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Second-order statistics of natural frequencies of smart laminated composite plates with random material properties

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Abstract. Nowadays developments in the field of laminated composite structures with piezoelectric have attracted significant attention of researchers due to their wide range of applications in engineering such as sensors, actuators, vibration suppression, shape control, noise attenuation and precision positioning. Due to large number of parameters associated with its manufacturing and fabrication, composite structures with piezoelectric display a considerable amount of uncertainty in their material properties. The present work investigates the effect of the uncertainty on the free vibration response of piezoelectric laminated composite plate. The lamina material properties have been modeled as independent random variables for accurate prediction of the system behavior. System equations have been derived using higher order shear deformation theory. A finite element method in conjunction with Monte Carlo simulation is employed to obtain the second-order statistics of the natural frequencies. Typical results are presented for all edges simply supported piezoelectric laminated composite plates to show the influence of scattering in material properties on the second order statistics of the natural frequencies. The results have been compared with those available in literature.

Keywords: free vibration; piezoelectric composite plates; random material properties; second-order statistics.

1. Introduction

The development of new class of smart composite materials has improved the performance and reliability of structural systems. In fact, it produces an electric field when the material changes dimensions as a result of an imposed mechanical force. Conversely, an applied electric field can cause a

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piezoelectric material to change dimensions. Such materials combine the superior mechanical properties of composite materials as well as incorporate the additional inherent capability to sense and adapt their static and dynamic response. Hence, the piezoelectric materials are incorporated in composite laminated plates to add sensing capability in them.

As plates form an essential part of many civil, aerospace, marine and automobile structures. Increased use of piezoelectric laminated composite plates in primary structures necessitates the development of accurate models to predict their response. It is impossible to have complete control over the manufacturing, fabrication and processing techniques of smart composites which inevitably creates uncertainties in the lamina properties of the smart composites which in turns leads to randomness in the structural responses. Hence, the deterministic analysis cannot provide the complete information on structural response.

Considerable results exist in literature on the distribution of ultimate tensile strength of fibre reinforced composites (e.g. Tenn 1981, Fukuda, *et al*. 1981, Maekawa, *et al*. 1994). Similar results on dispersion of the system properties are limited (Maekawa, *et al*. 1988). The uncertainties in several factors at micro level like the fibre volume fraction, fibre orientation, void volume, etc have significant effects on the response of fibre-reinforced composites. The uncertainties in the factors at micro level are in turn reflected on the characteristic of the lamina stiffness parameters like elastic modulii, shear modulii, Poisson’s ratio, etc. These can be treated as primary variables, as these are basic parameters of the laminate that are usually taken for formulation of any structural analysis problem. At this stage it is relevant to point out that point to point variations of these parameters over the structures particularly in composites made of prepegs are very small. These facts have been verified experimentally (Lin and Kam 2000). These considerations indicate the need for a more accurate probabilistic approach in the analysis of these sensitive composite structures.

Over the last several years, considerable work has been carried out in a broad range of research subjects concerning the macro-mechanical behavior of smart composites and limited investigations have been reported on free vibration of piezoelectric laminated plates and shells. Tzou and Tseng (1991) studied distributed structural identification and vibration control of continua using a newly developed piezoelectric finite element. Saravanos, *et al*. (1997) developed a layer wise mechanics for the dynamic analysis of smart composite plate structures with embedded piezoelectric sensors and actuators. The mechanics have capabilities to stimulate both sensory and active dynamic response of smart composite structures either at global structural or local laminates level. Correia Franco *et al*. (2000) presented an analytical closed form solution using higher order finite element formulations to study the mechanics of adaptive composite structures with embedded and/or bonded piezoelectric actuators and sensors. Chen, *et al*. (2000) studied the active vibration control of laminated composite plates using the active piezoelectric elements using Lagrange type of finite elements based on Reddy’s simplified composite plate theory. Huang and Wu (1996) presented a first order shear deformation theory for studying the response characteristics of hybrid composite and piezoelectric plates. Zhou, *et al*. (2000) developed a completely coupled Temp-Piezoelectric-Mechanical (T-P-M) theory based on higher order displacement field and the higher order temperature field to study dynamic responses of smart composite plates. Correia Franco, *et al*. (1997) presented the structural optimization of multi-laminated composite plate structures of arbitrary geometry and lay up, using single layer higher order shear deformation theory discrete models. Although much has been done on predicting the free vibration response characteristics of composite plates with various system uncertainties (e.g. Salim, *et al*. 1993, Singh, *et al*. 2000, Naveenthraj, *et al*. 1998), no work dealing with the stochastic free vibration behavior of smart structures has been reported. All of the aforementioned works on smart composite laminated plates are virtually deterministic since they did not account for the inherent uncertainties in the structures and assumed that all of the system parameters are completely deterministic.

This paper presents a probabilistic methodology for application of the FEM in conjunction with Monte Carlo simulation to the uncertain eigen value problem of free vibration arising from random variation of laminate material properties for smart composite plates taking into account the rotary inertia effect and the transverse shear strains using higher order shear deformation theory (HSDT). The lamina material properties are treated as independent random variables (Lin and Kam 2000), while the other system parameters are taken as deterministic. A C^0 finite element method proposed for the laminated composite plate based on the HSDT model by Sankara and Iyengar (1996) and Singh, *et al*. (2002) is extended for the smart composite laminate and then combined with Monte Carlo simulation (MCS) technique which does not put any limitation like mean centered first-order perturbation technique, etc. and considered to be exact method for probabilistic analysis are employed to determine the second-order statistics, i.e., mean and standard deviation (SD) of the natural frequencies of the piezoelectric laminated composite plates. Typical numerical results present the effect of individual and simultaneous variation of material properties on scattering of the first five natural frequencies for all edges simply supported plates.

2. General formulation

2.1. Displacement field model

Fig. 1 shows the piezoelectric laminated composite rectangular plate element in Cartesian coordinate system with *x*- and *y*- axes located in the middle plane and its origin placed at the corner of the plate. The displacement field along *x*, *y* and *z* directions based on higher order shear deformation model using the condition of zero transverse shear stresses on the top and bottom of the plate is expressed as (Reddy1984):

$$\bar{u} = u + z\psi_x - z^3 4/3h^2(\psi_x + \partial w/\partial x) = u + f_1(z)\psi_x + f_2(z)\theta_x;$$

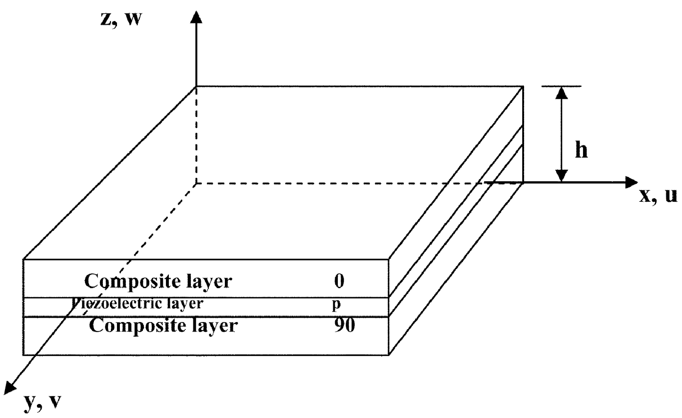

Fig. 1 Geometry of the laminated composite plate with piezoelectric layer

$$\bar{v} = v + z\psi_y - z^3 4/3h^2(\psi_y + \partial w/\partial y) = v + f_1(z)\psi_y + f_2(z)\theta_y;$$

$$\bar{w} = w; \tag{1}$$

where u, v and w denote the displacements of a point (x, y) on the mid plane, and ψ_x and ψ_y are the rotations of normal to the mid plane about the y and x axes, respectively. $\theta_x = \partial w/\partial x$, $\theta_y = \partial w/\partial y$, $f_1(z) = C_1 z - C_2 z^3$ and $f_2(z) = -C_4 z^3$ with constants, C_1=1 and C_2=C_4=4/3 h^2.

The displacement vector does not contain derivative, and hence a C^0 continuous element would be sufficient for the finite element analysis. In this study, a nine nodded isoparametric Lagrangian element has been used, leading to 63 degrees of freedom (DOFs) per element (Singh, *et al*. 2002). The displacement vector for the model is taken as $\{\Lambda\} = (u\ v\ w\ \psi_y\ \psi_x\ \theta_y\ \theta_x)^T$.

Induced electric potential is assumed to be quadratic and expressed in terms of thickness coordinate as

$$\phi(x, y, z) = \phi^{(0)}(x, y) + z\phi^{(1)}(x, y) + z^2\phi^{(2)}(x, y) \tag{2}$$

where ($\Phi^{(0)}$, $\Phi^{(1)}$, $\Phi^{(2)}$) are unknown electric potential parameters.

2.2. Strain-displacement relations

The strain-displacement relations are obtained by using small deformation theory. The strain vectors corresponding to the displacement field given by Eq. (1) are:

$$\varepsilon_{xx} = \varepsilon_1 = \partial\bar{u}/\partial x = \varepsilon_1^0 + z(k_1^0 + z^2 k_1^2);\ \ \varepsilon_{yy} = \varepsilon_2 = \partial\bar{v}/\partial x = \varepsilon_2^0 + z(k_2^0 + z^2 k_2^2)$$

$$\gamma_{xy} = \varepsilon_6 = \partial\bar{u}/\partial y + \partial\bar{v}/\partial x = \varepsilon_6^0 + z(k_6^0 + z^2 k_6^2)$$

$$\gamma_{yz} = \varepsilon_4 = \partial\bar{v}/\partial z + \partial\bar{w}/\partial y = \varepsilon_4^0 + z k_4^2;\ \ \gamma_{xz} = \varepsilon_5 = \partial\bar{u}/\partial z + \partial\bar{w}/\partial x = \varepsilon_5^0 + z^2 k_5^2 \tag{3}$$

where,

$$\varepsilon_1^0 = \partial u/\partial x,\ k_1^0 = \partial\psi_x/\partial x,\ k_1^2 = -4/3h^2(\partial\psi_x/\partial x + \partial\theta_x/\partial x)$$

$$\varepsilon_2^0 = \partial u/\partial y,\ k_2^0 = \partial\psi_y/\partial y,\ k_2^2 = -4/3h^2(\partial\psi_y/\partial y + \partial\theta_y/\partial x)$$

$$\varepsilon_6^0 = \partial u/\partial y + \partial v/\partial x,\ k_6^0 = \partial\psi_x/\partial y + \partial\psi_y/\partial x$$

$$k_6^2 = -4/3h^2(\partial\psi_x/\partial y + \partial\psi_y/\partial x + 2\partial\theta/\partial y)$$

$$\varepsilon_4^0 = \psi_y + \theta_y,\ k_4^2 = -4/h^2(\psi_y + \theta_y)$$

$$\varepsilon_5^0 = \psi_x + \theta_x,\ k_5^2 = -4/h^2(\psi_x + \theta_x) \tag{4}$$

The potential vectors in terms of the induced electric potential field are given as

$$E_x = -\frac{\partial\phi}{\partial x};\; E_y = -\frac{\partial\phi}{\partial x};\; E_z = -\frac{\partial\phi}{\partial z} \tag{5}$$

Using Eq. (2), Eq. (5) can further be written as

$$E_x = -\frac{\partial\phi}{\partial x} = -\left(\frac{\partial\phi^{(0)}}{\partial x} + z\frac{\partial\phi^{(1)}}{\partial x} + z^2\frac{\partial\phi^{(2)}}{\partial x}\right);\; E_y = -\frac{\partial\phi}{\partial y} = -\left(\frac{\partial\phi^{(0)}}{\partial y} + z\frac{\partial\phi^{(1)}}{\partial y} + z^2\frac{\partial\phi^{(2)}}{\partial y}\right)$$

$$E_z = -\frac{\partial\phi}{\partial z} = -(\phi^{(1)} + 2z\phi^{(2)}) \tag{6}$$

Eq. (6) can be expressed in matrix form as

$$\{E\} = [T_\phi][L_\phi]\{\phi\} \tag{7}$$

where $[T_\phi]$ is a matrix which is function of thickness coordinate, $[L_\phi]$ is a differential operator and $\{\phi\} = (\phi^{(0)}\ \phi^{(1)}\ \phi^{(2)})^T$.

2.3. Constitutive relations

General stress-strain relations that couples the deformation and electric fields in the smart composite plate are given as (Tiersten 1969)

$$\{\sigma\} = [\bar{Q}]\{\varepsilon\} - [e]\{E\}$$

$$\{D\} = [e]^T\{\varepsilon\} + [k]\{E\} \tag{8}$$

where $[\bar{Q}]$ are the reduced stiffness matrix (elastic modules), $\{e\}$ are the piezoelectric constants, $\{k\}$are the dielectric constants and $\{\varepsilon\}$ is the strain vector.

2.4. Potential energy of the smart laminate

The potential energy of a piezoelectric laminated composite plate undergoing small deformation is given as: Potential energy (U) = Strain energy – Electric energy

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV - \frac{1}{2}\int_V \{E\}^T\{D\}dV \tag{9}$$

Using Eqs. (3), (4), (7) and (8), the Eq. (9) can be expressed in terms of the displacement vector and the electric potential vector.

2.5. Kinetic energy of plate

The kinetic energy of a vibrating plate in bending is given as

$$T = \frac{1}{2}\int_A \{\dot{\Lambda}\}^T[m]\{\dot{\Lambda}\}dA \tag{10}$$

where, $[m]$ is the inertia matrix and $\{\Lambda\}$ is the velocity vector.

3. Methods of solutions

3.1. Finite element formulation

3.1.1. Potential energy

The displacement vector and electric potential vector can be expressed as

$$\{\Lambda\}^{(e)} = \sum_{i=1}^{NN} [N_i]\{\Lambda_i\}; \{\phi\}^{(e)} = \sum_{i=1}^{NN} [N_{\phi i}]\{\phi_i\} \tag{11}$$

where N_i and $N_{\phi i}$ represent shape functions at i^{th} node of the e^{th} element. $\{\Lambda_i\}$ and $\{\phi_i\}$ are the displacement and electric potential vectors of i^{th} node.

Using Eq. (11) , Eq. (9) can be expressed as

$$U^{(e)} = \frac{1}{2}\{q\}^{(e)T}[K]^{(e)}\{q\}^{(e)} - \frac{1}{2}\{q\}^{(e)T}[K_1]^{(e)}\{q_\phi\}^{(e)} - \frac{1}{2}\{q_\phi\}^{(e)T}[K_1]^{(e)}\{q\}^{(e)} - \frac{1}{2}\{q_\phi\}^{(e)T}[K_2]^{(e)}\{q_\phi\}^{(e)} \tag{12}$$

where, $\{q\}^{(e)} = \sum_{i=1}^{NN} \{\Lambda_i\}$, the element bending stiffness matrix, the coupling matrix, $[K_1]^{(e)} = \int_{A^{(e)}} [B]^{(e)T}[D_1][B]^{(e)} dA$ and the dielectric stiffness matrix, $[K_2] = \int_{A^{(e)}} [B]^{(e)T}[D_3][B_\phi]^{(e)} dA$. Here $[D_1]$, $[D_2]$ and $[D_3]$ are the laminate stiffness matrices. $[B]^{(e)} = [L][N]^{(e)}$ and $[B_\phi]^{(e)} = [L_\phi][N_\phi]^{(e)}$.

3.1.2. Kinetic energy

In terms of the elemental values, the kinetic energy (T) is written as

$$T = \sum_{e=1}^{NE} T^{(e)} \tag{13}$$

where, NE is number of elements used for messing the plate and the elemental kinetic energy (Eq. 10) is expressed as

$$T^{(e)} = \frac{1}{2}\int_{A^{(e)}} \{\dot{\Lambda}\}^{(e)T}[m]\{\dot{\Lambda}\}^{(e)} dA \tag{14}$$

Using Eq. (11), the elemental kinetic energy (Eq. 14) can also be expressed as

$$T^{(e)} = \frac{1}{2}\{\dot{q}\}^{(e)T}[M]^{(e)}\{\dot{q}\}^{(e)} \tag{15}$$

where, $[M]^{(e)}$ is the mass matrix of the e^{th} element.

3.1.3. Governing equation

The governing equation for free vibration of piezoelectric laminated composite plate can be derived using Variational principle, which is generalization of the principle of virtual displacement. Lagrange equation for a conservative system can be written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial U}{\partial q_i} = 0 \qquad \text{for } i = 1,2,\ldots\ldots; \tag{16}$$

where, q_i are the generalized coordinates and $\dot{q}_i$ represents the generalized velocities.

Using Eqs. (12) and (15) , the Eq. (16) yields

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \{\ddot{q}_\phi\} \end{Bmatrix} + \begin{bmatrix} [K] & [K_1] \\ [K_1^T] & [K_2] \end{bmatrix} \begin{Bmatrix} \{q\} \\ \{q_\phi\} \end{Bmatrix} = 0 \tag{17}$$

where, $[M] = \sum_{e=1}^{NE} [M]^{(e)}$ is the global mass matrix, $[K] = \sum_{e=1}^{NE} [M]^{(e)}$ is the global elastic stiffness matrix, $[K_1] = \sum_{e=1} [K_1]^{(e)}$ and $[K_1]^T$ are the coupling matrices between elastic mechanical and electrical effects and $[K_2] = \sum_{e=1}^{NE} [M_2]^{(e)}$ is the dielectric stiffness matrix.

Assuming the system vibrates in principal mode with respect to time with natural frequency, ω and eliminating $\{q_\phi\}$ from Eq. (17), one obtains the generalized eigen-value problem,

$$[K^*]\{q\} = \lambda[M]\{q\} \tag{18}$$

where, $[K^*] = [K] - [K_1][K_2^{-1}][K_1^T]$ and $\lambda = \omega^2$.

The stiffness matrix $[K^*]$ is random in nature, being dependent on the material properties. Consequently, the initial free responses are also rendered random. A computer code in MATLAB 6.1 was developed to solve equations and obtain the second order statistics of natural frequencies.

3.2. Monte Carlo simulation

Numerical methods that are known as Monte Carlo simulation (MCS) can be loosely described as statistical simulation methods, where statistical simulation is defined in quite general terms to be any method that utilizes sequences of random numbers to perform the simulation. Monte Carlo methods have been used for centuries, but only in the past several decades has the technique gained the status of a full-fledged numerical method capable of addressing the most complex applications. The technique is considered to be exact method for random analysis and also it does not put any limitation regarding degree of the uncertainty in properties. However, this technique is computationally expensive. In MCS approach, the samples for the random parameters are obtained by generating a set of random numbers of given sample size to fit the desired mean and standard deviation (SD). For the present work, MATLAB inbuilt command is used for generating random numbers corresponding to mean values of the material property to be varied assuming normal distribution.

The formulae for the mean and SD of property x to be varied are:

Mean, $$\mu = \frac{\sum_{i=1}^{n} x_i}{n} \tag{19}$$

SD, $$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}} \tag{20}$$

where, $i = 1, 2, 3, \ldots, n$.

4. Results and discussion

The method outlined has been used to obtain the second order statistics of the first five natural frequencies of smart laminated composite plates with random material properties. All the lamina are assumed to be of same thickness and made up of the same material. Though the approach does not put any restrictions, the assumption reduces the size of the problem in many folds in calculation efforts. The results have been compared with those available in the literature. A nine noded isoparametric element, with 63 degrees of freedom (DOFs) per element for the HSDT model has been used for discretizing the laminated plate. These elements are found to be quite stable. The results have been computed by Gauss-quadrature with employing the full (3×3) integration rule for thick panels and reduced (2×2) integration rule for the thin plate. On the basis of convergence study, a (5×5) mesh has been used throughout the study. The details of the convergence study are presented in next sub-section. The following nondimensionalized mean natural frequency has been used in this study.

$$\varpi = (\omega a^2 \sqrt{\rho / Ett}) / h$$

Typical results for the first five nondimensionalized natural frequencies of laminated composite square plates with and without piezoelectric layers with various stacking sequences with all edges simply supported are presented for the ratio of the SD to mean of material properties varying from 0 percent to 20 percent (Liu, *et al*. 1986) for a/h=10 & a/h=100. The lamina material properties have been taken as random variable (Lin and Kam 2000). These random variables (RVs) are sequenced as

$$d_1 = E_{ll},\ d_2 = E_{tt},\ d_3 = G_{lt},\ d_4 = G_{lz},\ d_5 = G_{tz} \text{ and } d_6 = \nu_{lt}$$

Convergence study for number of random values to be generated for variation in material properties has been also done. Based on this convergence study, throughout this work 8100 random values have been generated for variation in each material property. The details of the random number convergence study are also presented in next sub-section.

The performance of the Matlab code has also been examined in the present study. It is observed that the performance of Matlab when more refined meshes are used for obtaining the solution of the problem is not good. It takes more CPU time. In the present work, the Monte Carlo simulation has also been used as a probabilistic method to handle randomness in the material properties and 8100 random numbers generated as explained in the previous paragraph for input random variables have been used to obtain the second order statistics of the response. The Matlab become further very slow in case of this. In our opinion, it is not advisable to use MatLab especially for random analysis.

The following materials used for present investigations are Graphite/Epoxy composite material and Lead Zirconate Titanate (PZT-4) piezoelectric material (Singh, *et al*. 2001, Saravanos, *et al*. 1997).

Material-1: $E_{ll} = 25\, E_{tt}$, $G_{lt} = G_{lz} = 0.5\, E_{tt}$, $G_{tz} = 0.2\, E_{tt}$, $E_{tt} = 10.3$ GPa, $\nu_{lt} = 0.25$.

Material-2:

Elastic Constants: $E_{ll} = 132.38$ GPa, $E_{tt} = 10.76$ GPa, $G_{lt} = G_{lz} = 5.65$ GPa, $G_{tz} = 3.61$ GPa, $\nu_{lt} = 0.25$.

Piezoelectric Coefficients (10^{-12} m/V):

$$e_{31} = e_{32} = 0.0,\ e_{33} = 0.0,\ e_{24} = 0.0$$

Electric permittivity:

($\varepsilon_0 = 8.85 \times 10^{-12}$ Farad/m)

$$d_{11}/\varepsilon_0 = 3.5,\ d_{22}/\varepsilon_0 = d_{33}/\varepsilon_0 = 3.0,$$

PZT-4:

Elastic Constants: $E_{ll} = E_{tt} = 81.3$ GPa, $G_{lt} = 30.6$ GPa, $G_{lz} = G_{tz} = 25.6$ GPa, $\nu_{lt} = 0.33$.

Piezoelectric Coefficients (10^{-12} m/V):

$$e_{31} = e_{32} = -122.0,\ e_{33} = -285,\ e_{24} = 0.0$$

Electric permittivity:

($\varepsilon_0 = 8.85 \times 10^{-12}$ Farad/m)

$$k_{11}/\varepsilon_0 = k_{22}/\varepsilon_0 = 1475.0,\ k_{33}/\varepsilon_0 = 1300.0.$$

The boundary conditions for all edges simply supported for the present investigation are given as: $u = w = \theta_x = \psi_x = 0$, for $y = 0, b$; $v = w = \theta_y = \psi_y = 0$, for $x = 0$, a.

4.1. Convergence study

Table 1 shows the non-dimensional fundamental frequency of the plate with various mesh densities for the laminated composite plate with piezoelectric layers for Material-1 and PZT-4. It is observed that there is very small change in non-dimensional fundamental frequency ϖ for increasing mesh from (5×5) to (6×6), hence it can be concluded that for mesh (5×5) the value of non-dimensional fundamental frequency ϖ becomes stable.

The ratio of SD (σ) to Mean (μ) of the natural frequency square (ω^2) have been obtained for Material-1 and PZT-4 by allowing composite material properties E_{ll}, E_{tt}, G_{lt}, G_{lz}, G_{tz} and n_{lt} varying simultaneously.

Table 1 Convergence study for nondimensionalized fundamental frequency $\varpi = (\omega a^2\sqrt{\rho/Ett})/h$ for a [0/p/90] laminated composite plate with all edges simply supported (material-1, PZT-4, b/a=1)

Mesh	a/h = 10	a/h = 100
1x1	12.1688	12.8147
2x2	11.4886	12.7840
4x4	11.0210	11.7753
5x5	10.9956	11.6599
6x6	10.9861	11.6418

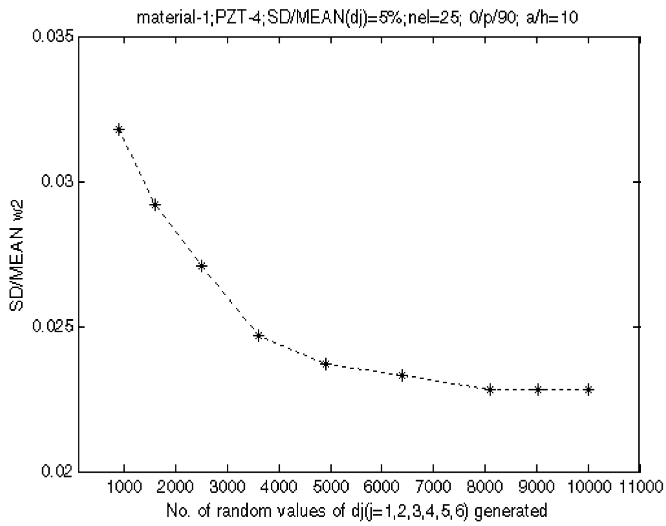

Fig. 2 Convergence study for random numbers to be generated for random variables with material properties varying simultaneously (dj)

The convergence for random numbers to be generated for various elastic composite material properties has been shown in Fig. 2. It is observed that the curve of the ratio of SD (σ) to Mean (μ) of the natural frequency square (ω^2) versus number of random values of dj becomes nearly straight line at value 8100 in x-axis. So, plot shows convergence for 8100 random values to be generated for elastic material properties of composite plate.

4.2. Validation

Table 2 shows a comparison of normalized mean fundamental frequency for piezoelectric laminated composite square plate obtained by outlined approach with that obtained by Saravanos et al. (1997) and with that exact piezoelectricity results obtained by Heyliger ad Saravanos (1995). All layers have been assumed to have equal density, ρ = 1 kg/m^3.The laminate configuration consists of a [0/90/0] Gr/Epoxy cross ply laminate (Material-2) with composite plies each having 0.267h thick, where h is plate thickness. Two continuous PZT-4 layers of thickness 0.1h each are also bonded to upper and lower surfaces of the laminate. The results are in good agreement.

4.3. Numerical results: Second order statistics

The SD/mean of the natural frequencies has an almost close to linear variation with the change in the

Table 2 Validation study for nondimensionalized fundamental frequency, $\omega a^2/hr^{1/2}\times 10^3$ Hz (Kg/m)$^{1/2}$ of a [p/0/90/0/p] laminated composite plate with all edges simply supported (material-2, PZT-4, b/a=1)

a/h = 4			a/h = 50		
Saravanos, *et al*. (1997)	Heyliger and Saravanos (1995): [Exact]	Present	Saravanos, *et al*. (1997)	Heyliger and Saravanos (1995): [Exact]	Present
145.323	145.339	146.861	236.833	245.941	243.192

Table 3 Nondimensionalized mean natural frequencies, $\varpi = (\omega a^2 \sqrt{\rho / Ett})/h$ for smart laminated composite plates with all edges simply supported (Material -1, *b/a*=1)

Mode	[0/90/p/90/0]		[0/p/90]	
	a/h = 10	*a/h* = 100	*a/h* = 10	*a/h* = 100
1	13.7854	15.2992	10.9956	11.6599
2	28.6106	45.0626	23.4267	28.4415
3	28.6160	45.0762	26.8840	33.1332
4	33.0242	63.2933	36.5545	46.7890
5	33.2180	104.1973	38.7663	65.6864

material properties. The rate of scatter depends on the thickness ratio, edge condition and the material properties being considered. The behavior also shows sensitivity to the lay up sequencing.

Table 3 presents the nondimensionalized mean natural frequencies of a [0/90/p/90/0] and a [0/p/90] square laminates with all edges simply supported for *a/h* = 10 & 100, material-1 and PZT-4. It is observed that with increase in thickness ratio the mean natural frequencies of free vibration increases. It is also observed that for *a/h* = 100, the first five natural frequencies for [0/90/p/90/0] laminate are more as compared to [0/p/90] laminate. However, in case of *a/h* = 10, only the first three natural frequencies are higher.

Figs. 3(a) and (b) show a comparison between scatterings of fundamental frequency square (ω^2) for a [0/p/90] and a [0/90] square laminated composite plates, i.e., with and without piezoelectric layers, with SD of all the material properties varying simultaneously allowing the ratio of SD (σ) to Mean (μ) to vary from 0 percent to 20 percent for material -1 with *a/h* = 10 and *a/h* = 100, respectively. The nondimensionalized mean values computed for a [0/90] laminate are 9.7734 and 11.0253 for *a/h* = 10 and *a/h* = 100, respectively. And the nondimensionalized mean values for a [0/p/90] laminate are given in Table 1. It is observed that the laminated composite plate with piezoelectric layers shows less

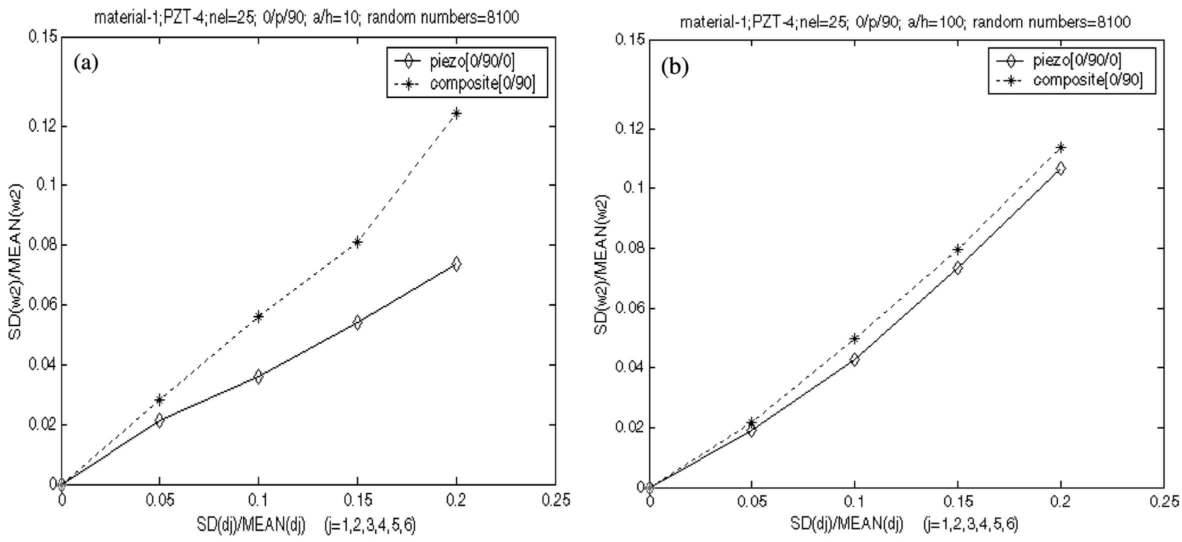

Fig. 3 Influence of SD of basic material properties on the SD of the fundamental frequency with all basic material properties changing at a time

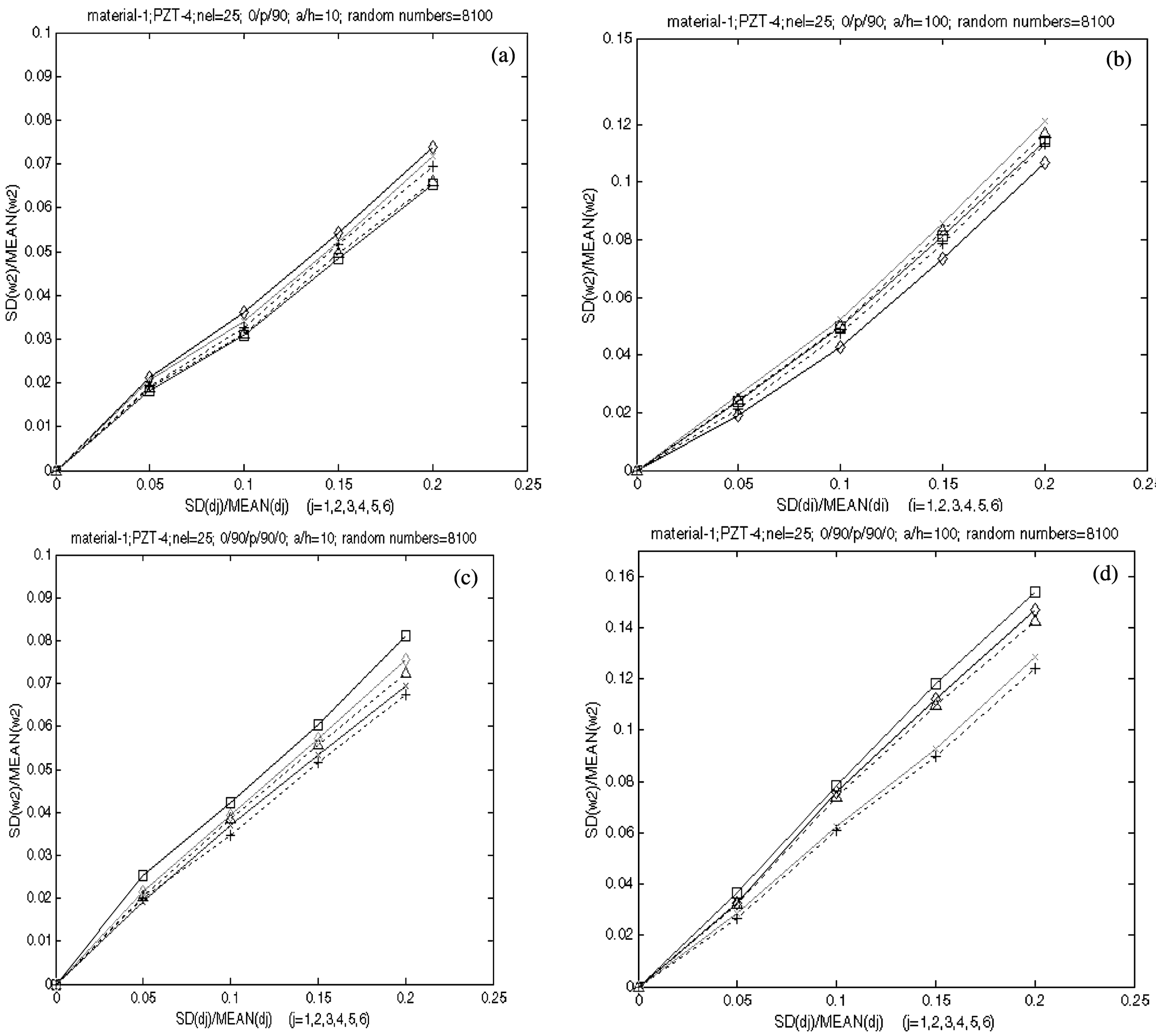

Fig. 4 The effect of varying all material properties simultaneously on $\sigma\omega^2/\mu\omega^2$ of the first five mean frequencies of vibration. (a) *a/h* = 10 and [0/p/90], (b) *a/h* = 100 and [0/p/90], (c) *a/h* = 10 and [0/90/p/90/0], (d) *a/h* = 100 and [0/90/p/90/0]. Key: ◊ : first mode, +: second mode, □: third mode, X: fourth mode and Δ: fifth mode

scattering as compared to that of laminated composite plate without piezoelectric layers. It also shows that for *a/h* = 100, the fundamental frequency shows more scattering than for *a/h* = 10.

From application point of view, it is appropriate to consider the case where all the properties vary simultaneously. Figs. 4(a) & (b) present normalized SD of the normalized natural frequencies with SD of all the material properties varying simultaneously each assuming the same value for the ratio of its to mean for SD for a [0/p/90] square laminate for material-1 with *a/h* = 10 and *a/h* = 100, respectively. Figs. 4(c) & (d) represent corresponding behavior for [0/90/p/90/0] square laminate. It is observed that the fundamental frequency of vibration is more sensitive to *a/h* = 10 as compared to *a/h* = 100. It is also observed that fundamental frequency of vibration is more sensitive for stacking sequence [0/90/p/90/0]

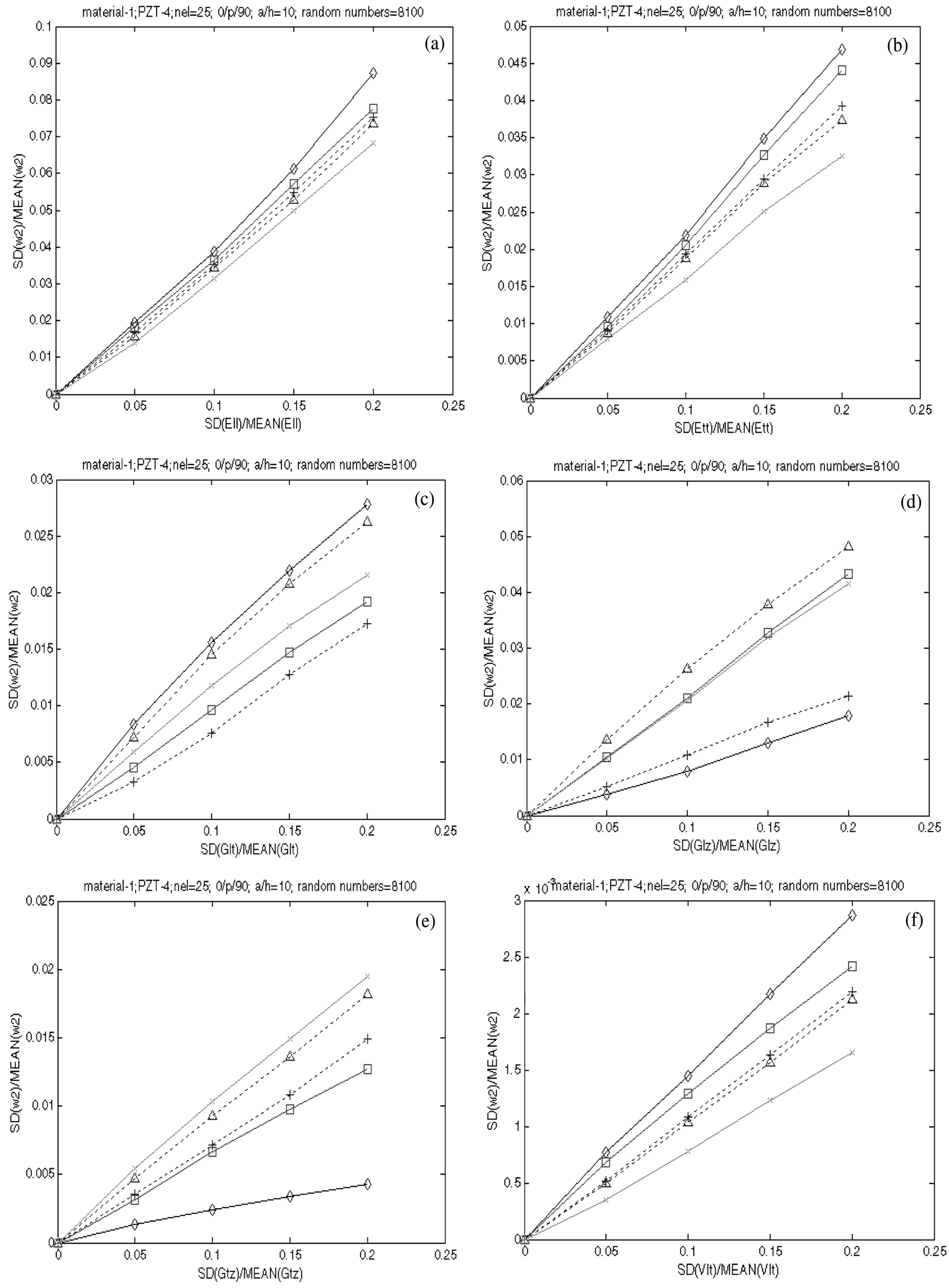

Fig. 5 Influence of the SD of basic material properties on the SD of the first five natural frequencies, for [0/p/90] laminate. (a) only E_{ll} varying, (b) only E_{tt} varying, (c) only G_{lt} varying, (d) only G_{lz} varying, (e) only G_{tz} varying, (f) only ν_{lt} varying, Key: Same as in Fig. 4.

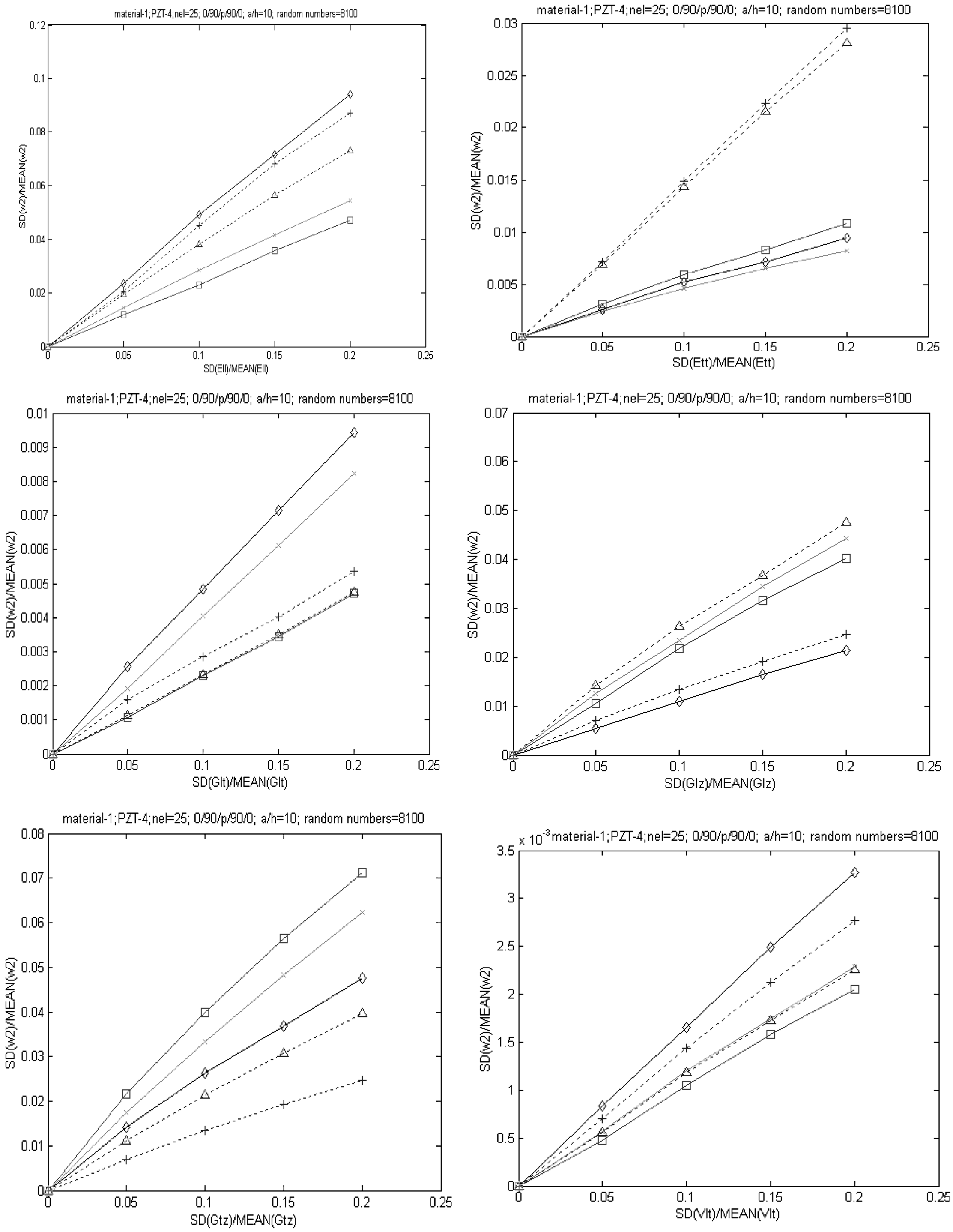

Fig. 6 Influence of SD of basic material properties on SD of the first five natural frequencies, for [0/90/p/90/0] laminate (a) only E_{ll} varying, (b) only E_{tt} varying, (c) only G_{lt} varying, (d) only G_{lz} varying, (e) only G_{tz} varying, (f) only ν_{lt} varying, Key: Same as in Fig. 4.

laminate as compared to [0/p/90] laminate.

Fig. 5(a) to 5(f) show influence of normalized SD of basic material properties on normalized SD of the first five natural frequencies for material-1, a/h=10 and stacking sequence [0/p/90] for individual variation of E_{ll}, E_{tt}, G_{lt}, G_{lz}, G_{tz} and ν_{lt}, respectively. It is observed that the individual variation of E_{ll}, E_{tt}, G_{lt}, and ν_{lt} has dominant effect on the scattering of the fundamental frequency as compared to any other frequencies. It is also observed that G_{lz} and G_{tz} have dominant effect on the fifth and second natural frequencies, respectively.

Fig. 6(a) to 6(f) shows influence of SD of basic material properties on SD of the first five natural frequencies for material-1, a/h=10 and stacking sequence [0/90p/90/0] for individual variation of E_{ll}, E_{tt}, G_{lt}, G_{lz}, G_{tz} and ν_{lt} respectively. It is observed that fundamental frequency of vibration is more sensitive to change in E_{ll}, G_{lt}, and ν_{lt}. For variation in E_{tt} and G_{lz} the frequency for fifth mode of vibration is most sensitive. And the frequency for third mode of vibration is most sensitive to variation in G_{tz}. Also, the scattering in natural frequency is more for stacking sequence [0/p/90] than stacking sequence [0/90/p/90/0].

5. Conclusions

The second order statistics of free vibration response of piezoelectric laminated composite plate using FEM in conjunction with MCS has been obtained for different thickness ratio (*a/h*), material and stacking sequences of cross ply laminates. The following conclusions are noted from the results for PZT-4 and graphite/epoxy laminated plates with all edges simply supported:

(i) The laminated composite plates with piezoelectric layers shows less scattering in frequency for variation in material properties as compared to plates without piezoelectric layers.

(ii) The influence of SD of natural frequencies shows different sensitivity to different material properties. The sensitivity changes with the mode of vibration, the stacking sequence, the *a/h* ratio and the material.

(iii) The dispersion in natural frequency is the most affected with scatter in longitudinal elastic modulus and the least affected with scatter in Poisson's ratio.

Reference

Chen, X. L., Hua, H. X. and Liu, Y. J. (2000), "A higher-order FEM for vibration control of composite plates with distributed piezoelectric sensors and actuators", *Proceedings of* ICES 2000 *Advances in Computational Engineering & Sciences*, Los Angeles, July.

Correia Franco, V. M., Aguiar Gomes, M. A., Suleman, A, Mota Soares, C. M. and Mota Soares, C. A. (2000), "Modelling and design of adaptive composite structures", *Comput. Meth. in Appl. Mech. and Eng.*, **185**, 325-346.

Correia Franco, V. M., Mota Soares, C. M. and Mota Soares C. A. (1997), "Higher order models on the eigen frequency analysis and optimal design of laminated composite structures", *Compos. Struc.*, **39**(3-4), 237-253.

Fukuda, H., Chou, T. W., and Wata, K. K. (1981), "Probabilistic approach on the strength of fibrous composites, composite materials", *Proceedings Japan-US Conference*, Tokyo, 181-193.

Heyliger, P. R. and Saravanos, D. A. (1995), "Exact free vibration analysis of laminated plate with embedded piezoelectric layers", *J. Acous. Soc. Am.*, **98**, 1547-57.

Huang, J. H. and Tseng, L. W. (1996), "Analysis of hybrid multilayered piezoelectric plates", *Int. J. Engr. Sc.*, **34**(2), 171-181.

Lin, S. C. and Kam, T. Y. (2000), "Probability failure analysis of transversely loaded composite plates using higher second order moment method", *J. Eng. Mech.*, **126**(8), 812-20.

Liu, W. K., Belytschko, T. and Mani, A. (1986), Random field finite elements", *Int. J. Num. Meth. I Eng*., **23**, 1831-45.

Maekawa, Z., Hamada, H., Yokoyama, A., Ishibashi, S. and Tanimoto, T. (1988), "Reliability evaluation of unidirectional CFRP", *Proceedings of the 4th US-Japan Conference on Composite Materials*, Washington D.C., 1025-1034.

Maekawa, Z., Hamada, H., Lee, K. and Kitagawa, T. (1994), "Reliability evaluation of mechanical properties of AS4/PEEK composites", *Composites*, **25**(1), 37-45.

Naveenthraj, B. N., Iyengar, N. G. R. and Yadav, D. (1998), "Response of composite plates with random material properties using FEM and Monte Carlo simulation", *Adv. Compos. Mat*., **7**(3), 219-237.

Reddy, J. N. (1984), "A simple higher-order theory for laminated composite plates", *J. Appl. Mech*., **51**, 745-752.

Salim, S., Yadav, D. and Iyengar, N. G. R. (1993), "Analysis of composite plates with random material characteristics", *Mech. Res. Commun*., **20**(5), 405-414.

Shankara, C. A. and. Iyengar, N. G. R. (1996), "A CO element for the free vibration analysis of laminated composite plates", *J. Sound Vib*., **191**(5), 721-738.

Saravanos, D. A., Heyliger, P. R. and Hopkins, D. A. (1997), "Layer wise mechanics and finite element for the dynamic analysis of piezoelectric composite plates", *Int. J. Solids and Struct*., **34**(3), 359-378.

Singh, B. N., Yadav, D. and Iyengar N. G. R. (2001), "Natural frequencies of composite plates with random material properties using higher-order shear deformation theory", *Int. J. Mech. Sci.*, **43**, 2193-2214.

Sngh, B. N., Iyengar, N. G. R. and Yadav, D. (2002), "A C^0 finite element investigation for buckling of shear deformable laminated composite plates with random material properties", *Struc. Eng. & Mech., An Int. J*., **13**, 53-74.

Tenn, L. F. (1981), "Statistical analysis of fibrous composite strength data, test methods and design allowable for fibrous composites", *ASTM STP 734*, *American Society for Testing and Materials*, pp. 229-244.

Tiersten, H. F. (1969), *Linear Piezoelectric Plate Vibrations*, Plenum Press, New York.

Tzou, H. S. and Tseng C. I. (1991), "Distributed modal identification and vibration control of continua: piezoelectric finite element formulation and analysis", *Trans. of the ASME Appl. Mech*., **113**, 500-505.

Zhou, X. and Chattopadhyay, H. G. (2000), "Dynamic responses of smart composites using a coupled thermo-piezoelectric mechanical model", *AIAA J*., **38**(10), 1939-1949.

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