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Non-spillover control design of tall buildings in modal space

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Abstract. In this paper, a new algorithm for active control design of structures is proposed and investigated. The algorithm preserves the decoupling property of the modal vibration equation and eliminates the spillover problem, which is the main shortcoming in the independent modal space control (IMSC) algorithm. With linear quadratic regulator(LQR) control law, the analytical solution of algebraic Riccati equation and the optimal actuator control force are obtained, and the control design procedure is significantly simplified. A numerical example for the control design of a tall building subjected to wind loads demonstrates the effectiveness of the proposed algorithm in reducing the acceleration and displacement responses of tall buildings under wind actions.

Key words: structural control; tall buildings; wind-induced vibration.

1. Introduction

The modern high-rise buildings are constructed to be more flexible and lightly damped than in the past. Therefore, the emphasis in the design of modern tall buildings has been shifted to design the buildings to satisfy lateral drift requirements. The level of oscillations of tall buildings during windstorm may not be significant enough to cause structural damage but may cause discomfort to the building occupants. The structural control, thus, play a more and more important role in reducing wind-induced vibrations during strong windstorms.

In recent years, great progress has been achieved in the field of active structural vibration control. A variety of control algorithms have been developed specifically for civil engineering structures (Yang, Akbarpour and Ghaemmaghami 1987, Yang, Li and Liu 1991, Yang, Li and Liu 1992) and a number of full-scale active control systems have been installed in actual structures and have performed well for the purposes intended (Soong, Reinhorn, Wang, and Lin 1991, Soong and Reinhorn 1993). The IMSC algorithm, in which the control design is carried out in the modal space, may be one of the simplest algorithms for structural control, and has been widely used in the control design. However, there are intrinsic shortcomings in this algorithm, in which one of them may be the spillover program, i.e., the control inputs play the similar role as external excitation for the higher modes when the control design is based on the lower modes. As a result, the responses of the controlled structure may be

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underestimated. The spillover problem of the IMSC algorithm has been extensively discussed by several investigators, i.e., Soong (1990), Soong and Chang (1982). In this paper, based on the LQR control law, a new control algorithm, non-spillover control law, is presented. In the proposed algorithm, not only the decoupling property of the vibration equation in the modal space is preserved, but also the vibration of higher uncontrolled modes is unrelated to the control forces. The response of the controlled structure, thus, can be estimated more efficiently and precisely. For specified weight matrices, the analytical solution of algebraic Riccati equation is obtained, the computation cost is thus reduced significantly. However, it should be pointed out that this study does not include the consideration of actuator dynamics since the main attention herein is paid to the investigation of the control algorithm.

2. Control algorithm

2.1. Comments on IMSC algorithm

The vibration equation of controlled MDOF system is

$$\boldsymbol{M}\ddot{z}(t)+\boldsymbol{C}\dot{z}(t)+\boldsymbol{K}z(t)=\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{W}(t) \tag{1}$$

Where, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are $n\times n$ mass, damping and stiffness matrices, respectively; $z(t)=[z_1, z_2, \cdots, z_n]^T$ is n-dimensional vector of displacements; $\boldsymbol{u}(t)$ is an m-dimensional control-force vector; $\boldsymbol{D}$ is an $n\times m$ matrix which denotes the location of the control forces; $W(t)=[w_1, w_2, \cdots, w_n]^T$ is an n-dimensional wind loading vector; the superscript T denotes vector or matrix transpose.

Let $z(t)=\boldsymbol{\Phi}\boldsymbol{q}(t)$, where $\boldsymbol{\Phi}$ is a modal matrix and $\boldsymbol{q}$ is the vector of modal displacements. Eq. (1) may be rewritten as

$$\boldsymbol{M}^*\ddot{\boldsymbol{q}}(t)+\boldsymbol{C}^*\dot{\boldsymbol{q}}(t)+\boldsymbol{K}^*\boldsymbol{q}(t)=\boldsymbol{H}\boldsymbol{u}(t)+\boldsymbol{E}(t) \tag{2}$$

where

$$\boldsymbol{H}=\boldsymbol{\Phi}^T\boldsymbol{D}\,,\quad \boldsymbol{E}(t)=\boldsymbol{\Phi}^T\boldsymbol{W}(t) \tag{3a}$$

$$\boldsymbol{M}^*=\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi},\qquad \boldsymbol{C}^*=\boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi},\qquad \boldsymbol{K}^*=\boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} \tag{3b}$$

Assume that the control design is only based on the first R modes, which are referred to controlled modes, and higher modes are referred to uncontrolled modes. Rewrite Eq. (2) as

$$\boldsymbol{M}_c^*\ddot{\boldsymbol{q}}_c(t)+\boldsymbol{C}_c^*\dot{\boldsymbol{q}}(t)+\boldsymbol{K}_c^*\boldsymbol{q}_c(t)=\boldsymbol{H}_c\boldsymbol{u}(t)+\boldsymbol{E}_c(t) \tag{4a}$$

and

$$\boldsymbol{M}_r^*\ddot{\boldsymbol{q}}_r(t)+\boldsymbol{C}_r^*\dot{\boldsymbol{q}}(t)+\boldsymbol{K}_r^*\boldsymbol{q}_r(t)=\boldsymbol{H}_r\boldsymbol{u}(t)+\boldsymbol{E}_r(t) \tag{4b}$$

where

$$\boldsymbol{H}=\begin{bmatrix}\boldsymbol{H}_c\\ \boldsymbol{H}_r\end{bmatrix}\quad \boldsymbol{E}=\begin{bmatrix}\boldsymbol{E}_c\\ \boldsymbol{E}_r\end{bmatrix}\quad \boldsymbol{M}^*=\begin{bmatrix}\boldsymbol{M}_c^* & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_r^*\end{bmatrix} \tag{5a}$$

$$C^* = \begin{bmatrix} C_c^* & 0 \\ 0 & C_r^* \end{bmatrix} \quad K^* = \begin{bmatrix} K_c^* & 0 \\ 0 & K_r^* \end{bmatrix} \tag{5b}$$

Obviously, $\boldsymbol{H}=\boldsymbol{\Phi}^T$ if all the DOF of the system are controlled.

The causal and linear controller $\boldsymbol{G}$ is designed to generate the control signal $\boldsymbol{u}(t)$, according to

$$\boldsymbol{u}(t) = -\boldsymbol{G}\begin{bmatrix} \boldsymbol{q}_c \\ \dot{\boldsymbol{q}}_c \end{bmatrix} = -\boldsymbol{G}_1\boldsymbol{q}_c - \boldsymbol{G}_2\dot{\boldsymbol{q}}_c \tag{6}$$

in which $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are sub-matrices of $\boldsymbol{G}$ with dimension $m \times R$.

Eqs. (4a,b) and be rewritten as

$$\boldsymbol{M}_c^*\ddot{\boldsymbol{q}}_c(t) + (\boldsymbol{C}_c^* + \boldsymbol{H}_c\,\boldsymbol{G}_2)\dot{\boldsymbol{q}}_c(t) + (\boldsymbol{K}_c^* + \boldsymbol{H}_c\,\boldsymbol{G}_1)\boldsymbol{q}_c(t) = \boldsymbol{E}_c(t) \tag{7a}$$

and

$$\boldsymbol{M}_r^*\ddot{\boldsymbol{q}}_r(t) + \boldsymbol{C}_r^*\dot{\boldsymbol{q}}_r(t) + \boldsymbol{K}_r^*\boldsymbol{q}_r(t) = -\boldsymbol{H}_r\,[\boldsymbol{G}_1\boldsymbol{q}_c(t) + \boldsymbol{G}_2\dot{\boldsymbol{q}}_c(t)] + \boldsymbol{E}_r(t) \tag{7b}$$

In general the matrices $\boldsymbol{H}_c\boldsymbol{G}_2$ and $\boldsymbol{H}_c\boldsymbol{G}_1$, in Eq. (7a), are not diagonal matrices, thus the governing equations of the controlled modes are a set of coupling equations, this means that the decoupling characteristics of the modal decomposition can not be kept in the conventional IMSC algorithm. It can also be seen, from Eq. (7b), that both the external loading and the control forces make contributions to the response of the uncontrolled modes, i.e., the responses of uncontrolled modes are related to both the response of controlled modes and the external loading. Fang *et al.* (1997) pointed out that, if the control design is based on the first mode, the modal control force could increase the root mean square response of modal acceleration by 4.22% for a base excited controlled structure. The similar conclusion has been also made by them for wind excited structures. The structural acceleration response, thus, may be significantly underestimated when the response estimation is based on the controlled modes due to the spillover problem.

2.2. Non-spillover IMSC algorithm

Suppose that $\boldsymbol{G}$ is a linear and causal controller, both $\boldsymbol{H}_c\boldsymbol{G}_1$ and $\boldsymbol{H}_c\boldsymbol{G}_2$ are diagonal matrices and $\boldsymbol{H}_r\boldsymbol{G}_1 = \boldsymbol{H}_r\boldsymbol{G}_2 = \boldsymbol{0}$ in Eqs. (7a,b), one yields

$$\boldsymbol{HG}_1 = \begin{bmatrix} s_1 & & \\ & \ddots & \\ & & s_R \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{0} \end{bmatrix}_{n\times R} \qquad \boldsymbol{HG}_2 = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_R \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{d} \\ \boldsymbol{0} \end{bmatrix}_{n\times R} \tag{8}$$

Note that $\boldsymbol{D} \equiv \boldsymbol{I}$, i.e., $\boldsymbol{H} = \boldsymbol{\Phi}^T$ when all DOF of the system are controlled.

Eqs. (7a,b) thus can be rewritten as

$$M_c^* \ddot{q}_c(t) + (C_c^* + d)\dot{q}_c(t) + (K_c^* + s)q_c(t) = E_c(t) \tag{9a}$$

and

$$M_r^* \ddot{q}_r(t) + C_r^* \dot{q}_r(t) + K_r^* q_r(t) = E_r(t) \tag{9b}$$

Eq. (9a) is a set of decoupling equations, and Eq. (9b) shows that the governing equations of the uncontrolled modes are unrelated to the control forces. Therefore, the non-spillover problem can be eliminated if the causal controller satisfies Eq. (8).

Consider a configuration in which all the DOF are controlled, i.e., $H = \Phi^T$ as mentioned above (Note that $m \geq R$ must be satisfied for the proposed algorithm, see appendix I). Without losing the generality, the following assumptions are made

$$G_1 = M\Phi F_1 \qquad G_2 = M\Phi F_2 \tag{10}$$

Substituting Eq. (10) into Eq. (8) and using the orthogonality of mode shapes, one yields

$$F_1 = [M^*]^{-1}\begin{bmatrix} s \\ 0 \end{bmatrix} \qquad F_2 = [M^*]^{-1}\begin{bmatrix} d \\ 0 \end{bmatrix} \tag{11}$$

Then

$$G_1 = M\Phi_c\,[M_c^*]^{-1}s \qquad G_2 = M\Phi_c\,[M_c^*]^{-1}d \tag{12}$$

where $\Phi_c = (\Phi)_{n\times R}$, i.e., Φ_c is a partition matrix of Φ formed by the first R modes.

It is easy to verify that the responses of the higher uncontrolled modes are unrelated to the actuator control force because

$$H_r G_1 = \Phi_r^T M\Phi_c\,[M_c^*]^{-1}s = 0 \qquad H_r G_2 = \Phi_r^T M\Phi_c\,[M_c^*]^{-1}d = 0 \tag{13}$$

The entries of matrices $G_1 = (g_{ij})_{n\times R}$ and $G_2 = (\bar{g}_{ij})_{n\times R}$ are obtained from Eq. (12) as

$$g_{ij} = \frac{m_i\,\phi_{i,j}\,s_j}{m_j^*} \qquad \bar{g}_{ij} = \frac{m_i\,\phi_{i,j}\,d_j}{m_j^*} \qquad (i = 1, 2, \cdots n;\ j = 1, 2, \cdots, R) \tag{14}$$

Eqs. (10) and (14) show that the feedback gain matrices can be obtained indirectly from the diagonal matrices s and d. In the following study, an attempt is made to propose an algorithm for calculation the diagonal matrices s and d.

Rewrite Eq. (4a) as a state-space equation as follows

$$\dot{Y} = AY + DE_c(t) + Bu(t) \tag{15}$$

in which

$$Y = [q_c\,,\ \dot{q}_c]^T, \quad A = \begin{bmatrix} 0 & I \\ -\operatorname{diag}(\omega_j^2) & -\operatorname{diag}(2\xi_j\,\omega_j) \end{bmatrix} \tag{16a}$$

$$D = \begin{bmatrix} 0 \\ [M_c^*]^{-1} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ [M_c^*]^{-1}\,\Phi_c^T \end{bmatrix} \tag{16b}$$

and diag $(\omega_j^2)=[M_c^*]^{-1}[K_c^*]$, diag$(2\xi_j\ \omega_j)=[M_c^*]^{-1}[K_c^*]$, respectively.

The quadratic performance index is

$$J=\int_0^{\infty}(Y^T QY+u^T Ru)\,dt \tag{17}$$

The weight matrix Q is as the form of $Q=\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$, in which the sub-matrices Q_1 and Q_2 are diagonal matrices. The control force vector $u(t)$ is obtained as

$$u(t)=-R^{-1}B^T PY \tag{18}$$

in which, P satisfies the following algebraic Riccati equation

$$PA+A^T P-PBR^{-1}B^T P+Q=0 \tag{19}$$

Matrix P has the apartition form $P=\begin{bmatrix} P_{11} & P_{21} \\ P_{21} & P_{22} \end{bmatrix}$, then Eq. (19) can be rewritten as

$$P_{21}[M_c^*]^{-1}K_c^*+[M_c^*]^{-1}K_c^*P_{21}+P_{21}[M_c^*]^{-1}\Phi_c^T R^{-1}\Phi_c\,[M_c^*]^{-1}P_{21}-Q_1=0 \tag{20a}$$

$$P_{22}[M_c^*]^{-1}C_c^*+[M_c^*]^{-1}C_c^*P_{22}+P_{22}[M_c^*]^{-1}\Phi_c^T R^{-1}\Phi_c\,[M_c^*]^{-1}P_{22}-2P_{21}-Q_2=0 \tag{20b}$$

and the control force vector is

$$u(t)=-R^{-1}\Phi_c[M_c^*]^{-1}P_{21}q_c-R^{-1}\Phi_c\,[M_c^*]^{-1}P_{22}\dot{q}_c \tag{21}$$

By comparison of Eq. (10) with (21), one yields

$$M\Phi_c[M_c^*]^{-1}s=R^{-1}\Phi_c\,[M_c^*]^{-1}P_{21} \tag{22}$$

$$M\Phi_c[M_c^*]^{-1}d=R^{-1}\Phi_c\,[M_c^*]^{-1}P_{22} \tag{23}$$

The weight matrix R is expressed as

$$R^{-1}=\frac{1}{R_1}M \tag{24}$$

where R_1 is a constant which can be chosen according to the control condition.

Substituting Eq. (24) into Eqs. (22) and (23), one obtains

$$P_{21}=R_1 s \qquad P_{22}=R_1 d \tag{25}$$

The solutions of Eqs. (20a,b), thus, are

$$s_j=k_j^*\left(\sqrt{1+\frac{Q_{1j}m_j^*}{R_1(k_j^*)^2}}-1\right) \tag{26}$$

$$d_j = c_j^* \left(\sqrt{1 + \frac{L_j m_j^*}{R_1 (c_j^*)^2}} - 1 \right) \tag{27}$$

where

$$L_j = \frac{m_j^* Q_{1j}}{2k_j^* + s_j} + Q_{2j} \qquad (j = 1, 2, \cdots, R) \tag{28}$$

Therefore, the feedback gain matrices $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ can be obtained from Eq. (14) by using Eqs. (26) and (27).

3. Application of the proposed algorithm

3.1. Specifications of a building

The building used as an example for the control design is specified in Tsukagoshi, Tamura, Sasaki and Kanai (1993). This high-rise building has a square plan, an aspect ratio H/D=4 (H= 160 m, both length and width are 40 m) and 40 storey as shown in Fig. 1. The building is modelled as a mass-spring-damping system of 10 degrees of freedom. The stiffness is assumed to have a trapezoidal distribution with the ratio of the value at the top to that at the bottom of 1:2, and the fundamental damping ratio to the critical value is set as 1% on the assumption that the damping ratio varies proportionally with the stiffness. Table 1 shows the specifications of the building model.

3.2. Simulation of wind force

The wind velocity profile along the vertical direction is written as

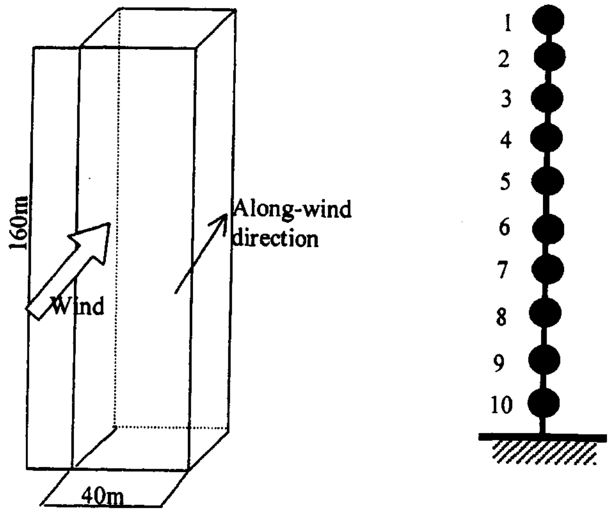

Fig. 1 The configuration of a building

Table 1 Specifications of the building model

Mass No.	Height (m)	Projected area (m^2)	Mass (t)	Spring constant (KN/m^2)
1	152	640	3920	2.60×10^5
2	136	640	3920	2.89×10^5
3	120	640	3920	3.18×10^5
4	104	640	3920	3.47×10^5
5	88	640	3920	3.76×10^5
6	72	640	3920	4.05×10^5
7	56	640	3920	4.34×10^5
8	40	640	3920	4.63×10^5
9	24	640	3920	4.91×10^5
10	8	640	3920	5.20×10^5

$$V(z, t) = \bar{V}(z) + v(z, t) \tag{29}$$

where $\bar{V}(z)$ denotes the mean wind speed at height z and is assumed to follow the power law as

$$\bar{V}(z) = \bar{V}_{10}\,(z/10)^{\alpha} \tag{30}$$

in which $\bar{V}_{10}$ is the mean wind speed at 10 m height, α is a constant, and $v(z, t)$ is the fluctuating component of wind velocity with a specified non-white noise spectrum.

The horizontal wind force acting on the ith floor is the drag force and can be expressed as Simiu and Scanlan (1996)

$$W_D(z_i, t) = \frac{1}{2}\rho C_D A(z_i) V^2(z, t) = \frac{1}{2}\rho C_D A(z_i)[\bar{V}(z_i) + v(z_i, t)]^2 \tag{31}$$

Since in high winds $v(z_i, t)/\bar{V}(z_i)$ rarely exceeds 0.2 (Simiu and Scanlan 1996), $v^2(z_i, t)$ may generally be neglected with small error yielding

$$W_D(z_i, t) = \bar{W}_D(z_i) + \tilde{W}(z_i, t) \tag{32}$$

where $\bar{W}_D(z_i)$ and $\tilde{W}(z_i, t)$ are the steady and turbulent components of the drag force, respectively

$$\bar{W}_D(z_i) = \frac{1}{2}\rho A^2(z_i) C_D [\bar{V}^2(z_i) + v^2(z_i, t)] \tag{33a}$$

$$\tilde{W}_D(z_i, t) = \rho \bar{V}(z_i) A^2(z_i) C_D\, v(z_i, t) \tag{33b}$$

in which $\bar{V}(z_i)$ is mean wind speed at elevation z_i, $v(z_i, t)$ is turbulent component of wind speed, ρ is air density(1.225 kg/m^3), C_D is drag coefficient that equals to 1.18 in the current calculation, $A(z_i)$ is structural area (reference area) perpendicular to the wind at height z_i.

The method developed by Shinozuka and Deodatis (1988) for simulating the time series of the turbulent component of wind speed is as

$$v(z, t) = \sqrt{2}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sqrt{2S(\omega_i, k_j)\Delta\omega\Delta k}\,[\cos(\omega_i t + k_j z + \theta_{ij}) + \cos(\omega_i t - k_j z + \phi_{ij})] \tag{34}$$

where $S(\omega_i, k_j)$ is spectral density function of turbulent wind; $\Delta\omega = \omega_u/N_1$; $\Delta k = k_u/N_2$, ω_u and k_u

are cut-off frequency and cut-off wave number, respectively; N_1 and N_2 are samples number in the frequency domain and wave number domain, respectively; θ_{ij}, ϕ_{ij} are random phase angles which are uniformly distributed between 0 and 2π.

The spectral density function suggested by Davenport (1961, 1962) for turbulent wind was expressed as

$$S(\omega, k)=\frac{K'\Phi^2}{2\pi^2}\frac{|\omega|}{[1+\Phi^2\omega^2/(2\pi\bar{V}_{10}^2]^{4/3}}\frac{\varepsilon|\omega|}{\pi(\varepsilon^2\omega^2+k^2)} \tag{35}$$

where Φ= scale of turbulence, K' is surface drag coefficient, ε is a constant. Fig. 2 shows a simulated wind speed time history at 10 meter height with $\varepsilon=0.046$ m/s, $\Phi=470$ m, $\alpha=0.16$, $\bar{V}_{10}=30$ m/s and $K'=2.5\times10^{-3}$, respectively.

3.3. Control design

The governing equations of the controlled modes are

$$\ddot{q}_j(t)+2\xi_j\,\omega_j\,\dot{q}_j(t)+\omega_j^2 q_j(t)=\sum_{i=1}^{n}\Lambda_{ij}\,[u_i(t)+F(z_i\,,t)] \qquad (i=1,2,\cdots,R) \tag{36}$$

in which $\Lambda_{ij}=\phi_{i,j}/m_j^*$, and $\phi_{i,j}$ is the ith element of the jth modal shape.

Let R=1, i.e., the control design is based on the first mode, Eq. (36) may be rewritten as

$$\dot{\boldsymbol{Y}}(t)=\boldsymbol{AY}(t)+\boldsymbol{DF}(t)+\boldsymbol{Bu}(t) \tag{37}$$

in which

$$\boldsymbol{Y}=[q_1,\dot{q}_1]^T,\quad \boldsymbol{A}=\begin{bmatrix}0 & 1\\ \omega_1^2 & 2\xi_1\omega_1\end{bmatrix} \tag{38}$$

$$\boldsymbol{B}=\boldsymbol{D}=\begin{bmatrix}0 & \cdots & 0 & 0\\ \phi_{1,1}/m_1^* & \cdots & \phi_{n-1,1}/m_1^* & \phi_{n,n}/m_1^*\end{bmatrix} \tag{39}$$

Let $\boldsymbol{Q}=\begin{bmatrix}Q_1 & 0\\ 0 & Q_2\end{bmatrix}$, and $\boldsymbol{R}^{-1}=R_1^{-1}\boldsymbol{M}$, Eq. (14) is as

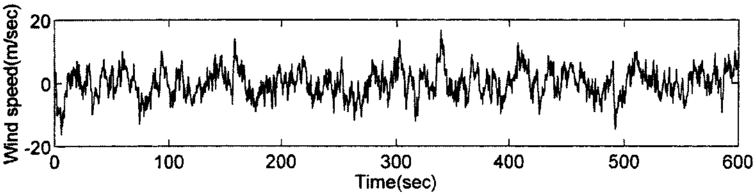

Fig. 2 The time history of fluctuating wind speed

$$g_i = \frac{m_i \,\phi_{i,1} s_1}{m_1^*} \qquad \bar{g}_i = \frac{m_i \,\phi_{i,1} d_1}{m_1^*} \qquad (i = 1, 2, \cdots, n) \tag{40}$$

and s_1, d_1 are

$$s_1 = k_1^* \left(\sqrt{1 + \frac{Q_1 m_1^*}{R_1 (k_1^*)^2}} - 1 \right) \tag{41}$$

$$d_1 = c_1^* \left(\sqrt{1 + \frac{L_1 m_1^*}{R_1 (c_1^*)^2}} - 1 \right) \tag{42}$$

where

$$L_1 = \frac{m_1^* Q_1}{2k_1^* + s_1} + Q_2 \tag{43}$$

Eq. (37) is then rewritten as

$$\ddot{q}_1(t) + \left[2\xi_1 \omega_1 + \frac{\sum_{i=1}^{n} \phi_{i,1} \bar{g}_i}{m_1^*} \right] \dot{q}_1(t) + \left[\omega_1^2 + \frac{\sum_{i=1}^{n} \phi_{i,1} g_i}{m_1^*} \right] q_j(t) = \sum_{i=1}^{n} \Lambda_{i1} F(z_i, t) \tag{44}$$

Let $Q_1 = Q_2 = 10^5$ and $R_1 = 0.1$, one gives $d_1 = 2.049 \times 106$ and $s_1 = 2.005 \times 10^5$. The feedback gain matrices, from Eq. (40), are

$G_1 = [\,1.062, 2.167, 3.293, 4.421, 5.524, 6.570, 7.520, 8.328, 8.938, 9.284\,]^T \times 10^4$

$G_2 = [\,1.086, 2.216, 3.367, 4.520, 5.648, 6.717, 7.688, 8.514, 9.138, 9.492\,]^T \times 10^5$

3.4. Results analysis

Fig. 3 and Fig. 4 show the comparison of the responses (in the first 5 minutes) of Mass No. 1 and Mass No. 2 corresponding to the controlled and uncontrolled configurations. It can be seen that, due to the control, the maximum displacement and maximum acceleration are reduced significantly. The peak acceleration and displacement responses corresponding to each mass are shown in Fig. 5. For the top mass, the maximum acceleration is decreased by 65.8%, and the maximum displacement is reduced by 50%, respectively. The peak acceleration and displacement corresponding the second mass are decreased by 50% and 76.9%, respectively, as shown in Table 2. These results show that the vibration of the building has been suppressed significantly when the control design is carried out based on the proposed algorithm. For comparison purpose, the control design of the same building is re-carried out using the conventional IMSC algorithm. The responses of the building are given in Table 2. It can be seen

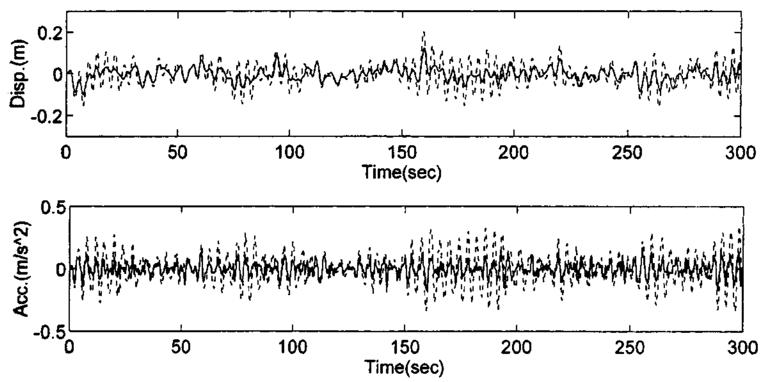

Fig. 3 The comparison of responses (Mass No. 1) (Uncontrolled ········ Controlled ———)

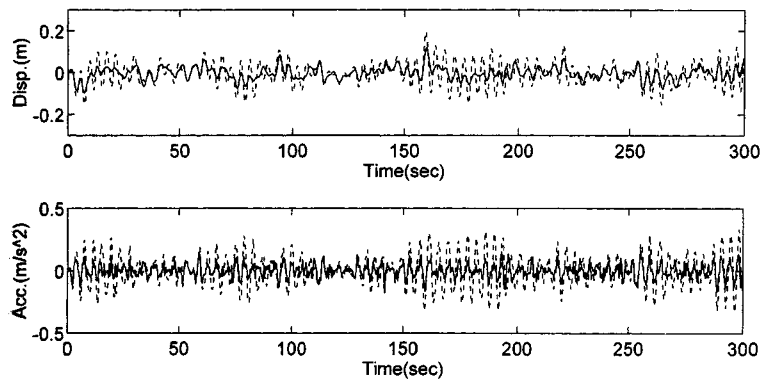

Fig. 4 The comparison of responses (Mass No. 2) (Uncontrolled ········ Controlled ———)

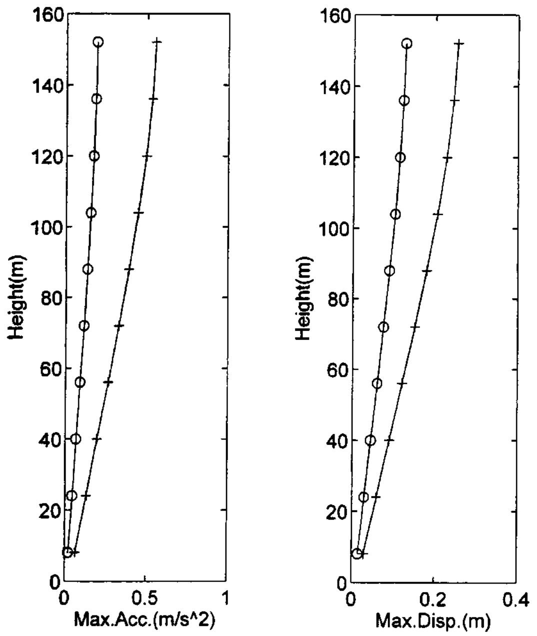

Fig. 5 The peak responses of the building (Uncontrolled + Controlled o)

Table 2 The peak values of the responses

		Controlled	Uncontrolled	Reduced ratio (%)	IMSC
Acceleration	No. 1 A_{max}	0.1888	0.5522	65.8	0.2017
(m/s^2)	No. 2 A_{max}	0.1270	0.2540	50.0	0.1302
Displacement	No. 1 D_{max}	0.1817	0.5617	67.6	0.1846
(m)	No. 2 D_{max}	0.1223	0.5317	76.9	0.1296

Table 3 The comparison of the control forces

	Mass No.	IMSC	Proposed	Increasing ratio(%)
Control force	No. 1	221.1	238.6	7.91
(KN)	No. 2	183.5	196.2	6.92

that the proposed algorithm is more efficient than the conventional IMSC algorithm. In order to compare the energy consuming of the two algorithms, the peak values of the control forces acting on the mass 1 and mass 2 are given in Table 3. The control forces required by the proposed algorithm are greater than those needed by the conventional IMSC algorithm. This is due to the fact, for the proposed algorithm, that the feedback matrices have to satisfy Eq. (8), there is less freedom in selecting the weight matrices in the proposed algorithm. However, the increase of control forces in the proposed algorithm is not significant. Considering the control efficiency, this increase is acceptable. It should be pointed out that the control of the cross-wind vibration is not studied herein because the scope of this paper is focused on the algorithm studies.

4. Conclusions

In this paper, a new algorithm for structural control implementation is proposed on the basis of the elimination of spillover effects of the IMSC algorithm. The decoupling property of the governing equations of the controlled structure in modal space is preserved since the proposed algorithm is based on the diagonal matrix. With LQR control design, the analytical solution of algebraic Riccati equation and the optimal actuator control force have been obtained for the proposed algorithm. Thus, the proposed algorithm significantly simplifies the control design procedure. A numerical example for control design of a tall building subjected to wind loading has been given for verifying the effectiveness of the proposed algorithm. The results of the numerical simulations have demonstrated the effectiveness of the proposed algorithm in reducing the acceleration and displacement responses of tall buildings under wind actions.

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Appendix

Proposition: When the control design is carried out by the proposed algorithm, the following relation must be satisfied

$$m \geq R \tag{I1}$$

where m is the number of the control forces and R is the number of the controlled modes

Proof:

In the MIMSE, Eq. (36) must be held, then, one has

$$\operatorname{rank}(\boldsymbol{H}\boldsymbol{G}_1)=\operatorname{rank}(\boldsymbol{\Phi}^T \boldsymbol{D}\boldsymbol{G}_1)=R \tag{I2a}$$

$$\operatorname{rank}(\boldsymbol{H}\boldsymbol{G}_2)=\operatorname{rank}(\boldsymbol{\Phi}^T \boldsymbol{D}\boldsymbol{G}_2)=R \tag{I2b}$$

rank($\boldsymbol{\Phi}^T$)=n since $\boldsymbol{\Phi}^T$ is a nonsingular matrix, there are finite elementary matrices $\boldsymbol{L}_i$ ($i = 1, 2, \cdots r$) to make the following equation bold

$$\boldsymbol{\Phi}^T = \boldsymbol{L}_1 \boldsymbol{L}_2 \cdots \boldsymbol{L}_r \tag{I3}$$

therefore

$$\boldsymbol{\Phi}^T \boldsymbol{D} = \boldsymbol{L}_1 \boldsymbol{L}_2 \cdots \boldsymbol{L}_r \boldsymbol{D} \tag{I4}$$

Because the rank of the matrix remains the same when it is multiplied by the elementary matrices, then

$$\operatorname{rank}(\boldsymbol{\Phi}^T \boldsymbol{D})=\operatorname{rank}(\boldsymbol{L}_1 \boldsymbol{L}_2 \cdots \boldsymbol{L}_r \boldsymbol{D})=\operatorname{rank}(\boldsymbol{D})=m \tag{I5}$$

If $m<R$, then rank $(\boldsymbol{G}_1)$ = rank $(\boldsymbol{G}_2) = m$, so that

$$\operatorname{rank}(\boldsymbol{\Phi}^T \boldsymbol{D}\boldsymbol{G}_1)=m < R \tag{16a}$$

$$\operatorname{rank}(\boldsymbol{\Phi}^T \boldsymbol{D}\boldsymbol{G}_2)=m < R \tag{16b}$$

Eqs. (I2) is not satisfied, so as to $m \geq R$ must be satisfied, as claimed.

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