Smart Structures and Systems, Vol. 8, No. 2 (2011) 205-217

DOI: http://dx.doi.org/10.12989/sss.2011.8.2.205

Direct assignment of the dynamics of a laboratorial model using an active bracing system

C. Moutinho*, A. Cunha and E. Caetano

Faculty of Engineering of University of Porto (FEUP), Portugal

(Received June 18, 2010, Accepted May 17, 2011)

Abstract. This article describes the research work involving the implementation of an Active Bracing System aimed at the modification of the initial dynamics of a laboratorial building structure to a new desired dynamics. By means of an adequate control force it is possible to assign an entirely new dynamics to a system by moving its natural frequencies and damping ratios to different values with the purpose of achieving a better overall structural response to external loads. In Civil Engineering applications, the most common procedures for controlling vibrations in structures include changing natural frequencies in order to avoid resonance phenomena and increasing the damping ratios of the critical vibration modes. In this study, the actual implementation of an active system is demonstrated, which is able to perform such modifications in a wide frequency range; to this end, a plane frame physical model with 4 degrees-of-freedom is used. The Active Bracing System developed is actuated by a linear motor controlled by an algorithm based on pole assignment strategy. The efficiency of this control system is verified experimentally by analyzing the control effect obtained with the modification of the initial dynamic parameters of the plane frame and observing the subsequent structural response.

Keywords: vibration control; system dynamics; pole assignment; laboratorial structure; active control; active bracing system.

1. Introduction

Many Civil Engineering structures have vibration problems in terms of the serviceability and ultimate limit states due to several transient or periodic dynamic loads, e.g., footbridges subjected to pedestrians actions, road bridges and railway bridges excited by traffic loads and tall buildings exposed to wind and seismic forces. In these situations, the implementation of control systems can improve the structural performance by reducing the vibration levels to acceptable values, defined for each case. To achieve this objective, several types of control systems can be used such as passive, active or semi-active devices.

It is generally accepted that active systems, despite constituting a powerful control scheme, are not an interesting solution for many structural problems, especially when dealing with large structures. This stems from the fact that active control demands sophisticated technology, high costs and high maintenance and are less reliable than passive systems (Soong 1990). This is why passive devices are still the most widely used solution, although potentially less efficient than active systems. In order to solve some of the problems of passive control systems, especially the difficulty

*Corresponding Author, Assistant Professor, E-mail: moutinho@fe.up.pt

in operating in a wide frequency range, the use of semi-active systems constitutes an intermediate and more realistic solution between passive and active control (Casciati *et al.* 2006). In the last decade, some interesting devices of this type have been investigated and implemented worldwide (Spencer Jr. *et al.* 2003).

However, given the potential of active systems, active control can still be an attractive solution, particularly for small structures and very flexible systems whose dynamics is dominated by the contribution of several vibration modes shapes (Fujino 2002). In this case, active systems can operate effectively in a wide frequency bandwidth and control the dynamics of the most significant modes of the system (Preumont 1997).

In this context, this article attempts to demonstrate how active control can be used to artificially reshape the initial dynamics of a structural system to a new dynamics, supposedly more advantageous in terms of the overall structural behavior. In practical cases, the implementation of such a control system could be applied to lightweight and flexible structures where there is interest in assigning an entirely new dynamics in terms of a new set of natural frequencies and damping ratios.

To achieve this goal, several control strategies can be used. One of the possible approaches is to define a control law based on the optimization of a certain performance index, like optimal control (Meirovich 1990). In this case, the dynamics of the system is characterized by the new natural frequencies and damping ratios established according to the adopted optimal criteria. Although this procedure provides the mathematical solution to the control problem, it also causes a loss in sensitivity to the physical meaning of the changes implemented in the system.

On the other hand, as is the case in many Civil Engineering structures, the desired dynamic characteristics of the system can be clearly defined according to the required structural response. As an example, in the occurrence of resonance problems associated with some vibration modes of the structure, the amount of damping necessary to reduce the structural response to certain values can be set according some basic structural dynamics calculations. Similarly, it is easy to quantify the changes in the natural frequencies of the system in order to avoid resonance problems with known external excitations. In these situations, the desired dynamic parameters of the system can be previously defined, which means that the control problem under analysis can be solved by directly assigning the system properties.

Moreover, in some practical cases there is only interest in adding damping to the system without changing natural frequencies because the stiffness forces are much larger than damping forces. A procedure for directly assigning damping ratios to the vibration modes of a structure without modifying natural frequencies is presented in this work.

2. Control strategy

The control strategy used to modify the dynamics of the system to a new dynamics defined according to a specific design scenario is the classical pole assignment technique (Ogata 1996). According to this approach, given a system with natural frequencies $f_1, f_2,\ldots, f_n$ and damping ratios ξ_1, $\xi_2,\ldots$, ξ_n, it is possible to directly assign a new dynamics characterized by the new parameters $f'_1, f'_2,\ldots, f'_n$ and ξ'_1, $\xi'_2,\ldots$, ξ'_n, using a single control force. To achieve this, the system must be considered linear in the displacements range of interest, and be completely controllable and fully completely observable. Otherwise, not all parameters can be directly assigned.

The requirement that the system must be linear stipulates that this strategy is only applicable to structures during the serviceability stage. If the system behaves as nonlinear, which may happen under ultimate limit state conditions, or if the system experiences a significant variation of its parameters over time, a different robust strategy should be used instead.

The application of classical pole assignment strategy starts by establishing the new dynamic for the system and calculating the values of the closed-loop poles in correspondence with these desired values. Afterwards, given the system state matrix and the input matrix, the gain matrix is calculated using some known method, like the Ackermann's formula (Ogata 1996) or simply Matlab's "place" command. As a result, using the obtained gain matrix in a closed-loop control system, the structure will behave according to the new dynamics, artificially induced by the control force.

In many practical cases, increasing damping ratios of the significant vibration modes of the structure without changing natural frequencies is preferable because damping forces are generally lower than stiffness forces, while having a more favorable phase angle in terms of reducing the motion of the structure. However, the methods available in the literature require the definition of both parameters, damping ratios and natural frequencies. The assignment of damping ratios alone is not so common, but it makes perfect sense in the case of Civil Engineering structures.

On the other hand, the assignment of both parameters results in a gain matrix containing non-zero elements, which necessarily involves the calculation of stiffness forces. In this context, a method that can be used to assign only damping factors to the system is proposed, as summarized in the following steps:

(a) Definition of the damping ratios required for all vibration modes;

(b) Definition of the current (unchanged) natural frequencies of the system in correspondence with the same vibration modes;

(c) Calculation of the closed-loop poles using these dynamic parameters;

(d) Obtaining the gain matrix (K) using any available method;

(e) Set to zero the semi-matrix of the gain matrix that feeds backs the system displacements (K_d) and maintain the semi-matrix associated with velocities (K_v), obtaining the sought gain matrix (K'):

$$K = [K_d \; K_v] \quad K' = [0 \; K_v] \tag{1}$$

Using this final gain matrix, the damping ratios will move to the desired values while the natural frequencies remain approximately the same.

3. Description of the model, equipment and sensors

3.1 Laboratorial structure and shaking table

The laboratorial structure used to implement a real control test in terms of changing the initial dynamics of the system to another predefined dynamics is the three-storey plane frame physical model supported on a shaking table shown in Fig. 1(a). The structure of a building was selected not because it is a potential receiver of an active system, but because it can be easily constructed and numerically modeled.

The plane frame is comprises by rigid iron masses connected to each other through aluminum columns. As shown in Fig. 1(b), the shaking table has springs connecting it to the ground which adds one degree-of-freedom to the system, resulting in a 4-DOF building model. The total mass of

(a) (b)

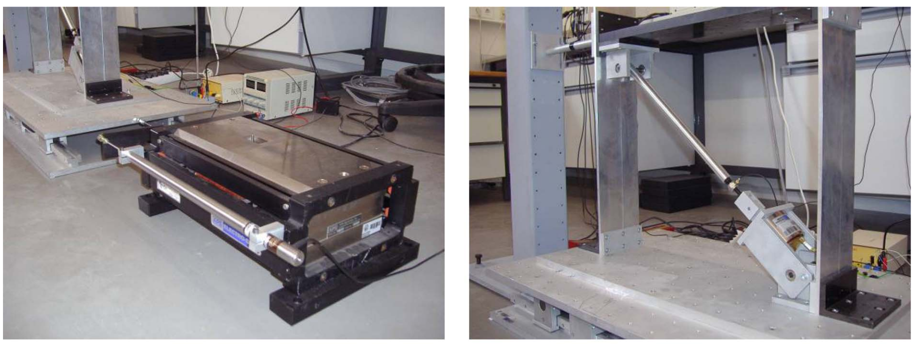
Fig. 1 (a) Plane frame physical model supported by a shaking table and (b) detail of the shaking table

the system at the shaking table level is $m_1 = 44.6$ kg and the others masses, at higher levels, are m_2=15.7 kg, m_3=15.0 kg and m_4=12.8 kg, respectively. The aluminum columns are 400 mm high, 120 mm wide and 7 mm thick, and are clamped at each level. The aluminum modulus of elasticity was estimated at about 60 Gpa.

The total stiffness of the auxiliary springs of the shaking table is 76.9 kN/m. This sliding platform of dimensions 0.54×0.94 m^2 is connected to an electromagnetic shaker, used to apply perturbation forces at the base of the model (see Fig. 2(a)). This device can apply maximum forces of about 445 N in the frequency range of interest and the electric current powered by the respective amplifier can be measured by an internal sensor, which is directly related to the applied force.

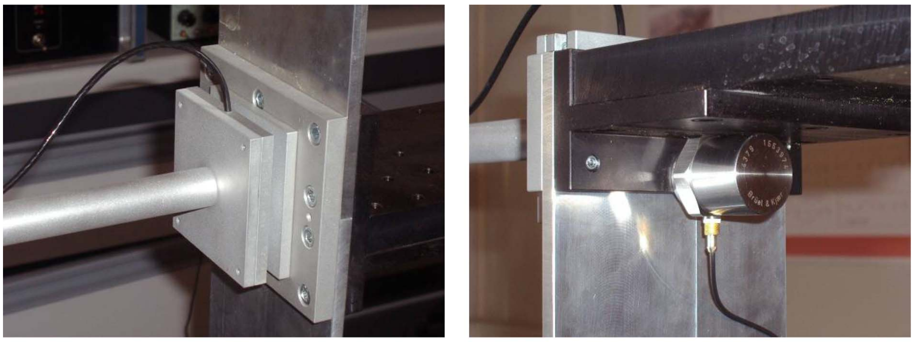
3.2 Active bracing system

In order to achieve the objective of the control, an Active Bracing System was installed at the bottom level of the structure (see Fig. 2(b)). This device consists of a voice coil motor which was adapted to a diagonal structural bar, and is able to apply a pair of control forces in the connection nodes. The vertical components of this pair of forces are reacted by the vertical columns which are stiff enough to avoid pier buckling, and the horizontal components correspond to the effective control action. The voice coil motor can apply forces up to 45 N in the frequency range of interest, using 50 W of power.



(a) (b)

Fig. 2 (a) Electromagnetic shaker connected to the shaking table and (b) active bracing system



(a) (b)

Fig. 3 (a) Non-contact displacement sensor and (b) accelerometer at the top level

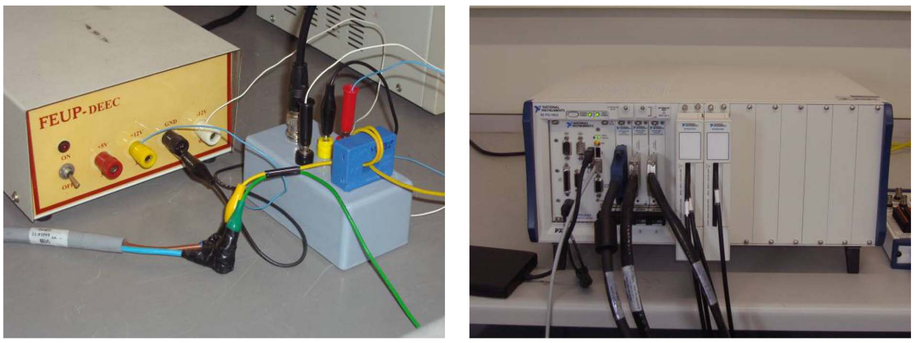
3.3 Sensors

The dynamic response of the physical model was continuously measured by non-contact displacement sensors positioned at each frame level, as shown in Fig. 3(a). This kind of transducers is particularly adequate for operating with small models without affecting its dynamics. The system response at shaking table level was measured by a displacement transducer integrated in the shaker functioning. A piezoelectric accelerometer positioned at the top floor was also used (see Fig. 3(b)).

The control force applied by the active bracing system was measured by a load cell aligned with the diagonal bar axis, as well as the electric current supplied by the motor amplifier using a non-contact sensor (see Fig. 4(a)).

3.4 Controller and fourier analyzer

The control algorithm developed was implemented in a Real-Time controller comprising an NI PXI computer (see Fig. 4(b)), which includes a specific operating system dedicated to ensuring determinism in control loops. This controller works as a target computer controlled by a host computer used to develop software and viewing data from the Real-Time device. All software created to operate with this system was developed with Labview from National Instruments. When the controller is



(a) (b)

Fig. 4 (a) Detail of the electric current transducer and (b) NI PXI Controller functioning, it executes several tasks according to some defined levels of priority. Naturally, tasks like calculating the control force and tasks involving safety procedures are of priority compared to tasks like saving or viewing data.

Besides the controller, a Fourier analyzer was used, which allows estimating the Frequency Response Functions (FRFs) of the physical model and, consequently, extracting the dynamic parameters of the system, like natural frequencies and damping ratios. This equipment was also used to measure some quantities not directly related to the control algorithm, like electric current supplied to the voice coil motor, effective control forces and accelerations at top level, as well as to acquire time signals for complementary evaluation of the system damping ratios.

4. Characterization of system dynamics

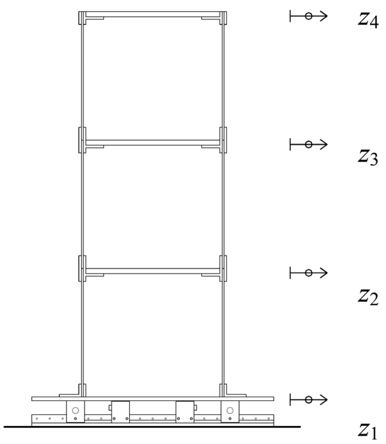
4.1 Numerical model

In order to characterize the system dynamics, a numerical model of the plane frame was developed, based on the physical characteristics described in last section and in some previous experimental tests. The numerical model is based on the assumption that the plane frame has a 4-DOF shear building behavior, with the following well-known equation of motion

$$M_s\ddot{z} + C_s\dot{z} + K_s z = f \tag{2}$$

where M_s, C_s and K_s are the mass, damping and stiffness matrices, respectively, $\ddot{z}$, $\dot{z}$ and z are the accelerations, velocities and displacements vectors, and f is the load vector. The mass and stiffness matrices of the physical model are summarized in Fig. 5 and are in correspondence with the degrees-of-freedom indicated in the plane frame drawing.

The damping properties of the system were evaluated by previous experimental tests. The method used to extract the damping coefficients of the structure consists of exciting the physical model with a resonant harmonic force and, by suddenly stopping the excitation, measuring the free vibration response. The estimation of the respective modal damping ratio can be obtained by performing an



$$\mathrm{M_s} = \begin{bmatrix} 43.55 & 0 & 0 & 0 \\ 0 & 15.67 & 0 & 0 \\ 0 & 0 & 14.99 & 0 \\ 0 & 0 & 0 & 12.76 \end{bmatrix} \mathrm{kg}$$

$$\mathrm{K_s} = \begin{bmatrix} 154.5 & -74.5 & 0 & 0 \\ -74.5 & 149.0 & -74.5 & 0 \\ 0 & -74.5 & 149.0 & -74.5 \\ 0 & 0 & -74.5 & 74.5 \end{bmatrix} \mathrm{kN/m}$$

Fig. 5 Identification of structure degrees-of-freedom and corresponding mass and stiffness matrices

Table 1 Analytical vs. Experimental natural frequencies and damping ratios

Vibration mode	Analytical	Experimental	
	Natural frequency(Hz)	Natural frequency(Hz)	Damping ratio(%)
1	3.86	3.80	1.78
2	8.67	8.65	2.21
3	14.96	15.75	2.31
4	20.40	22.25	0.85

exponential fitting of the free decay response envelope. The damping matrix can be determined using the superposition of the modal damping matrices method (Chopra 1995), resulting in this case

$$C_s = \begin{bmatrix} 104.2 & -19.7 & -15.4 & 0.8 \\ -19.7 & 46.1 & 1.9 & -16.4 \\ -15.4 & 1.9 & 29.9 & -10.46 \\ 0.8 & -16.4 & -10.46 & 31.5 \end{bmatrix} \text{Ns/m}$$

The summary of the dynamic parameters of the structure is listed in Table 1. A more convenient numerical model for control purposes was obtained from the previous formulation using space state analysis, resulting in the following well-known space state and output equations

$$\dot{x} = Ax + Bu; \; y = Cx + Du \tag{3}$$

where A, B, C and D are the state matrix, input matrix, output matrix and direct transmission matrix, respectively, and x, u and y are the state vector, input vector and output vector, respectively. In the case of mechanical systems, they are given by

$$A = \begin{bmatrix} I & 0 \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M_s^{-1}J \end{bmatrix} \quad C = I \quad D = 0 \quad x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

where J the is the mapping matrix of the input force, I is the identity matrix and 0 is the null matrix.

4.2 Identification of modal parameters

In order to identify the experimental modal properties of the system, in particular natural frequencies, mode shapes and damping ratios, the model was subjected to several tests. The estimates obtained were compared to the analytical values (with the exception of the damping ratios, which can only be found via experimental results, specified in the previous section).

The natural frequencies of the system were evaluated with the help of a Fourier analyzer, which allows estimating the frequency response functions (FRFs) of a system. In the present case, the FRF of interest was evaluated by relating the input force applied by the shaking table and the output acceleration measured at the top floor. For this purpose, a frequency range from 0 to 25 Hz was stipulated and the average estimate of 15 FRFs was considered. Each time series has an acquisition time of 16s, which means that the frequency resolution achieved was 0.0625 Hz.

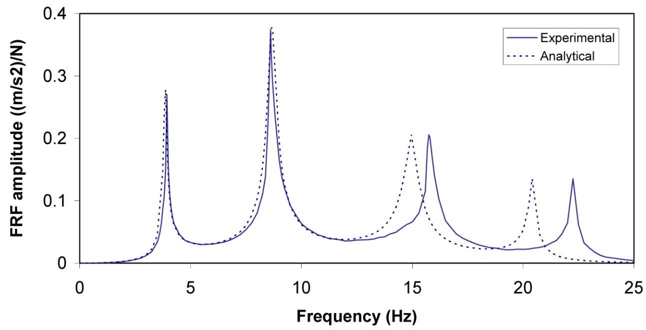


Fig. 6 Experimental vs. Analytical FRF

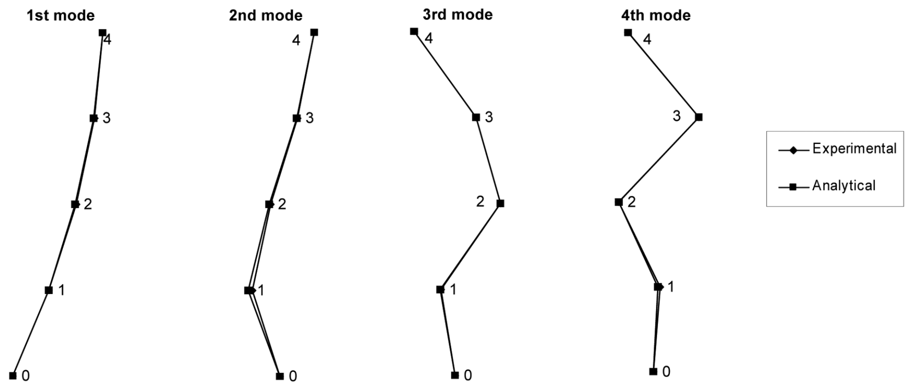


Fig. 7 Experimental vs. Analytical vibration mode shapes

Fig. 6 shows the magnitude of the FRF obtained as described, which allows to extract the experimental natural frequencies of the system, clearly identified by the peaks on the graph. In this figure, the analytical FRF is also plotted, which suggests a very good correlation between the measured and calculated natural frequencies associated with the first two modes of vibration. These results are summarized in Table 1.

The method used to identify the vibration mode shapes simply involved exciting the structure at resonant frequencies at the base of the model using the shaking table, and measuring the amplitude and phase angle of the system response at several degrees-of-freedom. Fig. 7 shows the graphical representation of the mode shapes in correspondence with the natural frequencies indicated in Table 1. An excellent agreement between identified and calculated vibration modes can be observed.

5. Design of the control system

5.1 Controllability and observability

The first issue considered in the design of the control system was controllability, which is directly related to the position of the actuator. Depending on the desired dynamics of the structure, the actuation system should be positioned in a section where the most significant vibration modes have important modal components.

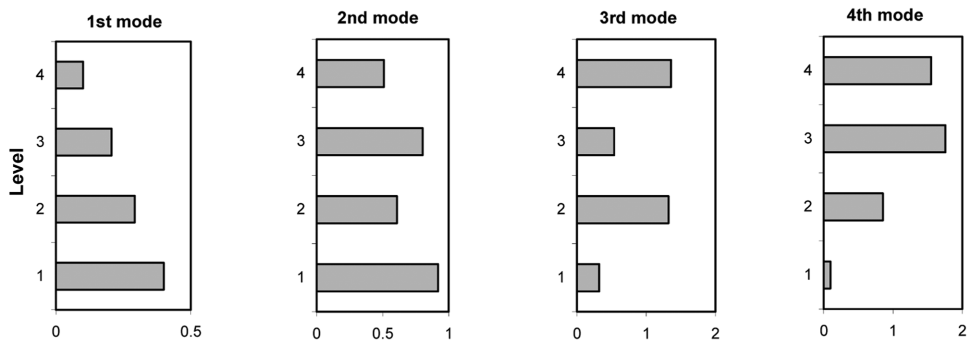


Fig. 8 Absolute values of relative modal inter-storey components

Because the control action is applied in terms of a pair of nodal forces, it is necessary to evaluate the modal inter-storey components associated with each vibration mode. Fig. 8 represents the absolute values of these quantities, which were extracted from Fig. 7. "1st level" is related to the space between the ground and the shaking table, while "4th level" corresponds to the space between 2nd and 3rd floor. From analyzing this figure, the choice of positioning the actuation system at the 2nd level (between shaking table and 1st floor) seems to be adequate. In fact, all the vibration modes have important relative modal components at this location, with particular emphasis for the first mode, which is often dominant in system response.

The system observability was assured by the direct measurement of all states by the displacement transducers. The corresponding velocities were obtained from displacements after removing noise with a suitable combination of analogue and digital filters which allowed getting a negligible time delay in the signal.

5.2 Definition of the new dynamics

As mentioned before, the goal of this control system is to change the initial dynamics of the structure to another dynamics, which is more interesting in terms of performance against external loads. In this particular work, a specific design scenario which could define those desired dynamic characteristics for the system was not considered. Instead, it was decided to consider a general case for which it would be advantageous to change its natural frequencies and damping ratios.

In this hypothetical scenario, the goal is to decrease the 2nd natural frequency from 8.67 Hz to 8.00 Hz and increase the 3rd natural frequency from 14.96 Hz to 15.5 Hz. Simultaneously, it is necessary to increase the damping ratio of the 1st vibration mode from 1.78% to 2.5% and increase damping of the 2nd vibration mode from 2.21% to 3.0%. All other dynamic parameters should maintain their initial values. This desired dynamics was established based on analytical natural frequencies and damping ratios, which differ slightly from the experimental values. This is because the control gain matrix will be defined using the numerical model. Furthermore, the changes required were not too demanding in order to take into account the force limitations of the actuating system.

The target natural frequencies and respective damping ratios corresponding to that new dynamics are summarized in Table 2. Fig. 9 plots the analytical FRF, as defined in the previous section, obtained without control, in comparison with the target FRF obtained with control system switched on.

Table 2 Target analytical system parameters

Vibration mode	Frequency(Hz)	Damping ratio(%)
1	3.86	2.50
2	8.00	3.00
3	15.50	2.31
4	20.40	0.85

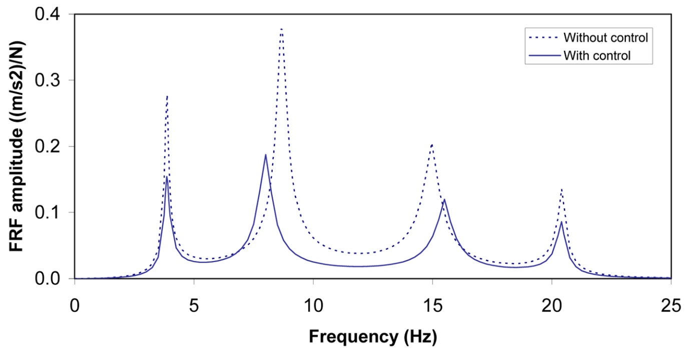


Fig. 9 Analytical FRF obtained with and without control

5.3 Implementation of the control system

The closed-loop poles associated with the dynamics required for the system are listed in Table 3. Given the system state matrix and output matrix calculated according to section 4.1, the gain matrix in correspondence with these poles can be calculated by the pole assignment technique, resulting in

$$K = [27661 \;\; 18391 \;\; -4273 \;\; -18919 \;\vdots\; 472 \;\; -253 \;\; -177 \;\; -204]$$

Table 3 Target closed-loop poles

Vibration mode	Closed-loop poles
1	0.606±24.245i
2	1.508±50.242i
3	2.250±97.363i
4	1.090±128.17i

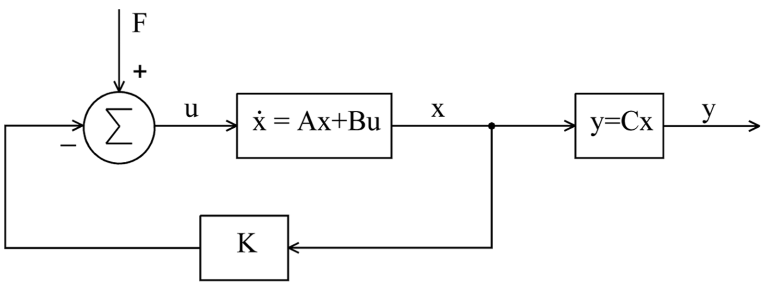


Fig. 10 Block diagram of the control system

The block diagram of the whole control system is represented in Fig. 10, where F is an external perturbation. The control action at each instant of time is obtained by $F_c = -Kx$ and is applied by the Active Bracing System to the structure. Although the control strategy is formulated in continuous time, its implementation is integrated in a digital control loop. This is possible because the frequency rate was set at 200 Hz, which can be considered adequate in order to get a digital signal very close to the analogue one.

6. Experimental results

6.1 Random loads

The efficiency of the proposed control system was tested using either random or harmonic loads. In the case of random loads, the implementation of the new dynamics in the structure was verified by calculating the experimental FRF with the control system switched on and comparing it to the former function obtained without control. These FRFs are represented in Fig. 10. These FRFs can be verified as being identical to the analytical ones plotted in Fig. 9, which denotes a good performance of the control system in achieving the predefined objectives. Moreover, when analyzing the new experimental system properties, it can be concluded that the desired system dynamics was effectively implemented in the controlled structure, despite the expected small deviations. In fact, the natural frequency of 8.65 Hz decreased to 8.0 Hz and the 15.75 Hz frequency increased to 16.12 Hz, which means that these frequencies have changed approximately in the same proportion as required by the analytical study. On the other hand, the damping ratio of the 1st vibration mode increased to 2.83% and the damping ratio of the 2nd vibration mode increased to 2.92%. The overall results of this analysis are condensed in Table 4, which should be compared to Tables 1 and 2.

Table 4 Modified system parameters

Vibration mode	Frequency (Hz)	Damping ratio (%)
1	3.90	2.83
2	8.00	2.92
3	16.12	2.10
4	22.35	1.05

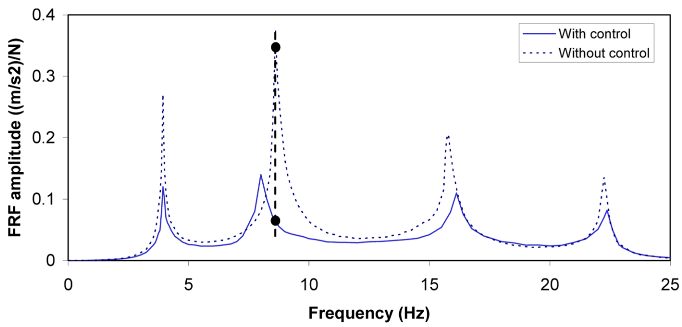


Fig. 11 Experimental FRF obtained with and without control

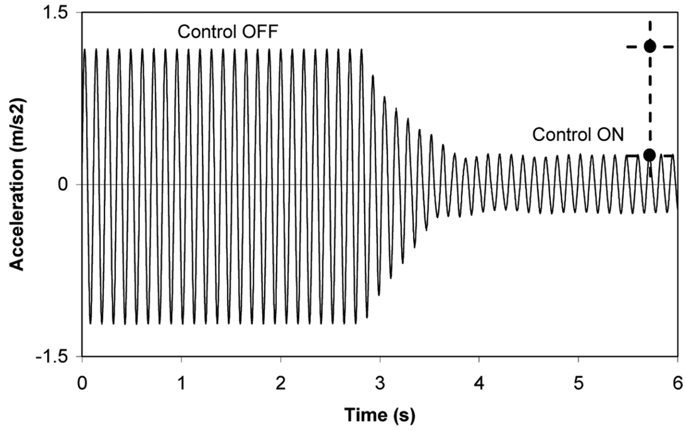


Fig. 12 System response to harmonic load of 8.65 Hz

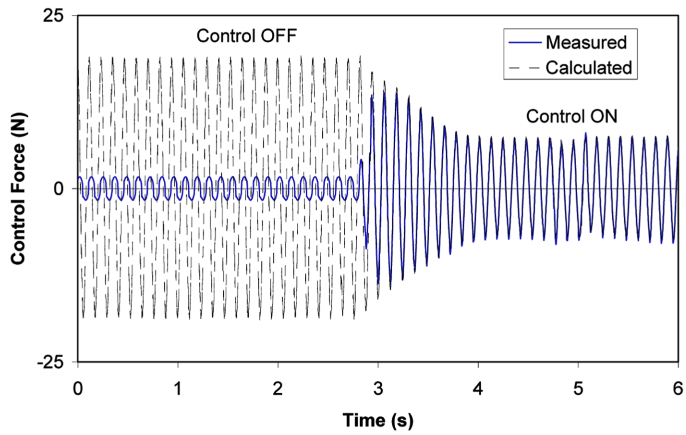


Fig. 13 Measured and calculated control force

6.2 Harmonic loads

The effect of the control system on the modification of the structure response to harmonic loads can be directly evaluated by analyzing the experimental FRFs of Fig. 10. In fact, since the FRF is a function that expresses the system response to harmonic excitations, the reduction achieved with the control can be obtained by consulting the FRF curves at a specific frequency of interest.

In order to verify the control efficiency in time domain, the physical model was initially excited without control with a harmonic force of 8.65Hz. Suddenly, the control system was switched on. The time history obtained at the top floor in terms of acceleration is represented in Fig. 11. It can be observed that the effect obtained is in good agreement with the reduction indicated in Fig. 10 (see dashed lines and dots).

This test also compared the calculated and measured control forces related to this harmonic excitation. Fig. 12 shows that the bracing system initially develops a residual force due to the passive functioning of the device. But when the control signal is gradually increased, the active system applies a force that fits quite well with the calculated control action.

7. Conclusions

This article describes the experimental work involving the implementation of an Active Bracing System to control the dynamics of a plane frame physical model. The control strategy adopted is based on the well-known pole placement design, which can be used to change the dynamic characteristics of the system to predefined values, aimed at the improvement of its response when excited by different kind of loads. This is possible because the system is considered to be linear and is completely controllable and observable.

In this particular work, a target dynamics was defined for the physical model based on a hypothetical design scenario. In this case, it was decided to change both natural frequencies and damping ratios as the general approach to the problem, knowing that in many practical cases there is only interest in changing damping ratios. The new dynamics was artificially imposed by the proposed active system.

In order to verify the control efficiency, the model was subjected to random and harmonic loads. In the case of random loads, the calculated FRFs before and after control clearly show that the desired dynamics was effectively implemented. The tests with harmonic loads also reflected that result. In the particular case of excitation with a resonant harmonic load of 8.65 Hz, it was possible to reduce 80% of the initial response by moving the corresponding natural frequency and damping ratio to different values.

References

Casciati, F., Magonette, G. and Marazzi, F. (2006), *Technology of semiactive devices and applications in vibration mitigation*, John Wiley & Sons.

Chopra, A. (1995), *Dynamics of structures - theory and applications to earthquake engineering*, Prentice-Hall.

Fujino, Y. (2002), "Vibration, control and monitoring of long-span bridges - recent research developments and practice in japan", *J. Constr. Steel Res.*, **58**(1), 71-97.

Meirovich, L. (1990), *Dynamics and control of structures*, Wiley &Sons.

Ogata, K. (1996), *Modern Control Engineering*, Prentice-Hall.

Preumont, A. (1997), "Vibration control of active structures", Kluwer Academic Publishers.

Soong, T. (1990), *Active structural control - theory & practice*, Longman Scientific & Technical.

Spencer Jr., B. and Nagarajaiah, S. (2003), "State of the art of structural control", *J. Struct. Eng.- ASCE*, **129**(7), 845-856.

FC