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Influence of the porosities on the free vibration of FGM beams

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Abstract. In this paper, a free vibration analysis of functionally graded beam made of porous material is presented. The material properties are supposed to vary along the thickness direction of the beam according to the rule of mixture, which is modified to approximate the material properties with the porosity phases. For this purpose, a new displacement field based on refined shear deformation theory is implemented. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. Based on the present refined shear deformation beam theory, the equations of motion are derived from Hamilton's principle. The rule of mixture is modified to describe and approximate material properties of the FG beams with porosity phases. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions. Illustrative examples are given also to show the effects of varying gradients, porosity volume fraction, aspect ratios, and thickness to length ratios on the free vibration of the FG beams.

Keywords: functionally graded beam; shear deformation theory; porosity; vibration

1. Introduction

Functionally graded materials (FGMs) have many advantages for use in engineering structural components. Unlike fiber-matrix laminated composites, FGMs do not have problems of de-bonding and delaminating that result from large inter-laminar stresses. The concept of FGMs was initially introduced in the mid-1980s by Japanese scientists. FGMs are microscopically inhomogeneous and spatial composite materials which are usually composed of two different materials such as a pair of ceramic-metal or ceramic-polymer. The composition of the material changes gradually throughout the thickness direction. As a result, mechanical properties are assumed to vary continuously and smoothly from the top surface to the bottom. Due to good characteristics of ceramics in heat and corrosive resistances combined with the toughness of metals or high elastic of polymers, the combination of ceramics and metals or polymers can lead to excellent materials. The FGMs are widely used in mechanical, aerospace, nuclear, and civil engineering. Consequently, studies devoted to understand the static and dynamic behaviors of FGM beams and plates have being paid more and more attentions in recent years (El Meiche *et al.* 2011, Benachour *et al.* 2011, Bourada *et al.* 2012, Tounsi *et al.* 2013, Saidi *et al.* 2013, Bessaim *et al.* 2013, Bouderba *et al.* 2013, Houari *et al.* 2013, Kettaf *et al.* 2013, Khalfi *et al.* 2014, Zidi *et al.*

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2014, Ait Amar Meziane *et al*. 2014, Fekrar *et al*. 2014, Belabed *et al*. 2014, Bousahla *et al*. 2014, Mahi *et al*. 2015, Bouchafa *et al*. 2015, Al-Basyouni *et al*. 2015, Hamidi *et al*. 2015).

Zhong and Yu provided an analytical solution for cantilever beams subjected to various types of mechanical loadings using the Airy stress function. Bending analysis of FG beams based on higher order shear deformation under ambient temperature was investigated by Kadoli *et al.* (2008). Li (2008) investigated the static bending and transverse vibration of FGM Timoshenko beams by including the rotary inertia and shear deformation.

Sallai *et al.* (2009) investigated the static responses of a sigmoid FG thick beam by using different beam theories. Benatta *et al*. (2009) presented a mathematical solution for bending of short hybrid composite beams with variable fibers spacing. Şimşek (2010a) studied the free vibration analysis of an FG beam using different higher order beam theories. In a recent study, Şimşek (2010b) has studied the dynamic deflections and the stresses of an FG simply-supported beam subjected to a moving mass by using Euler–Bernoulli, Timoshenko and the parabolic shear deformation beam theory. Giunta *et al*. (2011) used the Hierarchical theories for the free vibration analysis of functionally graded beams. Thai (2012) investigated the Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories. Hadji *et al*. (2014) studied the bending and vibration responses of FG beams via a higher shear deformation beam theory. Ould Larbi *et al*. (2013) presented an efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams. Bourada *et al.* (2015) examined the flexure and free vibration responses of FGM beams using a new shear and normal deformation theory. The beauty of this theory (Bourada *et al.* 2015) is that, in addition to modeling the displacement field with only 3 unknowns as in Timoshenko beam theory, the thickness stretching effect ($\varepsilon_z \neq 0$) is also included in the present theory. Larbi Chaht *et al*. (2015) studied the bending and buckling behaviour of FGM nanobeams by including the thickness stretching effect.

However, in FGM fabrication, micro voids or porosities can occur within the materials during the process of sintering. This is because of the large difference in solidification temperatures between material constituents (Zhu *et al*. 2001). Wattanasakulpong *et al*. (2012) also gave the discussion on porosities happening inside FGM samples fabricated by a multi-step sequential infiltration technique. Recently, Ait Yahia *et al*. (2015) shown that the porosity have a considerable effect on the wave propagation in FGM plate. Therefore, it is important to take in to account the porosity effect when designing FGM structures subjected to dynamic loadings.

In this paper, a variationally consistent shear deformation theory is developed using a new displacement field for thick FG beams having porosities. The rule of mixture is modified to describe and approximate material properties of the FG beams with porosity phases. Based on the present refined shear deformation beam theory, the equations of motion are derived from Hamilton's principle. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions. Illustrative examples are given also to show the effects of varying gradients, porosity volume fraction, aspect ratios, and thickness to length ratios on the free vibration of the FG beams.

2. Problem formulation

Consider a functionally graded beam with length L and rectangular cross section $b \times h$, with

b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction.

2.1 Effective material properties of metal ceramic functionally graded beams

The properties of FGM vary continuously due to the gradually changing volume fraction of the constituent materials (ceramic and metal), usually in the thickness direction only. The power-law function is commonly used to describe these variations of materials properties. The expression given below represents the profile for the volume fraction.

A FG beam made from a mixture of two material phases, for example, a metal and a ceramic. The material properties of FG beams are assumed to vary continuously through the thickness of the beam. In this investigation, the imperfect beam is assumed to have porosities spreading within the thickness due to defect during production. Consider an imperfect FGM with a porosity volume fraction, $\alpha(\alpha \prec\prec 1)$, distributed evenly among the metal and ceramic, the modified rule of mixture proposed by Wattanasakulpong and Ungbhakorn (2014) is used as

$$P = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right) \tag{1}$$

Now, the total volume fraction of the metal and ceramic is : $V_m + V_c = 1$, and the power law of volume fraction of the ceramic is described as

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{2}$$

Hence, all properties of the imperfect FGM can be written as

$$P = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m - (P_c + P_m)\frac{\alpha}{2} \tag{3}$$

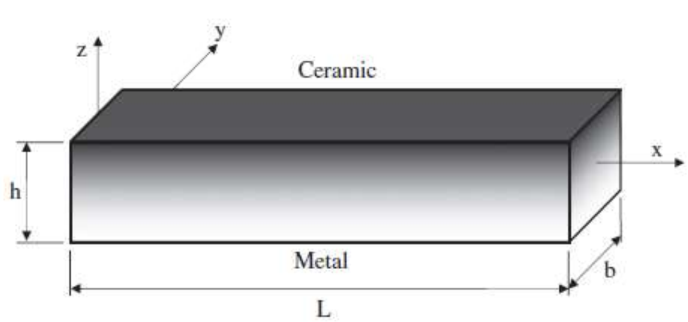

Fig. 1 Geometry and coordinate of a FG beam

It is noted that the positive real number $k\ (0 \le k \le \infty)$ is the power law or volume fraction index, and z is the distance from the mid-plane of the FG plate. The FG beam becomes a fully ceramic plate when k is set to zero and fully metal for large value of k.

Thus, the Young's modulus (E) and material density (ρ) equations of the imperfect FGM beam can be expressed as

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\alpha}{2} \tag{4}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \rho_m - (\rho_c + \rho_m)\frac{\alpha}{2} \tag{5}$$

However, Poisson's ratio (ν) is assumed to be constant. The material properties of a perfect FG beam can be obtained when α is set to zero.

In addition, for another scenario of porosity distribution, it is possible to obtain imperfect FGM samples which have almost porosities spreading around the middle zone of the cross-section and the amount of porosity seems to be on the decrease to zero at the top and bottom of the cross-section. Based on the principle of the multi-step sequential infiltration technique that can be employed to fabricate FGM samples (Wattanasakulpong *et al.* 2012), the porosities mostly occur at the middle zone. At this zone, it is difficult to infiltrate the materials completely, while at the top and bottom zones, the process of material infiltration can be performed easier and leaves less porosity. Consider this scenario, the equations of Young's modulus (E) and material density (ρ) in Eqs. (5) and (6) are replaced by the following forms

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\alpha}{2}\left(1 - \frac{2|z|}{h}\right) \tag{6}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \rho_m - (\rho_c + \rho_m)\frac{\alpha}{2}\left(1 - \frac{2|z|}{h}\right) \tag{7}$$

2.2 Basic assumptions

The assumptions of the present theory are as follows:

(i) The origin of the Cartesian coordinate system is taken at the median surface of the FG beam.

(ii) The displacements are small in comparison with the height of the beam and, therefore, strains involved are infinitesimal.

(iii) The transverse displacement w includes two components of bending $\mathrm{w_b}$, and shear $\mathrm{w_s}$. These components are functions of coordinates x, y only.

$$\mathrm{w(x,z,t) = w_b(x,t) + w_s(x,t)} \tag{8}$$

(iv) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x.

(v) The axial displacement u in x-direction, consists of extension, bending, and shear components.

$$u = u_0 + u_b + u_s \tag{9}$$

The bending component u_b is assumed to be similar to the displacements given by the classical beam theory. Therefore, the expression for u_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \tag{10}$$

The shear component u_s gives rise, in conjunction with w_s, to the parabolic variation of shear strain γ_{xz} and hence to shear stress τ_{xz} through the thickness of the beam in such a way that shear stress τ_{xz} is zero at the top and bottom faces of the beam. Consequently, the expression for u_s can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \tag{11}$$

where

$$f(z) = z - \frac{1}{2}z\left(\frac{1}{4}h^2 - \frac{1}{3}z^2\right) \tag{12}$$

2.3 Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (2)-(6) as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{13a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{13b}$$

The strains associated with the displacements in Eq. (13) are

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x^b + f(z)\,k_x^s \tag{14a}$$

$$\gamma_{xz} = g(z)\,\gamma_{xz}^s \tag{14b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \tag{14c}$$

$$g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{14d}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x = Q_{11}(z)\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z)\,\gamma_{xz} \tag{15a}$$

where

$$Q_{11}(z) = E(z) \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{15b}$$

2.4 Governing equations and boundary conditions

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Thai and Vo 2012, Draiche *et al*. 2014, Nedri *et al*. 2014, Chattibi *et al*. 2015, Zemri *et al*. 2015)

$$\delta \int_{t_1}^{t_2} (U - T)\,dt = 0 \tag{16}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy and δT is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned} \delta U &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}\right) dz dx \\ &= \int_0^L \left(N \frac{d\delta u_0}{dx} - M_b \frac{d^2 \delta w_b}{dx^2} - M_s \frac{d^2 \delta w_s}{dx^2} + Q \frac{d\delta w_s}{dx} \right) dx \end{aligned} \tag{17}$$

where N, M_b, M_s and Q are the stress resultants defined as

$$(N, M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f)\,\sigma_x dz_{ns} \quad \text{and} \quad Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} g \tau_{xz}\, dz \tag{18}$$

The variation of the kinetic energy can be expressed as

$$\delta T = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\left[\dot{u}\delta\,\dot{u} + \dot{w}\delta\,\dot{w}\right] dz dx$$

$$= \int_0^L \Bigg\{ I_1\left[\dot{u}_0\delta\dot{u}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta\dot{w}_b + \delta\dot{w}_s\right)\right] - I_2\left(\dot{u}_0 \frac{d\delta\dot{w}_b}{dx} + \frac{d\dot{w}_b}{dx}\delta\,\dot{u}_0\right) \tag{19}$$

$$+ I_4\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right) - I_3\left(\dot{u}_0\frac{d\delta\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\delta\,\dot{u}_0\right) + I_6\left(\frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_s}{dx}\right)$$

$$+ I_5\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right)\Bigg\} dx$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_1, I_2, I_3, I_4, I_5, I_6) are the mass inertias defined as

$$\left(I_1, I_2, I_3, I_4, I_5, I_6\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1, z, f, z^2, zf, f^2\right)\rho(z)dz \tag{20}$$

Substituting the expressions for δU and δT from Eqs. (17) and (19) into Eq. (16) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the functionally graded beam are obtained

$$\delta u_0 : \ \frac{dN}{dx} = I_1\ddot{u}_0 - I_2\frac{d\ddot{w}_b}{dx} - I_3\frac{d\ddot{w}_s}{dx} \tag{21a}$$

$$\delta w_b : \ \frac{d^2M_b}{dx^2} = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\frac{d\ddot{u}_0}{dx} - I_4\frac{d^2\ddot{w}_b}{dx^2} - I_5\frac{d^2\ddot{w}_s}{dx^2} \tag{21b}$$

$$\delta w_s : \ \frac{d^2M_s}{dx^2} + \frac{dQ}{dx} = I_1(\ddot{w}_b + \ddot{w}_s) + I_3\frac{d\ddot{u}_0}{dx} - I_5\frac{d^2\ddot{w}_b}{dx^2} - I_6\frac{d^2\ddot{w}_s}{dx^2} \tag{21c}$$

Eq. (21) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (13), (14), (18) and (20) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_b}{\partial x^3} - B_{11}^s\frac{\partial^3 w_s}{\partial x^3} = I_1\ddot{u}_0 - I_2\frac{d\ddot{w}_b}{dx} - I_3\frac{d\ddot{w}_s}{dx} \tag{22a}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_b}{\partial x^4} - D_{11}^s\frac{\partial^4 w_s}{\partial x^4} = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\frac{d\ddot{u}_0}{dx} - I_4\frac{d^2\ddot{w}_b}{dx^2} - I_5\frac{d^2\ddot{w}_s}{dx^2} \tag{22b}$$

$$B_{11}^{s}\frac{\partial^3 u_0}{\partial x^3}-D_{11}^{s}\frac{\partial^4 w_b}{\partial x^4}-H_{11}^{s}\frac{\partial^4 w_s}{\partial x^4}+A_{55}^{s}\frac{\partial^2 w_s}{\partial x^2}=I_1(\ddot{w}_b+\ddot{w}_s)+I_3\frac{d\ddot{u}_0}{dx}-I_5\frac{d^2\ddot{w}_b}{dx^2}-I_6\frac{d^2\ddot{w}_s}{dx^2} \quad (22c)$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11},B_{11},D_{11},B_{11}^{s},D_{11}^{s},H_{11}^{s}\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)dz \quad (23a)$$

and

$$A_{55}^{s}=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{55}[g(z)]^2\,dz \quad (23b)$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_m \cos(\lambda x)\, e^{i\omega t} \\ W_{bm} \sin(\lambda x)\, e^{i\omega t} \\ W_{sm} \sin(\lambda x)\, e^{i\omega t} \end{Bmatrix} \quad (24)$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with m th eigenmode, and $\lambda = m\pi / L$.

Substituting the expansions of u_0, w_b, w_s from Eqs. (24) into the equations of motion Eq. (22), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}-\omega^2\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (25)$$

where

$$a_{11}=A_{11}\lambda^2,\ a_{12}=-B_{11}\lambda^3,\ a_{13}=-B_{11}^{s}\lambda^3,\ a_{22}=D_{11}\lambda^4,\quad a_{23}=D_{11}^{s}\lambda^4 \quad (26a)$$

$$a_{33}=H_{11}^{s}\lambda^4+A_{55}^{s}\lambda^2$$

$$m_{11}=I_1,\quad m_{12}=-I_2\lambda,\quad m_{13}=-I_3\lambda,\quad m_{22}=I_1+I_4\lambda^2$$

$$m_{23} = I_1 + I_5 \lambda^2 \qquad (26b)$$
$$m_{33} = I_1 + I_6 \lambda^2$$

4. Results and discussion

In numerical analysis, fundamental frequencies of simply supported perfect and imperfect FG beams are evaluated. The FG beam is taken to be made of aluminum and alumina with the following material properties:

Ceramic (P_C : Alumina, Al2O3): $E_c = 380$ GPa; $\nu = 0.3$; $\rho_c = 3800$ kg/m3.
Metal (P_M : Aluminium, Al): $E_m = 70$ GPa; $\nu = 0.3$; $\rho_m = 2700$ kg/m3.

And their properties change through the thickness of the beam according to power-law. The bottom surfaces of the FG beams are aluminum rich, whereas the top surfaces of the FG beams are alumina rich.

For convenience, the following dimensionless form is used

$$\bar{\omega} = \frac{\omega L^2}{h} \sqrt{\frac{\rho_m}{E_m}}$$

To validate accuracy of the proposed theory, the comparisons between the present results and the available results obtained by Simsek (2010a) and Sina *et al*. (2009) is shown in Table 1.

Indeed, in Table 1, the non-dimensional natural frequencies for the perfect FG beam with $k = 0.3$ for different length-to-height ratios. As can be seen the results of the present theory are in good agreement with the other shear deformation theories.

The first five dimensionless frequencies of perfect and imperfect FG beams are provided in Table 2. It should be noted that the materials properties are predicted using Eqs. (3) and (4). The results reveal that the frequency results decrease as the volume fraction of porosity (α) increases.

In Figs. 2 and 3, the effect of the porosity the fundamental frequencies of FG beams with two different types of porosity distribution is illustrated. It is noted that Solution I refers to the result of imperfect FG beams with evenly distributed porosities using Eqs. (3) and (4), while, Solution II is for the beams with another type of porosity distribution using Eqs. (6) and (7). It can be seen from Fig. 2 that the porosity leads to a decrease of frequency and hence this type of porosity distribution (Solution I) makes the beam flexible. However, the effect of porosity on fundamental frequencies (Fig. 3) using Solution II is reversed and this type of porosity distribution makes the beam stiffer.

In Fig. 4, the fundamental frequencies of imperfect FG beams with two different types of porosity distribution are plotted versus the power-law exponent (k). As observed, Solution II provides higher frequencies than those of Solution I; moreover, the frequencies increase with the increase of the power-law exponent (k) when this latter takes values more than 2.

Table 1 Comparison of non-dimensional fundamental frequencies of FG beams with $k = 0.3$ $\left(\bar{\omega} = \left(\omega L^2 / h\right)\sqrt{I_0 / \int_{-h/2}^{h/2} E(z)dz}\right)$.

Source	$L/h = 10$	$L/h = 30$	$L/h = 100$
$FSDBT^R$ (Simsek 2010a)	2.701	2.738	2.742
$FSDBT^S$ (Simsek 2010a)	2.701	2.738	2.742
$PSDBT^R$ (Simsek 2010a)	2.702	2.738	2.742
$PSDBT^S$ (Simsek 2010a)	2.702	2.738	2.742
$ASDBT^R$ (Simsek 2010a)	2.702	2.738	2.742
$ASDBT^S$ (Simsek 2010a)	2.702	2.738	2.742
Sina *et al*. (2009)	2.695	2.737	2.742
Present	2.702	2.738	2.743

Table 2 Five five Non-dimensional frequencies of FGM beam (L/h=5)

k	α	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_4$	$\bar{\omega}_5$
0.5	0	4.4107	15.4588	29.8383	45.6900	62.2054
	0.1	4.4042	15.4572	29.8721	45.7847	62.3858
	0.2	4.3928	15.4438	29.8933	45.8745	62.5647
1	0	3.9904	14.0099	27.0975	41.5839	56.7346
	0.1	3.9070	13.7552	26.6754	41.0218	56.0673
	0.2	3.7865	13.3831	26.0497	40.1813	55.0562
5	0	3.4012	11.5430	21.7157	32.6789	43.9940
	0.1	3.1479	10.6851	20.1236	30.3323	40.9081
	0.2	2.6962	9.2066	17.4579	26.4951	35.9587
10	0	3.2816	11.0240	20.5566	30.7218	41.1431
	0.1	3.0292	10.1034	18.7478	27.9411	37.3584
	0.2	2.5718	8.5021	15.7008	23.3593	31.2283

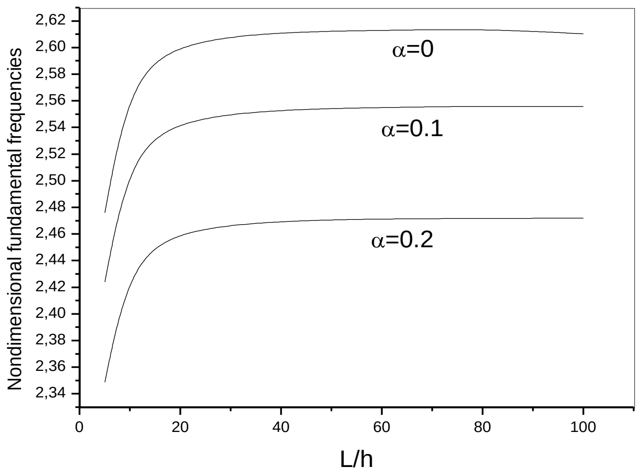

Fig. 2 Variation of the fundamental frequency $\left(\overline{\omega} = \left(\omega L^2 / h\right)\sqrt{I_0 / \int_{-h/2}^{h/2} E(z)dz}\right)$ of FG beams $(k = 1)$ with L/h ratio for various values of the porosity volume fraction by considering the first solution

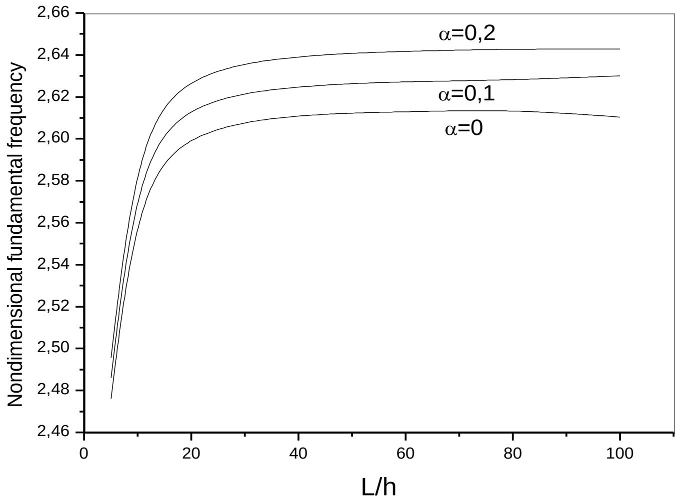

Fig. 3 Variation of the fundamental frequency $\left(\overline{\omega} = \left(\omega L^2 / h\right)\sqrt{I_0 / \int_{-h/2}^{h/2} E(z)dz}\right)$ of imperfect FG beams with power-law exponent k. $(L/h = 5, \alpha = 0.1)$.

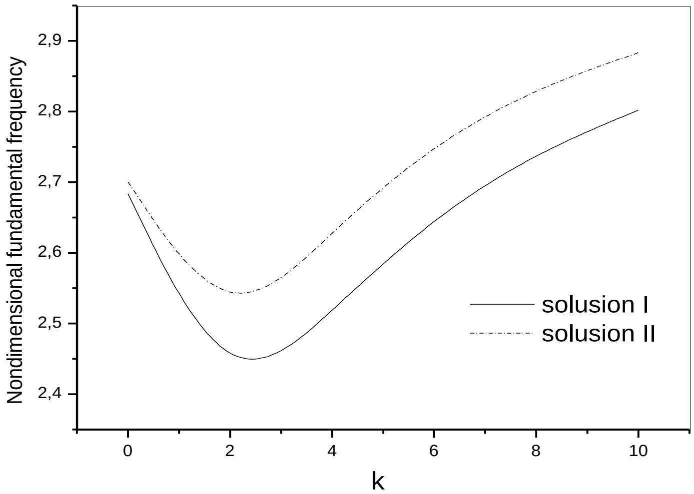

Fig. 4 Variation of the fundamental frequency $\left(\bar{\omega} = \left(\omega L^2 / h\right)\sqrt{I_0 / \int_{-h/2}^{h/2} E(z)dz}\right)$ of imperfect FG beams with power-law exponent k. $(L/h = 5, \alpha = 0.1)$.

5. Conclusions

A new shear deformation beam theory is proposed for free vibration of perfect and imperfect FG beams. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. The modified rule of mixture covering porosity phases is used to describe and approximate material properties of the imperfect FG beams. It is based on the assumption that the transverse displacements consist of bending and shear components. Based on the present beam theory, the equations of motion are derived from Hamilton's principle. The influence of the porosities on natural frequencies is then discussed. Numerical examples show that the proposed theory gives solutions which are almost identical with those obtained using other shear deformation theories.

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