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Proper orthogonal decomposition in wind engineering. Part 1: A state-of-the-art and some prospects

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Abstract. The Proper Orthogonal Decomposition (POD) is a statistical method particularly suitable and versatile for dealing with many problems concerning wind engineering and several other scientific and humanist fields. POD represents a random process as a linear combination of deterministic functions, the POD modes, modulated by uncorrelated random coefficients, the principal components. It owes its popularity to the property that only few terms of the series are usually needed to capture the most energetic coherent structures of the process, and a link often exists between each dominant mode and the main mechanisms of the phenomenon. For this reason, POD modes are normally used to identify low-dimensional subspaces appropriate for the construction of reduced models. This paper provides a state-of-the-art and some prospects on POD, with special regard to its framework and applications in wind engineering. A wide bibliography is also reported.

Keywords: aerodynamics; aeroelasticity; digital simulation; fluid dynamics; meteorology; proper orthogonal decomposition; structural dynamics; turbulence; wind engineering.

1. Introduction

All the meteorological phenomena which occur in the earth's atmosphere are caused by the solar radiation S (Fig. 1). This gives rise to atmospheric thermal states and pressure fields that produce complex air movements, usually classified depending on their space and time scales. The study of the wind actions and effects on structures starts from focusing on the wind velocity V at the structural site, defined symbolically as $V = \mu(S)$, where μ is a meteorological operator. Assuming the structure as initially fixed, i.e., restrained to avoid any motion, the undisturbed wind field produces a set of aerodynamic actions F_s; such actions may be defined symbolically as $F_s = \chi(V)$, where χ is an aerodynamic operator. Releasing the ideal restraint defined above, but assuming the structural motion so small as to identify the varied configuration with the initial one, the response R may be evaluated using the classical methods of structural dynamics; such response may be defined symbolically as $R = \psi(F_s)$, where ψ is a mechanical operator. However, if the motion is large, wind-

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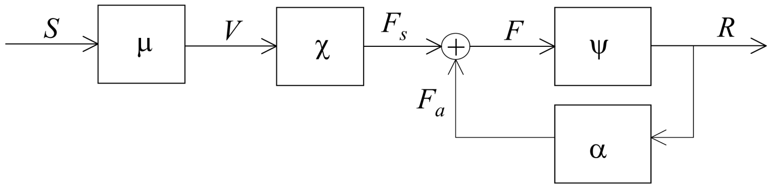

Fig. 1 Wind chain

structure interaction phenomena occur modifying the oncoming flow field, the aerodynamic wind actions and the dynamic response. Such phenomena are frequently idealised assuming that the total actions F are the sum of the aerodynamic actions F_s, exerted by wind on the fixed structure, and of the aeroelastic actions F_a, due to the structural motion, i.e., $F = F_s + F_a$. The aeroelastic actions F_a may be defined symbolically as $F_a = \alpha(R)$, α being an aeroelastic feedback operator.

The meteorological phenomena that occur in the earth's atmosphere and give rise to the wind chain represented in Fig. 1 are so complex as to cause air motions whose space and time variations, from climatology at the planetary scale, to turbulent fluctuations at the micro-scale, are governed by the probabilistic laws of the random fields. Thus, also aerodynamic wind actions on structures and wind-induced dynamic effects are random processes to be described through suitable space and time representations. Furthermore, the random quantities relevant to each component of the wind chain are characterised by the presence of organised or coherent structures, associated with specific physical phenomena dominated by different space and time scales, and by different energetic contents. For instance, meteorology and climatology deal with weather regimes at the planetary scale, cyclones and anti-cyclones at the synoptic scale, fronts, downburst and tornadoes at the mesoscale; fluid dynamics and turbulence studies recognise the existence, within the flow fields, of organised motions or characteristic eddies with different frequency and size; bluff-body aerodynamics is involved in pressure fluctuations caused by the different turbulent components in the upstream flow, by large scale wake unsteadiness, and by smaller scale unsteadiness in the separated flow regions; structural dynamics and aeroelasticity are usual to identify coherent vibrations and energy distributions related to structural natural modes.

The methods commonly applied to deal with the problems concerning each component of the wind chain are deeply affected by the existence of coherent structures, partially coherent or fully incoherent with each other. Thus, independently of whether the study is carried out experimentally, analytically or numerically, a correct and efficient representation of the correlation structure of the phenomenon is a key-point of the problem. In the experimental field, the use of many sensors able to make simultaneous measurements of the wind velocity, of the pressure field, and of the dynamic response is often necessary. Likewise, in the analytical field, it is focal to use spectral models able to reproduce, with enough precision, the coherence structure of the climatic phenomena, of the atmospheric turbulence, of the aerodynamic actions and of the dynamic response. In the numerical field, the burden of the solution and of the simulation algorithms depends on the representation of the correlation structure. Also, it is important to schematise and understand the link between the coherent structures that characterise the physical phenomena related to the sequential components of the wind chain. For instance, the correlation between the different turbulence components and the pressure field on bluff-bodies is relevant to the aerodynamic operator χ. Similarly important, for the mechanical operator ψ, is the correlation between the different components of the aerodynamic load, e.g. buffeting actions and vortex shedding, and the coherent structures of the response, e.g. vibration modes or Galerkin bases. Analogous concepts apply to the aeroelastic operator α and to the link between the different forms of aeroelastic instability, e.g., lock-in, galloping, divergence and flutter, and the dominant structural modes.

The proper orthogonal decomposition, defined in the technical literature by the acronym POD, is a statistical method particularly suitable and versatile to deal with all the above problems. POD represents a random process as a linear combination of the orthogonal eigenfunctions of the covariance function of the process itself. Such eigenfunctions, referred to as the modes of the process, are modulated by random variables, the POD principal components, which are uncorrelated with each other. POD owes its popularity to the attractive property, deriving from an optimality condition, that only few terms of the series are usually needed to capture the dominant components of the process, i.e., its most energetic coherent structures; thus, a link often exists between the dominant terms of the series and the main mechanisms of the overall physical phenomenon. Based on these peculiarities, POD modes are frequently used to identify a low-dimensional subspace on which the governing equations may be projected, for instance by means of Galerkin's method, to construct reduced models. Under this point of view, though POD is a linear procedure, no assumption is needed on the linearity of the problem involved; even if the data to which POD is applied originates from a nonlinear dynamic system, POD modes contain the information on its finite-dimensional attractors; in several cases a link exists between such attractors and the dominant modes.

Based on these remarks, to draw a state-of-the-art on the birth, the evolution and the most recent developments of POD is a hard task, since POD is probably the oldest, best known and most used procedure in random multi-variate analysis. Even more, POD was developed over the years in many fields, often in autonomous ways and with different names, discovering more and more times what other scholars already knew in different fields and, some times, also in the same one.

Faced by a problem so complex and articulated as to appear almost inextricable, this paper provides one of the infinite possible frameworks into which POD methods and applications can be grouped. Section 2 deals with three methods which play a focal role on the formulation of POD - Singular Value Decomposition (SVD), Principal Component Analysis (PCA), and Factor Analysis (FA) - and with the origins of Karhunen-Loeve Expansion (KLE), today well known as POD. Sections 3 to 7 provide a state-of-the-art on the use of POD in each component of the wind chain in Fig. 1, i.e., meteorology, fluid dynamics and turbulence, bluff-body aerodynamics, structural dynamics, and aeroelasticity. Section 8 discusses POD as a tool for simulating stochastic fields, a matter transversal to all the above topics. Section 9 provides some insights into the use of POD in other fields; it also describes a rather new method, Independent Component Analysis (ICA), which represents a noteworthy generalisation of POD. General remarks and some prospects are drawn in Section 10. A wide bibliography on POD is also provided at the end of this paper.

The following companion paper (Carassale, *et al.* 2007) takes this review as a starting point for focusing on some theoretic aspects of POD in a homogeneous and comprehensive framework, stressing the link between its general position and the prevalent uses in wind engineering. Referring to such framework, some applications developed at the University of Genoa are also revised and critically illustrated.

2. SVD, PCA, FA and KLE origins

Singular Value Decomposition (SVD), Principal Component Analysis (PCA) and Factor Analysis

(FA) play a crucial role in the foundation of Proper Orthogonal Decomposition (POD). Born between the end of the XIX century and the beginning of the XX century, they are characterised by strictly joined conceptual and mathematical properties.

SVD consists in expressing any $n \times p$ rectangular matrix **V** through the relationship:

$$\mathbf{V} = \mathbf{A}\mathbf{\Sigma}\mathbf{B}^* \tag{1}$$

where * represents conjugate transpose, **A** is the $n \times n$ unitary matrix of the left singular vectors, i.e., the eigenvectors of $\mathbf{VV}^*$, **B** is the $p \times p$ unitary matrix of the right singular vectors, i.e., the eigenvectors of $\mathbf{V}^*\mathbf{V}$, $\mathbf{\Sigma}$ is the $p \times n$ pseudo-diagonal matrix of the singular values, i.e., the square roots of the eigenvalues of both $\mathbf{VV}^*$ and $\mathbf{V}^*\mathbf{V}$ (Therrien 1992). Due to these properties, SVD is a fundamental method to solve undetermined and impossible linear systems of equations, providing the best solution in the mean squares sense.

PCA is a method by which an n-Variate (V) random vector **v** is expanded into a series of normal vectors provided by the relationship:

$$\mathbf{v} = \sum_{k=1}^{n} \boldsymbol{\phi}_k x_k \tag{2}$$

where $\boldsymbol{\phi}_k$ is the k-th eigenvector of the covariance matrix $\mathbf{C}_\mathrm{v}$ of **v**, x_k ($k = 1,..n$) are uncorrelated random coefficients, referred to as Principal Components (PC) of the vector **v**, whose variances are the eigenvalues of $\mathbf{C}_\mathrm{v}$. The central idea of PCA is that, ordering the PC in such a way to have decreasing variances, an accurate representation of **v** can be obtained retaining only the first few terms of the series (Jolliffe 2002). The link between PCA (Eq. 2) and SVD (Eq. 1) is apparent when considering **V** as a matrix whose p columns collect the observations of the random vector **v**, and $\mathbf{C}_\mathrm{v} = \mathbf{VV}^*/p$; in this case, unless the quantity p, **A** and $\mathbf{\Sigma}$ are the matrices of the eigenvectors and of the eigenvalues of $\mathbf{C}_\mathrm{v}$, respectively.

FA is a method by which an n-V random vector **v** can be expanded as:

$$\mathbf{v} = \sum_{k=1}^{\bar{n}} \boldsymbol{\psi}_k y_k + \mathbf{e} \tag{3}$$

where $\bar{n} \leq n$, $\boldsymbol{\psi}_k$ is a vector referred to as the k-th factor loading, $y_k(k = 1,..\bar{n})$ are weighting coefficients referred to as common factors, **e** is a vector of residual errors referred to as specific loads. Both PCA and FA attempt to achieve a dimensional reduction of a random vector; however, while PCA pursues this aim through operations concerning an intrinsic property of the random vector, its covariance matrix, FA implies the formulation of suitable models for the factor loadings, the common factors, and the specific loads (Mulaik 1972).

Starting from geometric researches conducted by Kronecker and Christoffel, Beltrami (1873) introduced the SVD of a square matrix in a form that underlies PCA. Independently of Beltrami, Jordan (1874) developed a similar method, starting from Kronecker's and Waierstrass' works. Sylvester (1889) gave a new contribution to the SVD of a square matrix in an exercise of abstract algebra where PCA appeared implicitly. Further advances were due to Schmidt (1907), Weyl (1912) and Autonne (1915), who provided the first general theorems on SVD. Eckart (1934) used SVD to diagonalise the quadratic forms which describe the kinetic and potential energy of molecular systems, to uncouple their eigenmotions. Few years later, working in psychometry, Eckart and

Young (1936, 1939) proposed the modern form of the SVD of a rectangular matrix.

It is generally acknowledged that PCA was introduced by Pearson (1901) in the biological field; concerned with finding the lines and planes that best fit a set of points in an *n*-dimensional space, he also provided a geometric interpretation of PCA. Also, it is generally acknowledged that FA was introduced by Spearman (1904) in psychology and psychometry. In the same field, Hotelling (1933) published the fundamental work on PCA, deriving this method as a special case of FA; in particular, starting from a set of correlated variables, he determined a smaller set of uncorrelated variables able to reproduce the initial one; with such an aim, he recalled the original factors of Spearman as PC, choosing these quantities in such a way as to maximise their successive contributions to the total variance of the original variables. The idea of PCA as independent of FA was elucidated by Hotelling (1936) and by Girschick (1936), together with the strategic role of SVD as a basic tool for actually finding the PC.

Two aspects deserve a special concern. Despite the bursting growth of new applications and theoretical developments, PCA and FA involved a so great computational burden, especially with reference to a large set of variables, as to make their use strongly confined up to the advent of electronic computers. Even more, all the above studies dealt with the observations as independent occurrences of random vectors; enlarging the horizon of the applications, e.g., when the data is derived from measurements of a physical phenomenon taken in a set of points in the space at different time instants, such observations could not be considered as independent, but characterised by a suitable correlation structure.

The extension of PCA to space/time series was clearly determinant for all the problems of the wind chain (Fig. 1), and represented the basis of modern POD. Independently of whether POD may be regarded as a special case of PCA (Jolliffe 2002) or as its generalisation (Ravindra 1999), it appeared in statistics, it seems independently, due to Kosambi (1943), Loeve (1945), Karhunen (1946), and Kac and Siegert (1947), being called later Karhunen-Loeve Expansion (KLE) (Loeve 1955, Papoulis 1965). While PCA expands a random vector **v** into a series of the eigenvectors of its covariance matrix, KLE expands a random process $v(t)$ into a series of normal functions provided by the relationship:

$$v(t) = \sum_{k=1}^{\infty} \phi_k(t) x_k \tag{4}$$

where $\phi_k(t)$ is the *k*-th eigenfunction of the covariance function $C_v(t, t')$ of $v(t)$, x_k ($k = 1,..n$) are uncorrelated random coefficients, referred to as Principal Components (PC) of the process $v(t)$, whose variances are the eigenvalues of $C_v(t, t')$. The generalisation of Eq. (4) to multi-variate and multi-dimensional processes was elucidated by Lumley (1967).

3. EOF, PCA AND POD in meteorology

While PCA and POD developed in many sectors, often without recognising their common bases, meteorology discovered these methods, between the 50's and the 60's, in an attempt to compact the representation of large data basis including sets of variables measured at discrete time instants, in a grid of points distributed in space. This led to express the data as the sum of an average term plus an expansion of Empirical Orthogonal Functions (EOF), weighted by coefficients determined using a least square residual condition and a condition of finality, i.e., imposing that a minimum number of terms of the series provided a description of the meteorological field with given accuracy. Inevitably, this gave rise to POD, here called EOF.

These applications began at Massachusetts Institute of Technology (MIT) when, following Richardson's pioneering research, Wadswoth, *et al.* (1948) made a short-term prediction for the atmospheric pressure on the northern hemisphere; to pursue this aim, they used orthogonal decomposition to search for the eigenvalues and eigenvectors of a 91×91 covariance matrix. Involving many persons for many weeks in hand calculations, they obtained the first 7 eigensolutions and demonstrated that the corresponding PC carried 89% of the total variance. This study pointed out the potentiality of PCA but, also, its unmanageable use with reference to numerical aspects.

The situation changed in 1955, when the Whirlwind general purpose electronic computer was available at MIT. Using the new theories developed by Charney and von Neumann on the hydrostatic and geostrophic approximations, Lorenz (1956) resumed the previous work and made new predictions for the 500 mb height anomaly field in Januarys 1947-1952, over a grid of 64 points. Applying EOF to a 64×64 covariance matrix, he showed that the first 9 PC accounted for 88% of the variance for a 1-day prediction. This paper, still judged as a milestone in meteorology, originated new fundamental researches in short term forecasting (Sellers and Shorr 1957, Lorenz 1959).

Almost contemporarily, Russian scientists developed EOF independently. Especially Obukhov (1947, 1954, 1960), Pougachev (1953) and Bagrov (1959) made relevant advances on KLE methods. In particular, they were most concerned, using statistical techniques, on the daily variations of the barotropic pressure field as a function of height. Similar methods were used later by Roukhovets (1963) and Koprova and Malkevich (1965) for analysing the vertical distribution of several meteorological variables.

Based on this impressive activity, White, *et al.* (1958) used EOF to make a 24-hour sea level pressure forecast over US. Grimmer (1963) produced space filtering of monthly anomalies of the surface air temperature, using EOF to eliminate noise from records. Holstrom (1963) applied PCA to weather diagnosis and forecasting, producing spectacular modes of the wind field over the earth's northern hemisphere. Freiberger and Grenander (1965) made critical remarks on the rational use of EOF in statistical approaches to diagnosis and forecasting. Rinne and Karhila (1975) improved the barotropic model based on EOF proposed by Sellers (1957). Hayden and Smith (1982) used EOF for the first prediction of cyclone frequencies. Schubert (1986) applied EOF in the wave domain to determine the modes of atmospheric variability. Obled and Creutin (1986) used EOF for mapping, archiving and simulating geophysical fields related to meteorological parameters. Bouhaddou, *et al.* (1987) proposed an interpolation technique for the implementation of a continuous version of PCA.

Presendorfer and Mobley (1988) provided the first state-of-the-art on PCA in meteorology and oceanography, dedicating much space to its continuous form, called functional PCA, and used this method to explore the link between EOF and the dominant modes of variation of atmospheric systems; however, while this book made a short reference to KLE, POD was never mentioned. Selten (1993) described the low-dimensional space of the attractors that govern the atmospheric evolution in terms of EOF, stressing the use of this method to study chaotic dynamical systems. EOF applications to earth's atmospheric climatology were carried out by Achatz and Branstator (1999) and by Achatz and Opsteegh (2003). A similar research on Martian atmosphere was developed by Whitehouse, *et al.* (2005), using the term POD instead of the classic EOF of meteorological literature.

4. POD in fluid dynamics and turbulent flows

The use of POD in fluid dynamics and turbulent flows originated from a paper where Lumley (1967) proposed an attractive definition of organised or coherent structures, the characteristic eddies, and a method for their extraction from a stochastic turbulent velocity field. Such field was expanded into the sum of the eigenfunctions of its two-point covariance tensor, and the largest eddies were identified with the most energetic eigenfunctions. This method was first applied to measured velocity fields by Payne and Lumley (1967), with reference to the one-dimensional covariance tensor in the wake of a circular cylinder, and by Bakewell and Lumley (1967), with reference to the streamwise covariance of a single velocity component in the wall region of a pipe flow.

Three years later Lumley (1970) published a fundamental contribution to POD. He began studying the representation of random processes to find some deterministic function whose structure is typical of the realisations of the process in some statistical sense. Using a classical variational method, he showed that such a function is an eigenfunction of the covariance function of the process, and that two situations should be distinguished: the cases in which the process has finite or infinite total energy. The first case was treated as a KLE, where the process was represented as a series of orthogonal modes, the eigenfunctions of the covariance function, weighted by random coefficients uncorrelated with each other; the energy content of each mode was quantified by the related eigenvalue, and the sum of the eigenvalues provided the total energy of the process. In the second case, Lumley highlighted that the eigenfunctions are classic harmonic functions, while KLE becomes a generalised Fourier transform. Based on this framework, Lumley extended this formulation to m-Dimensional (D) processes, solving the delicate case of incomplete homogeneity; especially, he put the bases of the classic concepts of turbulent flows, where velocity is usually assumed as stationary with respect to time, but homogeneous or non-homogeneous, depending on different spatial coordinates, due to the presence of border conditions. Finally, he put the basis to apply POD to n-V random processes. It is worth noting that Lumley's book of 1970 already included, even if masked by rather complex mathematical developments and notations, most of the forthcoming advances developed on POD, in following years, in any context.

Lumley (1981) came back to homogeneous processes, noting that Fourier modes are common to any process with infinite energy and thus cannot represent characteristic eddies. However, he showed that localised coherent structures can be represented by a suitable combination of Fourier modes with appropriate complex coefficients: the spectrum of the eigenvalues supplies the modules; the phase relationships yielding instantaneous events can be determined through a generalisation of the shot-noise decomposition (Rice 1944).

In despite of the impressive Lumley's work, the use of POD in turbulent flows was late in asserting its potentiality: in part this was due to the lack of experimental devices able to perform simultaneous field measurements and to detect three-dimensional coherent structures; in part this was due to the laborious nature of the method which remained lengthy unsuitable for dealing with large data sets. Moin (1984) was the first who made use of the full covariance tensor produced through a large-eddy numerical simulation of a turbulent channel flow. Herzog (1986) made a three-dimensional study on the wall layer of a turbulent flow in a pipe, measuring streamwise and spanwise velocity components simultaneously; the third component was computed using incompressibility; this experimental activity led to a data base which was considered, for many years, a basic case study for POD applications.

Sirovich (1987) suggested the method of snapshots or strobes, a numerical technique that represented a major turning point to save computer time in using POD and make fully three-dimensional flows accessible to POD treatment. Moin and Moser (1989) used Herzog's data to elucidate and develop Lumley's approach, and especially the use of his generalised shot-noise decomposition (Lumley 1981) related to flows which are homogeneous streamwise and spanwise, but not homogeneous in the third direction. Sirovich and Park (1990) first studied the POD modes of the joint velocity and temperature fields to analyse the thermal convection of turbulent flows in heated cavities. Aubry, *et al.* (1993) used Herzog's data to construct a low-dimensional model and demonstrated the need of preserving the symmetry of the POD expansion, even if modes with zero energy have to be retained. Berkooz at al (1993) and Holmes, *et al.* (1996) overcame the view to extract coherent structures, using POD and Galerkin's method in order to provide a relevant set of basis functions to identify a low-dimensional subspace used to project the governing equations and to construct Reduced Order Models (ROM). Bonnet, *et al.* (1994) studied the relationship between POD and Linear Stochastic Estimation (LSE) (Adrian and Moin 1988), another method addressed to identify coherent structures.

As a result of these new prospects, the use of POD in fluid dynamics and turbulent flows exhibited an impressive growth since middle 90's. Only to give some examples, Rempfer and Fasel (1994) studied the transition in a flat plate boundary layer where the developing flow is non-homogeneous in all spatial directions, using POD modes to probe the spatial-temporal evolution of coherent structures that propagate downstream. Delville, *et al.* (1999) carried out extensive experimental measurements in a turbulent plane mixing layer, using POD to detect large coherent structures and to construct dynamical ROM (Ukeiley, *et al.* 2001). Lumley and Blossey (1998) and Ravindran (2000) developed methods for the optimal active control of turbulent fluids embedding POD into ROM; Prabhu, *et al.* (2001) and Gunes and Rist (2004) investigated the issue of an adaptive POD basis to overcome the limitation of using POD modes related to an uncontrolled flow to construct an optimal ROM of the controlled flow field. Following the pioneering approach of Sirovich and Park, Podvin and Le Quéré (2001) and Gunes (2002) used POD, the snapshot method and Galerkin expansion to construct efficient ROM of the thermal convection in turbulent flows. POD was also increasingly used to interpret the results provided by Particle Image Velocimetry (PIV) analyses (Pedersen and Meyer 2002, Kostas, *et al.* 2005), Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES) (Pedersen and Meyer 2002, Alfonsi, *et al.* 2003). Mokhasi and Rempfler (2004) applied POD to determine the best position of sensors able to provide the most efficient representation of urban atmospheric flows. A wide tutorial addressed to the numerical application of POD to turbulent flows was published by Smith, *et al.* (2005).

A different line of research consists in applying POD to the modelling and simulation of atmospheric turbulence fields in the planetary boundary layer, to determine the wind-induced response of structures (Sections 6 and 7). This approach, introduced by Panofsky and Dutton (1984) and developed by several authors (Shinozuka, *et al.* 1990, Di Paola 1998, Benfratello and Muscolino 1999, Carassale and Solari 2002, Solari and Tubino 2002, Tubino and Solari 2005), represents the discretised turbulence field as an n-V random process, stationary with respect to time, and interprets POD as a linear transformation. This transformation was applied in two parallel forms (Solari and Carassale 2000): a weak form, referred to as Covariance Proper Transformation (CPT), where the target process was expressed as a linear combination of modes, the eigenvectors of the covariance matrix at the zero time lag, modulated by random processes, the covariance PC, uncorrelated at the zero time lag only; a strong form, referred to as Spectral Proper Transformation (SPT), where the generalised Fourier transform of the process was given by a linear combination of frequency dependent modes, the eigenvectors of the power spectral density matrix (psdm), modulated by random processes, as functions of frequency, whose inverse generalised Fourier transforms, called spectral PC, are uncorrelated for any time lag. It was demonstrated that CPT and SPT coincide under some conditions on the statistical properties of the process (Carassale 2005).

5. POD in bluff-body aerodynamics

Bluff-body aerodynamics prevalently uses POD to compact the pressure data deriving from wind-tunnel or full-scale measures, to construct reduced aerodynamic models, and to interpret the dominant mechanisms of the wind loading on structures. This aim is normally pursued by representing the aerodynamic load as an n-V random process, stationary with respect to time, and interpreting POD as a linear transformation. In this context, the pressure data and the force distribution were usually expanded into a series of modes represented by the eigenvectors of the covariance matrix at the zero time lag (CPT). More recently, however, another form of transformation was increasingly used based on the eigenvectors of the psdm (SPT) (Section 4).

Armitt (1968) pioneered this application by analysing full-scale pressure measurements on a cooling tower; one sentence deserves special attention with regard to future debates: "... there is no reason to suppose that the spatial variation of the pressure fluctuations due to one physical cause are necessarily orthogonal with respect to that due to another cause. The mathematical constraints caused by orthogonality condition could therefore mean that in some cases a unique physical cause cannot be associated with each eigenvector".

Without any reference to Armitt, Lee (1975) represented the covariance matrix of the circumferential pressures derived from wind tunnel tests on a two-dimensional square cylinder in uniform and turbulent flows in terms of POD modes. Best and Holmes (1983) applied POD to measured pressures along the centre-line of a low-rise building model, using results to estimate the quasi-static loading effects and to show that pressure fluctuations were mainly connected with turbulence in the incident flow. Kareem and Cermak (1984) studied the pressure field on a square building model in boundary-layer flows, showing that the main portion of the fluctuating pressure energy along a circumference was associated with the first POD mode corresponding to vortex shedding. Holmes (1990) analysed the end bay pressure of a low-rise building model, discussing the links between POD modes and the underlying physical phenomena; he also expressed some pioneering concepts on using the eigensolutions of the psdm and on the compilation of a catalogue of dominant modes and PC for the commonly used structural shapes and terrain roughness. MacDonald, *et al.* (1990) applied POD to the circumferential pressure on models of circular storage bins, silos and tanks, isolated and grouped, showing relevant links between POD modes and the longitudinal and lateral turbulence components of the oncoming flow. Letchford and Mehta (1993) analysed full-scale pressure tests, showing POD modes for two orthogonal beams of the roof and along a line in a roof corner. Bienkiewicz, *et al.* (1993) studied the roof corner of a low-rise building model, showing the reconstruction of the pressure field on increasing the number of POD modes. All these researches were deeply conditioned by limited sets of simultaneous pressure measures.

This trend changed in 1995, when Bienkiewicz, *et al.* (1995) studied a low-rise building model in a boundary-layer wind-tunnel, using simultaneous point pressure measurements at 494 taps distributed on the building surface; applying POD to the correlation matrix, they showed that the first mode resembled the distribution of the pressure mean square, while the second resembled the mean pressure derivative with respect to wind direction. Tamura, *et al.* (1997) perfected this study analysing the correlation between the POD decomposed pressure and the approaching wind; they showed that the first and second modes were linked with the longitudinal and lateral turbulence components, respectively. An analogous study carried out by Kikuchi, *et al.* (1997) on a square plan tall building model demonstrated that the generalised alongwind and crosswind forces could be reconstructed by very few modes, while the generalised torque required a longer expansion; the spectral analysis of the PC clarified the link between different modes and the exciting mechanisms.

Holmes, *et al.* (1997) returned to Armitt's original remark to note that POD modal shapes were constrained by the requirements of orthogonality, and hence the physical interpretation of modes could be misleading and probably fictitious in many cases. Also Tamura, *et al.* (1999) discussed POD in critical terms, highlighting its geometrical interpretation, the different peculiarities of calculating covariance or correlation modes, and the presence of singular conditions leading to repeated eigenvalues. Baker (2000) provided another discussion on whether or not POD modes have physical meanings, concluding that most energetic modes are likely to reflect different fluctuating flow mechanisms, though no mode can be associated with just one flow mechanism and vice versa; less energetic modes are likely to represent interactions between different flow mechanisms and are significantly affected by the number of pressure taps, by their distribution, and by measurement errors; such remarks were based on full-scale measurements carried out on a wall, on a cube, and on a pitched roof agricultural building; POD was also used to interpret natural ventilation and cladding loads. Katsumura, *et al.* (2004) proposed a universal wind load distribution based on a series of covariance POD modes whose coefficients were calibrated to reproduce a set of specified load effects.

Two critical comparisons between POD and analogous methods deserve a special mention. The first was carried out by Chen, *et al.* (2003) comparing POD and LSE with the aim of reducing the pressure data storage requirement for database-assisted design of low-rise buildings. The second concerns the so-called Bi-Orthogonal Decomposition (BOD), a deterministic tool developed by Aubry, *et al.* (1991) in the context of fluid dynamics to represent a space-time signal as a linear combination of products of space orthogonal modes, called topos, and time orthogonal modes, called chronos; each topos may be associated with an instantaneous coherent structure with a temporal evolution defined by the corresponding chronos. Hemon and Santi (2003) compared POD and BOD with the aim of identifying the most efficient methods to decompose the pressure field on bluff-bodies and to simulate spatially correlated turbulence fields.

A particular application of the covariance POD in bluff-body aerodynamics was proposed by Taniguchi, *et al.* (1996), with the purpose of representing the vortex structures travelling on the flat roof of a low-rise building. In such application, the authors evaluated the eigenvectors of a covariance matrix obtained including a time-lag between the different signals, equal to the time required to the mean flow to move between the corresponding pressure taps. At least in authors' knowledge, such application remained unique.

The first applications of SPT in bluff-body aerodynamics date back to 2004. De Grenet and Ricciardelli (2004) compared the CPT and SPT modes of the pressure field associated with a vibrating bridge deck section. Carassale, *et al.* (2004) developed a similar research using pressure measurements on a tall building model (Kikuchi, *et al.* 1997); likewise De Grenet and Ricciardelli (2004), they showed that, while covariance modes are often related to one physical phenomenon, spectral modes tend to be linked with single excitation mechanisms within given frequency ranges; on changing the frequency, each spectral mode is able to capture more mechanisms.

6. POD in structural dynamics

The use of POD to study the dynamic response of structures is rather recent but already rich of applications. They can be framed into two lines, namely, POD models of loading to facilitate the calculation of the dynamic response, and POD models of the dynamic response itself. This second line can be further subdivided into two classes of methods: the first includes POD-based Galerkin projections to generate lower dimensional models, to reduce the analytic or numeric burden, and to interpret the nonlinear behaviour of structures; the second, related to vibration testing of models and structures, uses POD as an identification tool.

The first POD model of loading aimed at facilitating the calculation of the dynamic response, and more in general, the first POD application in structural dynamics, dates back to Traina, *et al.* (1986); they carried out the spectral decomposition of the covariance function of a non-stationary process, and represented the truncated set of the time dependent eigenfunctions as an expansion of orthogonal Chebyshev polynomials, whose form allowed a simple analytical derivation of the covariance function of the linear response of an 1-Degree-Of-Freedom (1-DOF) system. Masri, *et al.* (1990, 1998) applied this procedure to evaluate the seismic non-stationary response of 1-DOF and *M*-DOF systems. A similar concept was applied by Gullo, *et al.* (1998), who schematised a turbulent field as an *n*-V stationary process, decomposed it into the sum of incoherent fully coherent component processes (Section 7), approximated the frequency dependent eigenvalues and eigenvectors of the psdm by polynomial expansions, and found the spectral moments of the wind-excited response of an *M*-DOF linear system in closed form.

Carassale, *et al.* (1998, 1999, 2001) introduced the Double Modal Transformation (DMT), a method based on the joint expansion of the Lagrangian motion coordinates, by classic modal analysis, and of the loading random process, using CPT and SPT. By means of DMT, the dynamic response was expressed as a double sum of structural and loading modes weighted by Structural Principal Coordinates (SPC) and Loading Principal Components (LPC), respectively. In principle, each SPC is excited by each LPC. Actually, due to modal truncation, only few SPC and LPC contribute to the dynamic response and to the loading process, respectively. Moreover, cross-modal orthogonality properties often apply, which make several SPC unexcited by several LPC. It follows that the dynamic response can be generally expressed by retaining only a few structural and loading modes. Wind engineering applications of DMT were provided in all the above papers, and by Solari and Carassale (2000), Chen and Kareem (2005) and Solari and Tubino (2005). A further research carried out by Tubino and Solari (2007) used DMT to introduce the concept of effective actions, i.e. that part of the actions reconstructed by using those SPT modes that really excite the structural response; such actions involve characteristics quite different from actual actions, providing new prospects on the physical interpretation of the loading mechanism and on the numerical solution of the dynamic response.

Benfratello, *et al.* (1998) used a Volterra series expansion up to the second order to study the role of the quadratic turbulence terms on the alongwind response of *M*-DOF systems, neglecting aeroelastic terms; the SPT of the turbulence field was embedded in such a procedure to reduce the computational burden of multiple integrals. Carassale and Kareem (2002) used a Volterra series to determine the nonlinear response of *M*-DOF systems; a key-point of this method was decoupling the loading by SPT, making the *M*-DOF case a rather straightforward generalisation of the 1-DOF case. Carassale and Piccardo (2003, 2004) analysed the wind-induced response of cables by Volterra series and Monte Carlo simulation taking into account the geometrical nonlinearity of the cable, and the nonlinear terms of the quasi-steady expansion of the aerodynamic and aeroelastic actions; structural displacements and turbulence components were expanded by a Galerkin and a POD series, respectively. Di Paola, *et al.* (2004) analysed a similar case by Monte Carlo method, without including nonlinear aerodynamic and aeroelastic terms; however, using first DMT in a nonlinear environment, they stressed the efficiency of retaining few Galerkin and POD modes.

The use of POD in nonlinear dynamics originated from Ghanem and Spanos (1993) and from Cusumano and Bai (1993) to identify a low-dimensional subspace onto which the equations of motion may be projected by Galerkin's method to construct a reduced model. The former paper, related to other contributions in stochastic finite elements (Spanos and Ghanem 1989, Ghanem and Spanos 1991), applied POD and polynomial chaos expansions of both loading and response, to solve the equations of motion through a limited set of ordinary differential equations. The latter paper used POD to estimate the dimensionality of the active states in chaotic attractors. Following these contributions, POD spread in nonlinear dynamics, bifurcation and chaos (Kreuzer and Kust 1996, Georgiou and Schwartz 1999, Kappagantu and Feeny 1999, Vasta and Schueller 2000, Rathinam and Petzold 2003). None of these papers is related to wind engineering problems.

The use of POD in the dynamic identification of models and structures subjected to vibration testing followed two parallel lines involving the CPT and the SPT of the response, respectively.

CPT-based identification originated from Feeny and Kappagantu (1998) who demonstrated that, for a linear undamped free-vibrating system with a mass matrix proportional to the identity matrix, CPT modes tend to classic modes on increasing the number of sensors; if otherwise the mass matrix does not satisfy the identity condition, classic modes can be derived from POD modes provided the knowledge of the mass matrix; in all the other cases, including damped systems, forced vibrations, and nonlinear modes, POD modes provide an approximation of the natural modes of vibration. This idea was developed by Kerschen and Golinval (2002) and by Lenaerts, *et al.* (2003), who provided insights into the use of SVD to interpret POD modes to represent linear and nonlinear modes of vibration. Han and Feeny (2002) and Riaz and Feeny (2003) tested POD identification by an experimental beam, using accelerometers and strain gauges, respectively.

SPT-based identification originated from Brincker, *et al.* (2001) who proposed a method, Frequency Domain Decomposition (FDD), closely related to classic peak picking; based on FDD, the psdm of the response of an M-DOF system can be decomposed into a set of 1-DOF systems, each corresponding to an individual mode; modes with close natural frequencies can be separated with great accuracy; very precise identifications can be carried out also in the case of strong noise contamination of signals. Two applications of FDD were reported by Yoshida and Tamura (2004) to identify the dynamic properties of a chimney and of a large span roof exposed to wind. Barroso and Rodriguez (2004) used FDD within a benchmark study on structural health monitoring of a four-story building model. Carassale and Percivale (2006) investigated the relationship between SPT of stationary n-V processes and FDD as well as its range of applicability in the identification of linear systems.

7. POD in aeroelasticity

The use of POD in aeroelasticity arose in aerospace engineering as a basic tool for improving and simplifying the construction of ROM of unsteady aerodynamic flows about two-dimensional isolated airfoils, cascades of airfoils, and three-dimensional aircraft wings (Hall 1994). This approach was pioneered by Dowell, Hall and Romanowski, who expanded the time-varying flow field, described by Navier-Stokes equations, into a series of POD fluid modes; initially, they stressed the potentiality of this new tool in the active control of aeroelastic and aeroacustic phenomena, likewise in aerospace aeroelastic analyses for flutter and gust response (Dowell, *et al.* 1997); later, they recommended a wider domain of applications related to the flow of blood through arteries, the response of bridges and tall buildings to winds, the vibration of turbine and compressor blades, and the oscillation of heat exchangers (Dowell and Hall 2001). As a response, the use of POD in aerospace engineering exhibited an impressive spread and growth, giving rise to a long series of studies using ROM and POD for system identification techniques, control models, shape optimization procedures, nonlinear oscillation analyses and limit cycles searches of wings, delta wings, panels, membranes and airfoils interacting with flows at various Mach numbers (Tang, *et al.* 2001, Thomas, *et al.* 2003, Beran, *et al.* 2004, Utturkar, *et al.* 2005). The joint use of POD and Volterra series expansion was also discussed as a powerful solution strategy (Lucia, *et al.* 2005).

The use of POD for analysing aeroelastic phenomena in structural and wind engineering is even more recent and still rather limited. Recalling the new research line opened by Dowell, *et al.* (1997), Hemon and Santi (2002) used POD to interpret the results of wind tunnel tests and CFD analyses of a rectangular vibrating cylinder; in particular, they pointed out the variation of the POD modes of the surface pressure when galloping occurs. Ricciardelli, *et al.* (2002) analysed the POD modes of the surface pressure field of a vibrating bridge deck section, and studied the evolution of the POD modes and of the harmonic content of the PC on varying the mean wind velocity; they noted different patterns related to different vibration regimes, namely vortex shedding, gust buffeting and flutter (Ricciardelli 2003). Carassale and Piccardo (2003, 2004) applied POD, Galerkin method and Volterra series expansion within the frame of a quasi-steady approach, and evaluated the aeroelastic response of geometrically nonlinear cables. Amandolèse and Crémona (2005) used POD to analyse the surface pressure field of two vibrating rectangular cylinders and a bridge deck section interacting with wind; they also discussed the reconstruction of the aerodynamic derivatives using a limited number of POD modes and PC.

8. POD-based digital simulation

The digital simulation of n-V random processes is usually based on the superposition of harmonic waves with random phase angles, or on the filtering of uncorrelated band-limited white noises (ARMA algorithms). These methods involve, respectively, the decomposition of a psdm or of a covariance matrix. Among infinite different ways, this operation may be carried out by Choleski's decomposition ($\mathbf{R} = \mathbf{L}\mathbf{L}^*$, where $\mathbf{R}$ is the psdm or the covariance matrix, and $\mathbf{L}$ is a lower triangular matrix) or by spectral decomposition ($\mathbf{R} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^*$, where $\mathbf{\Psi}$ and $\mathbf{\Lambda}$ are, respectively, the matrix of the eigenvectors and of the eigenvalues of $\mathbf{R}$). The joint application of the harmonic-wave superposition method and of the spectral decomposition of the psdm involves a close relationship with POD and especially with SPT.

To establish a framework on the evolution of this matter, let us begin with the link between the decomposition of a psdm, and the expansion of the corresponding random vector into the sum of incoherent fully coherent component vectors. This concept was pioneered, rather implicitly, by Dodds and Robson (1975), and elucidated, independently, within a series of papers by Li and Kareem (1989, 1993, 1995, 1997) on stochastic decomposition. It is worth notice, however, that all these contributions discussed the above concept making recourse, prevalently, to Choleski's decomposition.

Again independently, Yamazaki and Shinozuka (1990) applied the spectral decomposition to the covariance matrix at the zero time lag of an n-V stationary process in an implicitly POD environment; neglecting the spectral structure of the process, this method was limited to quasi-static analyses. Shinozuka, *et al.* (1990) generalised this approach to the wind-excited response of structures, embedding the spectral decomposition of the psdm of the process into a Monte Carlo procedure.

Caddemi and Di Paola (1994) and Di Paola and Pisano (1996), independently of the previous papers, embedded the spectral decomposition of the psdm of an n-V random process into a Monte Carlo procedure aimed at simulating wind velocity and sea wave fields, respectively. Di Paola (1998) acknowledged the contribution of Li and Kareem, emphasising the link between stochastic and spectral decompositions; in this context, he provided an interpretation of the eigenvectors of the psdm of a turbulence field, referred to as "blowing mode shapes", and discussed their limited variation on wide frequency ranges. Based on this remark, Benfratello and Muscolino (1999) applied the spectral decomposition of the psdm of a turbulence field, calculating its eigenvectors in a suitable fixed frequency value; consequently, they modelled each eigenvalue of the psdm as the power spectral density function (psdf) of a process dealt with as the output of digital filters driven by independent Gaussian white noises. Di Paola and Gullo (2001) further improved this method dividing the frequency domain into p sub-intervals where each eigenvector was approximated as a polynomial of order q; in this way, each incoherent fully coherent component process was given by the combination of p incoherent fully coherent $(q+1)-\mathrm{V}$ processes, whose generation was embedded into an Auto-Regressive (AR) procedure. It is worth noting, however, that none of these papers explicitly spoke of POD.

Solari and Carassale (2000) provided a parallel frame of the CPT and SPT of two classes of processes, namely n-V processes, stationary with respect to time, and 2-D processes, stationary with respect to time and non-homogeneous with respect to a finite 1-D space domain, stressing the advantages of the second representation to solve the eigenvalue problem in closed form. Starting from this remark, Carassale and Solari (2002) improved a method introduced by Van Trees (1968), obtaining an analytical expression of the eigenvalues and eigenfunctions of the cross power spectral density function (cpsdf) of the three turbulence components, assumed as uncorrelated. Parallely, Solari and Tubino (2002) formulated a turbulence model based on PC, whose application provided a new expression of the two-point coherence function of the longitudinal and vertical turbulence components. This method was further extended by Tubino and Solari (2005), who took into account the correlation between different turbulence components and represented the turbulence field using a double POD method, namely a POD of the one-point turbulence vector joint with the POD of the multi-point PC; both steps were solved in closed form. Embedding these closed form solutions into a Monte Carlo framework gives rise to efficient simulation schemes, whose numerical burden reduces to the generation of only uncorrelated Gaussian variables.

In the same period, a research project was carried out in Singapore on numerical aspects of POD simulations. Huang, *et al.* (2001) analysed POD as a simulation tool for both stationary and non-stationary Gaussian processes, focusing on convergence and accuracy. Phoon, *et al.* (2002a, 2004a) discussed the use of Galerkin's method and wavelets to solve conveniently the Fredholm integral equation which underlies the eigenvalue problem implicit in POD. Phoon, *et al.* (2002b) simulated stationary and non-stationary non-Gaussian processes with given marginal distribution and covariance function, through an iterative POD procedure. Phoon, *et al.* (2004b) improved the non-Gaussian simulation technique by prescribing a fractile covariance function, rather than the usual product-moment covariance function.

Four papers are finally worth noting. Carassale (2005) discussed the representation and the simulation of an n-V Gaussian stationary process as the output of filters whose input are independent random processes or even white noises; in such a context, time domain filters were formulated, corresponding to spectral eigenvectors with any dependence on frequency. Chen and Kareem (2005) embedded an analogous method into an AR procedure; besides, they provided a broad point of view and bright remarks on the implicit periodicity of a discretised band-limited psdm, and on the related trend of POD modes towards Fourier modes. Burlando, *et al.* (2005) carried out the first simulation of a non-stationary wind velocity process, studying the effective velocity histories experienced by an aircraft during a landing or take-off route. Carassale and Solari (2006) developed advanced numerical techniques for the POD simulation of turbulence fields in complex terrains and on complex domains.

9. Other POD applications and ICA

Section 2 highlighted how much broad was the use of SVD, PCA and FA, up to the 50's, in geometry and abstract algebra, physics and mechanics, psychology and psychometry. Sections 3 to 8 stressed the wealth of the applications of EOF, KLE and POD in meteorology, fluid dynamics and turbulence, bluff-body aerodynamics, structural dynamics, aeroelasticity and simulation, since the 60's. In reality, especially due to the advent of electronic computers, these procedures invaded most fields. Just as examples, POD was applied to data compaction and reduction (Ahmed and Rao 1975), anatomical measurements (Blackith and Reyment 1971), pattern recognition (Devijver and Kittler 1982), plantation growth (Basilewsky and Hum 1979), picture processing (Rosenfeld and Kak 1982), chemical reactions (Graham and Kevrekidis 1996), and electrical power grids (Rathinam and Petzold 2002).

Another aspect worthy of attention was the proliferation of alternative names to represent PCA, EOF, KLE and POD (Jolliffe 2002), e.g., Eigenvector Analysis (EA) and Latent Vector Analysis (LVA). Moreover, depending on the specific problems dealt with, POD was referred to as Singular Spectrum Analysis (SSA) (Elsner and Tsonis 1996), Principal Oscillation Pattern (POP) (Hasselmann 1988), Complex Hilbert EOF (Rasmusson, *et al.* 1981), and Frequency Domain PCA (Brillinger 1981).

This series of names, in practice synonimous of POD, shall not be confused with a new method, Independent Component Analysis (ICA), which represents a truly new approach and, even more, a relevant prospect for future developments and generalisations of PCA and POD. While PCA expresses a random vector $\mathbf{v}$ as a series of normal modes modulated by uncorrelated PC, ICA expresses $\mathbf{v}$ as a series of modes ζ_k ($k = 1,..n$), in general not orthogonal, modulated by independent PC z_k ($k = 1,..n$):

$$\mathbf{v} = \sum_{k=1}^{n} \zeta_k z_k \tag{5}$$

As this may be not possible, the vector $\mathbf{z} = [z_1 \ldots z_n]^T$ is selected in such a way as to maximise a contrast function; the contrast of $\mathbf{z}$ is maximum when its components are fully independent. Of course, ICA (Eq. 5) coincides with PCA (Eq. 2) for Gaussian random vectors.

To find the origin of ICA is rather difficult. However, Giannakis, *et al.* (1989) addressed this issue imposing appropriate conditions on cumulants up to the third order. Cardoso (1991) generalised this method to cumulants up to the fourth order. Comon (1994) provided a state-of-the-art and mathematical bases of this procedure. Meanwhile, ICA was developed by an increasing number of scholars, using again different names such as Blind Deconvolution (Bellini and Rocca 1990), Source Separation Problem (Fort 1991), Blind Separation of Sources (Jutten and Herault 1991), Blind Identification (Tong, *et al.* 1991), Blind Estimation of Multiple Independent Sources (Tong, *et al.* 1993), Blind Source Deconvolution (Haykin 1994), or Blind Signal Separation (Cardoso 1998). These methods spread to several sectors including airport surveillance (Chaumette, *et al.* 1993), wireless communications and medical diagnoses like ECG and EEG (Amari and Cichocki 1998), and image processing (Cichocki and Amari 2002). A general framework of this complex matter was given by several books which converge on the common name of ICA (Girolami 2000, Hyvärinen, *et al.* 2001, Roberts and Everson 2001, Stone 2004).

At least in authors' knowledge, the first contribution to ICA in wind engineering came from Gilliam, *et al.* (2004). They studied the conical vortices in the roof corner of a low-rise building, noting that POD covariance modes are very efficient for stationary flows, but less appropriate for intermittent conditions where the orthogonality constraint precludes a clear relationship with flow mechanisms. Thus, taking non-Gaussianity into account, the pressure field was expanded into a series of non-orthogonal modes, weighted by the uncorrelated coefficients that maximise independence. However, no explicit reference to ICA and to its unlimited world was reported, and the proposed method was referred to as Non-Orthogonal Decomposition (NOD).

10. Conclusions and some prospects

Few mathematical methods attracted theoretic and applied researches, both in the scientific and humanist fields, as POD made throughout the last century. Unfortunately, the different fields often developed POD in autonomous forms, discovering several times what other fields already knew, coining a long series of names and acronyms which certainly did not facilitate the homogeneity of this matter. This situation originated a broad band of fragmentary and variegated methods and applications, whose collocation requires working out a comprehensive viewpoint on the problem.

The historical and mathematical interpretation of POD shows that it was born as SVD at the end of the XIX century, it transformed into PCA at the beginning of the XX century, it evolved into KLE in the 40's, it exploded in the 60's thanks to the advent of the electronic computer, it exhibited a burst of tendency towards ICA in the 90's. This process pervaded almost the totality of sectors and, naturally, wind engineering, where POD represents a basic tool in meteorology, fluid dynamics and turbulence, bluff-body aerodynamics, wind-excited and aeroelastic response, and simulation.

It is curious noting that the chronologic development of POD in wind engineering followed the logic development of the wind chain in Fig. 1: POD is strategic in meteorology since 1956; it is applied in fluid dynamics and turbulent flows since 1967; it represents a basic tool in bluff-body aerodynamics since the 80's; it was used to study the wind-excited response of structures since the 90's; the first applications of POD in aeroelasticity date back to the 2000's. Also, it is worth noting that the search for the covariance and spectral modes of multi-variate stationary processes, which covered over 99% of the applications in wind engineering, represent a limited topic in the broad class of POD tools (Carassale, *et al.* 2006).

Looking at the prospects of POD in wind engineering, four lines seem to offer particularly attractive possibilities of development. The first deals with the use of tools still little applied, as the analysis of non-stationary and/or not homogeneous processes, for instance typical of aeolic phenomena at small temporal and/or spatial scale. The second refers to those sectors which, till now, used POD very rarely, first of all aeroelasticity. The third involves the joint application of POD to the subsequent rings of the wind chain in Fig. 1 and, in perspective, to the whole chain simultaneously. The fourth is obviously linked with the growth of ICA and with the new horizons, mostly unexplored, which this method offers to all the fields of wind engineering.

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