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A comparative study on the subspace based system identification techniques applied on civil engineering structures

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Abstract. The Subspace based System Identification Techniques (SSIT) have been very popular within the research circles in the last decade due to their proven superiority over the other existing system identification techniques. For operational (output only) modal analysis, the stochastic SSIT and for operational modal analysis in the presence of exogenous inputs, the combined deterministic stochastic SSIT have been used in the literature. This study compares the application of the two alternative techniques on a typical school building in Istanbul using 100 Monte Carlo simulations. The study clearly shows that the combined deterministic stochastic SSIT performs superior to the stochastic SSIT when the techniques are applied on noisy data from low to mid rise stiff structures.

Keywords: subspace based system identification; structural health monitoring; ambient vibration; operational modal analysis; buildings.

1. Introduction

Vibration monitoring of civil engineering structures for the purposes of damage detection is becoming increasingly popular due to the technological advances in sensors and data acquisition systems as well as the improvements in robust system identification techniques. Damage in structures results in changes in their modal parameters such as the eigenfrequencies, the damping ratios and the mode shapes (Celep 2001). Very successful vibration monitoring projects have been carried out on bridges by Bayraktar *et al.* (2009a, 2009b, 2009c, 2010), Wiberg *et al.* (2009), Magalhaes *et al.* (2009) and on buildings by Safak (1993), Kohler *et al.* (2005), Yoshimoto *et al.* (2005) and Ventura *et al.* (2003). Many system identification algorithms are used in the literature for identifying the modal parameters of structures (Zhou and Yan 2006, Katkhuda *et al.* 2010, Peeters 2000). Subspace based System Identification Techniques (SSIT) are known to be the most powerful class of the system identification techniques used for modal parameter identification of aerospace, mechanical and civil structures. A detailed description of the unified theory of the SSI techniques can be found in the book by Van Overschee and De Moor (1996).

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During the past 30 years or so, the so-called experimental modal analysis (EMA) technique has been used to monitor and identify the modal parameters of structures (Ewins 2000). In EMA, the structure is subjected to a measured force, where the response of the structure is also recorded. From this input-output data, a system model can be identified by the deterministic SSIT.

EMA techniques are quite useful for laboratory testing on small specimens or mechanical structures. However, in large civil engineering structures, only a small portion of the response is due to the artificial excitation sources as it is practically very difficult to shake the large civil structures by shakers or drop weights to appreciable amplitudes (Peeters 2000). Furthermore, the mobilization of drop weights or shakers to civil engineering structures is rather expensive. This gave rise to a new testing technique known as the ambient vibration testing or operational modal analysis (OMA). The advantage of this technique is that only the output data due to the readily available ambient excitations such as wind and traffic is recorded and the modal parameters are identified using the stochastic SSIT which assumes that the input is zero (Van Overschee and De Moor 1995). The OMA technique has many advantages. It is cheaper than EMA and it is more practical.

In the literature, the combined deterministic stochastic SSIT by Van Overschee and De Moor (1996) has recently been popular due to the fact that both the unknown ambient excitations and the known input can be modeled efficiently with this technique. Consequently, it is argued that better predictions of the modal parameters are obtained. The analysis associated with the combined deterministic stochastic SSIT is called the operational modal analysis with eXogenous (or deterministic) inputs (OMAX) as suggested by Cauberghe (2004).

In 2008, a scientific project sponsored by the Scientific and Technological Research Council of Turkey (TUBITAK) has been started for the continuous monitoring of a typical school building in Istanbul, Turkey (Bakir 2008). Within the context of the project, a typical school building is instrumented with 18 sensors and monitored continuously. During the project, an important need has emerged for determining the best SSIT for stiff mid rise reinforced concrete buildings. This study is a modest attempt of identifying the best system identification technique for the project. In the literature, it has been over emphasized that the predictions of the stochastic algorithm are as accurate as the predictions of the combined algorithm. Most of these studies were carried out on numerical models of ideal structures where the number of degrees of freedom were substantially lower as compared to the number of degrees of freedom of large civil structures. Moreover, these idealized structures had undeformable rigid floors, where it sufficed to measure 3 degrees of freedom per floor and the sensors' degrees of freedom were at least 1/3 of the total degrees of freedom in the structure. However, in this study, the SSIT are applied on a real life civil structure with 18 sensor degrees of freedom and several thousand degrees of freedom in the finite element (FE) model. Moreover, the data is low pass filtered, which introduces additional numerical poles into the system and then noise is added to the data in such a way as to make the noise to signal ratio, 5%.

This paper is organized as follows. The next section presents the theory of the deterministic, stochastic and the combined deterministic stochastic SSIT. In section 3, the identification method is explained. In section 4, the application of the techniques is shown on noisy data from a mid-rise reinforced concrete building and a Monte Carlo analysis is presented consisting of 100 runs. Within this context, a discussion is provided regarding the results from the parametric studies. Section 5 summarizes the conclusions. The results show that the combined algorithm yields better estimates than the stochastic algorithm.

2. Subspace based system identification

In this section, the three different types of the SSIT are explained, namely, deterministic, stochastic and the combined deterministic stochastic SSIT.

2.1 Deterministic SSIT

Deterministic SSIT represents the ideal case, where the input and the output are known and are free of noise. Naturally, this is not really possible in real life measurements, which makes the deterministic identification clearly an academic issue. The algorithm explained and used here is essentially the det-alt algorithm in Van Overschee and De Moor (1996). In deterministic identification, given s measurements of the input $u_k \in \mathbb{R}^m$ and the output $y_k \in \mathbb{R}^l$ at the time instant k, the idea is to determine the state space system matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{l \times n}$, $D \in \mathbb{R}^{l \times m}$ and the order n of the system in the following state space model

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

$$y_k = Cx_k + Du_k \tag{2}$$

where, l is the number of outputs, x_k is the state at time instant k, m is the number of inputs and finally, A, B, C and D are the system matrices of the state space model.

2.1.1 Construction of block Hankel matrices

Block Hankel matrices play an important role in SSIT. They can be constructed from the input and output data as follows

$$U_{0|2i-1} = \left(\begin{array}{cccc} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \\ \hline u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{array}\right) = \left(\frac{U_{0|i-1}}{U_{i|2i-1}}\right) = \left(\frac{U_p}{U_f}\right) \tag{3}$$

$$U_{0|2i-1} = \left(\begin{array}{cccc} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \\ \hline u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{array}\right) = \left(\frac{U_{0|i}}{U_{i+1|2i-1}}\right) = \left(\frac{U_p^+}{U_f^-}\right) \tag{4}$$

where U represents the input block Hankel matrix, the subscripts of U represent the first and the last element of the first column, i is the number of block rows and j is the number of columns of the block

Hankel matrices. From a statistical point of view, j should be much larger than i. To ensure that all the samples are used, j is taken as equal to $s-2i+1$, where s is the number of samples of the measurement. $U_{0|i-1}$ is denoted by U_p and $U_{i|2i-1}$ by U_f, the subscripts p and f denote the past and the future, respectively.

The output block Hankel matrices $Y_{0|2i-1}$, Y_p, Y_f, Y_p^+, Y_f^- are defined similar to Eqs. 3 and 4. A block Hankel matrix $W_{0|i-1}$, consisting of inputs and outputs can be defined as

$$W_{0|i-1} = \begin{pmatrix} U_{0|i-1} \\ Y_{0|i-1} \end{pmatrix} = \begin{pmatrix} U_p \\ Y_p \end{pmatrix} = W_p \tag{5}$$

where W_p is the block Hankel matrix containing the past inputs and outputs.

2.1.2 Deterministic algorithm

The deterministic algorithm starts with the calculation of the oblique projection of the output data as follows

$$O_i = Y_f/_{U_f}\mathbf{W_p} \tag{6}$$

where the oblique projection of the row space of E along the row space of F on the row space of G is defined as follows

$$E/_F\mathbf{G} = [E/\mathbf{F}^{\perp}][G/\mathbf{F}^{\perp}]^{\dagger}G \tag{7}$$

where

$$E/\mathbf{F}^{\perp} = E\,\Pi_{F^{\perp}} \tag{8}$$

$$\Pi_{F^{\perp}} = I_j - \Pi_F \tag{9}$$

Here, Π_F shows the operator that projects the row space of a matrix onto the row space of the matrix F as follows

$$\Pi_F = F^T(FF^T)^{\dagger}F \tag{10}$$

where $(\bullet)^{\dagger}$ defines the Moore-Penrose pseudo-inverse of the matrix $(\bullet)$. In the second step, the singular value decomposition of the weighted oblique projection is calculated as

$$W_1 O_i W_2 = (U_1 \;\; \mathrm{U}_2)\begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1 S_1 V_1^T \tag{11}$$

where $U = (U_1 U_2)$ and $V = (V_1\ V_2)$ are the orthonormal matrices (not to be confused with the notation U used for the input block Hankel matrices) and S is a diagonal matrix that contains the singular values in the diagonal. The user defined weighting matrices $W_1 \in \mathbb{R}^{li \times li}$ and $W_2 \in \mathbb{R}^{j \times j}$ are such that W_1 is of full rank and *rank* $(W_p) = \mathrm{rank}\ (W_p W_2)$. The extended observability matrix Γ_i is computed from

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} \tag{12}$$

$$\Gamma_i^{\perp} = U_2^T W_1 \tag{13}$$

The state space system matrix A is calculated as follows

$$A = \underline{\Gamma_i}^{\dagger}\overline{\Gamma_i} \tag{14}$$

where $\underline{\Gamma_i}$ represents the extended observability matrix Γ_i without the last l rows, $\overline{\Gamma_i}$ represents the extended observability matrix Γ_i without the first l rows. The state space matrix C is calculated as the first l rows of Γ_i. With

$$(M_1 \ \ M_2 \ \ \ldots \ \ M_i) = \Gamma_i^{\perp} Y_f U_f^{\dagger} \tag{15}$$

$$(L_1 \ \ L_2 \ \ \ldots \ \ L_i) = \Gamma_i^{\perp} \tag{16}$$

B and D are solved from

$$\begin{pmatrix} M_1 \\ M_2 \\ \vdots \\ M_i \end{pmatrix} = \begin{pmatrix} L_1 & L_2 & \ldots & L_{i-1} & L_i \\ L_2 & L_3 & \ldots & L_i & 0 \\ L_3 & L_4 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ L_i & 0 & \ldots & 0 & 0 \end{pmatrix} \times \begin{pmatrix} I_1 & 0 \\ 0 & \underline{\Gamma_i} \end{pmatrix} \begin{pmatrix} D \\ B \end{pmatrix} \tag{17}$$

2.2 Stochastic SSIT

This technique computes the state space models only from the output data. The algorithm is essentially the sto-alt algorithm in Van Overschee and De Moor (1996).The stochastic model that will be identified is given by

$$x_{k+1} = Ax_k + w_k \tag{18}$$

$$y_k = Cx_k + v_k \tag{19}$$

with

$$\mathbf{E}\left[\begin{pmatrix} w_k \\ v_k \end{pmatrix} (w_l^T \ v_l^T)\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{kl} \tag{20}$$

where $Q \in \mathbb{R}^{n \times n}$, $S \in \mathbb{R}^{n \times l}$ and $R \in \mathbb{R}^{l \times l}$. The vectors and are unmeasurable, Gaussian distributed zero mean white noise vector sequences. Stochastic SSIT starts with the calculation of the orthogonal projection of the output data as follows

$$O_i = Y_f / \mathbf{Y_p} = Y_f \Pi_{Y_p} = Y_f Y_p^T (Y_p Y_p^T)^{\dagger} Y_p \tag{21}$$

Subsequently, the singular value decomposition of the weighted projection is calculated as

$$W_1 O_i W_2 = USV^T \tag{22}$$

In the third step, the extended observability matrix Γ_i takes the following form

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} \tag{23}$$

Hence the state space system matrix A is computed as

$$A = \underline{\Gamma_i}^{\dagger} \overline{\Gamma_i} \tag{24}$$

The state space matrix C is calculated as the first l rows of Γ_i.

2.3 Combined deterministic stochastic SSIT

The algorithm is essentially the com-alt algorithm in Van Overschee and De Moor (1996). Given s measurements of the input $u_k \varepsilon \mathbb{R}^m$ and the output $y_k \varepsilon \mathbb{R}^l$ of the unknown system of order n

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{25}$$

$$y_k = Cx_k + Du_k + v_k \tag{26}$$

with w_k and v_k zero mean, white vector sequences with covariance matrix

$$\mathbf{E}\left[\begin{pmatrix} w_k \\ v_k \end{pmatrix} (w_l^T \ v_l^T)\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{kl} \tag{27}$$

where $Q \in \mathbb{R}^{n \times n}$, $S \in \mathbb{R}^{n \times l}$ and $R \in \mathbb{R}^{l \times l}$, the idea is to determine the order of the system and the system matrices A, B, C, D, Q, S, R in such a way that the second order statistics of the output of the stochastic subsystem and of the stochastic part of the given output are equal. In the combined algorithm, the oblique and orthogonal projections given below are computed first

$$O_i = Y_f /_{U_f} \mathbf{W}_p \tag{28}$$

$$Z_i = Y_f / \begin{pmatrix} \mathbf{W}_p \\ \mathbf{U}_f \end{pmatrix} \tag{29}$$

$$Z_{i+1} = Y_f^- / \begin{pmatrix} \mathbf{W}_p^+ \\ \mathbf{U}_f^- \end{pmatrix} \tag{30}$$

where

$$W_p^+ = \begin{pmatrix} U_p^+ \\ Y_p^+ \end{pmatrix} \tag{31}$$

The singular value decomposition of the weighted oblique projection $W_1 O_i W_2 = USV^T$ is calculated next

$$W_1 O_i W_2 = (U_1 \ \ \mathrm{U}_2) \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1 S_1 V_1^T \tag{32}$$

The extended observability Γ_i and the extended observability matrices without the last l rows represented by $\underline{\Gamma}_i$ are computed as

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} \tag{33}$$

$$\Gamma_{i-1} = \underline{\Gamma}_i \tag{34}$$

The following equation is solved for *A*, *C* and *K*

$$\begin{pmatrix} \Gamma_{i-1}^{\dagger} Z_{i+1} \\ \hline Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ \hline C \end{pmatrix} \Gamma_i^{\dagger} Z_i + K U_f + \begin{pmatrix} \rho_w \\ \hline \rho_v \end{pmatrix} \tag{35}$$

The matrices *L*, *M* and *K* are defined as

$$L = \begin{pmatrix} A \\ \hline C \end{pmatrix} \Gamma_i^{\dagger} = \begin{pmatrix} L_{1|1} & L_{1|2} & \dots & L_{1|i} \\ \hline L_{2|1} & L_{2|2} & \dots & L_{2|i} \end{pmatrix} \in \mathbb{R}^{(n+l) \times li} \tag{36}$$

$$M = \Gamma_{i-1}^{\dagger} = (M_1 \;\, M_2 \;\, \dots \;\, M_{i-1}) \in \mathbb{R}^{n + l(i-1)} \tag{37}$$

$$K = \begin{pmatrix} K_{1|1} & K_{1|2} & \dots & K_{1|i} \\ \hline K_{2|1} & K_{2|2} & \dots & K_{2|i} \end{pmatrix} \tag{38}$$

where $L_{1|k}, M_k \in \mathbb{R}^{n+l}, K_{1|k} \in \mathbb{R}^{n+m}, L_{2|k} \in \mathbb{R}^{l+l}$ and $M_{2|k} \in \mathbb{R}^{l+m}$. The state space system matrices *B* and *D* are calculated from the following equations given *K*, *A*, *C*, Γ_i and Γ_{i-1} as

$$\begin{pmatrix} K_{1|1} \\ \vdots \\ K_{1|i} \\ \hline K_{2|1} \\ \vdots \\ K_{2|i} \end{pmatrix} = N \begin{pmatrix} D \\ B \end{pmatrix} \tag{39}$$

$$N = \left(\begin{array}{lllll} -L_{1|1} & M_1 - L_{1|2} & \dots & M_{i-2} - L_{1|i-1} & M_{i-1} - L_{1|i} \\ M_1 - L_{1|2} & M_2 - L_{1|3} & \dots & M_{i-1} - L_{1|i} & 0 \\ M_1 - L_{1|3} & M_3 - L_{1|4} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ M_{i-1} - L_{1|i} & 0 & \dots & 0 & 0 \\ \hline I_l - L_{2|1} & -L_{2|2} & \dots & -L_{2|i-1} & -L_{2|i} \\ -L_{2|2} & -L_{2|3} & \dots & -L_{2|i} & 0 \\ -L_{2|3} & -L_{2|4} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -L_{2|i} & 0 & \dots & 0 & 0 \end{array} \right) \times \begin{pmatrix} I_l & 0 \\ 0 & \Gamma_{i-1} \end{pmatrix} \tag{40}$$

The matrices Q, S and R can be computed from the residuals ρ_w and ρ_v as

$$\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} = \mathbf{E}\left[\begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix}(\rho_w^T \ \rho_v^T)\right] \tag{41}$$

2.4 Determining the modal parameters from the system matrices

The system matrix A can be decomposed as (Reynders *et al.* 2008)

$$A = \psi \Lambda_d \psi^{-1} \tag{42}$$

where $\psi \in C^{n \times n}$ is the eigenvector matrix and $\Lambda_d \in C^{n \times n}$ is a diagonal matrix that contains the discrete time eigenvalues μ_i. The eigenfrequencies are calculated from

$$\lambda_i = \frac{ln(\mu_i)}{\Delta t} \tag{43}$$

where λ_i denotes the continuous time eigenvalues and Δt is the sampling time. The damping ratio (in %) is computed from

$$\xi_i = -100\frac{\lambda_i^R}{|\lambda_i|} \tag{44}$$

where $|.|$ denotes the complex modulus and λ_i is expressed as

$$\lambda_i = \lambda_i^R + i\lambda_i^I \tag{45}$$

The mode shape V is computed from

$$V = C\psi \tag{46}$$

3. Identification method

In this section, the deterministic, the stochastic and the combined deterministic stochastic SSIT are applied on the response data obtained from the sensor degrees of freedom of the FE model of the school building subjected to dynamic analysis. The building is a five storey reinforced concrete structure with one basement as shown in Fig. 1. Since the building is very stiff in both directions, only the first four global modes are used for system identification. The rest of the modes are local and not important in terms of damage identification.

The data is first low pass filtered in order to mimic the real world measurements. Low pass filtering inevitably introduces numerical poles into the system and makes it more challenging for the algorithms to detect the system poles. Then, noise is added to the data such that the noise to signal ratio is 5%. It should also be noticed that the data is obtained from an extremely stiff structure which makes the identifications even more challenging. Monte Carlo simulations consisting of 100 runs are carried out on the response data obtained from the FE model and the dynamic analysis using the three SSIT alternatives. Discriminating real physical poles from the spurious numerical poles is a very difficult problem. This is solved automatically by detecting the poles using the toolbox AUTOM developed

Fig. 1 The instrumented school building (website of the instrumented building 2009)

by the first author (Bakir 2010) rather than picking the poles from the stabilization diagrams for each Monte Carlo simulation.

4. Identification results

The first four mode shapes of the building are shown in Figs. 2, 3, 4 and 5 (Bakir 2011). The first mode is the first bending mode in the long direction, the second mode is the first bending mode in the short direction, the third mode is the torsional mode and the fourth mode is the second bending mode in the long direction.

The modal frequencies calculated using the FE model are given in Table 1. It should be stated here that the damping ratios from the FE model are 2% for the first and the third mode, 1.88% for the second mode and 2.71% for the fourth mode. The true or real modal parameters are those obtained from the modal analysis using the FE model. The estimates are the modal parameters identified from the response data obtained from the FE model using the three alternative SSIT. These two set of modal parameters are compared with each other in this study. The ratios of the estimates for each mode to the true value obtained from the modal analysis of the FE model are shown in Figs. 6, 7 and 8 for the frequencies, mode shapes and the damping ratios, respectively for

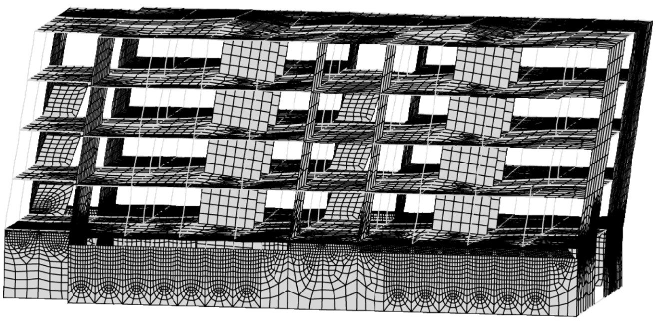

Fig. 2 The first vibration mode (the first bending mode in the longitudinal direction)

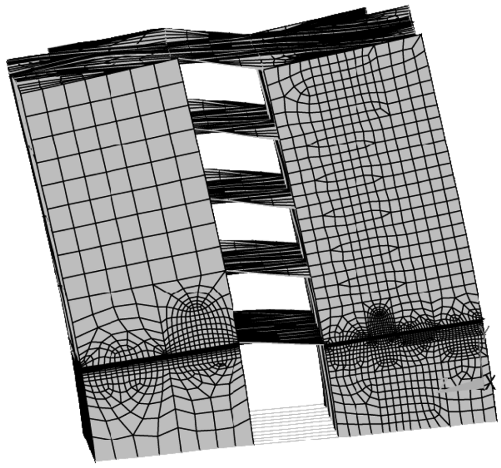

Fig. 3 The second vibration mode (the first bending mode in the transversal direction)

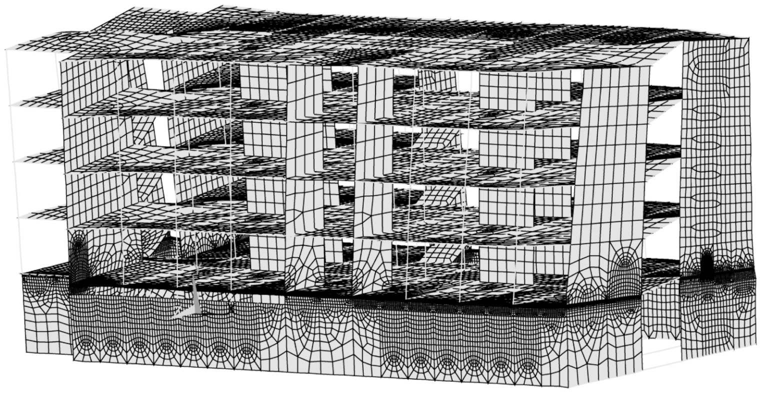

Fig. 4 The third vibration mode (the torsional mode)

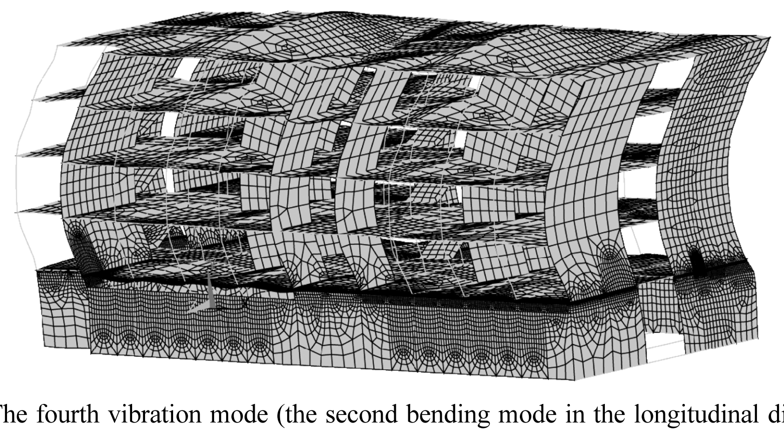

Fig. 5 The fourth vibration mode (the second bending mode in the longitudinal direction)

100 Monte Carlo simulations. Ideally, these ratios should approach 1 for the frequencies and the damping ratios and 100 for the Modal Assurance Criterion (MAC), which are shown with full lines. The dashed lines represents the ratio of the average value of the estimates from 100 Monte Carlo

Table 1 Modal frequencies calculated using the FE model

Number	Frequency (Hz)	Mode type
1	2.62	First bending mode in the z direction
2	4.84	First bending mode in the x direction
3	5.90	First torsion mode
4	10.007	Second bending mode in the z direction
5-12	10.133	Local modes of the ground storey beams
13	14.262	Local mode of the slabs
14	14.294	Local mode of the slabs
15	14.324	Local mode of the slabs
16	14.602	Local mode of the slabs
17	15.693	Local mode of the slabs
18	15.754	Local mode of the slabs
19	15.873	Local mode of the slabs
20	15.953	Local mode of the slabs

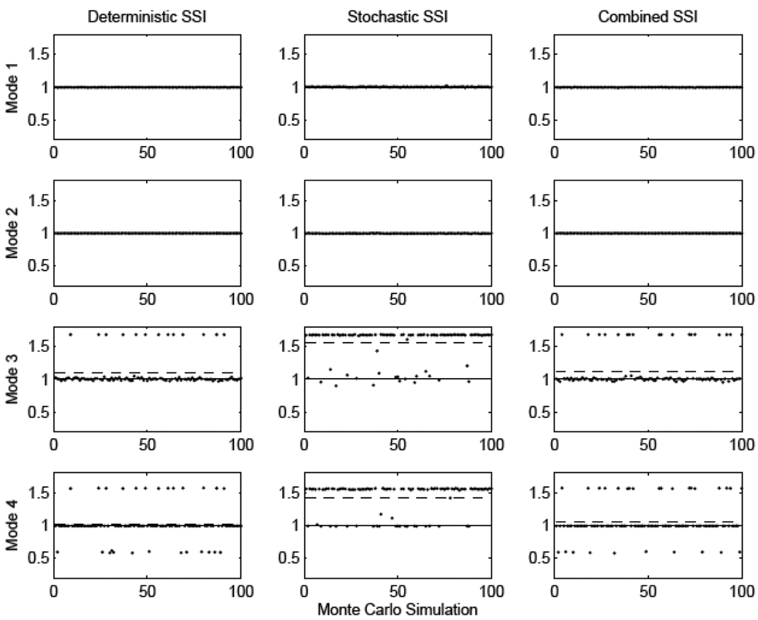

Fig. 6 Eigenfrequency estimation results (from 100 Monte Carlo simulations)

simulations to the real value of the modal parameters from the FE analysis corresponding to that particular mode. This quantity is a measure of the bias in the identifications. The parametric studies regarding the deterministic SSIT represent rather academic, idealized investigations as the algorithm does not consider the process and the measurement noise, which is not possible in real world applications. Thus, the algorithm is not further discussed and the comparison is made between the stochastic SSIT and the combined deterministic stochastic SSIT.

Fig. 6 reveals that all the algorithms estimate the frequencies corresponding to the bending modes in the two orthogonal directions very accurately. The real differences are in the estimates of the

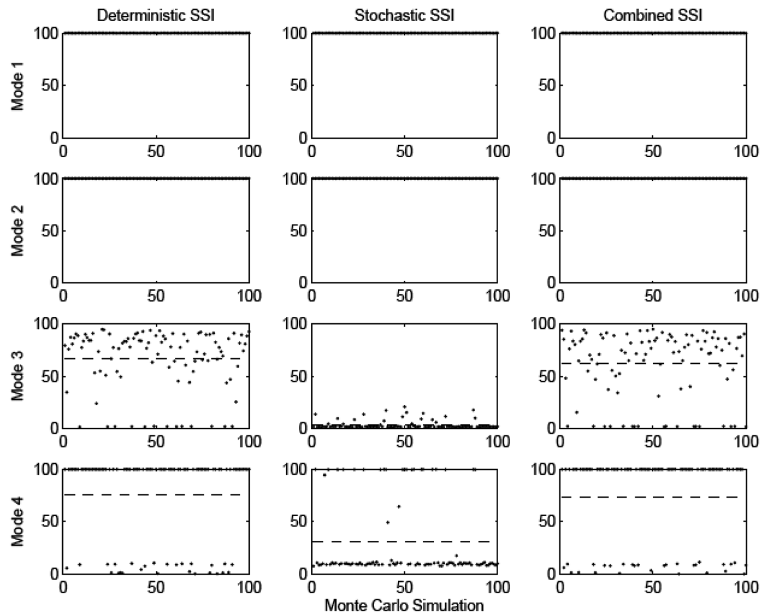

Fig. 7 Mode shape estimation results (from 100 Monte Carlo simulations)

torsional and the second bending modes, where the stochastic algorithm is overly and steadily biased in the identifications of the frequencies as compared to the combined deterministic stochastic SSIT and substantially overestimates the frequencies.

Fig. 7 shows that all the algorithms predict the mode shapes for the first two modes almost

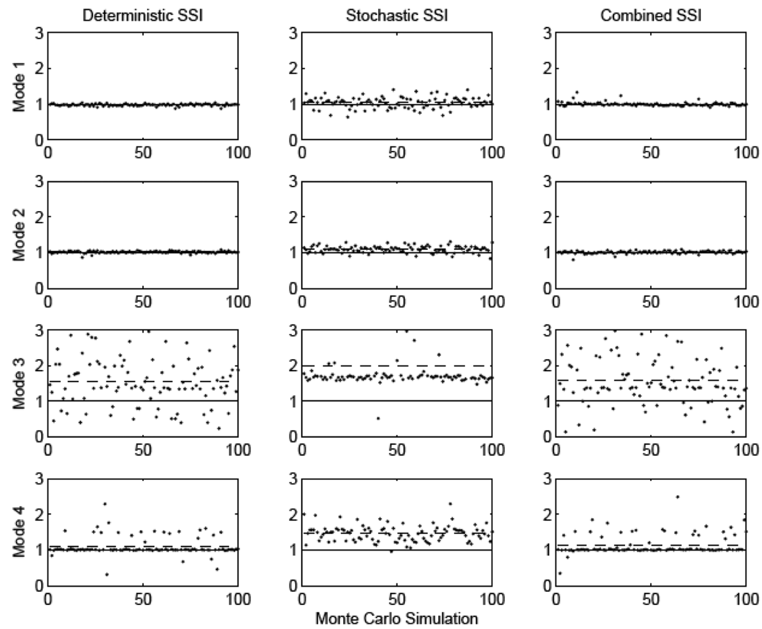

Fig. 8 Damping ratio estimation results (from 100 Monte Carlo simulations)

exactly. However, the stochastic SSIT can not identify the third and the fourth modes. The combined deterministic stochastic SSIT, on the other hand, adequately predicts these modes albeit a small amount of bias.

Fig. 8 shows that the combined deterministic stochastic SSIT accurately predicts the damping ratios for the first two modes without any bias. The stochastic algorithm also gives very good predictions for the first two modes, however, the standard deviations are higher than the combined algorithm. For the third mode, the bias of the estimates of the combined algorithm is smaller compared to that of the stochastic algorithm. However, its standard deviation is substantially higher, with a large variability of the estimates. The stochastic algorithm shows on the other hand a steady, very significant bias with a very small standard deviation. For the fourth mode, stochastic SSIT again gives a clear bias and the technique substantially overestimates the damping ratio. The combined algorithm, on the other hand, gives very accurate estimates with a very small bias value for this mode.

It should be stated here that these parametric studies are also repeated for a noise to signal ratio of zero although not shown here for the purposes of brevity. Even for the case without noise, the stochastic algorithm could not identify the mode shape for the third mode.

5. Conclusions

In this study, the application feasibility of the deterministic, the stochastic and the combined deterministic-stochastic SSIT algorithms on a stiff reinforced concrete school building is evaluated. For this purpose, a Monte Carlo analysis consisting of 100 simulations is carried out using the three alternative subspace system identification techniques. The results show that the stochastic algorithm falls short of accurately identifying the torsional and the second bending mode shapes of the structure. The technique also overestimates the eigenfrequencies and damping ratios substantially with pronounced bias values for the third and the fourth mode. The best results are certainly obtained when the combined deterministic stochastic SSIT is used for identifying the modal parameters of the structure. These identifications show very small bias and substantially better results as compared to the stochastic algorithm.

In the literature, it has been over emphasized that the estimates of the stochastic algorithm are as accurate as the estimates of the combined algorithm. Most of these studies were carried out on numerical models of ideal structures where the number of degrees of freedom were substantially lower as compared to the number of degrees of freedom of large civil structures. Moreover, these idealized structures had undeformable rigid floors, where it sufficed to measure 3 degrees of freedom per floor and the sensors' degrees of freedom were at least 1/3 of the total degrees of freedom in the structure. However, in this study, the techniques are applied on a real life civil structure with 17 sensor degrees of freedom and a total of several thousand degrees of freedom in the FE model. Moreover, the data is low pass filtered, which introduces numerical poles into the system and then noise is added in such a way that the noise to signal ratio is 5%. When the three algorithms are tested in such difficult conditions, it becomes apparent that the stochastic algorithm does not give equally accurate estimates as the combined algorithm. The combined algorithm's predictions clearly yield superior estimates compared to those of the stochastic algorithm. Thus, for real life measurements, it must be borne in mind that the stochastic algorithm is very susceptible to noise and modeling errors. It is therefore believed that the combined deterministic stochastic SSIT algorithm ought to be considered more frequently for system identification in mid rise reinforced concrete buildings.

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