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Locating the damaged storey of a building using distance measures of low-order AR models

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Abstract. The key to detecting damage to civil engineering structures is to find an effective damage indicator. The damage indicator should promptly reveal the location of the damage and accurately identify the state of the structure. We propose to use the distance measures of low-order AR models as a novel damage indicator. The AR model has been applied to parameterize dynamical responses, typically the acceleration response. The premise of this approach is that the distance between the models, fitting the dynamical responses from damaged and undamaged structures, may be correlated with the information about the damage, including its location and severity. Distance measures have been widely used in speech recognition. However, they have rarely been applied to civil engineering structures. This research attempts to improve on the distance measures that have been studied so far. The effect of varying the data length, number of parameters, and other factors was carefully studied.

Keywords: damage indicator; AR model; cepstral metric; pre-whitening filter; adaptive component weighting (ACW).

1. Introduction

Most of the literature about damage detection is based on modal parameters. One method uses frequency shifts to detect damage in composite materials (Cawley and Adams 1979), and an improvement to this method was shown to be sensitive to modal frequency changes (Stubbs and Osegueda 1990, Feng 2009). Mode shape information has also been used for localizing structural damage without the use a prior finite element model (West 1986). The author used the modal assurance criteria (MAC) to determine the level of correlation between modes from a test of an undamaged Space Shuttle Orbiter body flap and the modes from the test of the flap after it has been exposed to acoustic loading. A mode shape-based method for model error localization was revealed in 1991 (Mayes 1991). Structural translational and rotational error checking (STRECH) were used in this method. By taking ratios of relative modal displacements, STRECH assess the preciseness of the structural stiffness between two different structural DOF. The changes in the frequencies and mode shapes of a bridge were regarded as a function of damage (Doebling and Farrar 1997). This

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method estimates the statistics of the modal parameters by using Mote Carlo procedures to determine if damage has produced a statistically significant change in the mode shapes. An alternative to using mode shapes to obtain spatial information about sources of vibration changes is using mode shape derivatives (Pandey *et al*. 1991, Quek and Tua 2008). By varying the number of measurable target modes, sensitivity to changes in the mode shape can also be used to determine efficient locations to put accelerometers (Kwon *et al*. 2008). A method based on element modal strain energy was proposed to identify a structural damage (Zhang *et al*. 1998). This method uses measured mode shapes and modal frequencies from both damaged and undamaged structures as well as a finite element model to locate damage. An improved genetic algorithm based damage detection algorithm using a set of combined modal features is proposed to detect the damage (Kim *et al*. 2007). In this method, modal features such as natural frequency, mode shape, and modal strain energy are experimentally measured before and after damage in the test beams. Shin and Oh chose the stiffness and damping parameters to assess damage in a building (Shin and Oh 2007). All these methods get good results in special cases. However, the modal parameters are global properties of a structure, whereas damage is a local phenomenon. Modal parameters are insensitive to local damage, and this is the major obstacle to applying modal-domain damage indicators in practice.

Distance measures have been widely used in speech recognition (Tohkura 1986, Itakura and Umezaki 1987) but not in civil engineering. In this study, we used AR models to express the vibration response of the structure. The AR models were obtained for the undamaged state and unknown states. The sensitivity of the distance measure to the damage was strongly affected by the order of the AR models. We suggest a low-order model to maximize the effectiveness of the method. In addition, since the AR models are susceptible to noise, we use adaptive component weighting (ACW) (Assaleh and Mammone 1994), which can significantly reduce noise. We calculate the cepstral distance between the reference model and the new model by using the equation deduced from the metric for the AR model, which was first proposed by Martin (Martin 2000). We judge the damage according to the cepstral distance. A large distance means that the structure is damaged, and a small distance means that the structure is undamaged. One thing to keep in mind is that this cepstral distance is only effective when the input signals to drive the structure are mutually independent. Unfortunately however, most response signals are mutually dependent and correlated because of excitations acting on the structure, such as wind, traffic or earthquakes. This issue can be resolved by introducing a pre-whitening filter (Zheng and Mita 2007). We used numerical and experimental data to show the method's feasibility. The effects of varying the data length, damage severity, damage location and parameter uncertainty on recognition accuracy were carefully studied in this paper.

2. Proposed approach using low-order AR model and ACW

Zheng and Mita's paper (Zheng and Mita 2007) used distance measures for the purpose of damage detection in civil engineering. As an improvement to this method, we propose a scheme to calculate the cepstral distance between pole-zero models.

We find that low-order autoregressive models have advantages in terms of their computational efficiency, emphasis of high-energy frequency range, and less sensitivity to spectral peaks caused by noise (Wang *et al*. 1991). In addition, we introduce adaptive component weighting (ACW) to transform an AR model into a pole-zero model. Thus, we deduce a new equation for cepstral

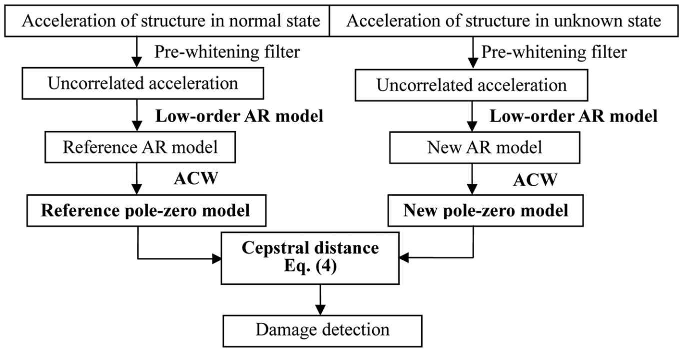

Fig. 1 Flowchart of damage detection

distances between pole-zero models based on Eq. (10) (Oppenheim and Schafer 1975) and the metric for AR model first proposed by Martin (Martin 2000). The flowchart of our damage detection scheme is shown in Fig. 1.

2.1 Cepstral distance

The cepstrum of a signal is defined as the inverse Fourier transform of the log magnitude spectrum, and it was originally used for detecting echoes (Bogert *et al.* 1963). In this study, the cepstral distance between pole-zero models is referred to as the damage indicator. First, we introduce the cepstral metric for the AR model, as proposed by Martin (Martin 2000)

$$D(\mathrm{M}^{(1)},\mathrm{M}^{(2)}) = \left[\sum_{n=1}^{\infty} n \left| c_n - c_n^{'} \right|^2 \right]^{1/2} \tag{1}$$

where c_n and c_n' are the cepstral coefficients of AR models $\mathrm{M}^{(1)}$ and $\mathrm{M}^{(2)}$ and D is cepstral distance between the two AR models. We then combine Eq. (1) with Eq. (2) (Oppenheim and Schafer 1975)

$$c_n = \begin{cases} \dfrac{1}{n}\left[\displaystyle\sum_{i=1}^{p} \alpha_i^n - \sum_{i=1}^{q} \beta_i^n \right], & n > 0 \\ \log \sigma^2, \quad n = 0 \end{cases} \tag{2}$$

where α_i and β_i mean the poles and zeros of the model, p and q are the numbers of the poles and zeros of the model respectively, σ is the variance of the white noise input, and by using the identity

$$\sum_{n}^{\infty} \alpha^n \equiv -\log(1-\alpha) \qquad (|\alpha| < 1) \tag{3}$$

we get the cepstral distance between pole-zero models as follows

$$D\left(\mathrm{M}^{(1)},\mathrm{M}^{(2)}\right)^2 = \log \frac{\prod_{i=1}^{p^{(1)}+p^{(2)}} \prod_{j=1}^{p^{(1)}+p^{(2)}} \left(1-\varphi_i^{(1)}\bar{\varphi}_j^{(2)}\right) \prod_{i=1}^{p^{(1)}+p^{(2)}} \prod_{j=1}^{p^{(1)}+p^{(2)}} \left(1-\varphi_i^{(2)}\bar{\varphi}_j^{(1)}\right)}{\prod_{i=1}^{p^{(1)}+p^{(2)}} \prod_{j=1}^{p^{(1)}+p^{(2)}} \left(1-\varphi_i^{(1)}\bar{\varphi}_j^{(1)}\right) \prod_{i=1}^{p^{(1)}+p^{(2)}} \prod_{j=1}^{p^{(1)}+p^{(2)}} \left(1-\varphi_i^{(2)}\bar{\varphi}_j^{(2)}\right)} \tag{4}$$

where D means the cepstral distance between models $M^{(1)}$ and $M^{(2)}$, $\varphi^{(1)} = [\alpha^{(1)}, \beta^{(2)}]$, $\varphi^{(2)} = [\alpha^{(2)}, \beta^{(1)}]$, α_i and β_i are the poles and zeros of the models, and $p^{(1)}$ and $p^{(2)}$ are the orders of the models $M^{(1)}$ and $M^{(2)}$, respectively.

2.2 Optimum order of AR models

The autoregressive (AR) model has been used to parameterize dynamical responses, typically the acceleration response (Mita 2003). In this study, the acceleration responses of the structures are modeled with the autoregressive (AR) model

$$x_n = \sum_{k=1}^{p} a_k x_{n-k} + e_n \tag{5}$$

where x_n is the response of the structure at sample index n, a_k is the AR coefficient to be estimated, p is the order of the AR model, and e_n is the prediction error term or residual.

The previous study (Zheng and Mita 2007) in which high-order autoregressive (AR) models were used to get the cepstral distance, however, we found that the sensitivity of the distance measure is strongly affected by the order of the AR models and the order determined by Akaike information criterion (Akaike 1974) is not the optimum AR order for the distance measure. Fig. 2 shows a simplified structure model. The mass of every floor and the lateral stiffness is assumed to be 100 kg

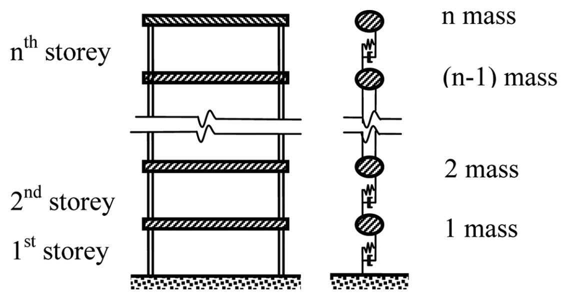

Fig. 2 Simplified structural model

and 1 MN/m, respectively. 3% is chosen as the damping ratio of all modes. Data sampling frequency is 200 Hz. We use the Gaussian white noise to simulate the input such as earthquakes and ambient vibration. The stiffness reduction is regarded as the damage to the structure. To get the optimum AR order for the distance measure, we examined AR models with different orders (3-19) for different multiple degrees of freedom (MDOF) systems (3-19-DOF) with different damage severities (8%, 16%, 24%, 32% and 40% lateral stiffness reduction) in different damage locations (1st, 2nd... nth storey). Because the number of results is very huge, we only show those of AR model orders (3, 4, 5, 6, 9 and 13) for 3, 5, 7 and 13-DOF systems with 8% damage severity in the 2nd storey, as listed in Table 1.

$$R1 = \frac{D_{3^{rd} mass}}{D_{1^{st} mass}} \tag{6}$$

$$R2 = \frac{D_{3^{rd} mass}}{D_{2^{nd} mass}} \tag{7}$$

Table 1 Cepstral distance (8% stiffness reduction in the 2nd storey, double underlined values: largest cepstral distance, single underlined values: second largest cepstral distance)

AR Order	3	4	5	6	9	13
	3 DOF (1st natural frequency: 7.1 Hz)					
1-mass	0.0663	0.1095	0.1508	0.1667	0.1042	0.2351
2-mass	0.0838	0.1125	0.1956	0.2399	0.5514	0.5131
3-mass	0.0350	0.0175	0.0069	0.0440	0.0519	0.1911
	5 DOF (1st natural frequency: 4.5 Hz)					
1-mass	0.0615	0.0910	0.1161	0.1210	0.1250	0.1223
2-mass	0.0946	0.1317	0.1370	0.1408	0.2525	0.2496
3-mass	0.0528	0.0137	0.0275	0.0133	0.0774	0.0809
4-mass	0.0242	0.0222	0.0569	0.0587	0.0821	0.0713
5-mass	0.0124	0.0056	0.8430	0.0164	0.0222	0.0600
	7 DOF (1st natural frequency: 3.3 Hz)					
1-mass	0.0416	0.0781	0.1192	0.1270	0.1290	0.1174
2-mass	0.1137	0.1295	0.1445	0.1203	0.2143	0.1895
3-mass	0.0469	0.0134	0.0269	0.0803	0.0259	0.1149
4-mass	0.0215	0.0196	0.0325	0.0318	0.0516	0.0829
5-mass	0.0080	0.0121	0.002	0.0162	0.0114	0.0164
6-mass	0.0116	0.0238	0.0249	0.0135	0.0223	0.0447
7-mass	0.0068	0.0072	0.0060	0.0013	0.0147	0.0191
	13 DOF (1st natural frequency: 1.9 Hz)					
1-mass	0.0542	0.0969	0.1206	0.1141	0.1333	0.1269
2-mass	0.1078	0.1326	0.1500	0.1363	0.2049	0.1775
3-mass	0.0368	0.0182	0.0253	0.0797	0.0608	0.1118
4-mass	0.0260	0.0182	0.0309	0.0315	0.0482	0.0691
5-mass	0.0072	0.0085	0.0067	0.0104	0.0217	0.0244
6-mass	0.0019	0.0043	0.0055	0.0087	0.0159	0.0351
7-mass	0.0033	0.0089	0.0037	0.0133	0.0158	0.0169
8-mass	0.0016	0.0059	0.0032	0.0087	0.0144	0.0109
9-mass	0.0046	0.0146	0.0060	0.0194	0.0194	0.0135
10-mass	0.0052	0.0095	0.0044	0.0139	0.0150	0.0206
11-mass	0.0032	0.0039	0.0037	0.0065	0.0107	0.0158
12-mass	0.0195	0.0359	0.0190	0.0288	0.0230	0.0274
13-mass	0.0024	0.0103	0.0038	0.0100	0.0266	0.0137

where D means the cepstral distance of the mass.

The cepstral distance was chosen as the damage indicator because it increases as the difference between two models increases. After damage occurs in the 2nd storey, the acceleration response of the 1st and 2nd mass is affected more than the others and is shown as an increase in the cepstral distance of the 1st and 2nd mass. Thus, the cepstral distance of the 1st and 2nd mass should be much larger than the cepstral distances of the other masses when damage occurs in the 2nd storey. Table 1 lists the cepstral distances of masses for different AR orders. The double and single underlined

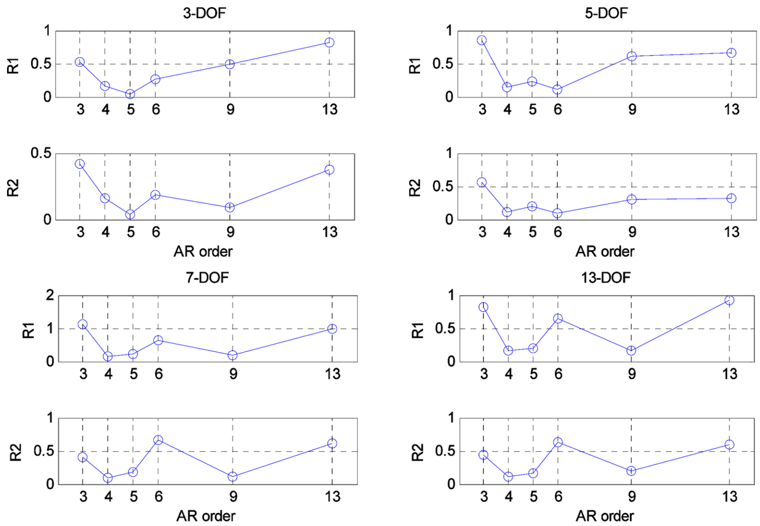

Fig. 3 Ratio of cepstral distances

values mean the largest and second largest cepstral distances, respectively. If the cepstral distances of the 1st and 2nd masses are not much larger than the others, it indicates that the damage can't be shown clearly and the corresponding AR order is not the optimum one. If the cepstral distances of the 1st and 2nd masses are smaller than the others, it means that the result gives the wrong damage location, and thus the order of the AR model is wrong.

The acceleration of masses closer to the damage will be the most affected. In the current case, damage to the 2nd storey, let us consider the ratios of the cepstral distance of the 3rd mass to the 1st mass and the cepstral distance of the 3rd mass to the 2nd mass, Eqs. (6) and (7), as the index for choosing the proper AR order:

The smaller $R1$ and $R2$ are, the better the recognition results mean. If $R1$ and $R2$ are larger than 1, it means that the AR order is wrong. Fig. 3 shows that increasing the AR order doesn't improve the recognition results. AR orders of 4 and 5 are more stable than others. The results for cases in which different storeys suffered damage of different severities (not listed here) also indicate that 4 and 5 are the most stable AR orders. We also tested taller structures and got similar results. So in this study, low-order AR models of order 4 were used.

2.3 Adaptive component weighting (ACW)

AR models are susceptible to noise. To reduce the effect of noise, we use adaptive component weighting (ACW), as is done in speaker identification. ACW modified the linear prediction (LP) spectrum so as to emphasize the formant structure.

$$\frac{1}{A(z)}=\frac{1}{1-\sum_{k=1}^{p}a_k z^{-k}}=\frac{1}{\prod_{k=1}^{p}(1-f_k z^{-1})}=\sum_{k=1}^{p}\frac{r_k}{1-f_k z^{-1}} \tag{8}$$

where f_k for $1\le k\le p$ represent the poles of the AR model, and r_k are the residues of the poles. It has been shown that the sensitivity of a pole to noise in the LP coefficients is proportional to the residues r_k

(Assaleh and Mammone 1994) and the variation caused by the residues r_k can be removed by setting all residues equal to a given constant such as unity. Hence, the ACW spectrum is given by

$$H(z) = \sum_{k=1}^{p} \frac{1}{1 - f_k z^{-1}} = \frac{N(z)}{A(z)} = \frac{N(z)}{1 - \sum_{k=1}^{p} a_k z^{-k}} \tag{9}$$

where
$$N(z) = \sum_{k=1}^{p} \prod_{i=1 \neq k}^{p} (1 - f_i z^{-1}) = p(1 - \sum_{k=1}^{p-1} b_k z^{-k}) \tag{10}$$

As mentioned in the previous section, we chose AR models of order 4 to examine whether ACW can reduce the effects of noise on the recognition results. We applied ACW to different MDOF structure systems (3-19-DOF) with damage of different severities (8%, 16%, 24%, 32% and 40% lateral stiffness reduction) and in different locations (1^{st}, 2^{nd}... n^{th} storey).

Here, we only show the results for the 5-DOF and 7DOF structures with 24% stiffness reduction in the 2^{nd} storey (Figs. 4 and 5). $R1$ means the ratio of cepstral distance of the 3^{rd} mass to the 1^{st} mass, and $R2$ means the cepstral distance of the 3^{rd} mass to the 2^{nd} mass. $R1$ and $R2$ are computed from Eqs. (6) and (7), respectively, and smaller values mean better recognition results. Fig. 4 shows the results for the 5-DOF system. The data length is 9000, and the signal-to-noise ratio (SNR) is 10. Application of ACW reduces $R1$ from 0.54 to 0.34 (37% reduction) and $R2$ from 0.50 to 0.32 (36% reduction). Fig. 7 shows the results for the 7-DOF system. The reductions in $R1$ and $R2$ are 41%

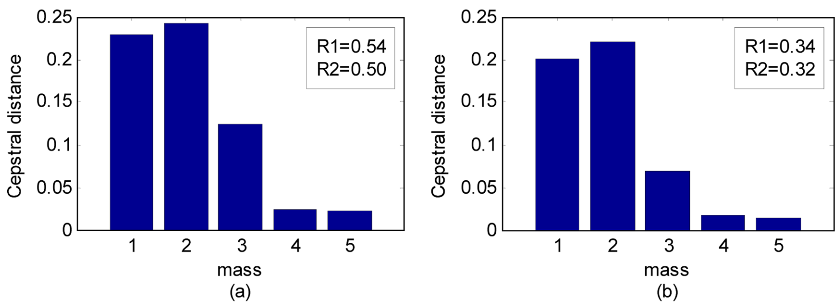

Fig. 4 5-DOF system with 24% stiffness reduction in the 2^{nd} storey (data length = 9000, SNR = 10): (a) without ACW and (b) with ACW

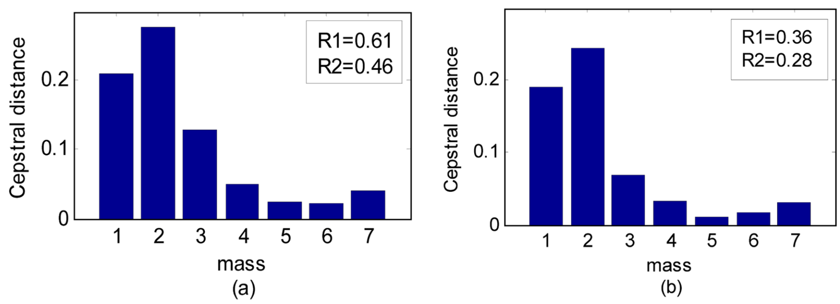

Fig. 5 7-DOF system with 24% stiffness reduction in the 2^{nd} storey (data length = 9000, SNR = 10): (a) without ACW and (b) with ACW and 46% in this case. Other results not listed here also show that ACW is feasible in reducing the effect of noise.

3. Performance verification by simulation

A simulation of a five-storey shear building model was performed to show the feasibility of the proposed scheme for damage detection. The building is simplified into a 5 DOF structural system.

The mass of every floor and the lateral stiffness were assumed to be 100 kg and 1 MN/m, respectively. 3% was chosen as the damping ratio of all modes. The data sampling frequency was 200 Hz. The undamaged natural frequencies of the structure were 4.5 Hz, 13.2 Hz, 20.8 Hz, 28.6 Hz and 30.5 Hz for the 1st mode, 2nd mode, 3rd mode, 4th mode and 5th mode, respectively. The 5-DOF system was assumed to be excited by the Gaussian white noise, which was used to simulate the input such as earthquakes and ambient vibration, and 10% noise was added to the acceleration responses of the structure. The storey stiffness reduction was regarded to be damage to the structure. Five damage cases (damage in the 1st storey, 2nd storey, 3rd storey, 4th storey or 5th storey) with five different damage severities (8%, 16%, 24%, 32% and 40% lateral stiffness reduction) were studied. Hence, there were 25 different damage scenarios in total.

To deal with the strong mutual correlation in the acceleration data, we applied a pre-whitening filter (Zheng and Mita 2007) to whiten the 5-dimensional signals. Then we constructed the AR models of order 4 to fit the acceleration responses of the structure in the 25 different scenarios. We used Burg's method to obtain the AR model because of its high frequency resolution and resulting stable AR model. The AR models were arranged in pairs (reference model and new model), and adaptive component weighting (ACW) was used to reduce the effect of noise by transforming the all-pole models (AR model) to pole-zero models. Finally, Eq. (4) was used to get the cepstral distance between every pair of models.

Figs. 6-8 show the recognition results for different data lengths. In order to show the damage more clearly, we squared the cepstral distance on the x axis. We can see that the change in the recognition results between these figures is related to the data length.

In Fig. 6 for which the data length is 6000, the recognition result when damage occurs in the 1st storey is not as good as the results for other damage to other storeys. However, it can be improved

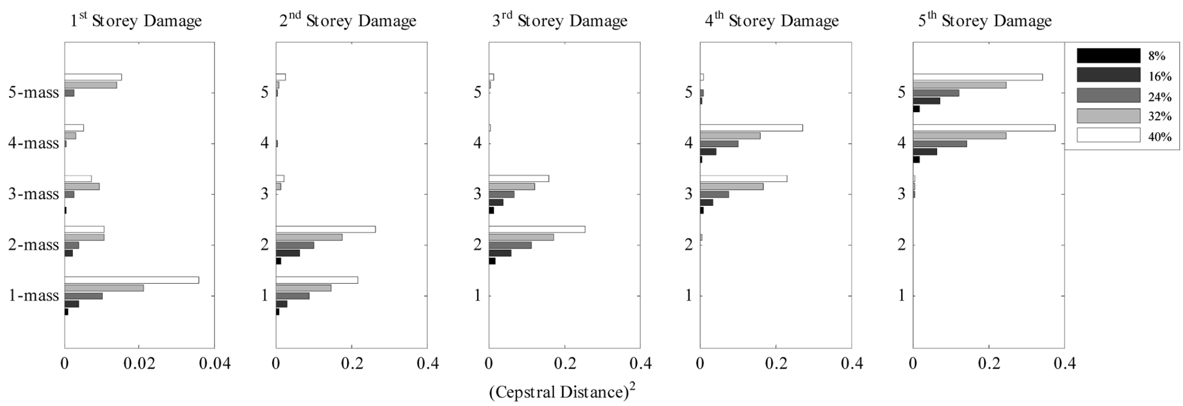

Fig. 6 Cepstral distance based on the pole-zero models (data length = 6000)

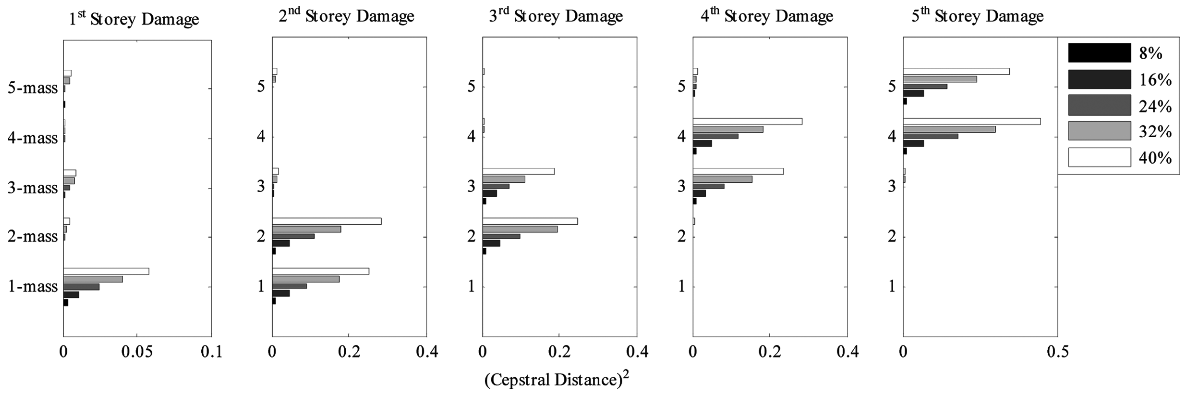

Fig. 7 Cepstral distance based on the pole-zero models (data length = 9000)

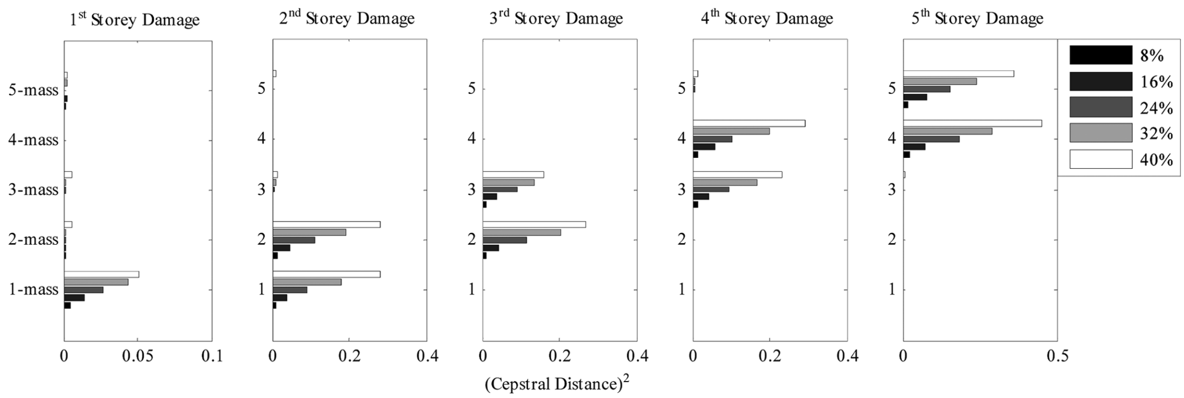

Fig. 8 Cepstral distance based on the pole-zero models (data length = 11000)

by increasing the data length (Figs. 7, 8). This phenomenon is mainly caused by the standard deviations of the AR coefficients, which are determined by the model covariance matrix (Ljung 1987), and thus, it is related to the data length. Another reason is that the damage indicators in this case are much smaller than those of the other damage cases and not very big compared with the standard deviations of the AR coefficients. Thus, the damage indicator for the 1^{st} storey suffers more from the standard deviations of the AR coefficients than those of the other cases. However, the standard deviations of the AR coefficients can be decreased by increasing the data length, and this in turn reduces the effect on the damage indicators and improves recognition results.

Next, we examined the standard deviations of the AR coefficients when the structure had suffered damage in different locations and with different severities (undamaged, 8%, 16%, 24%, 32% and 40% lateral stiffness reduction). Here we only list the standard deviations of the AR coefficients when the 1^{st} storey suffered no damage, and 8% and 40% lateral stiffness reductions. From Tables 2-4, it is clear that increasing the data length reduced the standard deviations, but changing the damage severity had almost no effect on them. Moreover, varying the damage location hardly affected the standard deviations. It is clear that the accuracy of the recognition result depends on the standard deviations of AR coefficients, especially when the standard deviation is not small enough compared with the damage indicator.

Table 2 Standard deviations of AR coefficients (undamaged)

	1st floor	2nd floor	3rd floor	4th floor	5th floor
			Data length 6000		
a1	0.0128	0.0126	0.0128	0.0119	0.0129
a2	0.0125	0.0160	0.0150	0.0153	0.0162
a3	0.0125	0.0160	0.0150	0.0153	0.0162
a4	0.0128	0.0126	0.0128	0.0119	0.0129
			Data length 9000		
a1	0.0104	0.0103	0.0104	0.0097	0.0105
a2	0.0102	0.0129	0.0122	0.0126	0.0134
a3	0.0102	0.0129	0.0122	0.0126	0.0134
a4	0.0104	0.0103	0.0104	0.0097	0.0105
			Data length 11000		
a1	0.0094	0.0093	0.0095	0.0088	0.0095
a2	0.0093	0.0117	0.0112	0.0114	0.0120
a3	0.0093	0.0117	0.0112	0.0114	0.0120
a4	0.0094	0.0093	0.0095	0.0088	0.0095

Table 3 Standard deviations of AR coefficients (8% lateral stiffness reduction, damage in the 1st storey)

	1st floor	2nd floor	3rd floor	4th floor	5th floor
			Data length 6000		
a1	0.0128	0.0126	0.0128	0.0120	0.0129
a2	0.0126	0.0157	0.0150	0.0154	0.0163
a3	0.0126	0.0157	0.0150	0.0154	0.0163
a4	0.0128	0.0126	0.0128	0.0120	0.0129
			Data length 9000		
a1	0.0104	0.0103	0.0104	0.0097	0.0105
a2	0.0102	0.0131	0.0123	0.0125	0.0133
a3	0.0103	0.0131	0.0123	0.0125	0.0133
a4	0.0104	0.0103	0.0104	0.0097	0.0105
			Data length 11000		
a1	0.0095	0.0093	0.0095	0.0088	0.0095
a2	0.0093	0.0119	0.0111	0.0114	0.0119
a3	0.0093	0.0119	0.0111	0.0114	0.0119
a4	0.0095	0.0093	0.0095	0.0088	0.0095

In addition to the single damage cases shown as above, multiple damage cases are studied in the paper. Similar to the single damage cases, we studied different multiple degrees of freedom (MDOF) systems (3-19-DOF) with different damage severities (8%, 16%, 24%, 32% and 40% lateral stiffness reduction) in different multiple damage locations. The data length is 9000 and the order of AR model is 4. Results also show that this method is available to the multiple damage cases. Because of the huge number of results, here we only list the result of a 5-DOF system when the damage with different damage severities (8%, 16%, 24%, 32% and 40% lateral stiffness reduction)

Table 4 Standard deviations of AR coefficients (40% lateral stiffness reduction, damage in the 1st storey)

	1st floor	2nd floor	3rd floor	4th floor	5th floor
			Data length 6000		
a1	0.0128	0.0124	0.0127	0.0117	0.0129
a2	0.0128	0.0163	0.0152	0.0157	0.0166
a3	0.0128	0.0163	0.0152	0.0157	0.0166
a4	0.0128	0.0124	0.0127	0.0117	0.0129
			Data length 9000		
a1	0.0105	0.0102	0.0104	0.0095	0.0105
a2	0.0105	0.0135	0.0126	0.0127	0.0136
a3	0.0105	0.0135	0.0126	0.0127	0.0136
a4	0.0105	0.0102	0.0104	0.0095	0.0105
			Data length 11000		
a1	0.0095	0.0092	0.0094	0.0087	0.0095
a2	0.0095	0.0122	0.0113	0.0115	0.0123
a3	0.0095	0.0122	0.0113	0.0115	0.0123
a4	0.0095	0.0092	0.0094	0.0087	0.0095

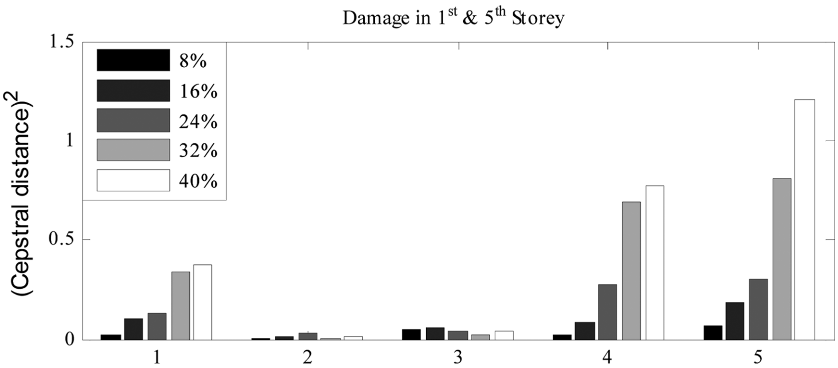

Fig. 9 Cepstral distance for the multiple damage case (data length = 9000)

in the 1st and 5th storey. In Fig. 9, the damage is localized correctly. The effect on the accuracy of identification results in multiple damage cases, which caused by the data length, the damage severities, damage locations and the standard deviations of AR coefficients, is similar to that in the single damage cases.

4. Experimental verification

We evaluated our methodology using actual data from a shake table test (Yoshimoto *et al*. 2002). A five-storey steel structure was tested (Fig. 10). The long side of every storey had removable central columns and braces. The weight of every storey was 2.57 tons, and the storey height was 1 m. The lengths of the long and short sides were 3 m and 2 m, respectively. The excitation signal was white noise with bandwidth of 0–200 Hz, in the long side's direction. Accelerometers were

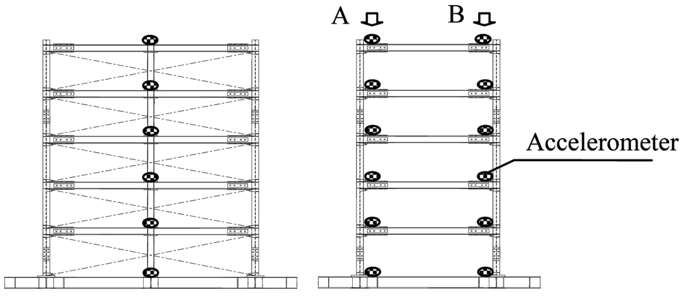

Fig. 10 Tested building model: (a) long-side and (b) short-side

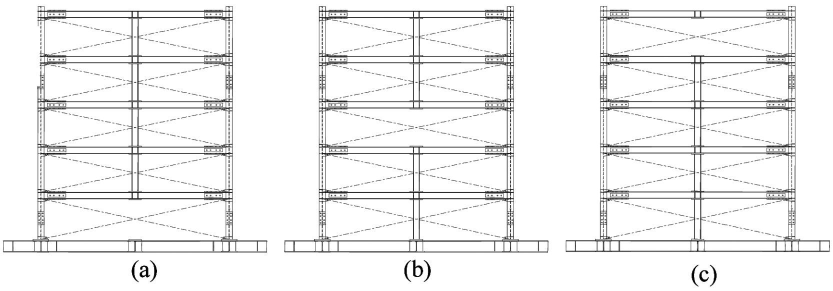

Fig. 11 Damage case: removing central column on the (a) first, (b) third and (c) fifth floor

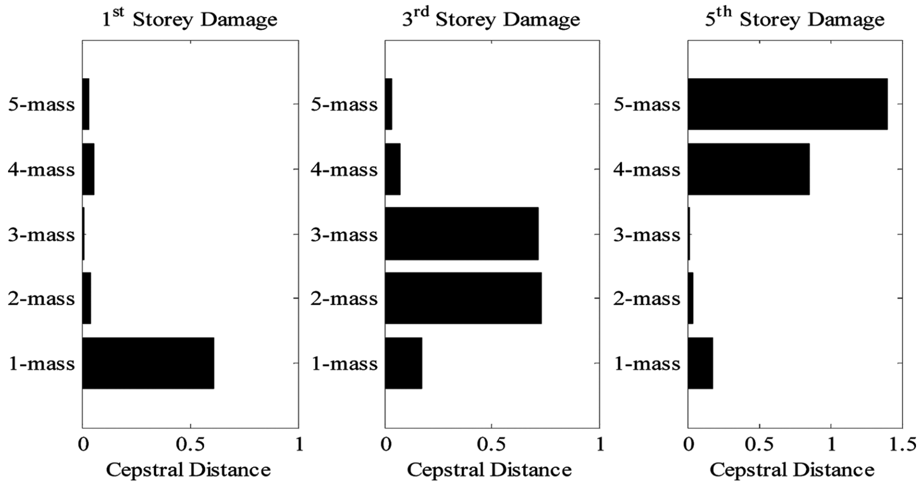

Fig. 12 Cepstral distances (the central columns were removed)

mounted on both long sides of every storey. The acceleration time histories were recorded with a 0.005 s sampling period. Damage to the building was simulated by removing the central columns and braces; the central columns on the first, third and fifth storey were removed as shown in Fig. 11, and the braces on the first, third and fifth storey were removed as shown in Fig. 13.

Figs. 12 and 14 are the recognition results, and they clearly show the damage locations. The length of data was 6000. Tables 5 and 6 list the standard deviations of the AR coefficients when

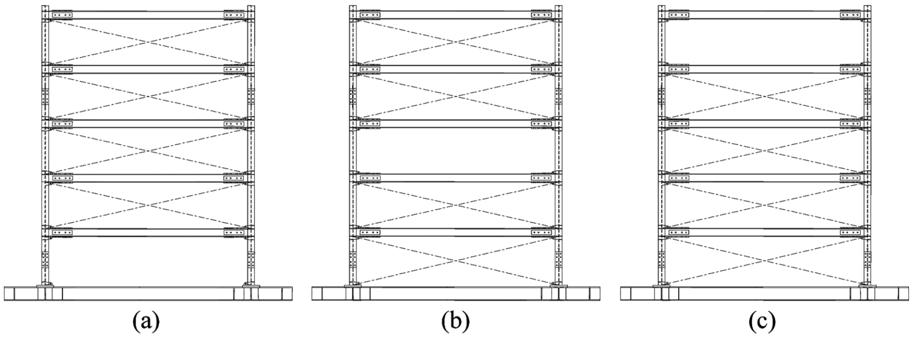

Fig. 13 Damage case: removing braces on the (a) first, (b) third and (c) fifth storey

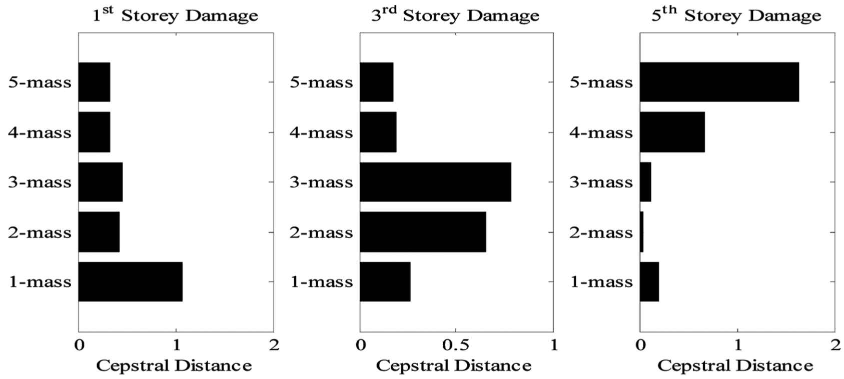

Fig. 14 Cepstral distances (the braces were removed)

Table 5 Standard deviations of AR coefficients (the central columns on the first storey were removed)

Data length 6000	1st floor	2nd floor	3rd floor	4th floor	5th floor
a1	0.0089	0.0078	0.0072	0.0087	0.0079
a2	0.0212	0.0176	0.0162	0.0200	0.0196
a3	0.0212	0.0176	0.0162	0.0200	0.0196
a4	0.0089	0.0077	0.0072	0.0087	0.0079

Table 6 Standard deviations of AR coefficients (the braces on the first storey were removed)

Data length 6000	1st floor	2nd floor	3rd floor	4th floor	5th floor
a1	0.0092	0.0090	0.0091	0.0095	0.0095
a2	0.0185	0.0160	0.0167	0.0173	0.0202
a3	0.0185	0.0160	0.0167	0.0173	0.0202
a4	0.0092	0.0090	0.0091	0.0095	0.0095

the central columns and the braces on the first storey were removed (the standard deviations for the other experimental conditions are similar). The damage indicators are very large compared with the standard deviations, which means that they are not influenced by the standard deviations very much. We thus obtained good recognition results, as shown in Figs. 12 and 14.

5. Conclusions

A new scheme based on distance measures of low-order AR models was proposed to identify damage to structures. The optimum order of the AR model for the distance measure was carefully studied. The standard deviations of the AR coefficients have a significant effect on the accuracy of identification, especially when the standard deviations are not small enough relative to the damage indicators; on the contrary, the severity and location of the damage have little effect on the standard deviations of the AR coefficients. To overcome this problem, we reduce the standard deviations of AR coefficients by increasing the data length. The results of simulations and an experiment show that the scheme is feasible for identifying the damage location. In simulation and experiment, buildings were assumed to be excited by Gaussian white noise. Gaussian white noise was used to simulate input such as earthquakes and ambient vibration. To apply the method, any excitation including earthquakes or ambient vibration can be used as long as the duration of excitation is long enough. However, there are still some problems unsolved. In this study, results with different damage severities show that larger cepstral distance is corresponding to a more serious damage severity and this indicate that there is a relation between the cepstral distance and the damage severity, but so far we can't quantify the damage severity by cepstral distance, and further study is needed. Pre-whitening is used to whiten the correlated signals which needs the vibration measurements at all DOFs, but for economy, generally the vibration measurements at a limited number of DOF are obtained, typically smaller than the number of DOF of structure. So far there has been no general method to determine the optimal order of the AR model for the distance measure for multiple degrees of freedom (MDOF) system. In this paper, we use simulation to decide the optimal order of the AR model and then expand to the real structure, which is similar to the model in simulation, to detect the damage. These unsolved problems will be studied in the following paper.

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