The filtering experiment on the BN is performed by predicting the probability that a component will work without failure for a mission time ‘t’ such as more than one year in a given environment and predicting the expected lifespan of a component according to the observations. To achieve this objective, we perform inference process on the variables of the BN model representing such a component at the mission time 10000 and 20000 hours for gearbox as a critical component of wind turbine. The results of the inference, as returned by the AgenaRisk tool, are reported in Table 7. It can be observed from obtained results that the probabilities of outage of the component gradually grow at 10000 and 20000 hours.

## 5.3 Smoothing

Smoothing as a kind of inference on BN model is useful for diagnosing the possible causes of the wind turbine failure. Assuming further observations of the wind turbine failure at times 800, 900, 1000h, we perform a smoothing experiment on the BN with the aim of computing the probabilities of the states of the selected components of the system.

The results are reported in Table 8. The variation in ratio of probabilities of failure for selected components can be considered that in several situations: from time 20000h to time 30000h, the reliability of generator is changed from 66% to 43% which shows a larger ratio of variation in comparison with reliability changes from time 10000h to 20000h.

## 5.4 ‘What-If’ analysis

The BN model can be used to analyze ‘what-if’ scenarios. For example, when new evidence regarding a component or climate condition becomes available, the expected change in the wind turbine condition can be traced. The performed what-if analysis in this research shows the result from assuming a significant improvement targeted exclusively on gearbox reliability due to applying a better maintenance strategy. Given the relatively criticality of gearbox, it can be observed from that an improvement in gearbox reliability from 77% to 91% affect the wind turbine reliability (from 67 % to 72%).

Furthermore, the real power of the BN model as a decision-support tool is in enabling backward ‘what if’ analysis, which can be used as a tool to analyze maintenance plans cost and benefit trade off given a desired wind turbine reliability.

Table 7 the results of predicting the probability of gearbox at 10000 and 20000 hours

Mission time	10000 h	20000 h
Reliability	77.111%	63.173%

Table 8 the results of smoothing process for wind turbine at 10000, 20000 and 30000 hours

Mission time	10000 h	20000 h	30000 h
Gearbox Reliability	77%	63%	48%
Generator Reliability	81%	66%	43%

## 5. Conclusions

The growing complexity of modern technological systems demands increasingly powerful tools providing accurate, flexible, and computationally feasible solution techniques. In order to address these issues, we have considered the applicability and advantages of BNs for reliability analysis. BNs are accompanied with an efficient calculation scheme, which makes them preferable to traditional methods like FTs. They provide a modeling framework particularly easy to use for interaction with both domain experts and historical data which makes it a useful tool in practice. Correlations between components failures are incorporated in the constructed BN.

The BN model is developed for reliability analysis of wind turbines considering their series and parallel modules. The BN model was then solved to compute system reliability. Several approaches consisting both forward and backward propagation such as smoothing, filtering, what-if analysis, and sensitivity analysis are applied to perform inference process on wind turbine condition.

The numerical result shows that the BN makes it applicable to the reliability assessment of complex structures, especially when new data on structural performance becomes available. Furthermore, BN enables a representation of the system and its components evolution in a more tangible way with respect to the abstract statistical models.

The inclusion of time slices into the model to construct a dynamic BN and trace system transition over time is an avenue for future work.

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