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Entropy-based optimal sensor networks for structural health monitoring of a cable-stayed bridge

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Abstract. The sudden collapse of Interstate 35 Bridge in Minneapolis gave a wake-up call to US municipalities to re-evaluate aging bridges. In this situation, structural health monitoring (SHM) technology can provide the essential help needed for monitoring and maintaining the nation's infrastructure. Monitoring long span bridges such as cable-stayed bridges effectively requires the use of a large number of sensors. In this article, we introduce a probabilistic approach to identify optimal locations of sensors to enhance damage detection. Probability distribution functions are established using an artificial neural network trained using *a priori* knowledge of damage locations. The optimal number of sensors is identified using multi-objective optimization that simultaneously considers information entropy and sensor cost-objective functions. Luling Bridge, a cable-stayed bridge over the Mississippi River, is selected as a case study to demonstrate the efficiency of the proposed approach.

Keywords: structural health monitoring (SHM); sensor networks; information entropy; cable-stayed bridge.

1. Introduction

A recent report by USA Today (July 2008) shared with the public the current state of the nation's infrastructure. USA Today's report indicated "*Billions needed to shore up bridges*". The current status of our infrastructure reflects the need for reliable and efficient monitoring strategies and techniques. Deploying efficient structural health monitoring (SHM) systems on bridges can provide early warning about potential damage. Moreover, continuous monitoring of bridges and critical infrastructure might enable us to move from the current schedule-based maintenance to condition-based maintenance while saving millions of dollars and focusing our resources (Adams 2007).

Using SHM systems on long span cable-stayed bridges represents a technical challenge where a large number of sensors shall be deployed. To design an efficient, reliable and economical sensor network, the type, number and location of sensors need to be identified. The type of sensors is directly related to the damage feature that differentiates between healthy and damage cases. In the past, many researchers used vibration-based damage features to detect damage occurrence in bridges. Such features included natural frequency (Natke and Cempel 1997, Doebling, *et al*. 1996), mode shapes (Stanbridge, *et al*. 1997) and curvature of mode shapes (Maeck and De Roeck 1999, Ho and Ewins 2000). These damage features can be extracted from time, frequency or wavelet domains using means of digital signal processing methods (Neild, *et al*. 2003, Pothisiri and Hjelmstad 2003, Reda Taha, *et al*. 2004).

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Researchers have examined methods to design optimal sensor and actuator networks (Kumar and Narayanan 2008 and Pulthasthan and Pota 2008). The Probability of Detection (POD), introduced by Thompson (1999), was used by Guratzsch and Mahadevan (2006) to find the optimal sensor network on a composite plate. POD was also used by Chang, *et al.* (2007) to identify the optimal sensor network needed to monitor structural composites. Achenbach (2007) and Beard, *et al*. (2007) utilized POD to identify optimal sensor network for damage detection. Parker, *et al*. (2006) showed the possible identification of optimal locations of the sensors using time invariant dynamic analysis. Schulte, *et al.* (2006) examined the use of the information content in measurement data to find the optimal sensor locations by maximizing the determinant of the Fisher information matrix after Fritzen and Bohle (2001). Furthermore, the use of information-entropy for sensor networks was suggested by different researchers (Papadimitriou, *et al*. 2000, Ntotsios, *et al.* 2006, Udwadia 1994 and Heredia-Zavoni and Esteva 1998). More recently, Rao and Anandakumar (2007) introduced a swarm optimization technique to solve the optimal sensor placement problem. Most methods suggested in the literature for determining optimal sensor network rely heavily on assumptions directly related to the damage feature used for detecting damage. This limits the usefulness of these methods if such damage feature cannot be used for detecting damage in other structures. An optimal sensor allocation method that is independent of the damage feature is needed by the SHM community.

In this article, we introduce an entropy-based method for optimal sensor allocation. A case study of the Luling Bridge, a cable-stayed bridge over the Mississippi River, demonstrates the ability of the proposed method to successfully identify the optimal number and location of sensors necessary to effectively monitor the bridge. An entropy-based probabilistic method can address the uncertainties existing in sensor allocation without the need to prior assumptions on the damage feature.

2. Methods

We suggest an integrated probabilistic and entropy-based technique to identify both the optimal sensor locations and number of sensors in the sensor network. We first explain a probabilistic approach to optimally allocate any number of sensors. We then demonstrate how the optimal number of sensors can be identified through multi-objective optimization. We start by defining a finite set of damage locations and severities such that damage at any location of the structure can be described as ξ_{ij} where $i=1,2,\ldots, m$ and $j = 1,2,\ldots, n$ where m, n represent number of possible damage locations and severities, respectively.

A set of Finite Element (FE) models can thus be created to include all possible combinations of damage locations and severities. The vibration response obtained from each FE model of the structure can be used to calculate the damage feature(s). The selected damage feature can be calculated at each sensor assumed at each FE node location. The damage feature shall be able to differentiate between healthy and damage states of the structure. The damage features computed at each node of the FE models are used to compute the weights of an artificial neural network (ANN). ANN inputs include damage feature values while its outputs are damage locations known *a priori*. ANN is trained therefore to pattern damage features and damage locations as shown in Fig. 1. In this figure, Φ_1 to Φ_n are damage features calculated at each node of the FE model and L_1 to L_m are the locations of damage. The weights of the trained ANN are used to demonstrate the relative importance of each sensor in the damage detection process. Normalizing the ANN weights can be used to establish the discrete probability distribution function (PDF), $g(n)$ as

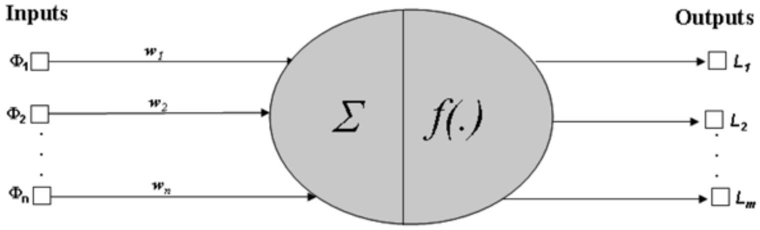

Fig. 1 Schematic representation of ANN to determine the PDF for distributing the sensors over the bridge

$$g(n) = \sum_{k=1}^{N} \frac{\gamma(k)}{\sum_{m=1}^{N} \gamma(m)} \delta(n - kr) \tag{1}$$

$g(n)$ is the PDF calculated at n locations of the structure, δ is a discrete impulse function and γ (k) is the absolute value of the kth weight of ANN, N is the number of sensors distributed throughout the structure and r is the finite element resolution. The continuous PDF can be constructed by interpolation. Since the continuous PDF demonstrates the importance of sensors as a function of structure dimension, by sampling the continuous PDF for any given number of sensors, the optimal locations of sensors can be identified.

2.1. Optimal number of sensors

The above approach will allow allocating N number of sensors to enhance the damage detection success rate. However, the above approach does not provide a tool for identifying the optimal (minimum) number of sensors. While minimizing the total number of sensors is of less interest in the case of monitoring relatively short bridges (25-100 m long), such number is important for significantly long bridges such as cable-stayed bridges (200-1000 m long).

We suggest here that the optimal number of sensors can be identified using multi-objective optimization system. Two objective functions can be realized in formulating the problem. <u>First</u>: the cost of sensors and their deployment and <u>Second</u>: the uncertainty associated with the sensor measurements. The optimal number of sensors shall enable reducing both objective functions simultaneously.

Information entropy was suggested as a scalar to quantify uncertainty in probabilistic based information systems (Ross 2004). The principles of information entropy introduced by Shannon (1948) can be used to quantify the uncertainty in damage features computed from a different number of sensor distributions. We suggest using Shannon entropy to quantify uncertainty in sensor measurements for a specific sensor distribution. Considering the fact that minimum uncertainty in damage detection can be associated with the case where sensors are allocated at all nodes, the difference between damage feature values of the case of interest and the case where sensors are allocated at all nodes of the FE model can be used as a measure of uncertainty in the monitoring system. Then the uncertainty objective function based on Shannon entropy can be calculated as

$$E = -\left(\sum_{i=1}^{N_{\max}} \Phi_i - \sum_{j=1}^{N} \Phi_j\right)^2 \left(\ln\left(\sum_{i=1}^{N_{\max}} \Phi_i - \sum_{j=1}^{N} \Phi_j\right)^2\right) \tag{2}$$

In Eq. (2), N_{max} is the maximum number of sensors where sensors are allocated in all nodes of the FE model and this number is governed by the resolution of the FE model and N is the number of sensors for that specific sensor allocation. Φ_i, Φ_j are damage features computed at nodes i, j respectively. As the number of sensors increase, information entropy will decrease and thus monitoring uncertainty will decrease. On the other hand, the number of sensors is strongly correlated to the cost of the sensor network. However, it is realized that the cost of the sensor network does not increase linearly with the number of sensors because sensor installation and implementation represents part of the sensor network cost. Installation cost does not change significantly as the number of sensors exceeds a specific threshold. The sensor network cost function "C_N" is defined as

$$C_N = \begin{cases} c_1 N & N \le N_1 \\ c_1 N + c_2(N - N_1) & N > N_1 \end{cases} \tag{3}$$

N is the number of sensors considered for the sensor network, N_1 is the number of sensors beyond which installation cost does not significantly increase. The function constants c_1, c_2, N_1 can be determined from field data.

By defining the two objective functions: entropy function "E" and sensor network cost function "C_N", a multi-objective optimization approach can be used to determine the optimal number of sensors. In multi-objective optimization, the design variable can be determined by establishing the Pareto front (Pareto 1971, Osyczka 1984 and Miettinen 1999). The Pareto front allows realizing the tradeoffs between different objective functions. A number of methods are discussed in the literature to perform such optimization by rank ordering the objective functions and performing the optimization in a hierarchical fashion or by defining a global objective function that combines both functions with varying weights (Osyczka 1984). We consider here equal weighted objective functions. In the above optimization problem, the optimal number of sensors (N) is the design variable. The optimization constraints include the maximum number of sensors (N_{max}) which is related to the number of finite element nodes and is governed by the FE model resolution. Once the optimal number of sensors (N) in the sensor network is determined, the probabilistic approach explained above for allocating these sensors can be implemented.

2.2. Redundancy of sensor network

The challenge with optimal sensor networks is the need to ensure network robustness. A robust sensor network shall operate efficiently even after losing one or more sensors. This goal can be achieved by identifying the location of the critical sensors. Redundant sensors shall be used at these critical locations. Here we suggest using 'leave one sensor out analysis' to examine sensor network sensitivity after Satelli, *et al*. (2000). In this analysis, the critical sensor location is related to the significance factor (ψ_i) defined as

$$\psi_i = \frac{|\Phi_{opt} - \Phi_i|}{\Phi_{opt}} \times 100 \tag{4}$$

Where Φ_{opt} is the mean value of the damage feature computed for the optimal sensor network with the minimum required number of sensors and Φ_i is the mean value of the damage feature computed for optimal sensor network after removing the ith sensor from the network. The critical sensors are those with the maximum significance in the network performance compared with its original performance.

2.3. Validation of the proposed method

To demonstrate the ability of the proposed method in enhancing damage detection, POD is defined as a probability that specific damage can be detected by the sensor network in the structure. We define a threshold value Φ_α for a given damage level (α). The damage feature is assumed to be normally distributed, thus the probability that the damage feature is less than the damage threshold ($\Phi \le \Phi_\alpha$) can be described as

$$P(\Phi \le \Phi_\alpha) = \int_0^{\Phi_\alpha} \frac{1}{\sigma_H \sqrt{2\pi}} e^{\frac{-(\Phi - \Phi_H)^2}{2\sigma_H^2}} \tag{5}$$

Where Φ_H and σ_H are the mean and the standard deviation for the healthy damage feature values and Φ_α is the threshold value that is a function of damage severity (α) level. Using the above approach, a damage feature threshold (Φ_α) for a specific level of damage can be computed. To validate the efficiency of the proposed optimal sensor network, the probability of detection (POD) is defined as

$$POD = \frac{N(\Phi_{mean} \ge \Phi_\alpha)}{N_{total}} \tag{6}$$

Where $N(\Phi_{mean} \ge \Phi_\alpha)$ is the number of simulations where the sensor network was capable of identifying the damage in the structure correctly (i.e. with a mean damage feature higher than the damage threshold). N_{total} is the total number of simulations performed.

3. Case study

The proposed method is applied to the Luling Bridge located over the Mississippi River in St. Charles Parish near New Orleans in Louisiana. The Luling Bridge has been in service since 1984. This bridge was selected to demonstrate the efficiency of such approach for long span cable-stayed bridges that would require large numbers of sensors. As-built drawings of the bridge were used to establish the FE model. The cable-stayed spans of the bridge, including three spans 151 m, 372 m and 155 m were modeled for the design of a SHM system. The bridge is 23 m wide. The bridge cross-section consists of two steel box girders that are 2.5 m high, 7 m and 3 m wide at top and bottom flanges, respectively. The thickness of the web is 12 mm and the thickness of flanges is 20 mm. A concrete deck (200 mm) is cast on the top of cross section to allow composite action. To hold the main 372 m span, 72 cables, attached to the top of two 122 m high towers, were installed. Each cable is a 7 (6.35 mm) wire strand cable with each wire developing an ultimate strength of 1665 MPa. Fig. 2 illustrates the structural configuration of Luling Bridge showing longitudinal and cross sections.

A FE model of the cable-stayed spans of the bridge was developed. The FE model includes 227 frame elements, each 3 m long. The frame elements were used to model each girder. Shell elements were used to model the bridge deck. Finally, cable elements were used to model the cables in the bridge. Fig. 3 shows the three-dimensional FE model of the Luling Bridge. A time history loading function with trapezoidal time-step shape that has 0.15 seconds duration was used to model the traffic loading on the bridge. The trapezoidal loading function simulates two HL-93 trucks according to the American Association of State Highway and Transportation Officials (AASHTO) (2006) moving with 35 km/h and 45 km/h in opposite directions on the bridge. The FE model was developed in SAP2000®. The

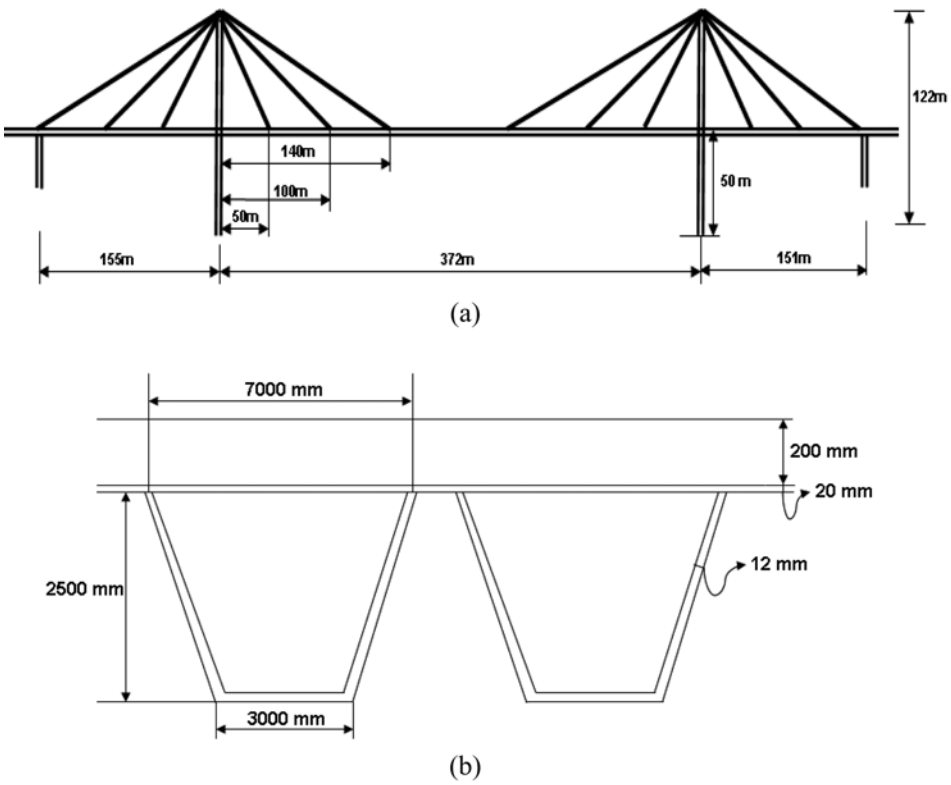

Fig. 2 Luling Bridge (a) configuration and (b) cross section

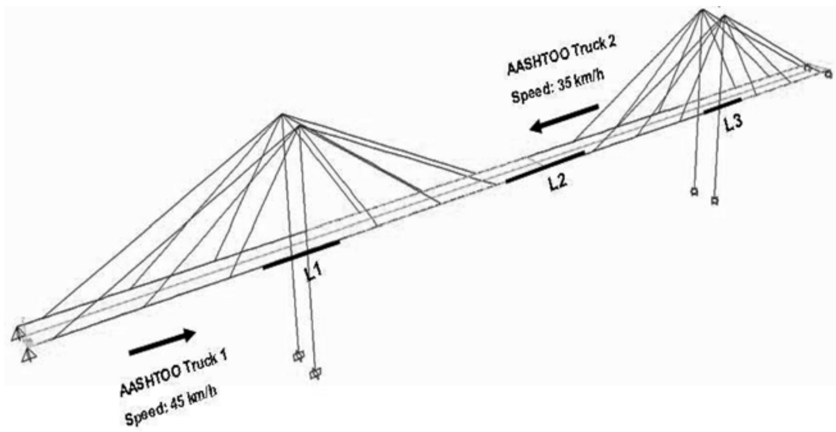

Fig. 3 FE model of the Luling Bridge with damage locations

acceleration signals at the structural nodes in z-direction were evaluated using the FE model. The energy of the acceleration signal released at the nodes (assumed to be the sensor location) was calculated. This energy has been shown by Reda Taha, *et al.* (2004) and Kumara, *et al*. (1999) to be related to the structural damage and thus can be used as a damage feature for damage detection. The energy of the acceleration signal can be calculated as

$$\Phi_i = \sum_{t=1}^{T} \varepsilon[a_{zi}^2(t)] + \varepsilon^2[a_{zi}(t)] \tag{7}$$

In Eq. (7), Φ_i is the energy of the *i*th accelerometer, $a_{zi}(t)$ is the z-axis direction acceleration measured at accelerometer i, ε is the expected value, t is time instant and T is the time window width. 5% noise was added to the FE simulation data to simulate field data. Three damage locations (L_1, L_2 and L_3) on the first Girder illustrated in Fig. 3 were considered as possible locations of damage on the bridge based on the maximum stress due to maximum bending moment. Two different levels of damage, D_1 and D_2 representing 40% and 50% loss in stiffness of the girder were considered at each damage location. Due to the size of the bridge considered in the case study, relatively large loss of stiffness of the first girder is required to enable damage detection using the sensor network. To train the ANN, the damage features Φ_i and the three associated locations for damage level D_2 were considered. The training process was repeated 10 times to obtain non-biased ANN weights. Non-normalized weights of ANN are shown in Fig. 4. The ANN weights were normalized to establish the discrete PDF. The continuous PDF was established by considering interpolation function. Fig. 5 illustrates the continuous PDF. The POD for the optimally allocated sensors is computed and compared to uniformly distributed sensors.

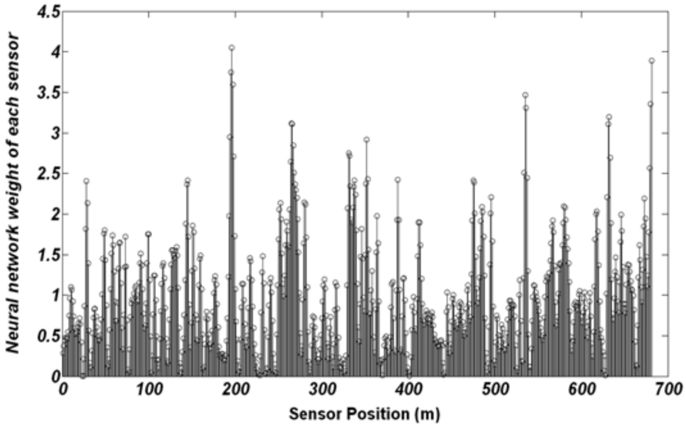

Fig. 4 Weights of each sensor obtained from ANN

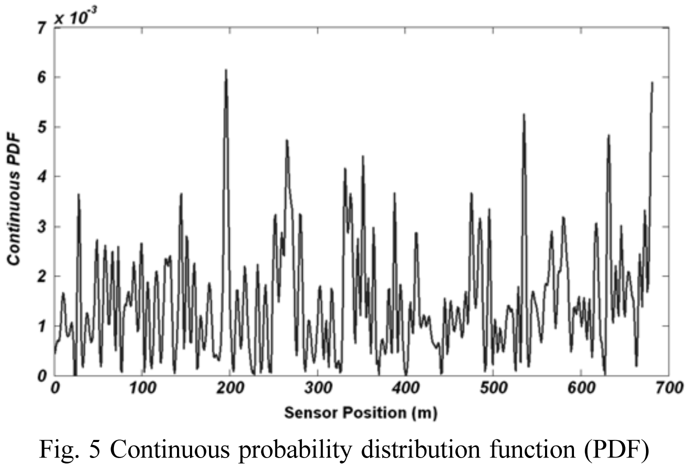

Fig. 5 Continuous probability distribution function (PDF)

4. Results and discussion

By sampling the continuous PDF shown in Fig. 4 for any given number of sensors, the optimal sensor locations on the first girder of the bridge can be identified. Table 1 presents the optimal sensor locations for 100 sensors (number arbitrarily chosen) distributed over the first girder. Fig. 6 represents the POD values for detecting severe damage for 50, 100, 150 and 200 sensors compared to uniform distribution of sensors. It is evident that the proposed method always achieved a higher POD than uniform distribution of sensors.

The two objective functions for the information entropy and the sensor network cost were evaluated. The Pareto front of the two objective functions is shown in Fig. 7. An optimal solution that satisfies both functions lies at the zone indicated in Fig. 7. The optimal solutions in the Pareto front achieve the balance between the two objective functions. It can be concluded from Fig. 7 that 85 sensors will be able to monitor the bridge efficiently while minimizing the sensor network cost and monitoring uncertainty. Sampling the continuous PDF with 85 sensors, the optimal locations of 85 sensors are presented in Table 2. We repeated the process by slightly changing the damage level from 50% to 40% and by considering a constant noise to the signal ratio. These changes did not affect the results of the optimization process. Finally, the critical sensor locations can be identified by calculating the significance

Table 1 Optimal locations of 100 sensors on the first girder of the Luling Bridge identified using the probabilistic method

Sensor Number	Sensor Location (m)									
S1-S10	3	6	9	15	27	30	33	36	45	48
S11-S20	54	60	63	75	78	81	84	87	114	117
S21-S30	123	126	132	135	138	141	144	156	159	162
S31-S40	165	168	174	177	186	195	210	213	216	225
S41-S50	246	255	264	282	285	291	294	303	324	336
S51-S60	345	351	369	375	381	405	408	411	414	420
S61-S70	423	450	453	459	465	468	474	477	492	501
S71-S80	507	510	513	516	519	522	525	528	531	537
S81-S90	540	570	573	576	579	582	585	588	594	597
S91-S100	606	609	618	621	627	630	639	645	654	666

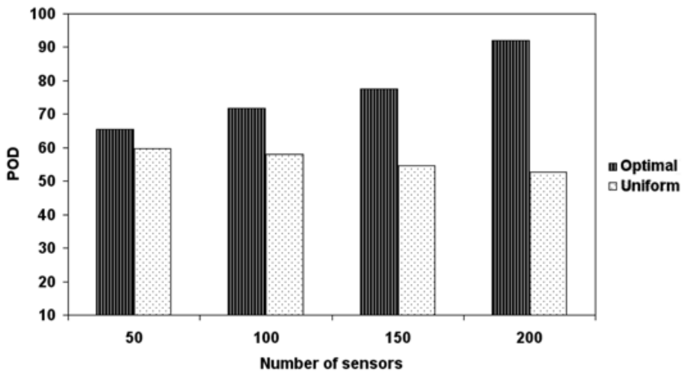

Fig. 6 Probability of detection (POD) versus number of sensors

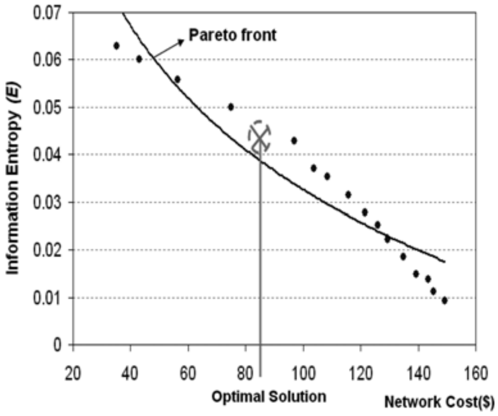

Fig. 7 Information entropy versus sensor network cost functions showing the Pareto optimal solutions in the marked area

Table 2 Optimal locations of 85 sensors on the first girder of the Luling Bridge identified using the above proposed method

Sensor Number	Sensor Location (m)									
S1-S10	6	12	21	33	36	39	42	84	87	93
S11-S20	96	99	105	108	126	129	132	135	138	153
S21-S30	165	180	183	201	204	219	222	225	231	240
S31-S40	246	270	273	276	291	297	300	318	327	333
S41-S50	336	339	342	348	354	357	363	366	369	378
S51-S60	381	399	417	423	438	447	450	471	474	480
S61-S70	483	489	492	513	516	528	537	543	546	549
S71-S80	552	555	558	564	567	588	591	594	597	624
S81-S85	636	645	654	663	669					

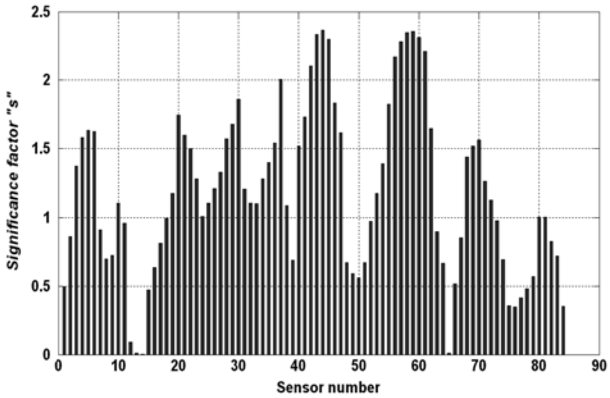

Fig. 8 Significance factor for each sensor of the sensor network showing critical sensors for sensor network robustness with high significance

factor using Eq. (4). Fig. 8 illustrates the significance factor for the 85 optimal sensors. It can be observed that sensors 43-45 and 55-80 seem to be the most critical sensors. It is also evident that sensors 11, 12, 13 and 65 seem to have minimal effect of the sensor network robustness. It is worth noting that redundant sensors shall be used in the locations of significant sensors.

5. Conclusions

An entropy-based multi-objective optimization approach was introduced to identify the optimal number of sensors for large sensor networks for SHM. The multi-objective optimization approach combines an entropy-based objective function to represent monitoring uncertainty and a sensor network cost function for cost limitations. The optimal number of sensors can be distributed on the structure using a probabilistic approach that is based on identifying location importance using knowledge on common damage locations. The proposed approach utilizes an artificial neural network which is trained based on *a priori* knowledge about damage locations and severities and selected damage features. Unlike other methods described in the literature, the proposed method does not rely on a specific damage feature and is also able to address the redundancy of the sensor networks. The proposed method was applied to find the optimal sensor network for the Luling Bridge, a long cable-stayed bridge over the Mississippi river. The optimal number of sensors and their locations are identified. The significance of sensors was also examined using 'leave one sensor out' analysis.

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