Smart Structures and Systems, Vol. 6, No. 2 (2010) 115-133
DOI: http://dx.doi.org/10.12989/sss.2010.6.2.115

The application of modal filters for damage detection

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(Received April 10, 2008, Accepted April 20, 2009)

Abstract. A modal filter is a tool used to extract the modal coordinates of each individual mode from a system's output. This is achieved by mapping the response vector from the physical space to the modal space. It decomposes the system's responses into modal coordinates, and thus, on the output of the filter, the frequency response with only one peak corresponding to the natural frequency to which the filter was tuned can be obtained. As was shown in the paper (Deraemecker and Preumont 2006), structural modification (e.g. a drop in stiffness or mass due to damage) causes the appearance of spurious peaks on the output of the modal filter. A modal filter is, therefore, a great indicator of damage detection, with such advantages as low computational effort due to data reduction, ease of automation and lack of sensitivity to environmental changes. This paper presents the application of modal filters for the detection of stiffness changes. Two experiments were conducted: the first one using the simulation data obtained from the numerical 7DOF model, and the second one on the experimental data from a laboratory stand in 4 states of damage.

Keywords: modal filter; structural health monitoring; damage detection; FRF synthesis.

1. Introduction

One of the groups of damage detection algorithms used in structural health monitoring is the group of the so-called vibration methods, i.e. methods that use data in the form of accelerations of vibrations in time or frequency domains. They are based on the observation of changes in the system's vibration responses, which result from damage occurrence. The methods' main idea is model-based diagnostics defined in the following way: the model of a particular system in an undamaged state is given, and this model is compared to the model identified from the data measured on the object in the current state. Differences between these two models indicate the object modification (e.g. stiffness or mass decrease), which may be caused by damage. The most frequently applied methods of this type include (Mendrok and Uhl 2004):

- methods based on perturbation of modal parameters (natural frequency, modal damping) (Żak, *et al.* 1999, Ettouney, *et al.* 1999, Kawiecki 2000),
- methods based on FRF (stiffness and compliance) variation detection (Agneni, *et al.* 2000, Lopes, *et al.* 2000, Balis Crema and Mastroddi 1998),
- methods based on mode shape analysis (Maeck, *et al.* 1998, Ho and Ewins 1999, Wang, *et al.* 2000, Ki-Young, *et al.* 2008),

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- methods based on detection of energy changes in modes (Carrasco, *et al*. 1997),
- methods based on Ritz vectors' variation detection, (Sohn and Law 2000)
- methods based on detection of regression model parameters (Heyns 1997, Bodeux and Golinval 2000),
- methods based on analysis of time-frequency characteristics (Gaul and Hurlebaus 2000, Naldi and Venini 1997)
- methods based on PCA and SVD analysis (Ruotolo and Surace 1997, Zimmerman 1999),
- methods based on FE model updating (Fritzen and Bohle 2000, Ruotolo, *et al*. 2000).

A detailed description of the methods listed above, together with sample content from the literature and verification of selected methods, can be found in Mendrok and Uhl 2004. A similar overview of the vibration based damage detection methods can be found in the articles: Doebling, *et al*. 1998 and Carden and Fanning 2004. The most convenient model, which can be applied in the described approach, is a modal model, i.e. a set of natural frequencies, modal damping coefficients and modal vectors describing the dynamics of the tested object. The modal model is used in methods no. 1, 3, 4 and 9 from the above list. The modal model is relatively easy to identify and, by means of operational modal analysis, may be identified only from response data; it is, therefore, very useful in diagnostics. The modal parameter perturbation (natural frequency, modal damping) is the simplest application of the modal model for damage detection. The results can be obtained from output-only data by means of operational modal analysis. In some cases it enables damage detection, but the detection strongly depends on object geometry, material and the nature of the damage itself. This technique does not provide information about the location of the damage, but the biggest disadvantage of the method is probably the fact that its effectiveness is dependent on environmental conditions. Methods based on mode shape analysis and methods based on detection of mode strain energy changes are much more sensitive. They can detect and locate damage at a relatively early stage, but are computationally very expensive. Methods based on FE model updating are used when there is no reference data (for undamaged structures) for comparison or when one wants to identify the type of damage caused. It requires an updated FE model of the monitored object and is also very time consuming.

Summarizing the above brief description of the methods, application of modal model-based diagnostics within damage detection has several limitations and faults. First of all, there is a serious problem distinguishing between the changes in parameters resulting from damage and those caused by environmental changes e.g. temperature or humidity. The changes in ambient temperature for civil engineering objects (bridges, viaducts, masts, tall buildings) may reach even tens of degrees in a relatively short period of time. It results in further changes in stiffness and finally in modal parameter variation. This effect is multiplied when the object is unevenly heated – e.g. when one side is exposed to the sun and the other is kept at a constant temperature due to its proximity to water. The influence of humidity variations is similar – concrete elements absorb moisture and this leads to increases in their mass and variations in modal parameters. Naturally, there are methods which enable the influence of environmental changes on the diagnostic procedure accuracy to be eliminated. In most applications, a lookup table is prepared in which modal parameters identified for different ambient temperatures and values of humidity are gathered together. Such a table is unique and independently prepared for every object as a result of a set of experiments. A more sophisticated method of eliminating the influence of weather conditions on the monitoring system's efficiency is application of an environmental filter (Peeters, *et al*. 2000). It is generally an autoregressive model with a moving average (ARMA) identified from a set of experimental data. When environmental changes are eliminated in this way, any modification of the object (e.g. lining another asphalt layer on a bridge deck) causes the necessity to repeat the entire set of measurements, to update the lookup table or the environmental filter.

A further limitation of modal model-based diagnostics involves difficulties with the automation of procedures. Despite the current development of autonomous modal analysis procedures in many scientific centers, the total independence of an engineer's interference is still problematic in practice. Additionally, diagnostic symptoms in the form of natural frequencies, modal damping coefficients and modal vectors are estimated periodically, and depend on the subjective assessment of a testing team.

There is also a method which uses vibration data as well as a modal model of the object and, in addition, enables damage to be distinguished from environmental influences. This is the advantage of modal filtering applied to the data recorded on the object.

The modal filter was used for the first time for damage detection and structural health monitoring by Shelley and Slater in 1993. They developed an approach to monitoring the health of a smart structure with sensor and actuation capability. Their approach is an integrated control and monitoring procedure whereby the sensors, which are assumed to be distributed spatially across the structure, are processed by a set of modal filters which automatically track the modal coordinates of specified modes, and similarly track changes in modal parameters (Shelley, *et al.* 1992). The adaptive modal filter is formulated and used to track the time varying behavior of the specified modes, thereby indicating the health of the structural system. The adaptive modal filter is insensitive to failures or calibration shifts in individual sensors and automatically ignores failed sensors. It can also be used to detect disturbances entering the system as well as to identify failed actuator locations.

A different approach to applying modal filtration to damage detection can be seen in Gawronski and Sawicki 2000. They use modal norms, calculated as Frobenius norms from the 1DOF frequency response functions (FRFs), as a damage indicator. 1DOF FRFs are obtained from the system characteristics by filtering them with a modal filter. The norms are calculated for each mode shape and for each measuring location, and then compared to their equivalents for an undamaged system. The method formulated in this way enables damage to be detected and localized.

Another way of applying modal filters in damage detection can be found in El-Ouafi Bahlous, *et al.* (2007). The suggested approach requires vibration data of the system in the undamaged and current stage along with an FE model parameterized by means of specified damage parameters. The modal filter with an asymptotic local approach is used to compute an improved, normally distributed residual. The covariance, mean and sensitivity of this residual are used for the generalized log-likelihood ratio test which enables damage to be detected.

One more way of using modal filtering for structural health monitoring is presented by Deraemaeker and Preumont in 2006. The frequency response function of an object filtered with a modal filter has only one peak corresponding to the natural frequency to which the filter is tuned. When a local change occurs in the object – in stiffness or in mass (this mainly happens when damage in the object arises), the filter stops working and, on the output characteristic, other peaks start to appear, corresponding to other imperfectly filtered natural frequencies. On the other hand, a global change in entire stiffness or mass matrix (due to changes in ambient temperature or humidity) does not corrupt the filter and the filtered characteristic still has one peak, although it is slightly moved in the frequency domain. Such an approach has also been applied in this paper, but the authors have added the pre- and post-processing algorithms to make the method more useful for experimental data. A different way of interpreting results is also presented.

2. Modal filters

The modal filter is a tool to extract the modal coordinates of each individual mode from the system outputs by mapping the response vector from the physical space to the modal space (Zhang, *et al.* 1990). It was first introduced by Baruh and Meirovitch in 1982 to overcome the spill-over problem within control of distributed parameter systems. Spill-over is a phenomenon in which the energy addressed to the controlled mode is pumped into the uncontrolled modes.

The construction of the r-th modal filter, which corresponds to r-th pole of the transfer function $H(\omega)$, starts with the assumption that the modal residue R_{rpp} is in the imaginary form:

$$R_{rpp} = j \cdot 1 \tag{1}$$

Next, the 1 DOF frequency response function $H_{pp}(\omega)$ is determined as follows:

$$H_{pp}(\omega) = \frac{R_{rpp}}{j\omega + \lambda_r} + \frac{R_{rpp}^*}{j\omega + \lambda_r^*} \tag{2}$$

where: λ_r - r-th pole of the system.

For the given frequency range, the above FRF is determined by the k values:

$$H_{pp}(\omega) = [H_{pp}(\omega_1)\ H_{pp}(\omega_2)\ldots H_{pp}(\omega_k)]^T \tag{3}$$

Assuming that a single excitation was used and the response signals were measured in N points, the experimental frequency response function matrix can be presented as the $k \times N$ matrix:

$$H_{kN}(\omega) = \begin{bmatrix} H_1(\omega_1)\ H_2(\omega_1) & H_N(\omega_1) \\ H_1(\omega_2)\ H_2(\omega_2) & H_N(\omega_2) \\ \vdots & \\ H_1(\omega_k)\ H_2(\omega_k) & H_N(\omega_k) \end{bmatrix} \tag{4}$$

The FRF matrix formed in this way is used to determine the reciprocal modal vector matrix Ψ_p:

$$\Psi_p = H_{kN}^+ \cdot H_{pp} \tag{5}$$

where: $^+$ - denotes a pseudo-inverse of the matrix

Reciprocal modal vectors should be orthogonal with respect to all the modal vectors except the one to which the filter is tuned, and, thanks to that, are applied to the decomposition of the system responses to the modal coordinates η_r.

$$\eta_r(\omega) = \Psi_p^T \cdot \{x(\omega)\} = \left(\frac{\{\phi_r\}^T}{j\omega - \lambda_r} + \{\psi_r\}^T \{\phi_r^*\} \frac{\{\phi_r^*\}^T}{j\omega - \lambda_r^*} \right) \tag{6}$$

where: ϕ_r - r-th modal vector

$\{x(\omega)\}$ - vector of system responses.

Now, scaling the modal coordinates η_r by the known input, it is possible to determine the FRF with all peaks, except r - th, filtered out.

3. Influence of damage and environmental changes on modal filters

Let us consider a dynamic system with proportional damping described by the equation of motion given in the matrix form:

$$M \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = f \tag{7}$$

where: M, C, K - mass, damping, stiffness matrices respectively,
$C = c_1 \cdot M + c_2 \cdot K$, c_1, c_2 - real constants
f - excitation forces vector

The modal model of the system given in Eq. (7) may be determined by solving the generalized eigenvalue problem formulated as follows (Uhl 1997):

$$\left(\left(\frac{s^2 + s \cdot c_1}{s \cdot c_2 + 1}\right) \cdot M + K\right) \cdot X(s) = 0 \tag{8}$$

As was shown in the previous section, the modal filter uses reciprocal modal vectors to decompose system responses on the components connected with consecutive modes. This happens because the r-th reciprocal modal vector ψ_r is orthogonal to all modal vectors except r-th. When damage occurs in the object, it results, in most cases, in a local drop of stiffness. The stiffness matrix is different and both eigenvalues (natural frequencies and modal damping coefficients) and eigenvectors (modal vectors) are changed. It also means that the reciprocal modal vectors obtained for the undamaged object are not perfectly orthogonal to the modal vectors and, consequently, the modal filter does not perfectly isolate the influence of other modes from the system response. So why don't environmental changes, such as temperature or humidity variations, affect the modal filter in this way? Let us now imagine that the temperature around the object increases and, due to that, the Young modulus of the object material decreases by a particular value. If we consider an ideal case, where the entire object has the same temperature and is made of a homogeneous substance, the change in the stiffness matrix can be defined as follows:

$$K_t = \alpha \cdot K \tag{9}$$

where: K_t - stiffness matrix for higher temperature,
α - coefficient relating stiffness changes to temperature

By solving Eq. (8), it is possible to obtain the diagonal matrix Λ containing the eigenvalues and matrix Φ of modal vectors. With such a solution the generalized eigenvalue problem for the undamaged system can be written down in the following form:

$$-K \cdot \Phi = M \cdot \Phi \cdot \Lambda \tag{10}$$

Since the mass matrix M is square and invertible, the generalized eigenvalue problem can be presented as a standard eigenvalue problem:

$$M^{-1} \cdot (-K) \cdot \Phi = \Phi \cdot \Lambda \tag{11}$$

The same equation can be formed for the system at higher temperatures:

$$M^{-1} \cdot (-\alpha \cdot K) \cdot \Phi_t = \Phi_t \cdot \Lambda_t \tag{12}$$

where: modal parameters for the system at higher temperatures are denoted with the index t

Dividing Eq. (11) by (12), and performing some simple mathematical operations, it is easy to derive the following:

$$\Lambda_t = \alpha \cdot \Lambda \tag{13}$$

Placing Eq. (13) into (12) and comparing it with (11), it is immediately visible that:

$$\Phi_t = \Phi \tag{14}$$

And this guarantees the further operation of the modal filter, i.e. there are no additional peaks on the filtered characteristic.

The assumption of a homogeneous change of temperature in the monitored object is, of course, due to the geometrical and material properties of the object. Additionally, the position and orientation of the object with respect to the heat source will disturb this assumption. The level of influence of the ambient temperature on modal filtration will be investigated in the following sections of the paper.

4. Diagnostic procedure for output-only data

As was already mentioned, the modal filter has great potential as a tool for structural health monitoring. Having a modal filter tuned to a system with no damage, it is possible to filter the FRFs of the object for the current state. The filtered FRFs are then tested for the existence of other peaks corresponding to the filtered mode shapes, with some maximum detection procedure. There are a number of peak picking algorithms, but for this type of application a method based on wavelet filtering seems to be the most suitable. Such a peak picking method would not only allow peak detection, but also peak recognition and differentiation between structural peaks as a result of imperfect modal filtration as well as operational peaks as a result of harmonic excitation. However, thorough and final evaluation of the peak detection methods remains a prospect for future research. Now, if the peaks from the other modes appear in the filtered FRFs, it means that some damage has occurred in the object. However its application to object diagnostics requires the solution of a few problems. First of all, the procedure deals perfectly only with FRFs. Naturally, modal filters also filter out the system response spectrum, but only from the peaks arising from structural vibration, not those coming from excitation. So the method based on the modal filter and maxima detection would work only for objects excited with white noise, which is not very common in everyday use. On the other hand, the application of FRFs requires active testing, i.e. testing with controlled and measured excitation. This is very inconvenient as, in order to perform active testing, the object operation must first be stopped. It is difficult to assume that operation of an object like a bridge or a turbo-generator will be stopped every hour and excited with some special equipment. The cost of such a monitoring system would be enormous. It is then obvious that the system has to work only with operational output data. Another problem is that the

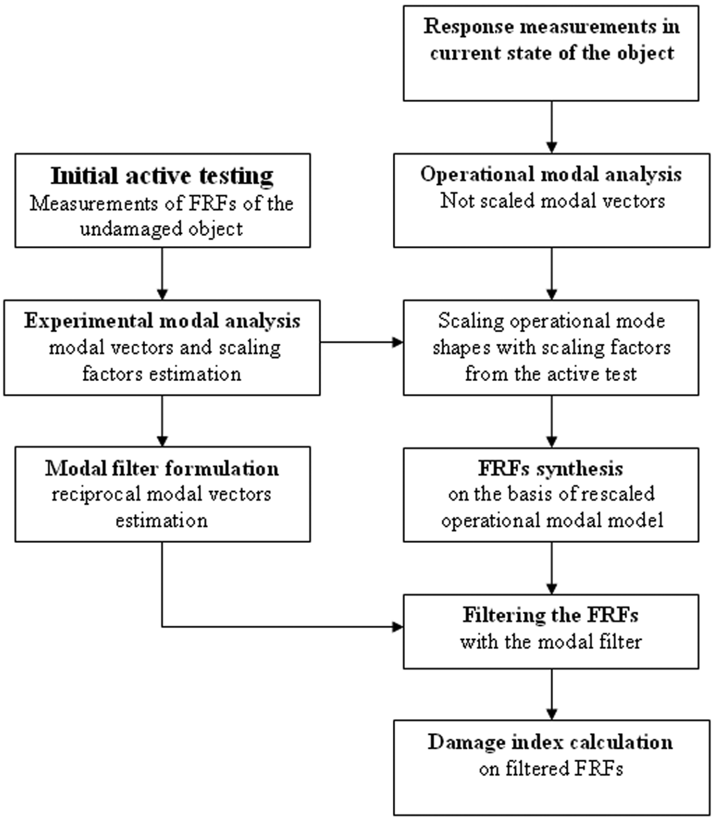

Fig. 1 Block diagram with damage detection procedure

modal filter does not work perfectly when noise appears in the measuring data. The maximum detection procedure is then not enough to fully automate the structural health monitoring system. An attempt to solve both of these problems is presented in Fig. 1.

To overcome problems with imperfectly filtered peaks which even appear in the reference data, the authors decided to use the damage index DI given by Eq. (15) instead of the peak picking algorithm.

$$DI = \frac{\int_{\omega_s}^{\omega_f} |x_i(\omega) - x_{ref}(\omega)|^2 d\omega}{\int_{\omega_s}^{\omega_f} x_{ref}(\omega)^2 d\omega} \tag{15}$$

where: $_s$ - starting frequency of the analyzed band,

$_f$ - closing frequency of the analyzed band

x_i - characteristic in the current state

x_{ref} - characteristic in the reference state

The DI is calculated for the entire frequency range of interest. If some additional peaks occur on the filtered characteristics this will be more clearly visible on the DI than when peak picking is applied, because the influence of all unfiltered maxima will be cumulated.

As presented in the block diagram in Fig. 1, modal filtering and the next damage index calculation are performed on the synthesized FRFs. The FRFs are synthesized from the operational modal model obtained for the object in the current state and rescaled with the scaling factors identified during the initial active experiment conducted on the object with no damage. The effectiveness of the FRFs synthesized in this way for the force identification purposes was shown in Mendrok and Uhl 2006. The initial test results are also used for the modal filter formulation as the object has no damage and refers to environmental conditions. Next, the synthesized FRFs are filtered with the modal filter and the DI is calculated.

5. Numerical example

Firstly, the authors decided to test the procedure on the data from numerical simulations. Two simulations were performed on two different models. The first simulation was performed on the simple 7 DOF model to check the correctness of the mathematical proof from Section 3. That is why the model was very simple, the temperature loading was homogeneous, and no thermal expansion was considered. The model is presented in Fig. 2.

The following notation was used: the stiffness between mass i and j - k_{ij}, the damping coefficient between mass i and j - c_{ij}. Proportional damping was applied while the modal filter works only for the normal modal vectors.

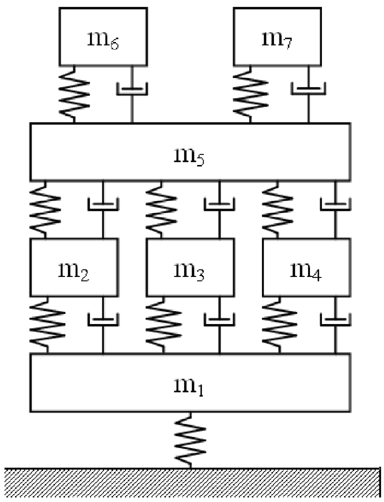

Fig. 2 Simulation system scheme

Physical parameters established for the model are gathered together in Table 1.

Table 1 Simulation model parameters

Mass [kg]	$m_1 = 5$; $m_2 = 1$; $m_3 = 1$; $m_4 = 1$; $m_5 = 4$; $m_6 = 2$; $m_7 = 2$;
Damping coefficients [N s/m]	$c_{01} = 26.6$; $c_{12} = 5$; $c_{13} = 5$; $c_{14} = 5$; $c_{25} = 4.95$; $c_{35} = 5$; $c_{45} = 5$; $c_{56} = 9$; $c_{57} = 9$;
Stiffness coefficients [N/m]	$k_{01} = 80000$; $k_{12} = 15000$; $k_{13} = 15000$; $k_{14} = 15000$; $k_{25} = 14800$; $k_{35} = 15000$; $k_{45} = 15000$; $k_{56} = 27000$; $k_{57} = 27000$

Table 2 Modal parameters of the simulation model

MS no.	Natural frequency [rad/sec]	Modal damping coefficients
1	41.73	0.0069
2	118.32	0.0194
3	123.27	0.0288
4	172.81	0.0288
5	173.20	0.0289
6	174.30	0.0253
7	216.37	0.0360

In order to determine the analytical model of the established system, its equation of motion was formed in the following matrix form:

$$\{f\} = [M] \cdot \{\ddot{x}\} + [C] \cdot \{\dot{x}\} + [K] \cdot \{x\} \tag{16}$$

For Eq. (16), the eigenvalue problem was solved assuming zero initial conditions for displacements and velocities. As a result, seven conjugated pairs of the system eigenvalues were obtained. On their basis, the modal parameters were calculated: natural frequencies, damping coefficients (presented in Table 2)

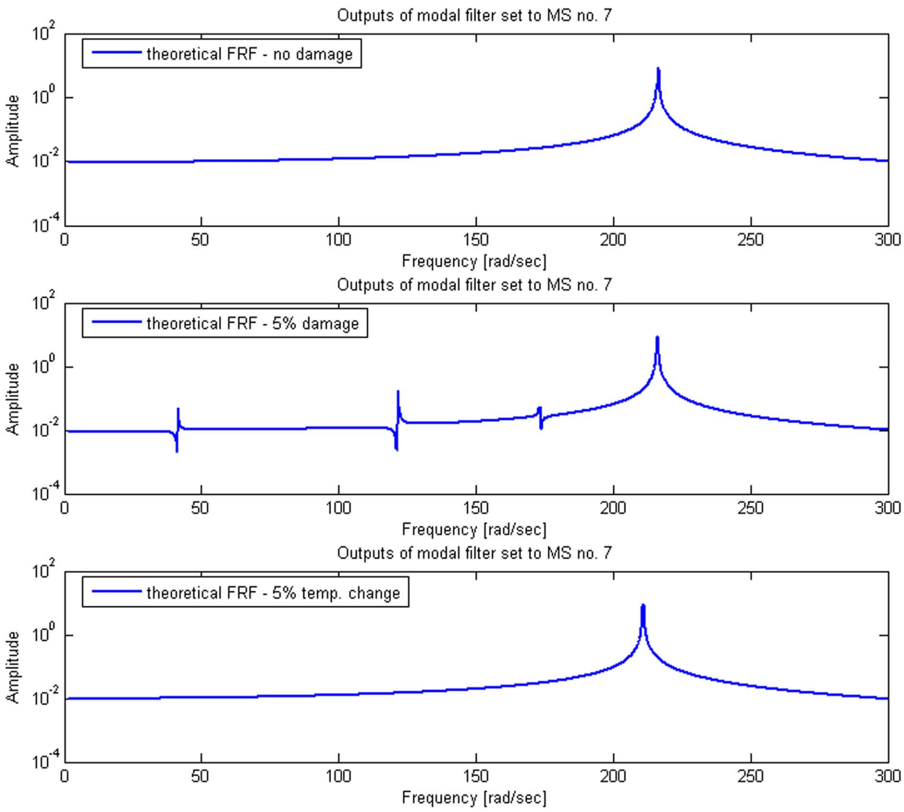

Fig. 3 Output of modal filter for theoretical example

and modal vectors.

Next, Mass No. 3 of the model was excited with a band noise signal of the frequency range 0 - 256 rad/sec. The response in all masses was then calculated and, with the use of an Hv estimate (Uhl 1997), the frequency response functions were computed. In the next step, the reciprocal modal vectors were calculated, i.e. the modal filter was formulated.

To check the mathematical proof correctness, two modifications were applied to the system. First, the stiffness k_{01} was decreased by 5% to model damage. Next, all stiffness values were decreased by 5% to model a change in ambient temperature. No noise was added to the data. The results of both tests are presented in Fig. 3. The results for the modal filter output set were similar to all mode shapes, and to save space, only one of them is presented in the paper. The next step was to repeat both tests but with noise added to the excitation and response signals. The noise had a uniform distribution, mean value equal to zero and amplitude equal to 10% of the corrupted characteristic amplitude. The results of the tests with noise data are presented in Fig. 4. The presence of noise means, as was already mentioned, that the filter stops working even for the data with no damage, but the amplitude of unfiltered peaks is lower for both undamaged systems and systems with different temperature than for systems with only 5% damage. Because, in physical applications, the measurements are always subject to noise, a peak

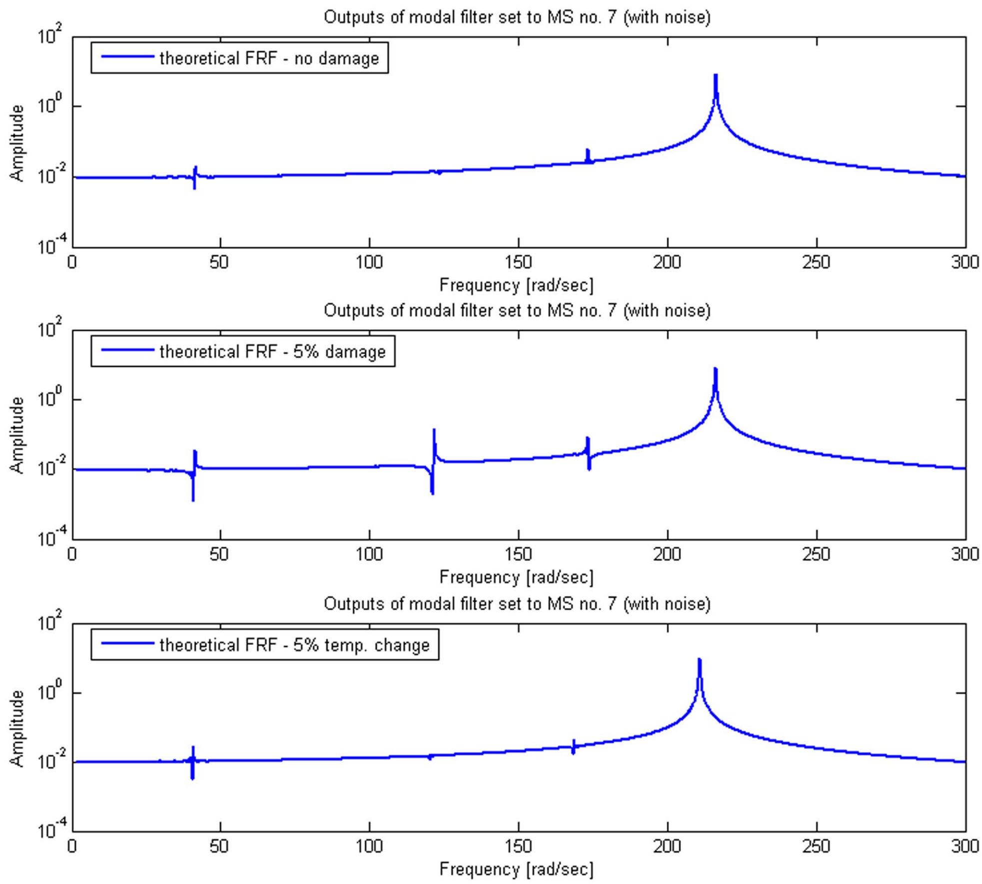

Fig. 4 Output of modal filter for theoretical example - noisy data

picking procedure applied on filtered system characteristics for the damage detection is insufficient. Therefore, the authors decided to replace it with the damage index given by Eq. (15).

The second part of simulation verification was performed on a more realistic model. The model had a complex geometrical shape, was non-homogeneous (steel - aluminum), had realistic boundary conditions and thermal expansion was included. The main goal of this simulation was to investigate the influence of different temperature loadings on the modal filtration and to compare it with the effect of damage.

First, the 3D model was performed in MSC.Patran and meshed with 10k hex-8 elements (the no. of nodes - approximately 14k). The obtained model is shown in Fig. 5.

In order to investigate the damaged frame behavior, a horizontal beam crack was modeled as node disconnectivity. There were two crack stadiums - 5% and 10% of the cross section area modeled. The material properties applied to the model are listed in Table 3.

Because the Young modulus varies with temperature, additional temperature-dependent material properties were applied. The material properties of aluminum were applied only to the horizontal beam. To take into account temperature-dependent material properties, each design scenario was carried out in

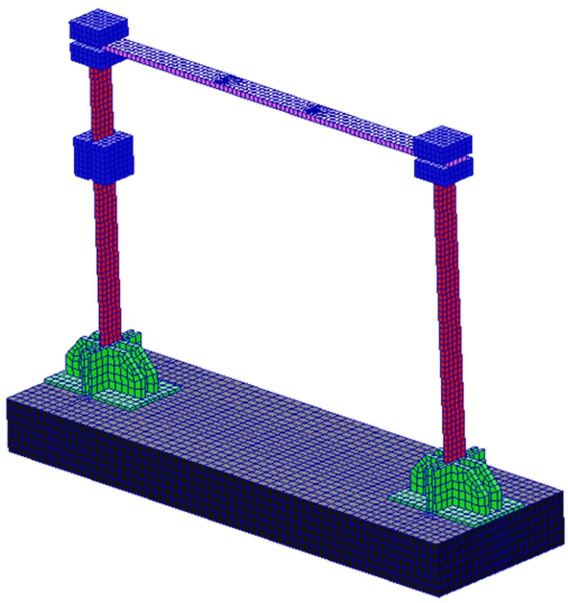

Fig. 5 Finite element model used for simulations

Table 3 Material properties selected for simulation

	Steel (20°C)	Aluminum 20 ºC)
Young modulus (E) [MPa]	$2.1 \cdot 10^5$	$0.69 \cdot 10^5$
Poisson's ratio	0.3	0.33
Density (ρ) [g/cm^3]	7.85	2.71
Thermal expansion coefficient (k) [1/K]	$1.5 \cdot 10^{-5}$	$2.35 \cdot 10^{-5}$
Thermal conductivity [W/m*K]	47	193
Specific heat capacity [J/kg*K]	420	880

Table 4 Simulated scenarios

No.	Tempera-ture load [°C]	Heating case	Frame state
1	20	Uniform temperature	Without crack
2	20	Uniform temperature	10% crack on upper beam
3	50	Upper beam heating (30 s)	Without crack
5	50	Local heating of the right vertical bar (30 s)	Without crack
7	50	Uniform temperature	Without crack
8	-10	Uniform temperature	Without crack

two sub-stages; first - coupled mechanical-thermal analysis to obtain the temperature distribution, second - normal mode analysis. The analyzed cases are listed in Table 4.

The modal model obtained for the first scenario was chosen as the reference one. The FRFs obtained from the consecutive simulations were modal filtered and the results of this filtration are in Figs. 6 and 7.

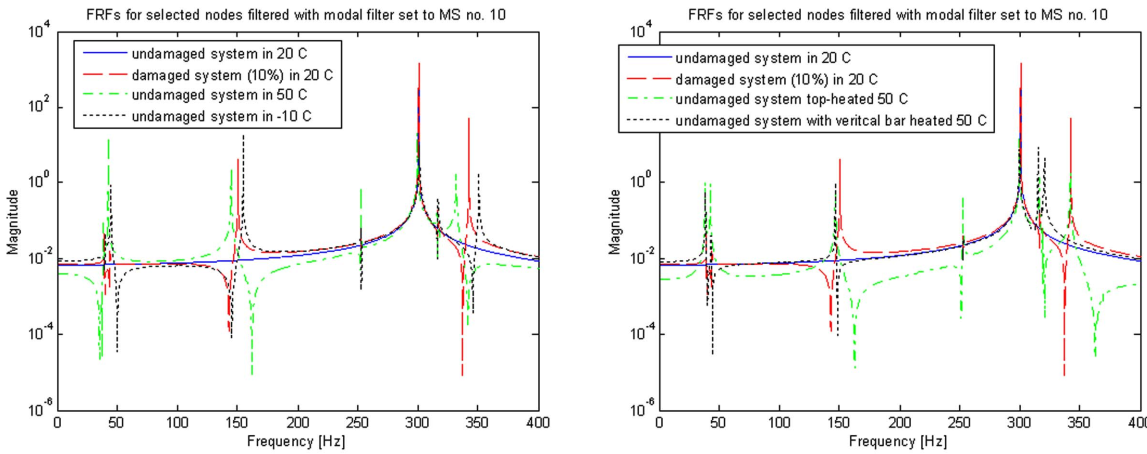

Fig. 6 Results of modal filtration - modal filter set to mode shape No. 1

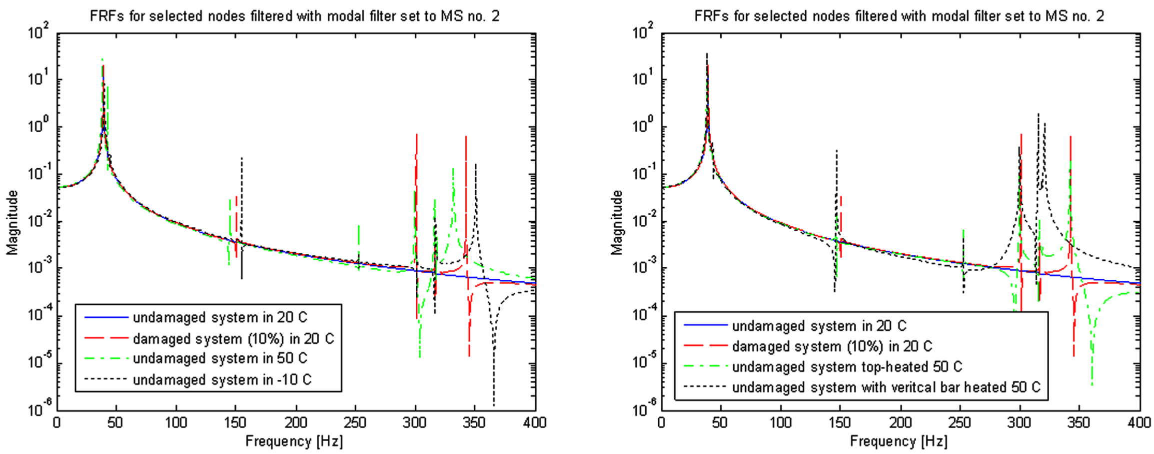

Fig. 7 Results of modal filtration - modal filter set to mode shape No. 2

The damage index values calculated for all the experiments are in Table 5.

Table 5 Damage index calculated for performed experiments

	No damage, temp. -10 ºC	No damage, temp. 50 ºC	No damage, temp. 50 ºC (top-heated)	No damage, local vertical bar heating temp. 50 ºC	Damage 10%, temp. 20 ºC
Damage index value (modal filter set to MS no. 3)	1.1103	3.1233	2.0602	4.2349	3.6607
Damage index value (modal filter set to MS no. 10)	0.9962	1.0042	1.0023	1.0028	9.1829

The best (modal filter set to Mode Shape 3) and the worst cases were chosen for result presentation. It can be seen that in the more realistic conditions the ambient temperature disturbs the modal filter. However its influence even for a 60 ºC difference between the lowest and highest temperatures is less than or comparable to the influence caused by relatively small damage. Additionally, the non-homogeneous distribution of the temperature field (heating the model from the top) did not introduce evident worsening of the method's efficiency. The worst case was for local heating. Here its effect on modal filtration was (for some modes) greater than the effect of the considered 10% damage. On the other hand, such local heating can also be interpreted as an emergency situation and its indication by the monitoring system could be considered as positive. Nevertheless, to be sure that the diagnostic system works properly, it would be advisable to build a bank of modal filters identified for different ambient temperatures and equip the diagnostic system with a temperature sensor.

Finally, it is worth mentioning that the best indication of damage occurred in the modal filters tuned to the mode shapes in which the modeled crack opened during deflection.

6. Measuring data example

The laboratory stand used for experimental validation of the proposed structural health monitoring procedure consists of a steel frame excited with an electrodynamic shaker. Vibrations were measured by accelerometers placed on the frame. A photo of the test setup without sensors and measuring equipment is presented in Fig. 8, and the network of measuring points is presented in Fig. 9. The frame

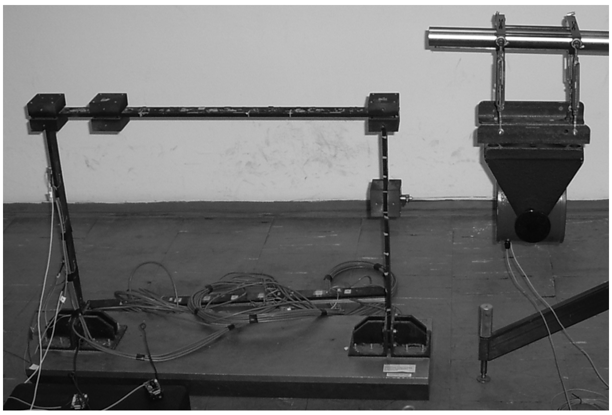

Fig. 8 The test setup (photograph)

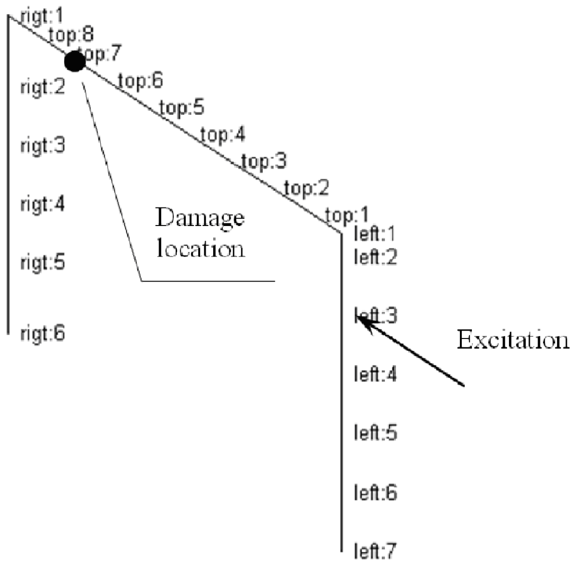

Fig. 9 Tested structure scheme - laboratory model of a frame

has been tested for different damage size. Damage in the frame has been introduced by nicking the upper bar at the measuring point 7.

- There were 4 modal tests carried out for different damage sizes:
- TEST 1 for an undamaged structure,
- TEST 2 for damage at point 7 with a 5 mm-deep gash (12 %),
- TEST 3 for damage at point 7 with a 14 mm-deep gash (35 %),
- TEST 4 for damage at point 7 with a 20 mm-deep gash (50 %).

Unfortunately, there are no data available for measurements taken at a different ambient temperature, so the SHM procedure could be tested only for damage detection.

In the course of each experiment, time histories of the excitation force and accelerations of vibrations at each measuring point were recorded. As was established in the procedure, the data from Test 1 were used to estimate scaling factors and reciprocal modal vectors; naturally, they were also reference data for the undamaged object. Table 6 presents the obtained modal parameters.

Table 6 Modal parameters of the laboratory stand

MS no	Test 1	
	N. f. [Hz]	Modal d.c.%
1	10.825	7.01
2	43.634	1.53
3	54.521	1.99
4	109.487	1.39
5	120.737	1
6	161.703	0.7
7	206.269	3.01
8	228.735	1.23

For the data from Tests 2 - 4, only output accelerations were used to estimate operational modal vectors. The modal vectors were then rescaled and used for FRFs synthesis.

Next, the FRFs were filtered with the modal filter formed for the undamaged object data. The results of filtering different damage level data are presented in Figs. 10 and 11.

As presented in the figures, the modal filters tuned to Mode Shapes 2 and 5 were chosen. The procedure was able to successfully detect all stages of damage starting from the smallest one (12% of the cross-section). Interestingly, for some mode shapes, the smallest damage provided higher peaks than damage at the level of 35%. Modal filtration with the use of modal filters tuned to other modes did not provide such clear results, especially for the smallest level of damage. This is due to the fact that not

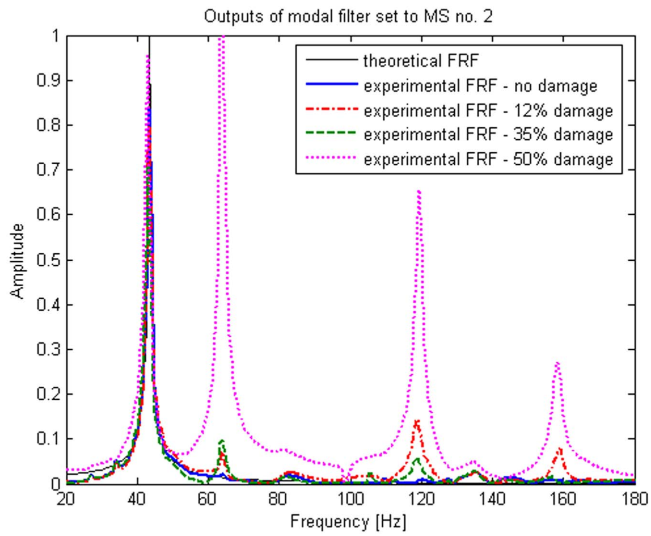

Fig. 10 Output of modal filter for laboratory tests - modal filter tuned to mode shape No. 2

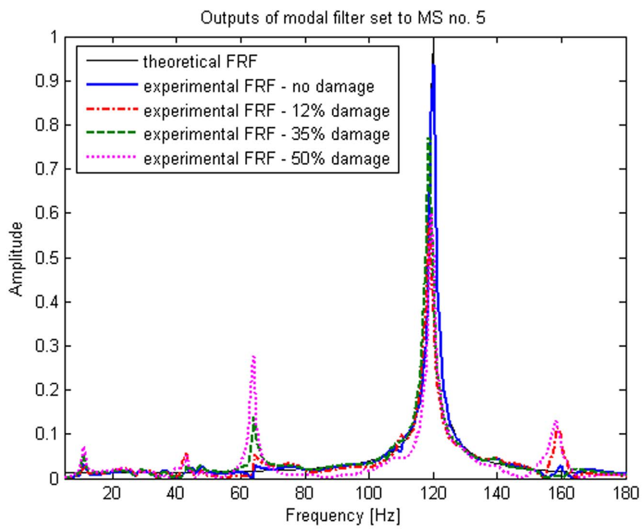

Fig. 11 Output of modal filter for laboratory tests - modal filter tuned to mode shape No. 5

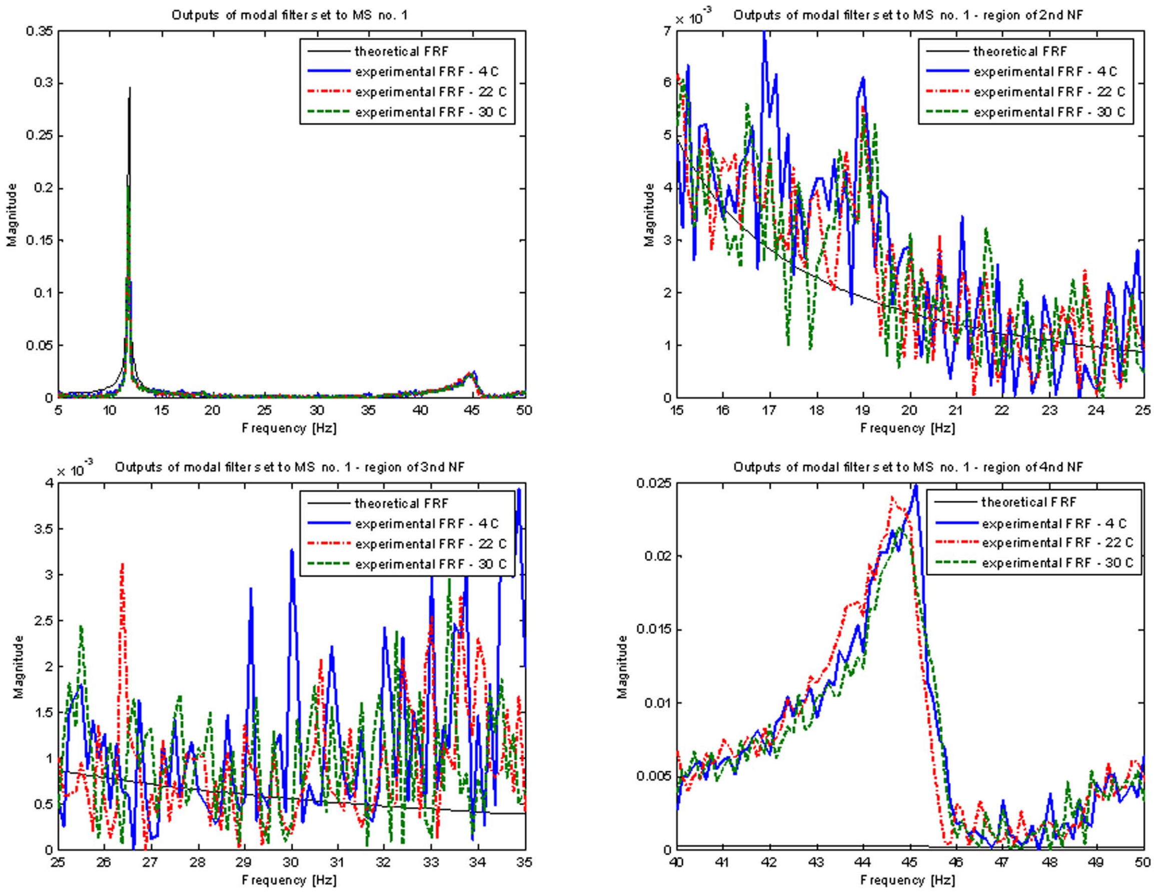

Fig. 12 Output of modal filter for laboratory tests (different ambient temperature) - modal filter tuned to mode shape No. 2

all mode shapes were disturbed by the damage considered in the experiment. However, for different types of failure, different modal filters would work more efficiently, which is why it is reasonable to analyze all the modal filter outputs from a given frequency range. The same data were used for testing several damage detection methods described in the literature. If one wants to compare the results obtained for modal filters with the efficiency of other methods, a description of performed analysis can be found in Mendrok and Uhl 2004. It is however worth noting that, from the vibration-based methods tested in the mentioned paper, the best results were obtained wherever the mode shapes were considered in the analysis, that is: testing of the correlation between modal vectors (MAC or CoMAC), analysis of mode shape curvature, and analysis of the deformation energy for particular natural vibration modes.

Additionally, the influence of temperature was investigated experimentally. The same testing object was used, but due to the fact that this test was performed some time later, it was slightly modified. The upper bar of the frame was replaced with a new one and the position of the steel boxes (see Fig. 5) was different. The reference measurements were performed at the ambient temperature 22 ^{o}C. Next the temperature in the laboratory was lowered to 4 ^{o}C and, after a period which guaranteed that the frame

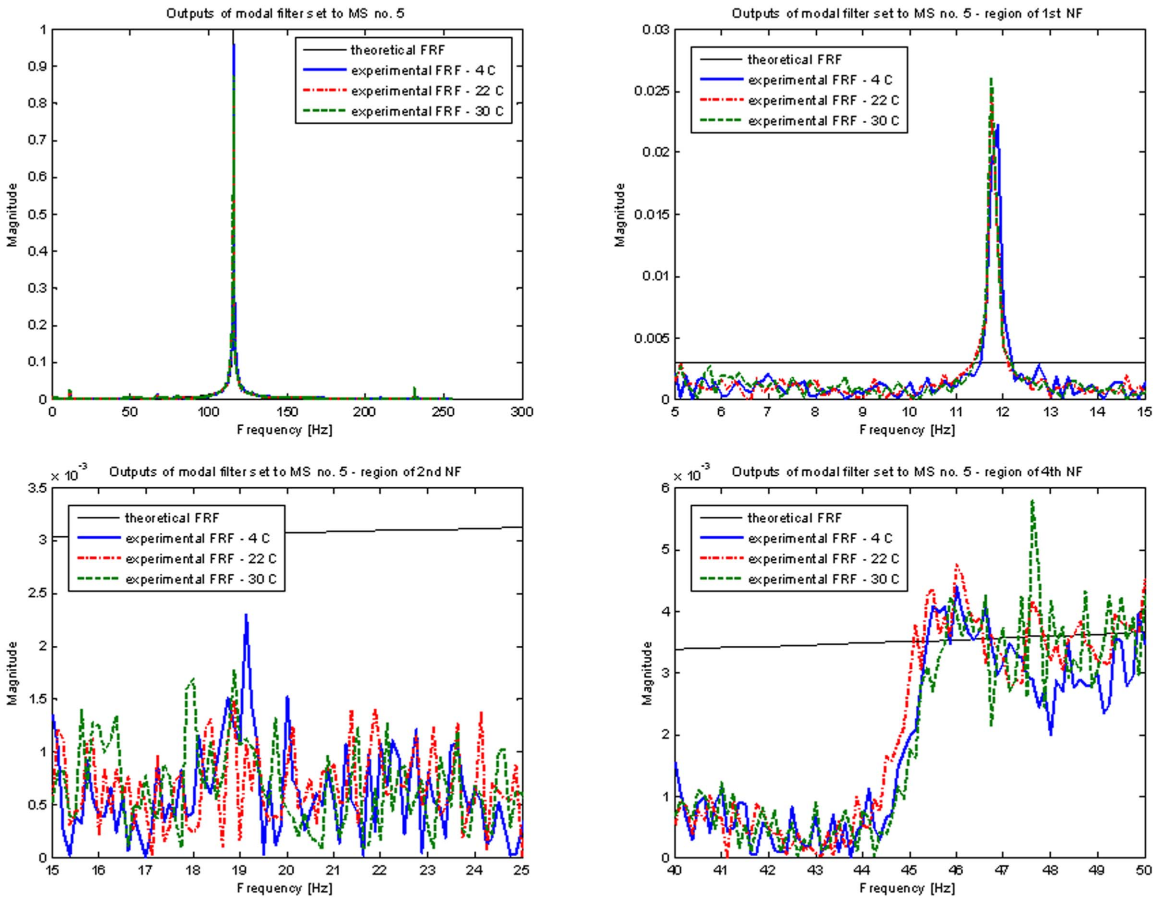

Fig. 13 Output of modal filter for laboratory tests (different ambient temperature) - modal filter tuned to mode shape No. 5

Table 7 Damage index calculated for performed experiments

	No damage, temp. 4 °C	No damage, temp. 30 °C	Damage 12%, temp. 22 °C	Damage 35%, temp. 22 °C	Damage 50%, temp. 22 °C
Damage index value (modal filter set to MS no. 1)	0.214	0.178	0.847	2.483	59.03
Damage index value (modal filter set to MS no. 5)	0.171	0.175	1.362	4.450	521.234

had the ambient temperature, the FRFs of the object were recorded. Similar measurements were taken at the laboratory temperature 30 °C. The diagnostic procedure with use of modal filtration was repeated and its results are presented in Figs. 12 and 13.

The results are shown for the Mode Shapes 1 and 5. It can be seen that the temperature influence on modal filtration is not as significant as the influence of damage in the previous experiment. For other modes there could be a problem with distinguishing between slight damage and temperature change. In Table 7 the damage index values calculated for all the experiments are placed.

7. Conclusions

The paper presents the procedure of damage detection based on modal filtering and FRF synthesis. The procedure was tested on data from the numerical example and, later, on data measured in the course of a laboratory experiment. The approach was successful for both of these tests, but the ambient temperature influence on the procedure was greater than had been expected. The necessity of applying FRFs as filtered characteristics was proposed to make the procedure independent from excitation. It, however, increases the computational cost of the procedure because the operational modal analysis and FRF synthesis have to be performed every single time. The application of an autonomous algorithm for modal analysis (Lisowski 2006) enables the procedure to work almost completely automatically. The authors plan to improve the method to allow for damage location and test it on real civil engineering structures.

Acknowledgement

Scientific research was financed from Polish Means for Science as Research Project No. R0301502.

References

Agneni, A., Crema, L. and Mastroddi, F. (2000), "Damage detection from truncated frequency response functions", *European COST F3 Conf. on System Identification and Structural Health Monitoring,* Madrid, Spain, 137-146.
Balis Crema, L. and Mastroddi, F. (1998), "Direct approach to updating and damage detection by using FRF data", *Proc. of ISMA23, Noise and Vibration Engineering*, Leuven, Belgium.
Bodeux, J.B. and Golinval, J.C. (2000), "ARMAV model technique for system identification and damage detection", *European COST F3 Conf. on System Identification and Structural Health Monitoring*, Madrid, Spain, 303-312.
Carden, P. and Fanning, P. (2004), "Vibration based condition monitoring: a review", *Struct. Health Monit.*, **3**(4), 355-377.
Carrasco, C., Osegueda, R., Ferregut, C. and Grygier, M. (1997), "Localisation and quantification of damage in a space truss model using modal strain energy", *Smart Systems for Bridges, Structures, and Highways, Proc. of SPIE*, **3043**, 181-192.
Deraemaeker, A. and Preumont, A. (2006), "Vibration-based damage detection using large array sensors and spatial filters", *Mech. Syst. Signal Pr.*, **20**(7), 1615-1630.
Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), "A summary review of vibration-based damage identification methods", *Shock Vib. Digest*, **30**(2), 91-105.
El-Ouafi Bahlous, S., Abdelghani, M., Smaoui, H. and El-Borgi, S. (2007), "Modal filtering and statistical approach to damage detection and diagnosis in structures using ambient vibrations measurements", *J. Vib. Control*, **13**(3), 281-308.
Ettouney, M., Daddazio, R., Hapij, A. and Aly, A. (1999), "Health monitoring of complex structures", *Smart Structures and Materials 1999: Industrial and Commercial Applications of Smart Structures Technologies, Proc. of SPIE*, **3326**, 368-379.
Fritzen, C.P. and Bohle, K. (2000), "Parameter selection strategies in model-based damage detection", *SHM 2000*, Palo Alto, CA.
Gaul, L. and Hurlebaus, S. (2000), "Wavelet transform to identify the location and force-time history of transient load in a plate", *Structural Health Monitoring 2000*, Stanford University, Palo Alto, CA, 851-860.
Gawronski, W. and Sawicki, J. (2000), "Structural damage detection using modal norms", *J. Sound Vib.,* **229**(1), 194-198.
Heyns, P.S. (1997), "Structural damage assessment using response-only measurements", *Structural Damage Assessment Using Advanced Signal Processing Procedures, Proc. of DAMAS '97*, Univ. of Sheffield, UK,

213-223.

Ho, Y.K. and Ewins, D.J. (1999), "Numerical evaluation of damage index", *Structural Health Monitoring 2000,* Stanford University, Palo Alto, CA, 995-1011.

Kawiecki, G. (2000), "Modal damping measurements for damage detection", *European COST F3 Conf. on System Identification and Structural Health Monitoring*, Madrid, Spain, 651-658.

Ki-Young, K., Jong-Jae, L., Chung-Bang, Y. and Jeong-Tae, K. (2008), "Damage detection in beam-like structures using deflections obtained by modal flexibility matrices", *Smart Struct. Syst.*, **4**(5), 605-628.

Lisowski, W. (2006), *Automation of Procedures within Experimental Modal Analysis; Selected Issues (in Polish),* AGH Publishers, Krakow, Poland.

Lopes, V. Jr., Pereira, J.A. and Inman, D.J. (2000), "Structural FRF acquisition via electric impedance measurement applied to damage location", *Proc. of SPIE*, **4062**, 1549-1555.

Maeck, J., Abdel Wahab, M. and De Roeck, G. (1998), "Damage detection in reinforced concrete structures by dynamic system identification", *Proc. of ISMA23, Noise and Vibration Engineering*, Leuven, Belgium.

Meirovitch, L. and Baruh, H. (1982), "Control of self-adjoint distributed parameter system", *J. Guid. Control Dynam.,* **8**(6), 60-66.

Mendrok, K. and Uhl, T. (2004), "Overview of modal model-based damage detection methods", *Proc. of 2004 ISMA*, Leuven, Belgium.

Mendrok, K. and Uhl, T. (2006), "Reconstruction of loading forces from response data measured in flight", *Proc. of 2006 ISMA*, Leuven, Belgium.

Naldi, G. and Venini, P. (1997), "Post-processing singular solutions by the wavelet transform", *Structural Damage Assessment Using Advanced Signal Processing Procedures, Proc. of DAMAS '97*, Univ. of Sheffield, UK, 109-120.

Peeters, B., Maeck, J. and De Roeck, G. (2000), "Dynamic monitoring of the Z24-Bridge: separating temperature effects from damage", *In Proc. of the European COST F3 Conf. on System Identification and Structural Health Monitoring*, 377-386, Madrid, Spain.

Ruotolo, R., Sorohan, S. and Surace, C. (2000), "Analysis of behaviour of three-dimensional truss structure", *European COST F3 Conf. on System Identification and Structural Health Monitoring*, Madrid, Spain, 169-178.

Ruotolo, R. and Surace, C. (1997), "Damage detection using singular value decomposition", *Structural Damage Assessment Using Advanced Signal Processing Procedures, Proc. of DAMAS '97*, Univ. of Sheffield, UK, 87-96.

Shelley, S.J., Freudinger, L.C. and Allemang, R.J. (1992), "Development of an on-line parameter estimation system using the discrete modal filter", *Proc. of the 10th Int. Modal Analysis Conf. (IMAC)*, San Diego, California, Feb. 3-8, pp. 173-183.

Slater, G.L. and Shelley, S.J. (1993), "Health monitoring of flexible structures using modal filter concepts", *Proc. of SPIE,* **1917**, 997-1008.

Sohn, H. and Law, K.H. (2000), "Extraction of ritz vectors from vibration test data", *Structural Health Monitoring 2000*, Stanford University, Palo Alto, CA, 840-850.

Uhl, T. (1997), *Computer-Aided Identification of Modal Models (in Polish),* WNT, Warsaw, Poland.

Wang, M.L., Xu, F.L. and Lloyd, G.M. (2000), "Systematic numerical analysis of damage index method used for bridge diagnostics", *Smart Structures and Materials 2000: Smart Systems for Bridges, Structures, and Highways*, *Proc. of SPIE*, Vol. 3988, Newport Beach, CA, 154-164.

Żak, A., Krawczuk, M. and Ostachowicz, W. (1999), "Vibration of a laminated composite plate with closing delamination; structural damage assessment using advanced signal processing procedures", *Proc. of DAMAS '99*, Dublin, Ireland, pp.17-26.

Zhang, Q., Allemang, R.J. and Brown, D.L. (1990), "Modal filter: concept and applications", *Proc. of Int. Modal Analysis Conf.*, pp. 487-496.

Zimmerman, D.C. (1999), "Looking into the crystal ball: the continued need for multiple viewpoints in damage detection", *Damage Assessment of Structures, Proc. of DAMAS 99*, Dublin, Ireland, 76-90.