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Effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory

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Abstract. In this paper, a shear deformation plate theory based on neutral surface position is developed for free vibration analysis of functionally graded material (FGM) plates. The material properties of the FGM plates are assumed to vary through the thickness of the plate by a simple power-law distribution in terms of the volume fractions of the constituents. During manufacture, defects such as porosities can appear. It is therefore necessary to consider the vibration behavior of FG plates having porosities in this investigation. The proposed theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The neutral surface position for a functionally graded plate which its material properties vary in the thickness direction is determined. The equation of motion for FG rectangular plates is obtained through Hamilton's principle. The closed form solutions are obtained by using Navier technique, and then fundamental frequencies are found by solving the results of eigenvalue problems. Numerical results are presented and the influences of the volume fraction index and porosity volume fraction on frequencies of FGM plates are clearly discussed.

Keywords: functionally graded material; porosities; free vibration; plate theory; neutral surface position

1. Introduction

Functionally graded materials (FGMs) are a new kind of materials exhibiting spatially continuous variation of material properties along one, two or three directions in a particular coordinate system. Since material interfaces are absent, the interfacial stress concentration phenomenon due to material mismatch as encountered in the conventional composite laminates or coated structures can be completely avoided. Primarily, FGMs were mainly developed as heat-resisting materials used in aerospace engineering (Liu *et al*. 2010, Houari *et al*. 2013, Fekrar

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et al. 2014, Bouchafa *et al*. 2015, Akbaş 2015, Kar and Panda 2015, Ait Atmane *et al*. 2015). Furthermore, FGMs have been widely used in many engineering applications, such as spacecraft industry, thermoelectric industry, power industry, human plants, civil engineering and so on (Miamoto *et al*. 1999, Lu *et al*. 2009, Bousahla *et al*. 2014, Liang *et al*. 2014, Mansouri and Shariyat 2014, Yaghoobi *et al*. 2014, Liang *et al*. 2015a,b, Hamidi *et al*. 2015, Arefi 2015, Larbi Chaht *et al*. 2015, Pradhan and Chakraverty 2015, Sallai *et al.* 2015, Tagrara *et al*. 2015, Bennai *et al*. 2015, Ebrahimi and Dashti 2015, Sofiyev and Kuruoglu 2015, Darılmaz 2015, Kirkland and Uy 2015, Cunedioglu 2015, Ebrahimi and Habibi 2016, Moradi-Dastjerdi 2016, Hadji *et al*. 2016). FGMs are now developed for the general use as structural components in high temperature environments, and consequently many studies on vibration characteristics of FGM plates are available in the literature, see, for example (Praveen and Reddy 1998, Yang and Shen 2001, 2002, Vel and Batra 2004).

Lin and Tseng (1998) analyzed free vibration of the polar orthotropic laminated circular and annular plates using the first-order shear-deformation theory and an eight node element. Liew and Yang (2000) and Hosseini Hashemi *et al*. (2008) employed Ritz method for three- dimensional free vibration analysis of thick annular plates with different edge conditions. Sundararajan *et al*. (2005) investigated the nonlinear free flexural vibrations of functionally graded rectangular and skew plates in thermal environments. Temperature is assumed to vary only in the thickness direction. The material properties of constituents are considered to be temperature dependent. Lee *et al*. (1998) analyzed free vibration and transient dynamic response of a rotating multi-layer annular plate using the finite element method. The governing equations of motion were derived using a zigzag theory with a higher-order shear deformation global and a linear local displacement fields. Li *et al*. (2009) analyzed free vibration of FGM rectangular plates in thermal environment based on three dimensional theory of elasticity. Both simply supported and clamped boundary conditions are considered. Results are obtained for different temperature distributions. Huang and Shen (2004) studied nonlinear vibration and dynamic response of FGM plates in thermal environments. The formulations are based on the high order shear deformation theory (HSDT) kinematics and general von-Karman type equation, which includes thermal effects.

Allahverdizadeh *et al*. (2008) developed a semi-analytical approach for nonlinear free and forced axi-symmetric vibrations of a thin circular FGM plates. The formulation is based on the classical plate theory (CPT) kinematics and the geometric nonlinearity is incorporated in von-Karman sense. Woo *et al*. (2006) provided an analytical solution for the nonlinear free vibration behavior of FGM plates. The governing equations for thin rectangular FGM plates are obtained using the von-Karman theory for large transverse deflection, and mixed Fourier series analysis is used to get the solution.

Shufrin and Eisenberger (2005) used Kantorovich method for stability and vibration analysis of plates. Based on the first and higher order shear deformation plate theory, they obtained the vibration and stability equations for an isotropic plate and proposed a method to obtain the frequencies and critical buckling load. Comparing the results of theories, they said that the first order theory is in fairly good agreement with higher order theory.

Based on the Reddy's third-order shear deformation plate theory, exact closed-form solutions in explicit forms are presented by Hosseini-Hashemi *et al*. (2011a) for transverse vibration analysis of rectangular thick plates having two opposite edges hard simply supported. Also, Baferani *et al.* (2011) presented an accurate solution for free vibration of functionally graded thick rectangular plates resting on elastic foundation. Dehghan and Baradaran (2011) solved the eigenvalue equations based on a mixed finite element (FE) and differential quadrature (DQ) method to obtain the natural frequency and buckling load parameters. Bessaim *et al*. (2013) proposed a new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for bending and vibration of FG plates. Hebali *et al*. (2014) proposed a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of FG plates. Mahi *et al.* (2015) developed a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Nguyen *et al.* (2015) studied the bending, vibration and buckling behavior of FG sandwich plates using a refined higher-order shear deformation theory. Bennoun *et al*. (2016) developed a novel five variable refined plate theory for vibration analysis of FG sandwich plates.

Amini *et al*. (2009) investigated the free vibration of FGM plates on elastic foundations by using three dimensional linear elasticity theory. Ait Atmane Hassen *et al*. (2010) also analysed the Free vibration of simply supported functionally graded plates (FGP) resting on a Winkler–Pasternak elastic foundation using a new higher shear deformation theory. Using a new four-variable refined plate theory, Hadji *et al*. (2011) investigated the free vibration analysis of functionally graded material (FGM) sandwich rectangular plates. The theory presented is variationally consistent and strongly similar to the classical plate theory in many aspects. It does not require the shear correction factor, and gives rise to the transverse shear stress variation so that the transverse shear stresses vary parabolically across the thickness to satisfy free surface conditions for the shear stress. Benachour *et al*. (2011) developed a model for free vibration analysis of plates made of functionally graded materials with an arbitrary gradient. Closed form solutions are obtained by using Navier technique, and then fundamental frequencies are found by solving the results of eigenvalue problems. El Meiche *et al*. (2011) developed a refined hyperbolic shear deformable plate theory for buckling and vibration of FGM sandwich plates. Zidi *et al.* (2014) studied the bending response of functionally graded material (FGM) plate resting on elastic foundation and subjected to hygro-thermo-mechanical loading. Tounsi *et al*. (2013) presented a refined trigonometric shear deformable plate theory for thermoelastic bending of FGM sandwich plates. The same theory was used to study the mechanical behavior of FG plates (Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Kettaf *et al.* 2013, Khalfi *et al.* 2014, Bakhti *et al.* 2013, Bouderba *et al.* 2013, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Nedri *et al.* 2014, Sadoune *et al.* 2014, Attia *et al.* 2015). Bouguenina *et al.* (2015) presented a numerical analysis of FG plates with variable thickness subjected to thermal buckling. Belkorissat et *al.* (2015) studied the vibration properties of FG nano-plate using a new nonlocal refined four variable model. Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling behavior of thick P-FG plates resting on elastic foundations. Tebboune et *al.* (2015) presented a thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Zemri *et al.* (2015) studied the mechanical response of FG nanoscale beam using a refined nonlocal shear deformation theory beam theory.

Since, the material properties of functionally graded plate vary through the thickness direction, the neutral surface of such plate may not coincide with its geometric middle surface. Therefore, stretching and bending deformations of FGM plate are coupled. Some researchers (Morimoto *et al*. 2006, Abrate 2008, Zhang and Zhou 2008, Saidi and Jomehzadeh 2009, Yahoobi and Feraidoon 2010, Bachir Bouiadjra *et al*. 2013, Eltaher *et al.* 2013, Bourada *et al*. 2015, Meradjah *et al*. 2015, Hadji and Adda Bedia 2015, Al-Basyouni *et al.* 2015, Meksi *et al*. 2015, Ait Atmane *et al*. 2016, Bellifa *et al*. 2016) have shown that there is no stretching-bending coupling in constitutive equations if the reference surface is properly selected.

In FGM fabrication, porosities can occur within the materials during the process of sintering. This is because of the large difference in solidification temperatures between material constituents (Zhu *et al*. 2001). Wattanasakulpong *et al*. (2012) gives the discussion on porosities happening inside FGM samples fabricated by a multi-step sequential infiltration technique. Wattanasakulponga and Ungbhakorn (2014) also investigate linear and non linear vibration problems of FGM beams having porosities. However in the open literature, it has not found any works on the behaviour of FGM plates with porosity. Recently, Ait Yahia et *al*. (2015) investigated the wave propagation in FG plates with considering the porosity effect.

The purpose of this paper is to obtain the analytical solution for free vibration of FGM plate with porosities. The analysis is based on hyperbolic shear deformation theory and the exact position of neutral surface. The present theory has only four unknowns and four governing equations, but it satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factors. The displacement field of the proposed theory is chosen based on a constant transverse displacement and hyperbolic variation of in-plane displacements through the thickness. The partition of the transverse displacement into the bending and shear parts leads to a reduction in the number of unknowns and governing equations, hence makes the theory simple to use. Also, the effect of porosities that can happen inside the FGM fabricated by multi-step sequential infiltration technique is taken into account. Numerical results for fundamental frequencies are investigated.

2. Problem formulation

Consider a rectangular plate made of FGMs of thickness h, length a, and width b made by mixing two distinct materials (metal and ceramic) is studied here. The coordinates x, y are along the in-plane directions and z is along the thickness direction. The top surface material is ceramic rich and the bottom surface material is metal rich. For such plates, the neutral surface may not coincide with its geometric mid-surface. The applied compressive force may be assumed to act at the mid-surface of the plate for all the practical purposes, but the in-plane stress resultants act along the neutral surface. The noncoincidence of line of action of stress resultant and applied compressive force results in a couple as schematically shown in Fig. 1.

Here, two different datum planes are considered for the measurement of z, namely, z_{ms} and z_{ns} measured from the middle surface, and the neutral surface of the plate, respectively (Fig. 1).The volume-fraction of ceramic V_C is expressed based on z_{ms} and z_{ns} coordinates (Fig. 1) as

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2} \right)^p = \left(\frac{z_{ns} + C}{h} + \frac{1}{2} \right)^p \tag{1}$$

Where p is the power law index which takes the value greater or equal to zero and C is the distance of neutral surface from the mid-surface. Material non-homogeneous properties of a functionally graded material plate may be obtained by means of the Voigt rule of mixture (Suresh and Mortensen 1998). Thus, using Eq. (1), all properties of the imperfect FGM can be written as (Wattanasakulponga and Ungbhakorn 2014).

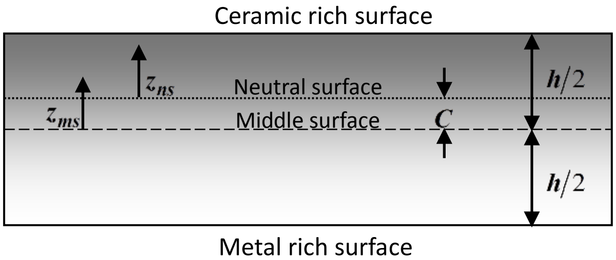


Fig. 1 The position of middle surface and neutral surface for a functionally graded plate

$$P(z) = (P_C - P_M)\left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^p + P_M - (P_C + P_M)\frac{\alpha}{2} \tag{2}$$

where $\boldsymbol{P_M}$ and $\boldsymbol{P_C}$ are the corresponding properties of the metal and ceramic, respectively. In the present work, we assume that the modulus of elasticity $\boldsymbol{E}$, and material density ρ, are described by Eq. (2), while Poisson's ratio $\boldsymbol{\nu}$, is considered to be constant across the thickness. The material properties of a perfect FG plate can be obtained when $\boldsymbol{\alpha}$ is set to zero.

The position of the neutral surface of the FG plate is determined to satisfy the first moment with respect to Young's modulus being zero as follows (Zhang and Zhou 2008, Bachir Bouiadjra *et al*. 2013, Bourada *et al*. 2015).

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms} - C)\,dz_{ms} = 0 \tag{3}$$

Consequently, the position of neutral surface can be obtained as

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms})\, z_{ms}\, dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms})\, dz_{ms}} \tag{4}$$

2.1 Theoretical model

Unlike the conventional shear deformation theory, the theory presented is variationally consistent. It has only four unknowns and four governing equations, but it satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factors.

2.2.1 Basic assumptions

Assumptions of the present theory are as follows

(i) The origin of the Cartesian coordinate system is taken at the neutral surface of the FGM plate.

(ii) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(iii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x, y only

$$w(x, y, z_{ns}) = w_b(x, y) + w_s(x, y) \tag{5}$$

(iv) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

(v) The displacements u in *x*-direction and v in *y*-direction consist of extension, bending, and shear components.

$$u = u_0 + u_b + u_s\,, \qquad v = v_0 + v_b + v_s \tag{6}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b = -z_{ns}\frac{\partial w_b}{\partial x}\,, \qquad v_b = -z_{ns}\frac{\partial w_b}{\partial y} \tag{7}$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses τ_{xz}, τ_{yz} through the thickness of the plate in such a way that shear stresses τ_{xz}, τ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s = -f(z_{ns})\frac{\partial w_s}{\partial x}\,, \qquad v_s = -f(z_{ns})\frac{\partial w_s}{\partial y} \tag{8}$$

Where

$$f(z_{ns}) = \frac{(h/\pi)\sinh\left(\frac{\pi}{h}(z_{ns}+C)\right) - (z_{ns}+C)}{[\cosh(\pi/2)-1]} \tag{9}$$

2.1.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (5)-(9) as

$$u(x, y, z_{ns}, t) = u_0(x, y, t) - z_{ns}\frac{\partial w_b}{\partial x} - f(z_{ns})\frac{\partial w_s}{\partial x}$$

$$v(x,y,z_{ns},t) = v_0(x,y,t) - z_{ns}\frac{\partial w_b}{\partial y} - f(z_{ns})\frac{\partial w_s}{\partial y}$$
$$w(x,y,z_{ns},t) = w_b(x,y,t) + w_s(x,y,t) \tag{10}$$

The kinematic relations can be obtained as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z_{ns}\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z_{ns})\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} \quad ; \quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z_{ns})\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{11}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}\boldsymbol{k}_x^b\\ \boldsymbol{k}_y^b\\ \boldsymbol{k}_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 \boldsymbol{w}_b}{\partial \boldsymbol{x}^2}\\ -\dfrac{\partial^2 \boldsymbol{w}_b}{\partial \boldsymbol{y}^2}\\ -2\dfrac{\partial^2 \boldsymbol{w}_b}{\partial \boldsymbol{x}\partial \boldsymbol{y}}\end{Bmatrix}, \quad \begin{Bmatrix}\boldsymbol{k}_x^s\\ \boldsymbol{k}_y^s\\ \boldsymbol{k}_{xy}^s\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 \boldsymbol{w}_s}{\partial \boldsymbol{x}^2}\\ -\dfrac{\partial^2 \boldsymbol{w}_s}{\partial \boldsymbol{y}^2}\\ -2\dfrac{\partial^2 \boldsymbol{w}_s}{\partial \boldsymbol{x}\partial \boldsymbol{y}}\end{Bmatrix}, \quad \begin{Bmatrix}\boldsymbol{\gamma}_{yz}^s\\ \boldsymbol{\gamma}_{xz}^s\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial \boldsymbol{w}_s}{\partial \boldsymbol{y}}\\ \dfrac{\partial \boldsymbol{w}_s}{\partial \boldsymbol{x}}\end{Bmatrix} \tag{12a}$$

and

$$\boldsymbol{g}(\boldsymbol{z}_{ns}) = 1 - \frac{\boldsymbol{df}(\boldsymbol{z}_{ns})}{\boldsymbol{dz}_{ns}} \tag{12b}$$

2.1.3 Constitutive relations

The linear constitutive relations are

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{pmatrix} = \frac{E(z)}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0\\ 0 & 0 & \dfrac{(1-\nu)}{2} & 0 & 0\\ 0 & 0 & 0 & \dfrac{(1-\nu)}{2} & 0\\ 0 & 0 & 0 & 0 & \dfrac{(1-\nu)}{2}\end{bmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{pmatrix} \tag{13}$$

where ($\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively.

2.1.4 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$0=\int_0^T(\delta U-\delta K)dt \tag{14}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate stated as

$$\delta U=\int_A\int(\sigma_x\,\delta\varepsilon_x+\sigma_y\,\delta\varepsilon_y+\tau_{xy}\,\delta\gamma_{xy}+\tau_{xz}\,\delta\gamma_{xz}+\tau_{yz}\,\delta\gamma_{yz})dAdz_{ns} \tag{15}$$

Substituting Eqs. (11) and (13) into Eq. (15) and integrating through the thickness of the plate, Eq. (15) can be rewritten as

$$\delta U=\int_A\left\{\begin{aligned}&N_x\frac{\partial\delta u_0}{\partial x}-M_x^b\frac{\partial^2\delta w_b}{\partial x^2}-M_x^s\frac{\partial^2\delta w_s}{\partial x^2}+N_y\frac{\partial\delta v_0}{\partial y}-M_y^b\frac{\partial^2\delta w_b}{\partial y^2}-M_y^s\frac{\partial^2\delta w_s}{\partial y^2}+N_{xy}(\frac{\partial\delta u_0}{\partial y}+\frac{\partial\delta v_0}{\partial x})\\&-2M_{xy}^b\frac{\partial^2\delta w_b}{\partial x\partial y}-2M_{xy}^s\frac{\partial^2\delta w_s}{\partial x\partial y}+S_{xz}^s\frac{\partial\delta w_s}{\partial x}+S_{yz}^s\frac{\partial\delta w_s}{\partial y}\end{aligned}\right\}dA \tag{16}$$

The stress resultants *N*, *M* and *S* are defined by

$$\begin{Bmatrix}\boldsymbol{N_x}, & \boldsymbol{N_y}, & \boldsymbol{N_{xy}}\\ \boldsymbol{M_x^b}, & \boldsymbol{M_y^b}, & \boldsymbol{M_{xy}^b}\\ \boldsymbol{M_x^s}, & \boldsymbol{M_y^s}, & \boldsymbol{M_{xy}^s}\end{Bmatrix}=\int_{-\boldsymbol{h}/2-\boldsymbol{C}}^{\boldsymbol{h}/2-\boldsymbol{C}}(\boldsymbol{\sigma_x},\boldsymbol{\sigma_y},\boldsymbol{\tau_{xy}})\begin{Bmatrix}1\\ \boldsymbol{z_{ns}}\\ \boldsymbol{f(z_{ns})}\end{Bmatrix}\boldsymbol{dz_{ns}}, \tag{17a}$$

$$\left(S_{xz}^s,S_{yz}^s\right)=\int_{-h/2-C}^{h/2-C}\left(\tau_{xz},\tau_{yz}\right)g(z_{ns})dz_{ns}. \tag{17b}$$

Using Eq. (13) in Eq. (17), the stress resultants of the FG plate can be related to the total strains by

$$\begin{Bmatrix}N\\ M^b\\ M^s\end{Bmatrix}=\begin{bmatrix}A & 0 & B^s\\ 0 & D & D^s\\ B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\ k^b\\ k^s\end{Bmatrix},\quad \boldsymbol{S}=\boldsymbol{A^s\gamma} \tag{18}$$

where

$$\boldsymbol{N}=\{\boldsymbol{N_x},\boldsymbol{N_y},\boldsymbol{N_{xy}}\}^{\boldsymbol{t}},\quad \boldsymbol{M^b}=\{\boldsymbol{M_x^b},\boldsymbol{M_y^b},\boldsymbol{M_{xy}^b}\}^{\boldsymbol{t}},\quad \boldsymbol{M^s}=\{\boldsymbol{M_x^s},\boldsymbol{M_y^s},\boldsymbol{M_{xy}^s}\}^{\boldsymbol{t}} \tag{19a}$$

$$\boldsymbol{\varepsilon}=\{\boldsymbol{\varepsilon_x^0},\boldsymbol{\varepsilon_y^0},\boldsymbol{\gamma_{xy}^0}\}^{\boldsymbol{t}},\quad \boldsymbol{k^b}=\{\boldsymbol{k_x^b},\boldsymbol{k_y^b},\boldsymbol{k_{xy}^b}\}^{\boldsymbol{t}},\quad \boldsymbol{k^s}=\{\boldsymbol{k_x^s},\boldsymbol{k_y^s},\boldsymbol{k_{xy}^s}\}^{\boldsymbol{t}} \tag{19b}$$

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12} & 0\\ \boldsymbol{A}_{12} & \boldsymbol{A}_{22} & 0\\ 0 & 0 & \boldsymbol{A}_{66}\end{bmatrix},\quad \boldsymbol{D}=\begin{bmatrix}\boldsymbol{D}_{11} & \boldsymbol{D}_{12} & 0\\ \boldsymbol{D}_{12} & \boldsymbol{D}_{22} & 0\\ 0 & 0 & \boldsymbol{D}_{66}\end{bmatrix} \tag{19c}$$

$$\boldsymbol{B}^{s}=\begin{bmatrix}\boldsymbol{B}_{11}^{s} & \boldsymbol{B}_{12}^{s} & 0\\ \boldsymbol{B}_{12}^{s} & \boldsymbol{B}_{22}^{s} & 0\\ 0 & 0 & \boldsymbol{B}_{66}^{s}\end{bmatrix},\quad \boldsymbol{D}^{s}=\begin{bmatrix}\boldsymbol{D}_{11}^{s} & \boldsymbol{D}_{12}^{s} & 0\\ \boldsymbol{D}_{12}^{s} & \boldsymbol{D}_{22}^{s} & 0\\ 0 & 0 & \boldsymbol{D}_{66}^{s}\end{bmatrix},\quad \boldsymbol{H}^{s}=\begin{bmatrix}\boldsymbol{H}_{11}^{s} & \boldsymbol{H}_{12}^{s} & 0\\ \boldsymbol{H}_{12}^{s} & \boldsymbol{H}_{22}^{s} & 0\\ 0 & 0 & \boldsymbol{H}_{66}^{s}\end{bmatrix} \tag{19d}$$

$$\boldsymbol{S}=\left\{\boldsymbol{S}_{yz}^{s},\boldsymbol{S}_{xz}^{s}\right\}^{t},\quad \gamma=\left\{\gamma_{yz},\gamma_{xz}\right\}^{t},\quad \boldsymbol{A}^{s}=\begin{bmatrix}\boldsymbol{A}_{44}^{s} & 0\\ 0 & \boldsymbol{A}_{55}^{s}\end{bmatrix} \tag{19e}$$

where $\boldsymbol{A}_{ij}$, $\boldsymbol{D}_{ij}$, etc., are the plate stiffness, defined by

$$\begin{Bmatrix}A_{11} & D_{11} & B_{11}^{s} & D_{11}^{s} & H_{11}^{s}\\ A_{12} & D_{12} & B_{12}^{s} & D_{12}^{s} & H_{12}^{s}\\ A_{66} & D_{66} & B_{66}^{s} & D_{66}^{s} & H_{66}^{s}\end{Bmatrix}=\int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} Q_{11}\left(1, z^{2}, f(z_{ns}), z_{ns}\ f(z_{ns}), f^{2}(z_{ns})\right)\begin{Bmatrix}1\\ \nu\\ \dfrac{1-\nu}{2}\end{Bmatrix} dz_{ns} \tag{20a}$$

where

$$Q_{11}=\frac{E(z_{ns})}{2(1+\nu)}, \tag{20b}$$

and

$$\left(\boldsymbol{A}_{22},\boldsymbol{D}_{22},\boldsymbol{B}_{22}^{s},\boldsymbol{D}_{22}^{s},\boldsymbol{H}_{22}^{s}\right)=\left(\boldsymbol{A}_{11},\boldsymbol{D}_{11},\boldsymbol{B}_{11}^{s},\boldsymbol{D}_{11}^{s},\boldsymbol{H}_{11}^{s}\right) \tag{20c}$$

$$A_{44}^{s}=A_{55}^{s}=\int_{-\frac{h}{2}-C}^{\frac{h}{2}-C}\frac{E(z_{ns})}{2(1+\nu)}\left[g(z_{ns})\right]^{2}dz_{ns}, \tag{20d}$$

The variation of kinetic energy is expressed as

$$\delta K=\int_{V}\left(\dot{u}\delta\,\dot{u}+\dot{v}\delta\,\dot{v}+\dot{w}\delta\,\dot{w}\right)\rho(z_{ns})dAd\,z_{ns} \tag{21}$$

$$\delta K=\int_{A}\left\{\begin{aligned}&I_{0}\left[\dot{u}_{0}\delta\dot{u}_{0}+\dot{v}_{0}\delta\dot{v}_{0}+(\dot{w}_{b}+\dot{w}_{s})\delta(\dot{w}_{b}+\dot{w}_{s})\right]-I_{1}\left[\dot{u}_{0}\frac{\partial\delta\dot{w}_{b}}{\partial x}+\frac{\partial\dot{w}_{b}}{\partial x}\delta\dot{u}_{0}+\dot{v}_{0}\frac{\partial\delta\dot{w}_{b}}{\partial y}+\frac{\partial\dot{w}_{b}}{\partial y}\delta\dot{v}_{0}\right]\\ &-J_{1}\left[\dot{u}_{0}\frac{\partial\delta\dot{w}_{s}}{\partial x}+\frac{\partial\dot{w}_{s}}{\partial x}\delta\dot{u}_{0}+\dot{v}_{0}\frac{\partial\delta\dot{w}_{s}}{\partial y}+\frac{\partial\dot{w}_{s}}{\partial y}\delta\dot{v}_{0}\right]+I_{2}\left[\frac{\partial\dot{w}_{b}}{\partial x}\frac{\partial\delta\dot{w}_{b}}{\partial x}+\frac{\partial\dot{w}_{b}}{\partial y}\frac{\partial\delta\dot{w}_{b}}{\partial y}\right]\\ &+K_{2}\left[\frac{\partial\dot{w}_{s}}{\partial x}\frac{\partial\delta\dot{w}_{s}}{\partial x}+\frac{\partial\dot{w}_{s}}{\partial y}\frac{\partial\delta\dot{w}_{s}}{\partial y}\right]+J_{2}\left[\frac{\partial\dot{w}_{b}}{\partial x}\frac{\partial\delta\dot{w}_{s}}{\partial x}+\frac{\partial\dot{w}_{s}}{\partial x}\frac{\partial\delta\dot{w}_{b}}{\partial x}+\frac{\partial\dot{w}_{b}}{\partial y}\frac{\partial\delta\dot{w}_{s}}{\partial y}+\frac{\partial\dot{w}_{s}}{\partial y}\frac{\partial\delta\dot{w}_{b}}{\partial y}\right]\end{aligned}\right\}dA \tag{22}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t ; $\rho(z)$ is the mass density; and $(I_0, I_1, J_1, I_2, J_2, K_2)$ are mass inertias defined as

$$\left(I_0, I_1, J_1, I_2, J_2, K_2\right) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left(1, z_{ns}, f, z_{ns}{}^2, z_{ns} f, f^2\right) \rho\left(z_{ns}\right) dz_{ns} \tag{23}$$

Substituting the expressions for δU and δK from Eqs. (16) and (21) into Eq. (14) and integrating by parts and collecting the coefficients of $\delta u_0, \delta v_0, \delta w_b$ and δw_s , the following equations of motion of the plate are obtained

$$\begin{aligned}
\delta u_0 &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x} \\
\delta v_0 &: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y} \\
\delta w_b &: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0(\ddot{w}_b + \ddot{w}_s) + I_1(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}) - I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s \\
\delta w_s &: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = I_0(\ddot{w}_b + \ddot{w}_s) + J_1(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}) - J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s
\end{aligned} \tag{24}$$

By substituting Eq. (18) into Eq. (24), the equations of motion can be expressed in terms of displacements $\left(u_0, v_0, w_b, w_s\right)$ as

$$A(\frac{\partial^2 u_0}{\partial x^2} + \frac{(1-\nu)}{2}\frac{\partial^2 u_0}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2 v_0}{\partial x \partial y}) - B\nabla^2 \frac{\partial w_b}{\partial x} - B^s \nabla^2 \frac{\partial w_s}{\partial x} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x} \tag{25a}$$

$$A(\frac{\partial^2 v_0}{\partial y^2} + \frac{(1-\nu)}{2}\frac{\partial^2 v_0}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2 u_0}{\partial x \partial y}) - B\nabla^2 \frac{\partial w_b}{\partial y} - B^s \nabla^2 \frac{\partial w_s}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y} \tag{25b}$$

$$B\nabla^2(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}) - D\nabla^4 w_b - D^s \nabla^4 w_s = I_0(\ddot{w}_b + \ddot{w}_s) + I_1(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}) - I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s \tag{25c}$$

$$B^s \nabla^2(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}) - D^s \nabla^4 w_b - H^s \nabla^4 w_s - A_s \nabla^2 w_s = I_0(\ddot{w}_b + \ddot{w}_s) + J_1(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}) - J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \tag{25d}$$

3. Navier solution for simply supported rectangular plates

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eqs. (25) for a simply supported FG plate. The following displacement functions are chosen to satisfy the boundary conditions of plate and are selected as Fourier series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t} \cos\alpha x \sin\beta y \\ V_{mn} e^{i\omega t} \sin\alpha x \cos\beta y \\ W_{bmn} e^{i\omega t} \sin\alpha x \sin\beta y \\ W_{smn} e^{i\omega t} \sin\alpha x \sin\beta y \end{Bmatrix} \tag{26}$$

Where $i=\sqrt{-1}$, $\alpha=\frac{m\pi}{a}$ and $\beta=\frac{n\pi}{b}$. w is the natural frequency. U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters to be determined. Substituting Eq. (26) into Eq. (25), the following eigen value equation is obtained

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \right) \begin{pmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

Where

$$k_{11} = A\alpha^2 + \frac{1-\nu}{2} A\beta^2,\ k_{12} = \frac{1+\nu}{2} A\alpha\beta,\ k_{13} = -B\alpha(\alpha^2+\beta^2), k_{14} = -B^s\alpha(\alpha^2+\beta^2),$$

$$k_{22} = \frac{1-\nu}{2} A\alpha^2 + A\beta^2,\ k_{23} = -B\beta(\alpha^2+\beta^2),\ k_{24} = -B^s\beta(\alpha^2+\beta^2),\ k_{33} = D(\alpha^2+\beta^2)^2,$$

$$k_{34} = D^s(\alpha^2+\beta^2)^2,\ k_{44} = H^s(\alpha^2+\beta^2)^2 + A^s(\alpha^2+\beta^2)$$

$$m_{11} = m_{22} = I_0,\ m_{13} = -\alpha I_1,\ m_{14} = -\alpha J_1,\ m_{23} = -\beta I_1,\ m_{24} = -\beta J_1,$$

$$m_{33} = I_0 + I_2(\alpha^2+\beta^2),$$

$$m_{34} = I_0 + J_2(\alpha^2+\beta^2),\ m_{44} = I_0 + K_2(\alpha^2+\beta^2) \tag{28}$$

4. Results and discussion

In this section, the free vibration analysis of simply supported FG plates by the present hyperbolic shear deformation theory is suggested for investigation. Navier solutions for the free vibration analysis of FG plates are presented by solving the eigenvalue equations.

The FG plate is taken to be made of aluminum and alumina with the following material properties:

Ceramic (Alumina, Al_2O_3) E_C = 380 GPa, ν = 0.3, and ρ_C = 3800 kg/m^3.

Ceramic (Zirconia, ZrO_2) E_C = 200 GPa, ν = 0.3, and ρ_C = 5700 kg/m^3.

Metal (Aluminium, Al) E_m = 70 GPa, ν = 0.3, and ρm = 2702 kg/ m^3.

For simplicity, the following non dimensional natural frequency parameter is used in the numerical examples.

$$\bar{\beta} = wh\sqrt{\frac{\rho_m}{E_m}}\,, \quad \hat{\beta} = wh\sqrt{\frac{\rho_C}{E_C}}\,, \quad \bar{w} = \frac{w a^2}{h}\sqrt{\frac{\rho_C}{E_C}} \tag{29}$$

4.1 Comparison studies

In this section, various numerical examples are described and discussed for verifying the accuracy of the present hyperbolic shear deformation plate theory in predicting the free vibration behaviors of simply supported FG plates. For the verification purpose, the results obtained by the present theory are compared with other theories existing in the literature, such as the classical plate theory (CPT), the first-order shear deformation plate theory (FSDPT) (Hosseini-Hashemi *et al.* 2011c), the 3-D exact solution (Vel and Batra 2004) and the high order shear deformation theory (HSDT) (Hosseini-Hashemi *et al.* 2011b, Matsunaga 2008). We also take the shear correction factor $K = 5/6$ in FSDPT.

Table 1 shows a good agreement by the comparisons of the fundamental frequency parameter $\bar{\beta}$ obtained by the present theory (with only four unknown functions) with other theories in the case of the FG perfect plates (α = 0). In general, the vibration frequencies obtained by CPT are much higher than those computed from the shear deformation theories. This implies the well known fact that the results estimated by CPT are grossly in error for a thick plate.

In addition, it should be noted that in the case of the imperfect FG plate ($\alpha = 0{,}1 \;\; \alpha = 0{,}2$) , in the present solution, the frequencies decrease as the porosity increase.

As a second example, a comparison of the natural frequency parameter $\hat{\beta}$ of Al/Al_2O_3 square plate with different thickness ratio (a/h) and power law index “p” using different plate theories are presented in Table 2. The results from the present hyperbolic shear deformation theory in the case of perfect plate (α = 0) are in good agreement with those from Refs (Hosseini-Hashemi *et al.* 2011a, b).

Also, the results show that the CPT overpredicts the natural frequency of FG plates, especially for the thick plate at higher modes of vibration. Moreover, it can be shown that the natural frequencies are increasing with the increase of the porosity parameter (α). As the volume fraction exponent increases for FG plates, the natural frequency will increase. These frequencies’ are also sensitive to the variation of a/h ratio.

Lowest four frequency parameters $\bar{\omega}$ of rectangular Al/Al_2O_3 perfect and imperfect plate for different values of the gradient indices “p” and thickness ratio (a/h) are presented in Table 3.

It can be observed that there is a little difference between the results; this is due the different approaches used to predict the natural frequencies. The first shear deformation theory (FSDT) presented by Hosseini-Hashemi *et al.* (2011c) have five kinematic unknowns contrary to the present theory which use a four unknown functions. In addition, the FSDT requires the use of shear correction factors to ensure the nullity of the stress at the top and bottom of the plate surfaces but the most interesting feature of the present theory is that it allows for parabolic distributions of transverse shear stresses across the plate thickness and satisfies zero shear stress conditions at the top and bottom surfaces of the plate without using shear correction factors.

Moreover, all frequencies parameters are decreasing with the existence of imperfection in the plate ($\alpha \neq 0$).

4.2 Parametric studies

After verifying the merit and accuracy of the present hyperbolic shear deformation theory, the following new results for the vibration analysis of rectangular FG plates, can be used as a benchmark for future research studies.

In Figs. 2 (a) and 2(b), variation of the frequency parameter with power law index p is given for a/h=5 and a/h=100 respectively. According to these figures the frequency parameter decreases with increasing index p and porosity parameter α .

Figs. 3 (a) and 3(b) depict the fundamental frequency parameters versus the thickness-side ratios of simply supported power-law FGM plate for p=1 and p=100 respectively. It is seen that the results increase as the thickness ratio of the plate increases. Moreover, the frequency parameter is approximately insensitive to a/h ratio after a/h > 30 for all cases (perfect and imperfect plate).

The variation of the frequency parameter with a/b ratio is given in Figs. 4 (a) and 4(b) for p = 1 and 100, respectively. It is observed that the frequency parameter increases for plates with higher aspect ratio, a/b. In addition, it is seen from the results that frequency parameter is very sensitive to the power- law index p.

Indeed, according to Fig. 4(a) frequencies are very close, and this regardless of the State of the plate (perfect or imperfect). However, for the case where *p* = 100, the difference between frequencies for the various cases of porosity is quite straightforward.

In Figs. 5, the variations of natural frequency parameter with the power law index p are given for different cases of porosity. It is seen from the figures that the increase of the power law index p produces a reduction of the natural frequency parameter and this regardless of the porosity.

The first nine frequency parameters of a square FG moderately thick plate (a/h=5) with various values of power law index and vibration modes are listed in table 4.

It is seen from the results that, regardless of mode number, frequency parameter increases by decreasing power law.

Table 1 comparison of fundamental frequency parameter $\bar{\beta}$ of Al/ZrO_2 square plate

		p = 1			a/h = 5		
Theory	porosity	a/h =5	a/h =10	a/h =20	p = 2	p = 3	p = 5
Vel and Batra (2004) 3-D	α = 0	0.2192	0.0596	0.0153	0.2197	0.2211	0.2225
Matsunaga (2008) HSDT		0.2285	0.0619	0.0158	0.2264	0.2270	0.2281
Hosseini- Hashemi *et al*. (2011b) HSDT		0.2276	0.0619	0.0158	0.2256	0.2263	0.2272
Hosseini- Hashemi *et al*. (2011c) FSDT		0.2276	0.0619	0.0158	0.2264	0.2276	0.2291
CPT		0.2479	0.0634	0.0159	0.2473	0.2497	0.2526
Present	α = 0	0.2276	0.0618	0.0158	0.2257	0.2263	0.2272
	α = 0.1	0.2258	0.0612	0.0156	0.2228	0.2233	0.2244
	α = 0.2	0.2231	0.0604	0.0154	0.2184	0.2186	0.2199

Table 2 Comparison of natural frequency parameter $\hat{\beta}$ of Al/ Al_2O_3 square plate

a/h	Mode (m,n)	Theory	porosity	P = 0.5	P = 1	P = 4	P = 10
5	(1,1)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.1807	0.1631	0.1378	0.1301
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.1805	0.1631	0.1397	0.1324
		CPT	α = 0	0.1959	0.1762	0.1524	0.1467
		Present	α = 0	0.1807	0.1631	0.1379	0.1301
			α = 0.1	0.1806	0.1599	0.1280	0.1195
			α = 0.2	0.1803	0.1552	0.1111	0.1009
	(1,2)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.3989	0.3607	0.2980	0.2771
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.3978	0.3604	0.3049	0.2856
		CPT	α = 0	0.4681	0.4198	0.3603	0.3481
		Present	α = 0	0.3988	0.3606	0.2982	0.2772
			α = 0.1	0.3991	0.3544	0.2776	0.2534
			α = 0.2	0.3991	0.3453	0.2428	0.2128
	(2,2)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.5803	0.5254	0.4284	0.3948
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.5779	0.5245	0.4405	0.4097
		CPT	α = 0	0.7184	0.6425	0.5478	0.5306
		Present	α = 0	0.5801	0.5253	0.4288	0.3950
			α = 0.1	0.5810	0.5171	0.4000	0.3601
			α = 0.2	0.5816	0.5050	0.3517	0.3018
10	(1,1)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.0490	0.0442	0.0381	0.0364
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.0490	0.0442	0.0382	0.0366
		CPT	α = 0	0.0502	0.0452	0.0392	0.0377
		Present	α = 0	0.0490	0.0441	0.0380	0.0363
			α = 0.1	0.0489	0.0432	0.0353	0.0336
			α = 0.2	0.0489	0.0418	0.0304	0.0285
	(1,2)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.1174	0.1059	0.0903	0.0856
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.1173	0.1059	0.0911	0.0867
		CPT	α = 0	0.1239	0.1115	0.0966	0.0930
		Present	α = 0	0.1173	0.1059	0.0902	0.0856
			α = 0.1	0.1172	0.1037	0.0837	0.0788
			α = 0.2	0.1170	0.1006	0.0724	0.0668
	(2,2)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.1807	0.1631	0.1378	0.1301
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.1805	0.1631	0.1397	0.1324
		CPT	α = 0	0.1959	0.1762	0.1524	0.1467
		Present	α = 0	0.1807	0.1631	0.1379	0.1301
			α = 0.1	0.1631	0.1599	0.1280	0.1195
			α = 0.2	0.1599	0.1552	0.1111	0.1009
20	(1,1)	Hosseini- Hashemi *et al.* (2011b) HSDT	α = 0	0.0125	0.0113	0.0098	0.0094
		Hosseini- Hashemi *et al.* (2011c) FSDT	α = 0	0.0125	0.0113	0.0098	0.0094
		CPT	α = 0	0.0126	0.0114	0.0099	0.0095
		Present	α = 0	0.0125	0.0113	0.0098	0.0094
			α = 0.1	0.0125	0.0110	0.0090	0.0087
			α = 0.2	0.0124	0.0106	0.0078	0.0074

Table 3 Comparison of frequency parameter $\bar{\omega}$ of Al/ Al_2O_3 rectangular plate (b=2a)

a/h	Mode (m,n)	Theory		porosity	1	2	5	8	10
5	(1,1)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	2.6473	2.4017	2.2528	2.1985	2.1677
		Present		α = 0	2.6476	2.3952	2.2285	2.1707	2.1414
				α = 0.1	2.5934	2.2740	2.0610	2.0009	1.9723
				α = 0.2	2.5150	2.0819	1.7655	1.6971	1.6703
	(1,2)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	4.0773	3.6953	3.4492	3.3587	3.3094
		Present		α = 0	4.0782	3.6812	3.3966	3.2987	3.2529
				α = 0.1	3.9982	3.4997	3.1417	3.0358	2.9893
				α = 0.2	3.8821	3.2118	2.6966	2.5724	2.5249
	(1,3)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	6.2636	5.6695	5.2579	5.1045	5.0253
		Present		α = 0	6.2664	5.6403	5.1481	4.9804	4.9085
				α = 0.1	6.1508	5.3723	4.7631	4.5748	4.4985
				α = 0.2	5.9821	4.9466	4.1001	3.8729	4.9804
	(2,1)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	7.7811	7.1189	6.5749	5.9062	5.7518
		Présente		α = 0	7.8762	7.0768	6.4153	6.1909	6.0995
				α = 0.1	7.7369	6.7490	5.9372	5.6808	5.5811
				α = 0.2	7.5330	6.2278	5.1208	4.8076	4.6922
10	(1,1)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	2.7937	2.5386	2.3998	2.3504	2.3197
		Present		α = 0	2.7937	2.5365	2.3920	2.3414	2.3112
				α = 0.1	2.7328	2.4031	2.2122	2.1643	2.1370
				α = 0.2	2.6452	2.1921	1.8894	1.8396	1.8189
	(1,2)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	4.4192	4.0142	3.7881	3.7072	3.6580
		Present		α = 0	4.4193	4.0092	3.7693	3.6855	3.2529
				α = 0.1	4.3243	3.8001	3.4859	3.4043	2.9893
				α = 0.2	4.1875	3.4693	2.9792	2.8922	2.5249
	(1,3)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	7.0512	6.4015	6.0247	5.8887	5.8086
		Present		α = 0	7.0516	6.3893	5.9790	5.8362	5.7590
				α = 0.1	6.9033	6.0604	5.5295	5.3858	5.3128
				α = 0.2	6.6891	5.5398	4.7305	4.5720	4.5085
	(2,1)	Hosseini- Hashemi *et al*. (2011c)	FSDT	α = 0	9.0928	8.2515	7.7505	7.5688	7.4639
		Present		α = 0	9.0935	8.2319	7.6772	7.4847	7.3845
				α = 0.1	8.9053	7.8123	7.1001	6.9022	6.8058
				α = 0.2	8.6331	7.1479	6.0788	5.8564	5.7685

Table 4 First nine frequency parameter ϖ of Al/Al_2O_3 square plate (a/h=5)

a /h	Mode no (m,n)	porosity	p 0.5	1	2	5	10	100
	1 (1,1)	α = 0	4.5181	4.0782	3.6812	3.3966	3.2529	2.8175
		α = 0.1	4.5158	3.9982	3.4997	3.1417	2.9893	2.5267
		α = 0.2	4.5096	3.8821	3.2118	2.6966	2.5249	2.0688
	2 (2,1)	α = 0	9.9714	9.0164	8.0925	7.3040	6.9318	6.1291
		α = 0.1	9.9783	8.8616	7.7240	6.7612	6.3365	5.4824
		α = 0.2	9.9797	8.6343	7.1378	5.8394	5.3223	4.4593
	3 (1,2)	α = 0	9.9714	9.0164	8.0925	7.3040	6.9318	6.1291
		α = 0.1	9.9783	8.8616	7.7240	6.7612	6.3365	5.4824
		α = 0.2	9.9797	8.6343	7.1378	5.8394	5.3223	4.4593
	4 (2,2)	α = 0	14.5049	13.1339	11.7508	10.4660	9.8768	8.8372
		α = 0.1	14.5261	12.9296	11.2462	9.6970	9.0043	7.8932
		α = 0.2	14.5423	12.6272	10.4411	8.4136	7.5460	6.3957
	5 (3,1)	α = 0	17.1907	15.5781	13.9180	12.3165	11.5907	10.4286
		α = 0.1	17.2224	15.3488	13.3393	11.4184	10.5546	9.3081
		α = 0.2	17.2502	15.0078	12.4146	9.9325	8.8382	7.5288
	6 (1,3)	α = 0	17.1907	15.5781	13.9180	12.3165	11.5907	10.4286
		α = 0.1	17.2224	15.3488	13.3393	11.4184	10.5546	9.3081
		α = 0.2	17.2502	15.0078	12.4146	9.9325	8.8382	7.5288
	7 (3,2)	α = 0	20.8537	18.9173	16.8757	14.8221	13.9024	12.5875
		α = 0.1	20.9019	18.6582	16.2029	13.7530	12.6443	11.2264
		α = 0.2	20.9481	18.2705	15.1256	12.0027	10.5816	9.0621
	8 (2,3)	α = 0	20.8537	18.9173	16.8757	14.8221	13.9024	12.5875
		α = 0.1	20.9019	18.6582	16.2029	13.7530	12.6443	11.2264
		α = 0.2	20.9481	18.2705	15.1256	12.0027	10.5816	9.0621
	9 (4,1)	α = 0	25.2274	22.9124	20.4125	17.7953	16.6351	15.1525
		α = 0.1	25.2978	22.6233	19.6365	16.5287	15.1139	13.5042
		α = 0.2	25.3690	22.1875	18.3910	14.4786	12.6451	10.8801

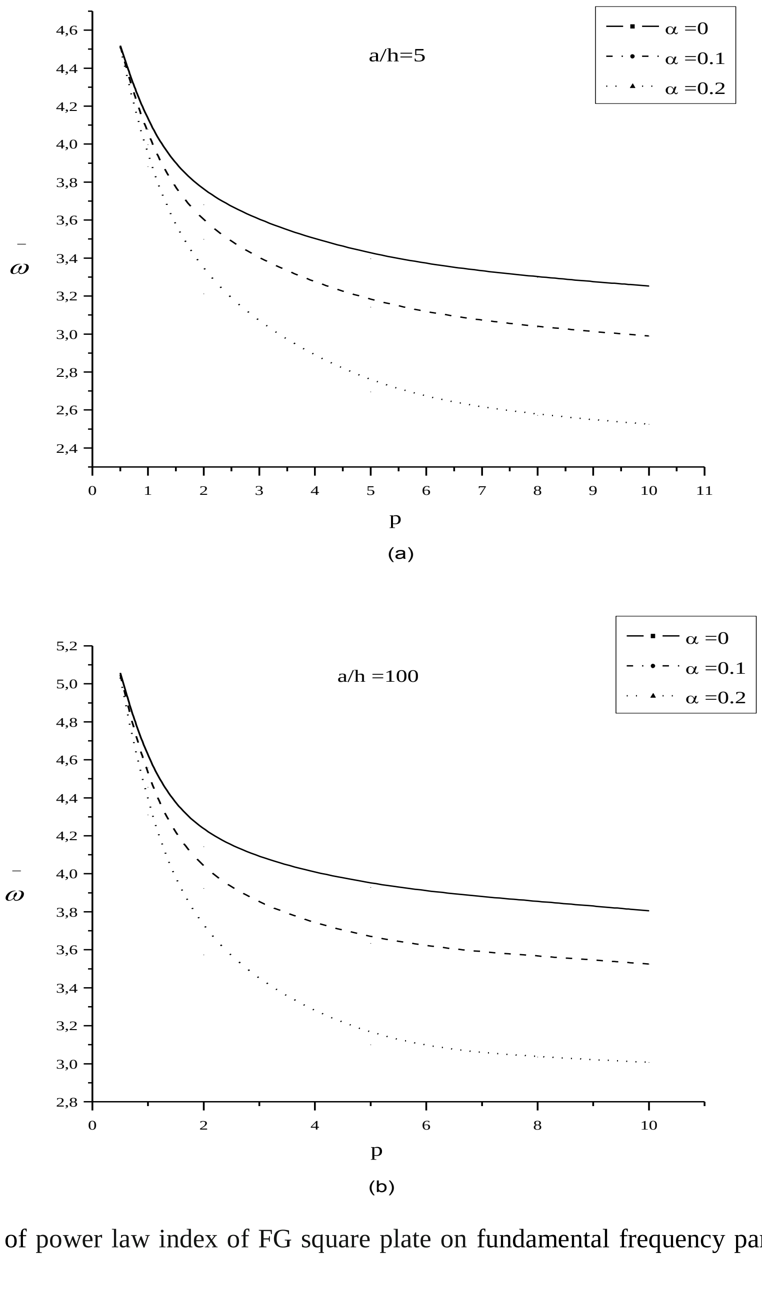


Fig. 2 The effect of power law index of FG square plate on fundamental frequency parameter (a) a/h=5 and (b) a/h=100

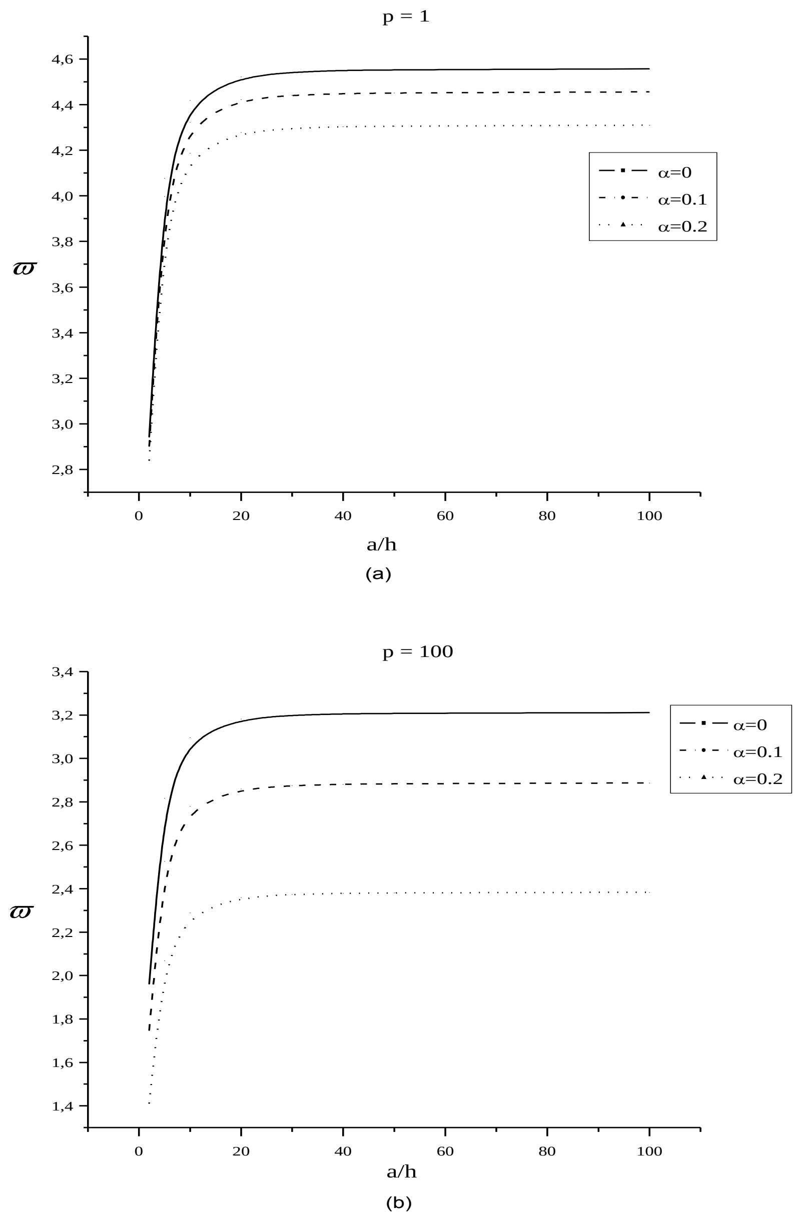
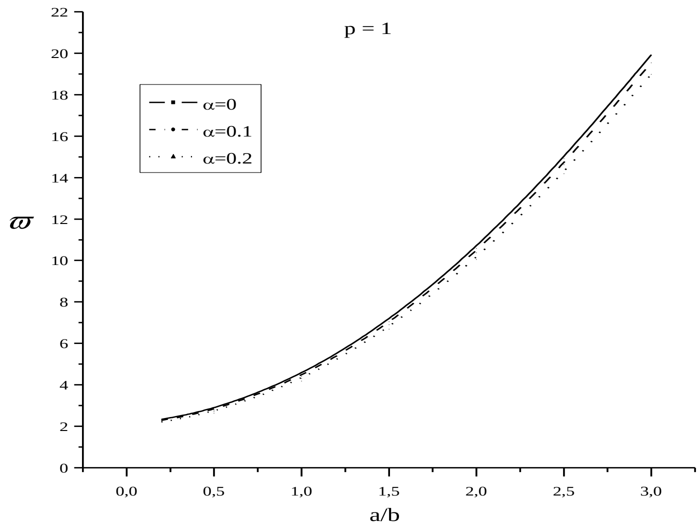
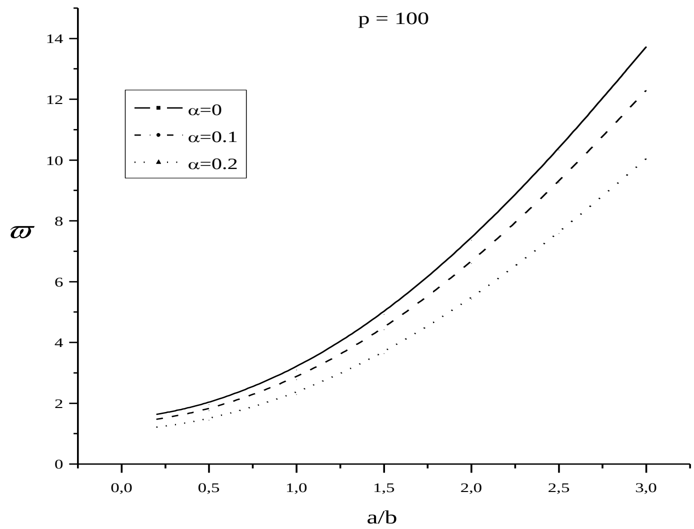


Fig. 3 The effect of thickness ratio on fundamental frequency parameter of square plate, (a) p=1 and (b) p=100



(a)



(b)

Fig. 4 The effect of aspect ratio on fundamental frequency parameter of rectangular plate. (a) p=1 and (b) p=100

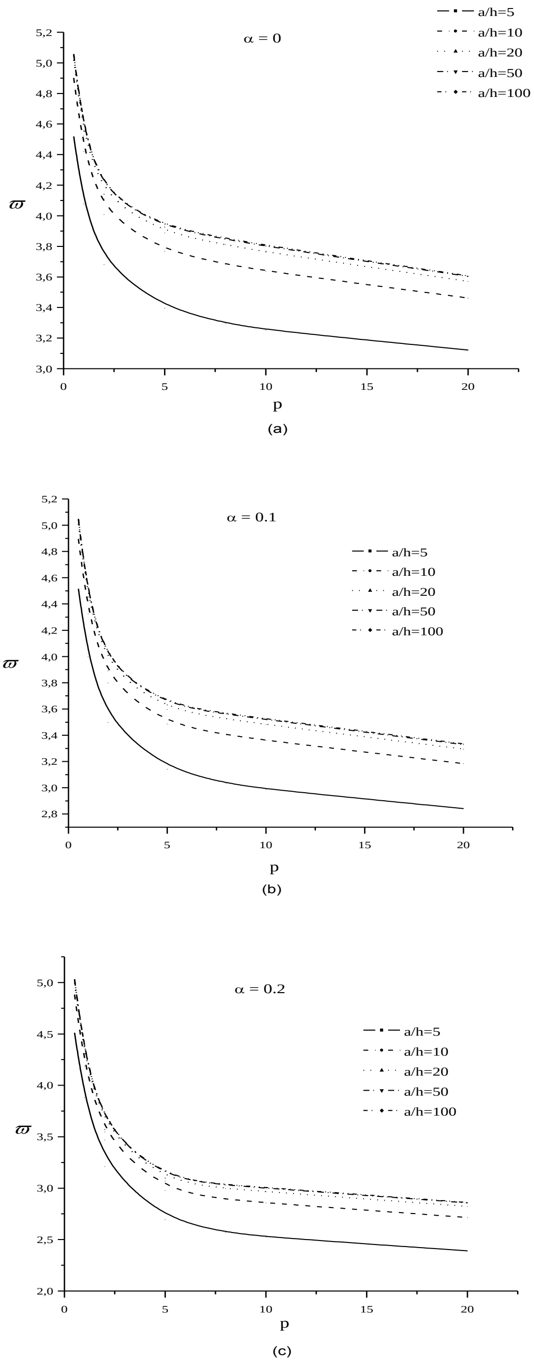


Fig. 5 The effect of the power law index on fundamental frequency parameter for different cases of porosity. (a) α = 0, (b) α = 0.1 and (c) α = 0.2

5. Conclusions

A four variable theory is developed for vibration analysis of rectangular functionally graded plates with porosities. The modified rule of mixture covering porosity phases is used to describe and approximate material properties of the imperfect FGM plate. The neutral surface position for such plates has been determined. An efficient hyperbolic plate theory based on exact neutral surface position has been used to find the basic equations of FG plates. The theory takes account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the FG plate, hence it is unnecessary to use shear correction factors. Unlike any other theory, the theory presented gives rise to only four governing equations resulting in considerably lower computational effort when compared with the other higher-order theories reported in the literature having more number of governing equations.

The accuracy of the present theory is ascertained by comparing it with other shear deformation theories where an excellent agreement was observed in all cases. Furthermore, the influences of plate parameters such as power law index, aspect ratio, porosities on the natural frequencies of FG rectangular plate have been comprehensively investigated.

The formulation lends itself particularly well to wave propagation in orthotropic non-homogeneous medium (Mahmoud *et al.* 2014), nanostructures (Besseghier *et al.* 2015, Chemi et *al.* 2015, Ould Youcef *et al*. 2015) and vibration of laminated composite plates (Draiche *et al.* 2014, Nedri *et al.* 2014, Chattibi et *al.* 2015, Ozturk 2015), which will be considered in the near future.

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