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Using harmonic class loading for damage identification of plates by wavelet transformation approach

S.B. Beheshti-Aval*[1], M. Taherinasab[1], and M. Noori[2]

[1]Faculty of Civil Engineering, Earthquake Department, K.N.Toosi University of Technology, Tehran, Iran
[2]College of Engineering, California Polytechnic State University, San Luis Obispo, California, USA

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Abstract. In this paper, the harmonic displacement response of a damaged square plate with all-over part-through damage parallel to one edge is utilized as the input signal function in wavelet analysis. The method requires the properties of the damaged plate, i.e., no information about the original undamaged structure is required. The location of damage is identified by sudden changes in the spatial variation of transformed response. The incurred damage causes a change in the stiffness or mass of the plate. This causes a localized singularity which can be identified by a wavelet analysis of the displacement response. In this study via numerical examples shown by using harmonic response is more versatile and effective compared with the static deflection response, specially in the presence of noise. In the light of the obtained results, suggestions for future work are presented and discussed.

Keywords: damage detection; Wavelet Transform; harmonic excitation.

1. Introduction

Based on the observations and experiments, existing structures are exposed to two serious threats namely, downfall of efficiency and destruction. To achieve the acceptable degree of information about the 'structural health', both destructive and nondestructive evaluation methods can be used. Numerous efforts have been carried out to develop a reliable and optimum system of 'Structural Health Monitoring' (SHM) over the last two decades. Systems should respond to the following questions:

(1) Where is the damage location and how much is the severity?
(2) How much is the remaining service longevity of the structure?
(3) What schemes have been required for renovation and improvement of the structure?

In references, SHM is defined as collection, evaluation and analysis of technical information in order to simplify the managerial decisions of the structural longevity. The important challenges in design of an efficient SHM system are recognition of variable changes and the relevant identification method. Variety of SHM methods developed are based on study of variations in natural frequency and relevant mode shapes. Damages and defects (like crack, notch and reduction in elasticity modulus) cause reduction in stiffness and increase in damping resulted by reduction in natural frequency.

But, in order to apply methods of damage identification based on modal analysis, accurate information resulting from vibration responses of structure are required before and after occurrence

*Corresponding Author, Assistant Professor, E-mail: beheshti@kntu.ac.ir

of the damage. Moreover the damage location and quantitative evaluation of damage severity requires complete dynamic analysis of healthy structure. Upon implementation of these methods several problems have appeared:

(1) In most cases, access to vibration response of a healthy structure before occurrence of damage is difficult or impossible.

(2) Sometimes, owing to the complexity of the structure, thorough dynamic analysis of structure is not practicable or can not lead to reliable results.

(3) Access to accurate mechanical properties of utilized material in structure is difficult or impossible.

Also research results reveal that these methods are sensitive to the error level of input data. By considering the above mentioned issues and existing problems in SHM systems during recent years, a great deal of research has been carried out to represent other methods in order to overcome existing problems. Two main aims of these studies are: (i) To present a method for Structural Health Monitoring that does not require the response of healthy structure (ii) To measure sensitivity and other errors.

The implementation of Wavelet Transform (WT) method having above characteristics has been growing in recent years.

Basically this method is based on observations of modal analysis i.e., influence of damage on vibration response. But instead of studying the natural frequency and its variation mode shapes, either static or dynamic response of the structures for damage detection is analyzed by WT.

The wavelet transform is capable of describing a signal in a localized time (or space) and frequency (or scale) domain. In the simplest term, 'wavelet transform' transfers a 2-D function $f(x)$, into a 3-D space with parameters x, $1/x$, i.e., $f(x, 1/x)$. In a time domain signal, x stands for time and $1/x$ for frequency. In a space domain signal (like static response deflection of a beam or plate), this mathematical analysis identifies the location of singularities, for instance caused by crack or point load, along the response of a beam or plate. The above property makes this method potentially reliable and cost-effective where it can be applied to the maintenance of the existing infrastructure (bridges, water and gas distribution pipelines, electrical and transmission towers, dams and other hydraulic structures).

1.1 Literature review

In recent years utilization of WT to monitor damages in structures has become increasingly popular among investigators. Hou and Noori (1999) and Hou *et al.* (2000) proposed a wavelet-based approach for structural damage detection and health monitoring. Hansang and Melhem (2004) presented the state-of-the-art of wavelet-based damage detection in structures. The sensitivity of WT to singularities caused by sudden changes of stiffness and its ability to increase the resolution of presenting a signal in time-frequency domain has led to considerable developments in utilizing wavelet-based damage detection methodologies.

Several papers have been published on the application of WT in the damage detection of beams. Deng and Wang (1998) used a discrete Haar WT to analyze the response signal of a simply supported beam under a static point load and a cantilever beam under a dynamic impact load. A notch with the ratio of depth (d) to the height of the beam (h) (d/h=0.1) simulated the damage. The finite difference method was used to obtain the analytical responses of beams. Quek *et al.* (2001) studied the utilization of suitable wavelet functions and the effect of size and shape of the damage. The deflection response of a damaged beam under static loading calculated by finite element method (FEM) and discrete wavelet transform (DWT) with Haar and Morlet as the mother wavelet was incorporated in that analysis. They concluded that: (a) the WT method can diagnose cracks within the range of d/h=0.027, (b) the method is applicable to both embedded cracks and surface cracks, and (c) support conditions do not affect the proposed wavelet-based damage detection. In the work reported by Chang and Chen (2003) the location of damage on a cantilever Timoshenko beam was identified by using a Gabor (Morlet) wavelet on the dynamic response. Torsional springs were used to simulate the cracks. They also investigated the effect of noise on the efficiency of WT. Ovenesova and Suarez (2004) utilized both CWT and DWT to calculate the dynamic and static response of a fixed-end beam. It was concluded by a rational explanation that Biorthogonal 6.8 mother wavelet is the best function for damage detection. The negligibility of singularity on the CWTs graphs due to the static point load was not explained by the authors. Gentile and Messina (2003) discussed the use of an analytical approach to model the damaged beam. The authors concluded that wavelets with higher vanishing moments are more suitable for damage detection in beams because they closely correlate to the higher order derivatives which are more sensitive to the presence of singularity in the response signal. Recently Zhong and Oyadiji (2011) proposed a new approach based on the difference of the CWTs of two sets of mode shape data of a cracked simply-supported aluminum beam. They provided a better crack indicator than the result of the CWT of the original mode shape data. The analytical and experimental results demonstrated that the proposed method had great superiority with respect to the methods which require the modal parameter of an uncracked beam as a baseline for crack detection. In experimental studies performed by Wu and Wang (2011), a high resolution laser profile sensor was employed to measure the static deflection profile of a cracked cantilever aluminum beam subjected to a static displacement at its end. De-noise processes of the original deflection signal given by the laser sensor and the surface polishing treatment of the beam was critical for an effective detection. The spatial wavelet transform on the beam with different crack depths was proven to be effective in identifying the damage area even for a minor crack.

Damage detection in plates using WT could be found in the works of several researchers (Lee 1992, Lee and Lim 1993, Wang and Deng 1999). Khadem and Rezaee (2000) developed an analytical approach for crack detection in rectangular plates. The flexibility of the crack was modeled as a line spring with varying stiffness along the crack. The obtained results show that moderate cracks have a minor effect on the natural frequencies of the cracked plate making accurate crack detection difficult. To improve the accuracy of their analysis, the authors used modified comparison functions for the prediction of natural frequencies in case of a plate with a crack of arbitrary length.

In another study by Douka *et al*. (2003), mode shapes were used as a basis for WT analysis. The advantage was attributed to the sensitivity of mode shapes to small-size damages in comparison to natural frequency variations. It was shown that the presence of a crack has less influence on lower modes which can be measured more easily. They also demonstrated that noise and the quality of sensors can affect the accuracy of the method. In order to Estimate crack depth, they established a relation between intensity factor and crack size. They could show that the intensity factor increases with increasing crack depth following a second order polynomial law.

In recent years applicability of 2-D WT to identify locations and shapes of damages in plate-type structures were examined. A concept of isosurface of 2-D wavelet coefficients was proposed by Fan and Qiao (2009) for damage identification in plats. The indication of the location and approximate shape or areas of the damages were inspected by using 2-D CWT-based approach. This algorithm was applied to the numerical vibration mode shapes of damaged cantilever plates to illustrate its effectiveness and viability. It demonstrated that the proposed algorithm was superior in noise immunity and robust with limited sensor data. In the other study performed by Huang *et al*. (2009), the spatially distributed 2-D CWT algorithm for a sensor network to automatically detect the local features around the damage sites was used. This method was suitable for both dense and sparse sensor networks. The advantageous features of this algorithm were its reliance on local data and limited communication and computation requirement.

1.2 Significance of the present work

Considering the studies that have been done on damage localization in plates using WT, in all the cases of dynamic loading, only the natural modal shapes have been used as input signal for wavelet analysis. But, in some recent works on using dynamic characteristics of plates for damage detection, it has been shown that using constrained vibration can be more useful. Schulz *et al*. (2003) showed that small damage can be detected more accurately using determined pattern for dynamic excitation resulting in constrained vibration deflection shapes. Khatam *et al*. (2007) also showed that vibration deflection shapes resulting in constrained dynamic excitation can detect damage location with high accuracy and high efficiency rather than static deflection shapes. Thus, the main idea of the present study is applying constrained vibration shapes in damage detection in plates using WT. We are interested in introducing the advantages of using dynamic response of structures as an input response function in a wavelet analysis to detect damages in plates. The dynamic response of a cracked plate is transformed by WT and the position of the crack is estimated by the variation of the spatial response signal along a line vertical to the crack due to the high resolution property of the wavelet transform.

The other objective of the present work is to study some aspects that have not been studied in previous work reported in the literature. The various effects of noise and the superiority of WT method under static or dynamic loading are studied. The ability of the WT method to detect a small damage in a plate and how to increase the efficiency and accuracy of wavelet-based damage detection in structures has also been presented. In this study, as mentioned, instead of utilizing the static response deflection or the mode shapes, the response of the plate subjected to a harmonic loading (with specified frequency and location) is used as the response signal in wavelet analysis. It is concluded that the application of this harmonic response approach for damage detection purposes, utilizing WT, has a few advantages over the application of static response. Less sensitivity to noise, more convenience, and better suitability in practical applications, as demonstrated, yield more reliable results in comparison with previous methods.

Prior to description of application in damage identification in plate, a brief mathematical explanation is given in the next section.

1.3 A brief overview of wavelet transformation

Ability in presenting both time and frequency content are two important properties of wavelet functions. A function $\psi(x)$ is a wavelet if and only if its Fourier transform $\Psi(\omega)$ satisfies

$$\int_{-\infty}^{+\infty} \frac{|\Psi_{(\omega)}|^2}{|\omega|^2} d\omega < +\infty \tag{1}$$

This condition implies that

$$\int_{-\infty}^{+\infty}\psi(x)dx = 0 \tag{2}$$

which means that a wavelet is an oscillating function with zero average value. This basic wavelet function $\psi(x)$ that is localized in both time and frequency domains is called the mother wavelet and used to create a family of wavelets $\psi_{a,b}(x)$ as following

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right) \tag{3}$$

where a and b are real numbers that dilate (scale) and translate the function $\psi(x)$, respectively. The CWT of function $f(x)$, where independent variable x, is time or space, is defined as

$$\mathrm{CWT}(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty}f(x)\,\psi\left(\frac{x-b}{a}\right)dx = \int_{-\infty}^{+\infty}f(x)\,\psi_{a,b}(x)dx \tag{4}$$

In translating Eq. (4) one might recognize the inner product of $f(x)$ with scaled and translated versions of the original wavelet function. In other words, the continuous wavelet transform (CWT) is the sum over all time of the signal multiplied by a scaled and shifted version of a mother wavelet. Large values of scale 'a' correspond to big wavelets and thus coarse features of $f(x)$, while low values of scale 'a' correspond to small wavelets and fine details of $f(x)$.

If, instead of using a continuum of dilations and translations, discrete values of 'a' and 'b' are used one can define DWT as has been explained in detail in Daubechies (1992) and Mallat (1999).

The results of the CWT are wavelet coefficients that show how well a wavelet correlates with the signal analyzed. Any singularity in the signal creates wavelet coefficients with large amplitudes. These singularities are easily detected in a graph of the coefficients of CWT or DWT. Wavelets are usually used to analyze signals in time domain, but, by replacing time with space or length, spatially distributed signals can also be analyzed by WT.

2. Harmonic responses as the base function

In comparison with static load, it seems that the harmonic loading can be more convenient, for the following reasons: (a) in some cases, obtaining a meaningful static response of a plate is difficult and (b) to effectively measure the deflection of a plate, a large amount of static load is required to have a usable signal with acceptable accuracy. However if the plate is already subjected to a large static loading, applying additional loads is not appropriate as it may adversely affect the structural stability and exceed the loading capacity. Therefore, by using harmonic excitation after the transient response is diminished, the vibration response of a plate can be measured by various kinds of sensors. The excitation could be produced via actuators installed on specified locations of a plate. Both strain gages and accelerometers can be appropriately used to store the large amount of data measured over a somewhat long period of time. Utilizing this approach, the dynamic response of the plate is recorded over time and the WT is applied to these measured dynamic response functions. For instance, having 10 signal records measured at different times and analyzing each by WT and comparing the results will help to gain a better assessment of the state of health of the plate. Moreover, this approach will allow us to identify the effect of noise, as a random signal, in the measured response while detecting the damage which has a constant effect on the signal.

The finite element analysis using a four-nodded isotopic element was utilized for analytical modeling of the damaged plates. Harmonic response analysis was applied to determine the steady-state response of the plates under harmonic loads. This forced vibration predicts the sustained dynamic behavior of the plates. One of the practical advantages of using harmonic excitation is the ability to change both the location and the frequency of excitation. In static loading, the only changeable factors are the location of the load and its magnitude, which has no effect on the normalized general shape of the static elastic response of the plate. However, the variation of the frequency can lead to completely different response shapes and analyzing each may lead to a more reliable final result. The frequency and the location of the excitation source are the two parameters influencing the harmonic response of a plate. By increasing the frequency, the higher modes will contribute more pronouncedly to the harmonic response of the plate. Obviously, detecting the damage by using higher mode shapes leads to more accurate results. Therefore, utilizing higher vibration frequency results in more reliable WT analysis. Many cases were analyzed to study these topics and are discussed in the following sections.

2.1 Analytical modeling

In this study, to obtain the harmonic and static response of the plate, FEM was used. It is preferred because of more practical and versatile than other analytical approaches.

However, using FEM adds likely extra noise and error to the subsequent dynamic response. On the other hand, the discrietization of the continuum media, resulting in noises in the signal, does not cause a serious problem, since in the physical media, noises from different sources are also present and inevitable in the measured signal. In the present research, the total number of 100 sample points was utilized for representing the response signal of the plate.

There is not any restrict prescription about minimum and also realistic number of sensors. However, it is well known that when the number of sample points is low, the singularities caused by the boundary condition in the region adjacent to supports emerged on CWTs graphs in low scales and make it more difficult to identify the singularities (perturbation in graph of CWT coefficients) caused by the probable damage. On the other hand, increase of sampling point's number causes increase of required scale to display damage location. Because of distinction of the frequency domain of defects, point plurality causes increase of scale limit pertaining to corresponding frequency domain.

In performing the analysis, the steady state harmonic response is used for measuring the deflection shape. The harmonic response analysis was used to determine the steady-state response and to ignore transient response at the beginning of the excitation; hence the response resembles more to the shape of the mode relevant to excitation frequency. In this study, The structures chosen to present numerical results include an elastic square plate of size $a \times a$ and thickness h, with simply supported over all four edges with an all over crack running parallel to one of its edges, as shown in Fig. 1.

Data acquisition from response deflection of sampling points across straight line perpendicular to damage is a rational assumption. Via this simple proposition, we are able to collect the 1D data from 2D modeling of the plate. Also the functionality of WT using harmonic class loading to detect damage identification in plates will be ascertained.

The usage of WT for identification of damages with finite length using 2D approaches is the subject of our ongoing investigation.

The geometric properties of the plate are as follows: the length a=1 m and the thickness h=0.01 m.

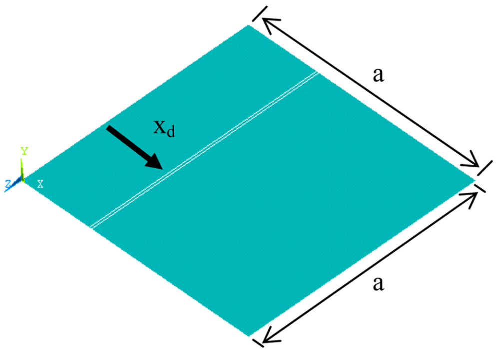

Fig. 1 Square plate with an all-over part-through crack with dimension of 100×100×1 cm, x_d = 30 cm

The plate contains a notch of width w and depth d at a distance of, x_d = 0.3 m away from the left hand edge of the plate and is considered to be open. Also x_v is the location of vibrator. The crack depth is varied during multiple analyses. Relative crack depth is defined as d/h. The material properties are as follows: Young's Modulus E = 200 GPa, mass density ρ = 7860 kg/m^3, and Poisson ratio, ν = 0.3. Damping coefficient is assumed to be 0.5%, which is a reasonable value for steel. Performing dynamic analysis, the first four natural frequencies of the plate are extracted as 0.78077, 1.9350, 1.9567, and 3.0498 Hz.

The input signal functions for the WT are harmonic responses of the plate. To apply harmonic loading, a vibrator was used at x_v = 65 cm; z_v = -50 cm and harmonic response of plate was obtained. First analysis was done with frequency of 0.1 Hz. Study of displacement response of plate at this frequency shows that; the first mode is the dominant response of plate. The location of dynamic concentrated load as a singularity causes perturbation on graph of coefficients of CWT. The wavelet families of Symlet are utilized throughout this study. This type of WT comprises high performance in identification of small damages (Gentile and Messina 2003). More explanations about choosing this type of mother wavelets are presented in Appendix A.

As shown in Fig. 2 this perturbation is similar to damage location, with the obvious difference

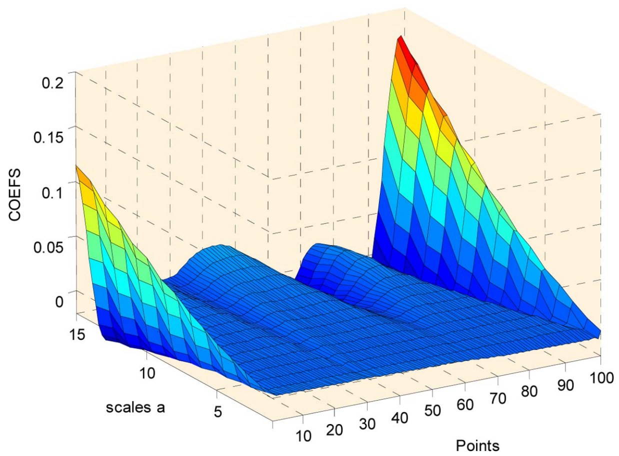

Fig. 2 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading of frequency 0.1 Hz (Symlet4), damage at x_d = 30 cm and vibrator at x_v = 65 cm

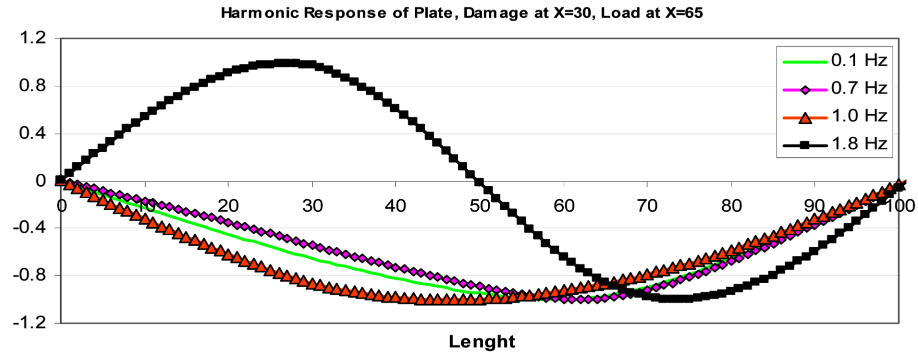

Fig. 3 Normalized displacement response of the damaged plate under harmonic loading of different frequencies 0.1, 0.7, 1, and 1.8 Hz, $x_d = 30$ cm, $x_v = 65$ cm

that, the locations of damage causes perturbation at lower scales rather than location of dynamic concentrated load. Further analyses were done using frequencies of 0.1, 0.7, 1, and 1.8 Hz. Fig. 3 shows the displacement responses of plate for these frequencies. Frequencies of 0.7 and 1 Hz are close to first natural frequency of plate and frequency of 1.8 Hz is located in the vicinity of the second natural frequency of plate.

Thus, from the first to third frequency, the first mode is the dominant response of the plate and at the fourth frequency, the second mode is dominanted.

In Fig. 4, graphs of coefficients of wavelet corresponding to the normalized displacement responses of the damaged plate shown in Fig. 3 are represented for several scales. Although it seems that the steady-state responses of the plate under harmonic loading with frequency of 0.1, 0.7 and 1 Hz have single-mode pattern, but these negligible differences between frequencies of loading cause different graphs of CWT.

It can be clearly seen, damage location from obtained harmonic response of plate at higher frequency (1.8 Hz) which is close to the second natural frequency, is more detectable than harmonic response of plate at lower frequency. This phenomenon is due to this fact that the higher modes have more influence on the harmonic response. However, measuring plate response under high frequency dynamic loading is more difficult. Therefore, the higher modes should only be used with attention to practical and implemental limitations.

2.2 Change in location of excitation

It is probable that the location of vibrator may be close to the location of the undetected damage, causing inaccurate results. However, in practice, the location of applied excitation on plate can be easily changed. Therefore the response of plate in several different positions can be measured. The efficiency of applied harmonic loading can not be influenced in this case, because by changing the location of excitation, new response which obviously distinguishes the damage location can be obtained more precisely.

Fig. 5 shows harmonic response of plate subjected to the harmonic loading of different frequencies when vibrator is at $x_v = 65$ cm. As shown in Fig. 6, the closeness of vibrator location to damage location not only has any negative influence on the resolution of perturbation resulted from damage at low scales, but also the, efficiency of applied excitation has been increased at all scales. Moreover

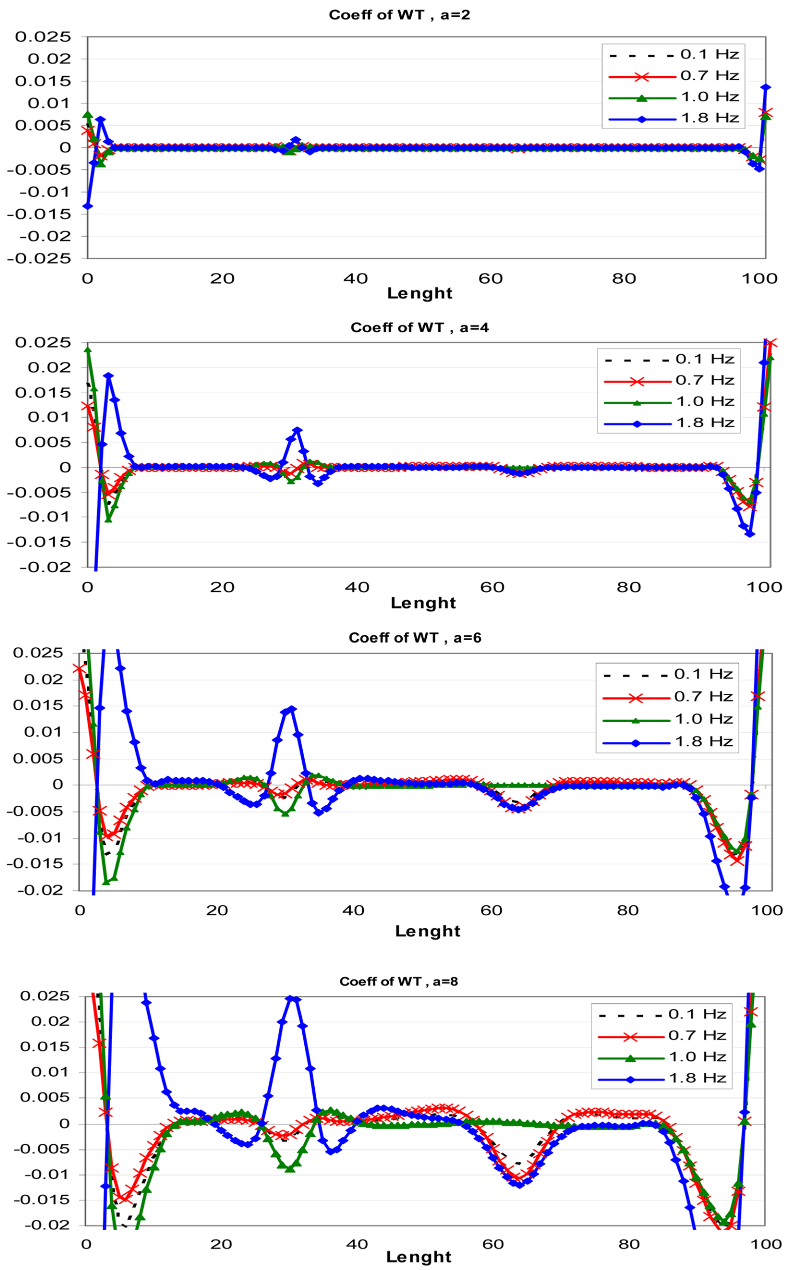

Fig. 4 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading of different frequencies (Symlet4), at different scales, $x_d = 30$ cm and $x_v = 65$ cm

at all scales from 2 to 8, perturbation caused by concentrated harmonic load has no effect on the graphs of coefficients.

Based on this result, the superiority of WT in various frequency segregations can be observed and without doubt the observed perturbations in low scales of the CWTs coefficient graph should be attributed to the damages of the plate.

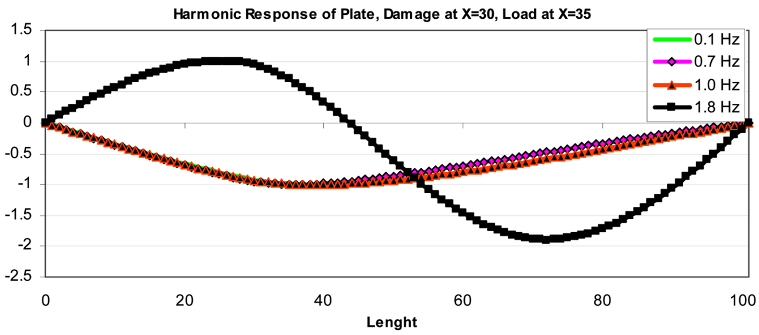

Fig. 5 Normalized harmonic response of the damaged plate under harmonic loading of different frequencies 0.1, 0.7, 1, 1.8 Hz, $x_d = 30$, $x_v = 35$ cm

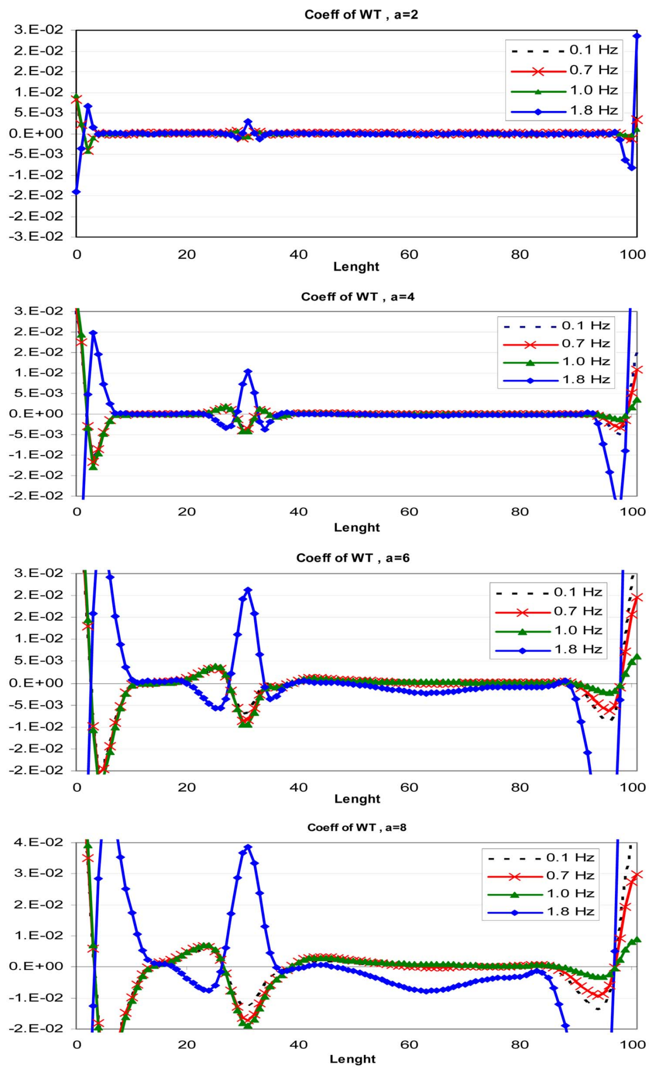

Fig. 6 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading of different frequencies (Symlet4), at different scales, $x_d = 30$ cm and $x_v = 35$ cm

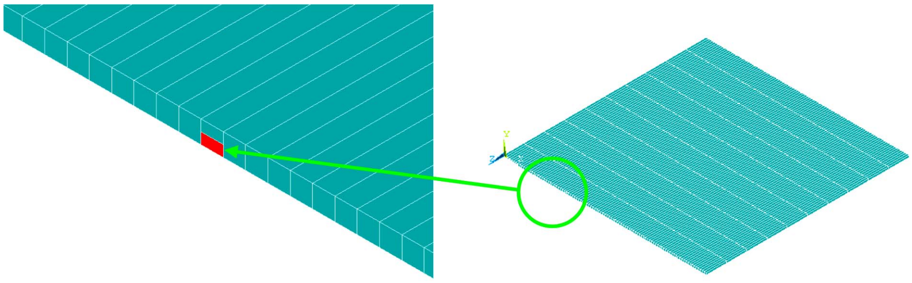

Fig. 7 Plate model containing a defect by reduction in elasticity modulus

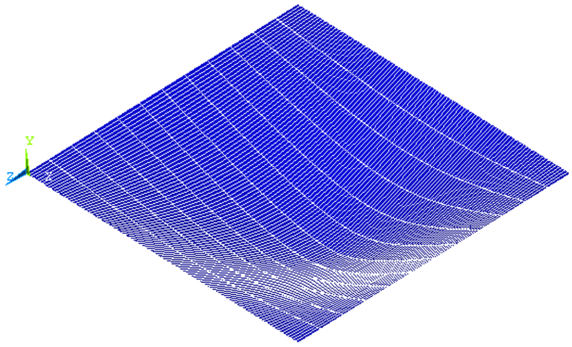

Fig. 8 Response of the damaged plate under harmonic loading with frequency 0.1 Hz

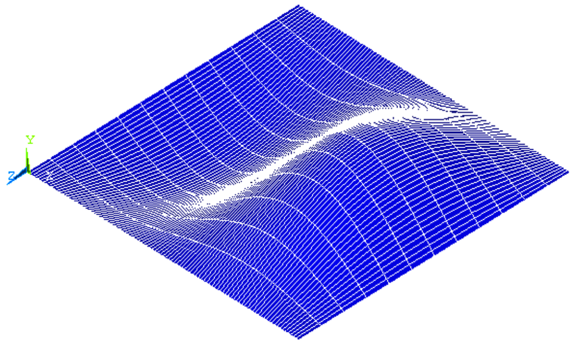

Fig. 9 Response of the damaged plate under harmonic loading with frequency 1.8 Hz

2.3 Detection pertaining to material disturbance

The efficiency of WT method in detection of defects pertaining to reduction in elasticity modulus is studied (Fig. 7). The reduction of 25% in young's modulus is assumed. A strip with rectangular section, 10×4 mm, parallel to one edge of plate is considered as a defect. Harmonic responses of plate under dynamic loading with frequencies of 0.1 and 1.8 Hz are shown in Figs. 8 and 9.

CWTs graphs corresponding to these responses are presented in Fig. 10. As can be seen, the location of reduction of 25% in young's modulus is clearly distinct. To study the effect of closeness of the location of applied excitation to the damage location, vibrator is placed at $x_v = 35$ cm and WT is applied on obtained harmonic responses. These new results are shown in Fig. 11. It seems that, in this state, the location of applied load has more negative effect on the resolution of the damage location.

As it can be seen in second graph of Fig. 10, the location of harmonic loading and damage are recognizable as two complete separated perturbations.

Second graph of Fig. 10 with frequency of 1.8 Hz is very clear on the contrary of first one. Both graphs have been shown in same scale, particularly in order to show that exciting with higher frequencies (1.8 Hz) are able to identify the location of damage more accurately than exciting with lower frequencies (0.1 Hz). It should be emphasized that recognize criterion of damage will be only the second graph of Fig. 10. In graphs of Fig. 11, the influence of closeness of harmonic loading to damage location has been considered. As can be seen, by approaching of the location of loading to the location of damage, the influence of these two perturbations has been mingled and it is possible that the location of perturbation is considered just as a result of the location of loading mistakenly

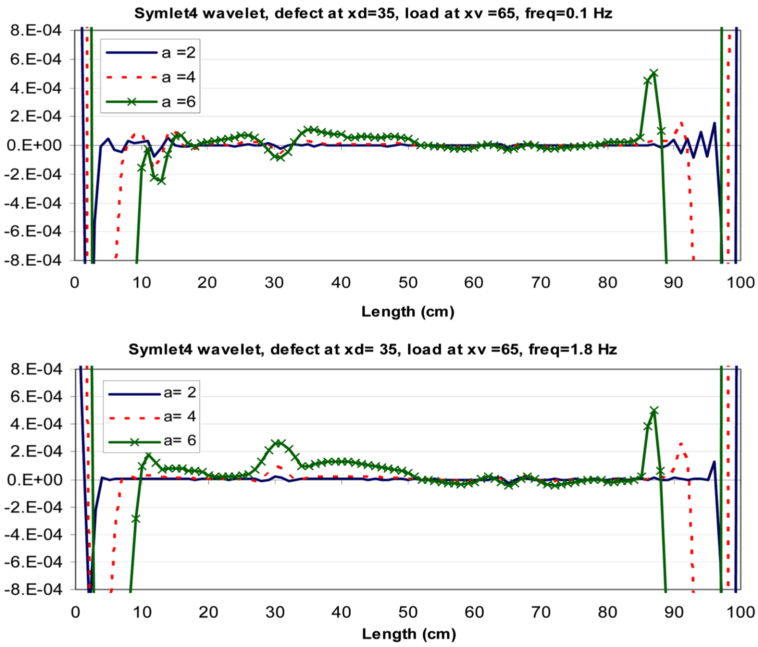

Fig. 10 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading of different frequencies (Symlet6), E1 = 0.75 E0, x_d = 35 cm, x_v = 65 cm

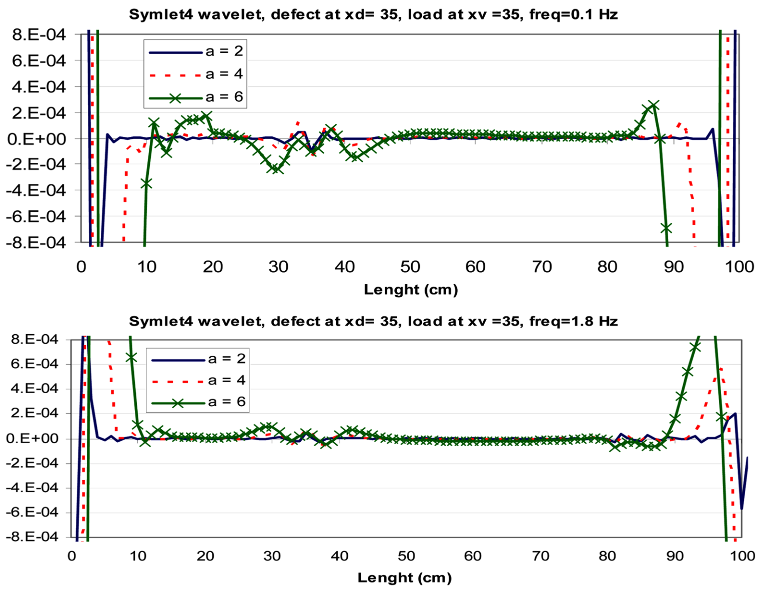

Fig. 11 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading of different frequencies (Symlet6), E1 = 0.75 E0, x_d = 35 cm, x_v = 35 cm

and the existence of damage be ignored. Therefore in use of harmonic loading for damage detection, loading should be applied in several points in purpose of the negative influence of closeness of harmonic loading to damage location be ignored.

3. Comparison of WT with derivative method

Considering that derivatives are a suitable method in identifying discontinuity (singularity) in measured signals, it is useful to compare CWTs graphs and derivative curves of harmonic response. For this purpose, harmonic response of cracked plate in Fig. 1 with $d/h = 0.1$ is used. The response derivatives of orders 1-6 under harmonic loading with frequencies of 0.1 and 1.8 Hz have been shown in Fig. 12.

As can be seen whenever frequency of 1.8 Hz is used, derivative curves with values greater than 2 exactly show the damage location. But when harmonic load with frequency of 0.1 Hz is used, even derivations with values of 5 and 6 act very weakly and are not able to identify the damage location. On the contrary, WT method can show damage location in this state (Fig. 13). This superiority of wavelet method will have more importance due to the fact that use of forced vibration with high frequency in experiment is more difficult compared to the low frequency.

Another issue of using derivatives is reduction in accuracy for the higher order derivatives. This issue does not produce any problems while keeping enough data by utilizing sampling points. But when the number of sampling points are low (for example 40 points), using higher order derivations cause reduction in accuracy of results.

Another remarkable advantage of WT method in comparison to direct derivatives is less sensitivity of WT to the noise and error in the analyzed signal. The existence of any noise and random deviation from real values, cause severe reduction in efficiency of derivatives during detection of discontinuities

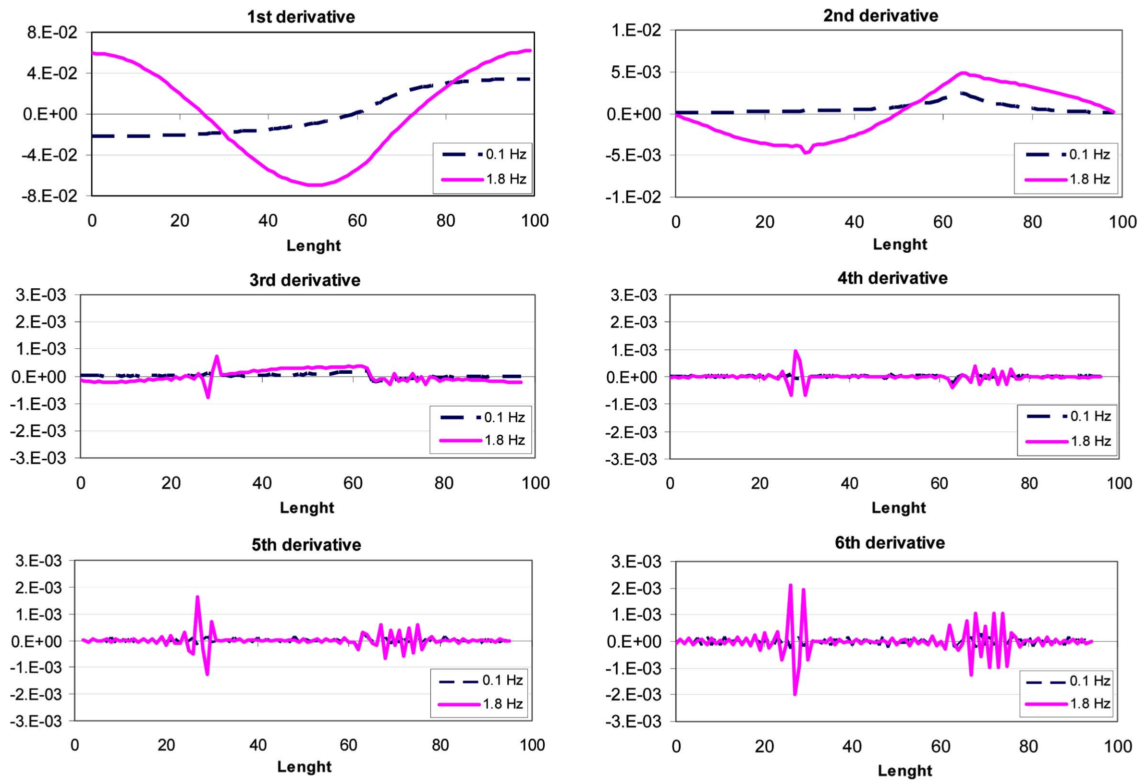

Fig. 12 The first six derivatives of the harmonic response of the damaged plate, x_d = 30 cm, x_v = 65 cm, d/h = 0.1

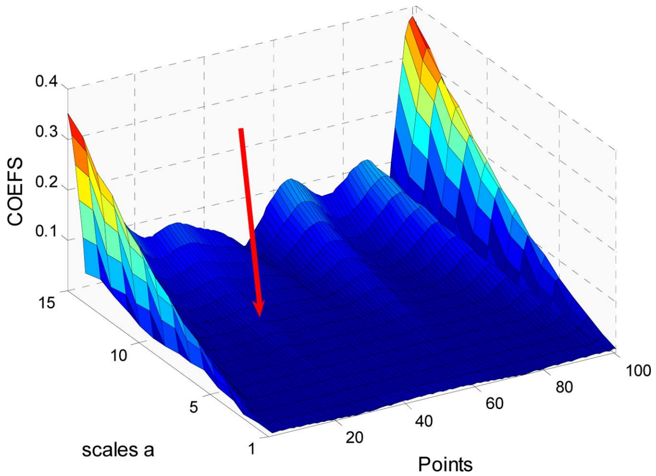

Fig. 13 The wavelet transformed harmonic response of the damaged plate subjected to the harmonic loading with frequency of 0.1 Hz (Symlet4), $x_d = 30$ cm, $d/h = 0.1$, $x_v = 65$ cm

resulted from structural defects. On the contrary, adding the noise to the signal that is under analysis has less effect on the WT results. The multiresolution characteristic of WT is the main reason of this property. As a result, the effect of noise only appears at low scales (high frequency). It can be said that there are two differences between the effect of noise and the effect of structural discontinuity on the CWTs graphs:

(1) Noise has various effects on CWTs graphs at different scales, whereas perturbation resulting from discontinuity (structural defect or damage) has a regular pattern so that this perturbation increases with the increasing of scale.

(2) The effect of noise is considerable at low scales (high frequency) which of course, depends on dominant frequency of noise.

In order to study the effect of noise, a random noise with Gaussian distribution is produced. The average value and the coefficient of variation of this error function with100 sampling points are equal to 0 and 1 respectively. The produced error function is shown in Fig. 14. By multiplying this error function with a coefficient indicating the amplitude of existence noise and after adding with normalized response of plate, polluted response is produced according to following equation

$$s(n) = f(n) + l.e(n) \tag{5}$$

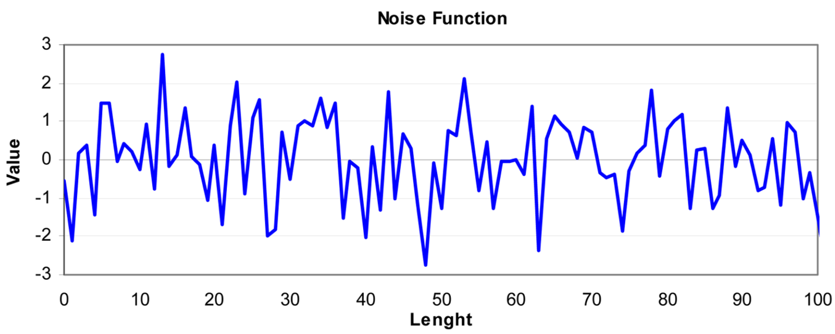

Fig. 14 Gaussian noise with average value and coefficient of variation equal to 0 and 1 respectively

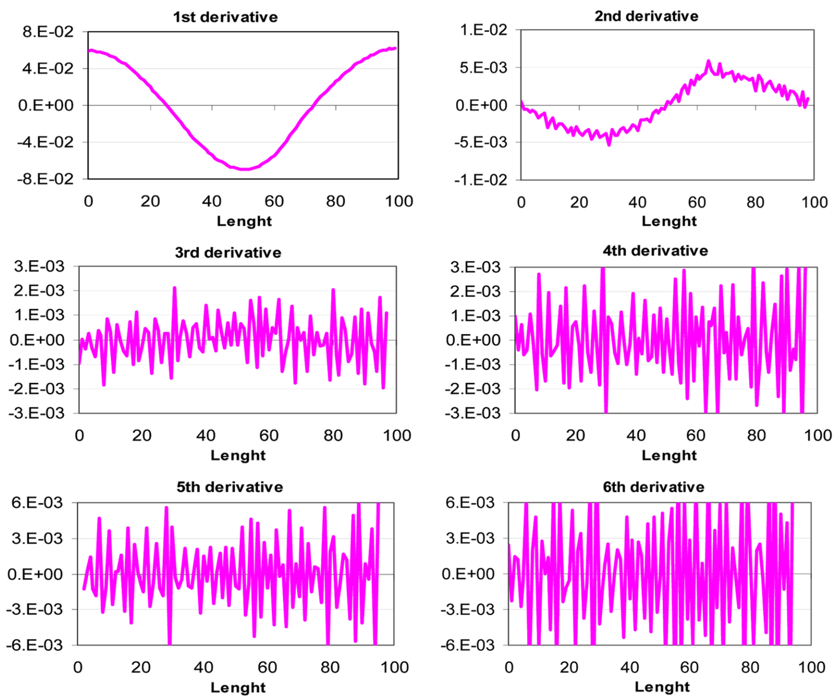

Fig. 15 The first six derivatives of the noisy harmonic response (1.8 Hz) of the damaged plate, damage at x_d = 30 cm, and x_v = 65 cm, $d/h = 0.1$. The damage location is completely blurry

where $f(n)$ is the real response signal (normalized to 1), $s(n)$ is the polluted signal, $e(n)$ is the noise, l is the amplitude, and n is the number of the data. In order to study the acceptable level of noise in each method, polluted response is analyzed with WT and derivative operation.

Whereas, the defect location could be diagnosed on WT coefficient graphs, regarding to value of irregularity, this revealed of noise may not immerse the perturbation caused by defect.

This algorithm was repeated for different levels of noise and defects. The results of applying derivative on displacement response of plates under 1.8 Hz dynamic loads are shown in Fig. 15. The crack consisting of $d/h = 0.1$ in the damaged area at $x_d = 30$ cm is also chosen in Fig. 1. The first time, the level of noise was assumed equal to 0.006%. The reason of this low level of noise is low severity of damage and possibility of assessment of damage in this example. As can be seen in Fig. 15, notwithstanding use of low level of noise, derivative curves with order of 1 to 6 lose the ability of detection of damage completely and however the order of derivative increases, the effect of noise dominates.

On the other hand, as can be seen in Fig. 16 damage position still is detectable on 3D graphs of CWTs. It is observed that the effect of noise in this level can not cover the effect of perturbation caused from crack location.

It seems that acceptable level of noise depends on the severity of damage to structure. To this end, harmonic response of a plate subjected to harmonic load with frequency of 1.8 Hz is analyzed with both WT and its derivative. Relative crack depth is assumed equal to 0.2 in the damaged area at $x_d = 30$ cm. The result of applying WT on displacement response of plate for $l = 0.02\%$ has been shown in Fig. 17. It can be seen that with increasing in severity of damage, the acceptable level of

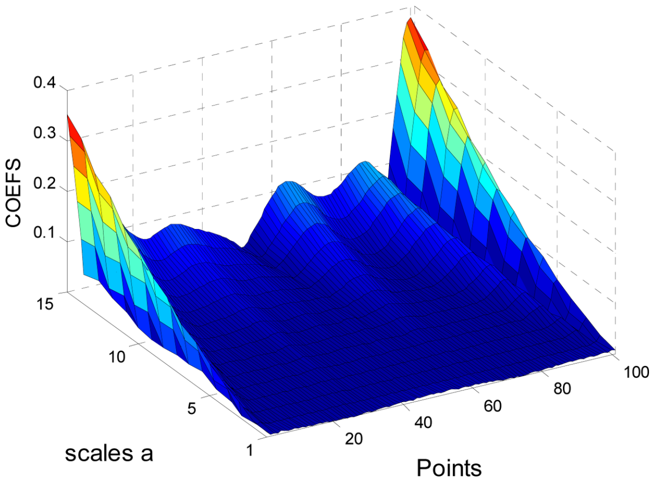

Fig. 16 The 3-D graph of CWTs for the noisy harmonic response (1.8 Hz) of the damaged plate, $x_d = 30$ cm, and $x_v = 65$ cm, $d/h = 0.1$. The location of this small damage can be seen in higher scales

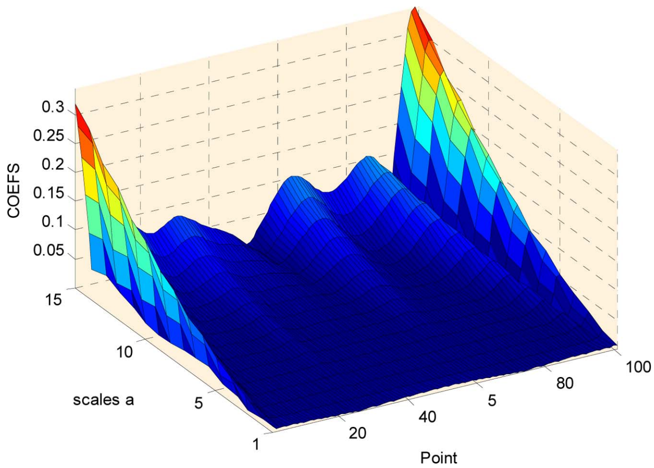

Fig. 17 The 3D graph of CWTs for the noisy harmonic response (1.8 Hz) of the damaged plate, $x_d = 30$ cm, $x_v = 65$ cm, d/h=0.2

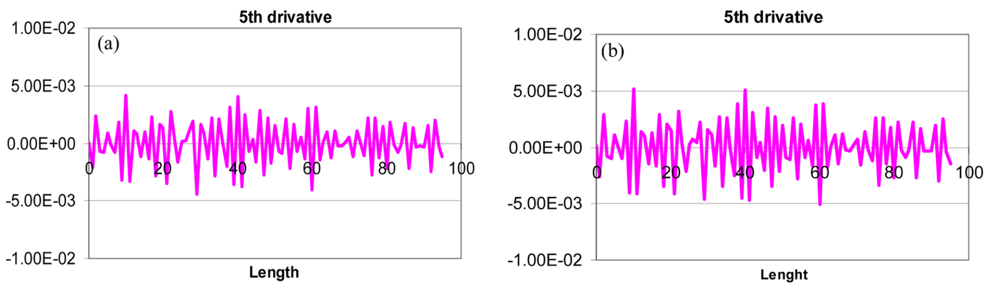

Fig. 18 Derivative curves with order of 5 with two level of noise (a) 0.01% and (b) 0.02%

noise also increases.

Also The results of applying derivative on displacement response of plate for $l = 0.01\%$ and

$l = 0.02\%$ have been shown in Fig. 18. The comparison of these Figures shows that WT method maintains its superiority from the aspect of acceptable error even in the most severe levels of damage too. While derivative loses its efficiency in the level of noise of 0.01%, WT method shows good efficiency even in high level of noise of 0.02% which is shown in Fig. 18(b).

Obviously, noise can not clear completely the singularity caused by a reduction of stiffness. Anyway, it can be concluded that the WT can detect the damages in the presence of noise.

4. Comparison of harmonic and static responses as a response input function

As discussed in the present work, it is apparent that both static and dynamic response of a plate can be applied for detection of defects in the WT approach. It is necessary to compare efficiency of these two methods in obtaining of plate response of plate. This comparison has been done under two criterions: i) intensity factor of coefficients and ii) less negative influence of the same noise amplitude in efficiency of both methods.

4.1 Comparison of intensity factor of coefficients

For comparison of proposed efficiency index, the dynamic response of the plate resulted from dynamic loading with frequencies of 0.5 Hz and also static response of the plate under uniform 2 kN/m^2 load have been normalized so that in both above signals the values of maximum displacement are scaled to one. In this manner, the result of wavelet transform of both signals will be comparable. Both signals were analyzed with "sym4" and where the graph of wavelet coefficient for $a = 2$ has been shown in Fig. 19. As it can be seen, the graph of wavelet coefficient resulted from dynamic response presents greater coefficients than static response at the vicinity of the crack ($x_d = 30$ cm). This result indicates that dynamic response is more efficient than static response in detection of the crack location.

4.2 Influence of the same noise amplitude

To study proposed efficiency index, three different responses of a damaged plate are chosen and the noise is added to them intentionally. Damaged plate of Fig. 1 with $d/h = 0.3$ is considered. 3D

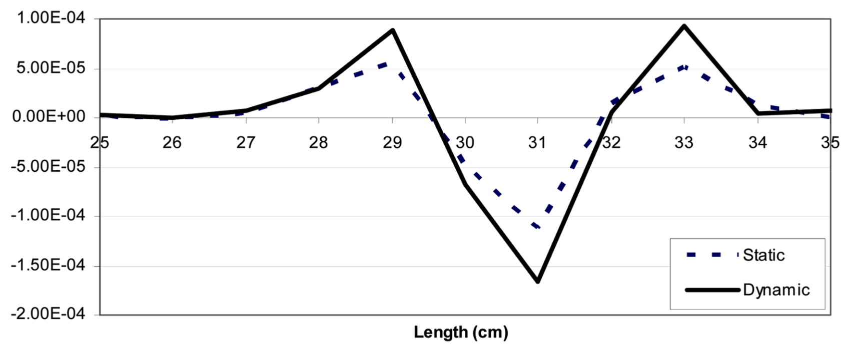

Fig. 19 Comparison between intensity factor of harmonic and static responses as efficiency index ($a = 2$)

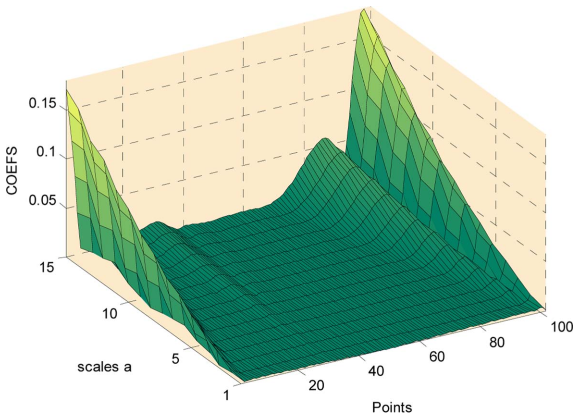

Fig. 20 Static response, $l = 0\%$

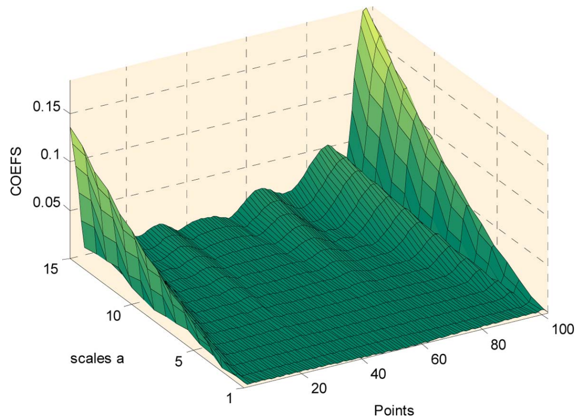

Fig. 21 Dynamic response, frequency is 0.5 Hz, $l = 0\%$

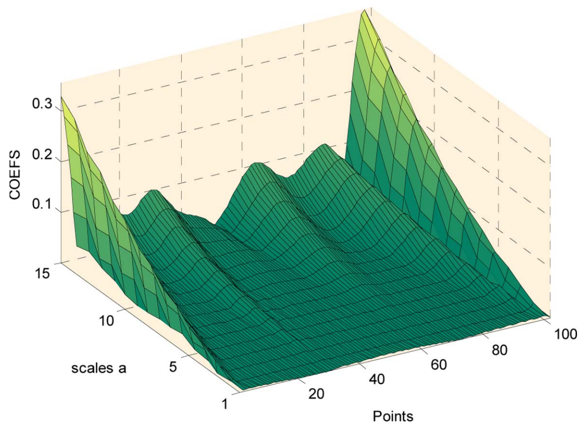

Fig. 22 Dynamic response, frequency is 1.8 Hz, $l = 0\%$

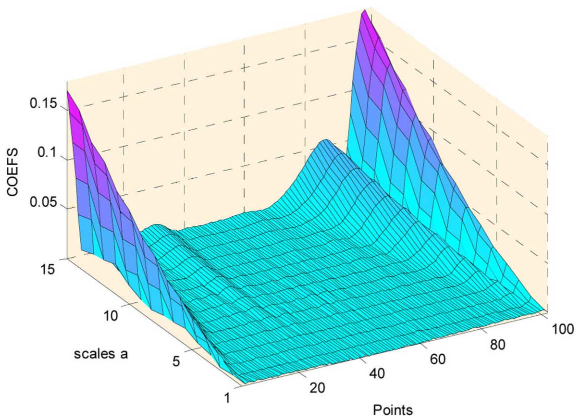

Fig. 23 Static response, $l = 0.006\%$

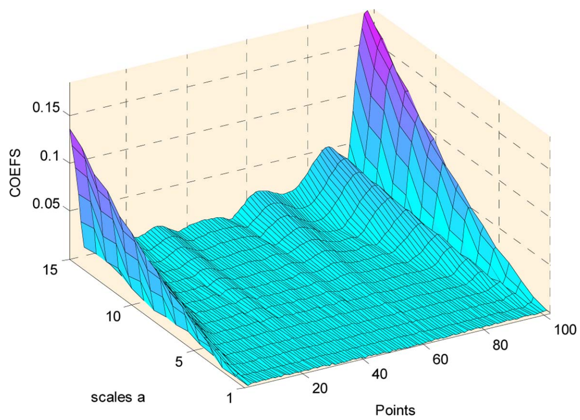

Fig. 24 Dynamic response, frequency is 0.5 Hz, $l =$ 0.006%

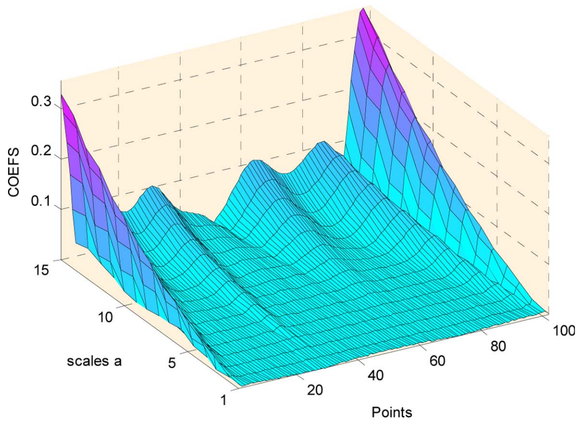

Fig. 25 Dynamic response, frequency is 1.8 Hz, $l =$ 0.006%

graphs of WT coefficient (sym4) without any noise in static and dynamic response is shown in Figs. 20, 21 and 22 respectively. The frequency of dynamic loading is 0.5 and 8 Hz. As can be seen in

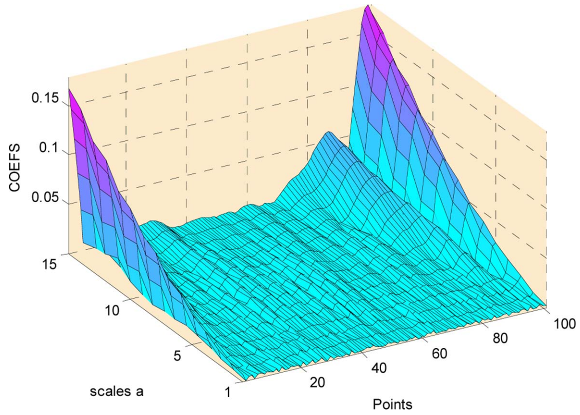

Fig. 26 Static response, $l = 0.3\%$

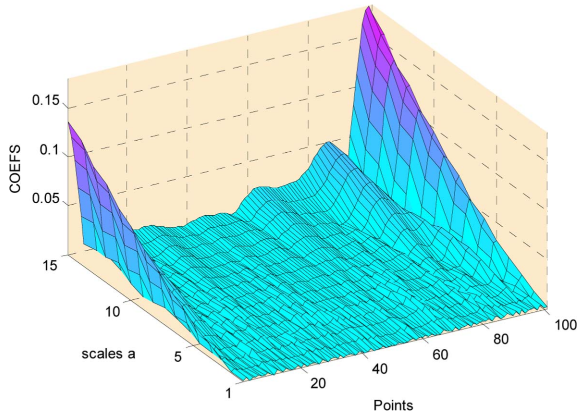

Fig. 27 Dynamic response, frequency is 0.5 Hz, $l = 0.3\%$

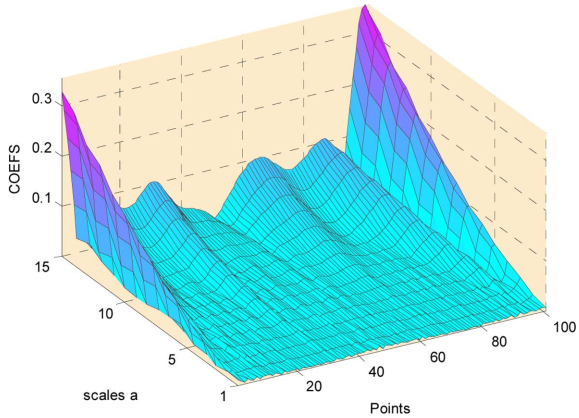

Fig. 28 Dynamic response, frequency is 1.8 Hz, $l = 0.3\%$

all graphs, the location of damaged area is detectable distinctively at $x_d = 30$ cm. Also the location of load is distinguished at $x_v = 65$ cm. To discover the damage location while using a noisy signal, the 3D graphs of CWTs (Symlet6) are shown in Fig. 23 to Fig. 28, because in the case of noisy response, 3-D graph has high efficiency to detect the crack location. To study the negative influence of the noise on efficiency of the static and dynamic responses, two noisy signal first with low amplitude, $l = 0.006\%$, and another with high amplitude, $l = 0.3\%$, are added to each of the two responses. Since all responses have been normalized to unit value, this method of comparison of influence of the noise seems logical. Figs 23, 24 and 25 shown the case of noise with low amplitude and Figs 26, 27 and 28 the case of noise with high amplitude. As can be seen, addition of the noise to dynamic response with high frequency (1.8 Hz) has a less influence than static response and dynamic response with low frequency (0.5 Hz). Another point is a similar approximately decreases of influence in the case of static response and dynamic response with low frequency (0.5 Hz).

5. Conclusions

In this study a powerful mathematical tool for crack identification in plate structures has been presented. The response of a cracked plate having an all-over part-through the crack parallel to one edge of the plate has been chosen as an input function in a WT analysis.

Damages and defects in a plate induce certain perturbation features in the structural response along lines perpendicular to the crack. Such perturbation features that are not obvious from the response data are, however, revealable as a singularity by analyzing the spatial response with continuous wavelet transform and with the desired resolution. Both static and dynamic loading can be used to gain the response of damaged plates. In this research, it was recognized that in order to damage detection, the harmonic loading is more effective than the static response analysis. In harmonic analysis, the response of a plate subjected to harmonic loading which is induced by a harmonic vibrator is obtained. The harmonic response of the plate is analyzed using WT. Local perturbations that correspond to the position of damages can be detected with the desired resolution by sudden changes in the spatial variation of the transformed response. Mentioned ability is due to the variable resolution property of the wavelet transform. This study demonstrates that the use of the harmonic response is more efficient than the static response for the following causes:

(1) Since, the frequency and the location of vibrators both have the ability of variation, in practical cases, harmonic loading is more versatile than static loading,
(2) The harmonic response is less sensitive than the static response to noise, in a WT analysis.
(3) In wavelet analysis, harmonic point loads have less negative influence on the coefficient of perturbation than the static loads
(4) Noise has the same efficiency in damage detection by WT analysis for both static and dynamic responses with low frequency.
(5) The efficiency and accuracy of damage detection procedure increases with the increase in harmonic vibration frequency.

More importantly, the present investigation cannot be considered a conclusive work. Further work is needed, however, to advance crack detection in plates using wavelet analysis. It is suggested to investigate the possibility of using of 2D wavelet in damage detection of plates. Also with attention to noise effects on CWTs graphs that in high amplitude affects similar to defects, the need for research in reduction of noise is necessary. Experimental tests would also provide a useful correlation with the study carried out in this work. Work is already under way on the above mentioned issues and will be the subject of a future publication.

References

Chang, C. and Chen, L. (2003), "Vibration damage detection of a Timoshenko beam by spatial wavelet based approach", *Appl. Acoust*., **64**(12), 1217-1240.

Daubechies, I. (1992), Ten Lectures on Wavelets, CBMS-NSF Conference Series, 61. Philadelphia, PA: SIAM.

Deng, X. and Wang, Q. (1998), "Crack detection using spatial measurements and wavelet", *Int. J. Fracture*., **91**, 23-28.

Douka, E., Loutridis, S. and Trochidis, A. (2003), "Crack identification in beams using wavelet analysis", *Int. J. Solids Struct*., **40**(13-14), 3557-3569.

Fan, W. and Qiao, Q. (2009), "A 2-D continuous wavelet transform of mode shape data for damage detection of plate structures", *Int. J. Solids Struct*., **46**(25-26), 4379-4395.

Gentile, A. and Messina, A. (2003), "On the continuous wavelet transforms applied to discrete vibrational data for detecting open cracks in damaged beams", *Int. J. Solids Struct*., **40**(2), 295-315.

Hou, Z. and Noori, M. (1999), "Application of wavelet analysis for structural health monitoring", *Proceedings of the 2nd Int. Workshop on Structural Health Monitoring*, Stanford, CA: Stanford University.

Hou, Z., Noori, M. and Amand, S. (2000), "Wavelet based approach for structural damage detection", *J. Eng. Mech-ASCE*., **126**(7), 677-683.

Hansang, K. and Melhem, H. (2004), "Damage detection of structures by wavelet analysis", *Eng. Struct*., **26**(3),347-362.

Huang, Y., Meyer, D. and Nemat-Nasser, S. (2009), "Damage detection with spatially distributed 2D Continuous Wavelet Transform", *Mech. Mater*., **41**, 1096-1107.

Khadem, S.E. and Rezaee, M. (2000), "An analytical approach for obtaining the location and depth of an allover part-through crack on externally in-plane loaded rectangular plate using vibration analysis", *J. Sound Vib*., **230**(2), 291-308.

Khatam, H., Golafshani, A.A, Beheshti-Aval S.B. and Noori, M. (2007), "Harmonic class loading for damage identification in beams using wavelet analysis", *Struct. Health Monit*., **6**(1), 67-80.

Lee, H.P. (1992), "Fundamental frequencies of annular plates with internal cracks", *Comput. Struct*., **43**, 1085-1089.

Lee, H.P. and Lim, S.P. (1993), "Vibration of cracked rectangular plates including transverse shear deformation and rotary inertia", *Comput. Struct*., **49**(4), 715-718.

Mallat, S.G. (1999), A wavelet tour of signal processing, 2nd ed. London, Academic Press.

Quek, S., Wang, Q., Zhang, L. and Ang, K. (2001), "Sensitivity analysis of crack detection in beams by the wavelet technique", *Int. J. Mech. Sci*., **43**(12), 2899-2910.

Ovanesova, A.V. and Suarez, L.E. (2004), "Applications of wavelet transforms to damage detection in frame structures", *Eng. Struct*., **26**(1), 39-49.

Schulz, M.J., Ghoshal, A., Sundaresan, M.J., Pai, P.F. and Chung, J.H. (2003), "Theory of damage detection using constrained vibration deflection shapes", *Struct. Health Monit*, **2**(1), 75-99.

Wang, Q. and Deng, X. (1999), "Damage detection with spatial wavelets", *Int. J. Solids Struct*., **36**(23), 3443-3468.

Wu, N. and Wang, Q. (2011), "Experimental studies on damage detection of beam structures with wavelet transform", *Int. J. Eng. Sci*., **49**, 253-261.

Zhong, S. and Oyadiji, S.O. (2011), "Detection of cracks in simply-supported beams by continuous wavelet transform of reconstructed modal data", *Comput. Struct*., 89(1-2), 127-148.

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Appendix A

Choosing Symlet wavelet family discussed in Douka *et al.* (2003). The comparative similar study by authors showed that using wavelet family with vanishing moments less than 4 (Harr and Gauss1, 2) for identification of very small defects had low performance. Otherwise using wavelet family with grater vanishing moments could not cause better performance. Moreover using such wavelet family as indicated existing one damage cause several perturbation in wavelet coefficients, then decrease the resolution of damage location (Fig. 29). Table 1 represents the sufficient mother wavelets and the number of vanishing moments related to them. The results of these studies showed that the *"Symmetrical 4"* wavelet could be chosen and used as analyzing wavelet throughout such works. This issue also addressed in Gentile and Messina (2003).

Table 1 Properties of mother wavelets

Mother wavelet	Number of V.M
symmetrical 4	4
Coiflet 2	4
Biorthogonal 6.8	5

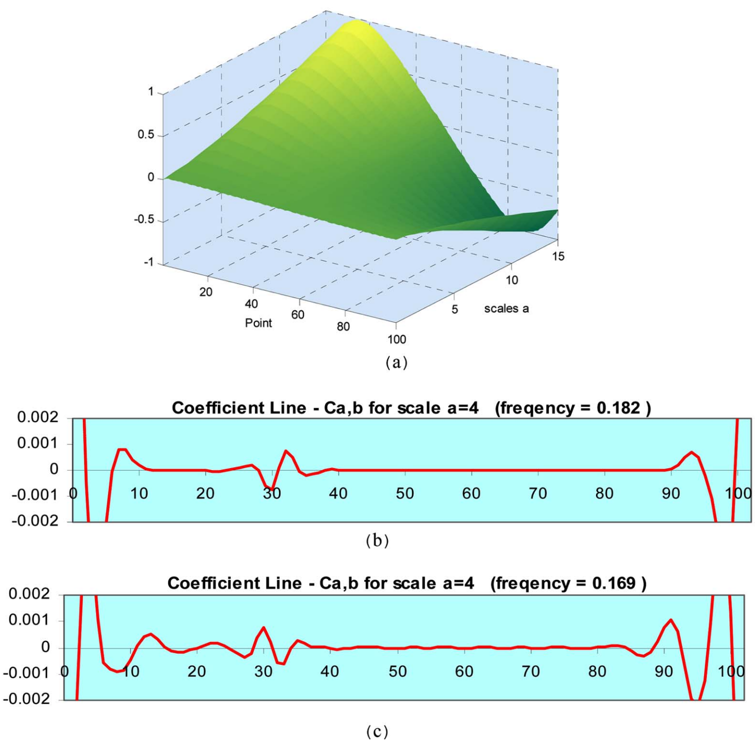

Fig. 29 Wavelet coefficient (a) Gauss 1 wavelet coefficient, (b) Sym4 and (c) Sym16