

Numerical buckling temperature prediction of graded sandwich panel using higher order shear deformation theory under variable temperature loading

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Abstract. The finite element solutions of thermal buckling load values of the graded sandwich curved shell structure are reported in this research using a higher-order kinematic model including the shear deformation effect. The numerical buckling temperature has been computed using an in-house specialized code (MATLAB environment) prepared in the framework of the current mathematical formulation. In addition, the mathematical model includes the excess structural distortion under the influence of elevated environment via Green-Lagrange nonlinear strain. The corresponding eigenvalue equation has been solved to predict the critical buckling temperature of the graded sandwich structure. The numerical stability and the accuracy of the current solution have been confirmed by comparing with the available published results. Thereafter, the model is extended to bring out the influences of structural parameters i.e. the curvature ratio, core-face thickness ratio, support conditions, power-law indices and sandwich types on the thermal buckling behavior of graded sandwich curved shell panels.

Keywords: FGM sandwich curved panels; HSDT; thermal buckling; FEM; MATLAB

1. Introduction

Sandwich structures are well known for their good mechanical properties (specific strength and stiffness), and temperature resistance capability as per the implemented material. In general, the sandwich arrangement associated with three different layers say the middle layer (core layer) is covered by top and bottom face sheets. The Functionally Graded Material (FGM) structures are the typical type of composite material developed with continuous grading of multiple materials i.e., the metal and ceramic constituent prepared from the different phases. The variation of material properties like Young's modulus and density along the thickness direction are mainly governed through the power law method. The closest similarity of Functionally Graded (FG) with the laminated composite, but possess a unique interface properties across the interlayer. The FGM structure or structural components have the diversified engineering industrial applications such as automobile, space, power and marine, ship construction because of the specific mechanical properties as per the application.

Further, change of the stresses at the interface of structure may happens due to the mechanical property variations at the boundaries of different material phases i.e., the ceramic and the metal which are utilized to make a regular sandwich arrangement. To address this issue, FGMs are one of the suitable candidates. This is because FGM permits the variation of properties through the thickness of structural panel. Further, the FGM sandwich cores (Aragh and Yas 2011, Alibeigloo and Liew 2014, Liu *et al.* 2015) and the face sheets (Shen and Li 2008, Wang and Shen 2013, Yaghoobi and Yaghoobi 2013) are modelled mathematically to compute the relevant structural responses to establish their applicability. Various works utilizing the FSDT (Bousahla *et al.* 2020, Draiche *et al.* 2019, Semmah *et al.* 2019) refined plate theories (two and four variables) (Balubaid *et al.* 2019, Tounsi *et al.* 2020, Abualnour *et al.* 2019, Belbachir *et al.* 2019) simple as well as nth-order shear deformation theory (Boussoula *et al.* 2020, Hellal and Bourada 2019), quasi-3D theories (Addou *et al.* 2019, Boukhelif *et al.* 2019, Boutaleb *et al.* 2019, Kaddari *et al.* 2020, Khiloun *et al.* 2019, Mahmoudi *et al.* 2017, Zaoui *et al.* 2018, Zarga *et al.* 2019) for the analytical (Meksi *et al.* 2017) and finite-element (Alimirzaei *et al.* 2019) solution of the buckling and vibration responses of ply laminated composite and soft core sandwich plates (Sahla *et al.* 2019), FG beams (Bourada *et al.* 2019), micro-beams (Tlidji *et al.* 2019), nano-beams (Berghouti *et al.* 2019), plates/beams on elastic foundations (Chaabane *et al.* 2019, Boulefrakh *et al.* 2019), nano-plates (Karami *et al.* 2019a) and nano-shells (Karami *et al.* 2019b, 2020) under thermo-mechanical and hygro-thermo-mechanical loading have been presented.

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Li *et al.* (2008) evaluated the natural frequencies using Ritz method and compared the final outcomes obtained using different two-dimensional plate theories of the FGM sandwich plate structure. Zenkour (2005a, b) analyzed the bending, buckling and the natural frequency parameters of the FG-sandwich plate component using the lower-order kinematic models. Additionally, the bending deflection values of the FGM viscoelastic sandwich beam including relation between the beam and its base reported by Zenkour *et al.* (2010). Recently, the bending, buckling and the natural frequencies of FG-porous micro-rectangular plates are obtained by Kim *et al.* (2019) via the lower-order kinematic models (Classical Laminated Plate Theory, CLPT; First-Order Shear Deformation Theory, FSDT). Thai *et al.* (2014) reported the analytical solutions of the bending, buckling and the free vibration frequencies of the graded sandwich structure. The structural motion equations are obtained via Hamilton's principle and solved for different end constraint conditions. Ghannadpour *et al.* (2012) implemented a finite strip method for the evaluation of buckling responses of the rectangular FG-plate under thermal environment. Sobhy (2013) implemented a new function for the mid-plane kinematics which satisfies different kinds of boundary conditions of exponentially graded sandwich structural plate. The optimal buckling temperature parameters of the laminated structural components are predicted numerically by Topal (2012, 2013) for different line supports considering the FSDT kinematic model. Wu and Liu (2014) carried out three-dimensional linear buckling analysis of simply supported FGM sandwich plate under biaxial compressive load. Meziane *et al.* (2014) studied the free vibration frequency and the buckling responses of Exponential FGM (E-FGM) sandwich plate under the different boundary conditions. They implemented variable types of kinematic models i.e., the Simple Deformation Theory (SDT) and compared with available lower-order (FSDT), as well as the higher-order shear deformation theory (HSDT) models. Vo *et al.* (2015) investigated vibration and buckling responses of FGM beam under different boundary condition by utilizing FEM model. Al-Basyouni *et al.* (2015) evaluated size-dependent bending response and vibration analysis of higher-order FG micro-beams by using MCST and neutral surface position. Kolahchi *et al.* (2015) studied bending response of FG nano-plate by implementing new sinusoidal shear deformation theory and then compared the results with that of with FSDT and HSDT while Kolahchi (2017) investigated the transverse and axial bending behaviour of nano-sandwich plate by using RZT (refined zigzag theory), SSDT, FSDT and CPT. Neves *et al.* (2013) evaluated deflection, frequency and stress values of FGM sandwich structure by implementing principle of virtual displacement. Tounsi *et al.* (2013, 2016) analyzed the bending, buckling and natural frequencies of the functionally graded sandwich shell panels exposed to the thermal environment (Tounsi *et al.* 2013) and FGM sandwich plates (Tounsi *et al.* 2016). They adopted refined-third-order shear deformation (Tounsi *et al.* 2013) model using three-unknown non-polynomial shear deformation theory (Tounsi *et al.* 2016). Beldjelili *et al.* (2016) analyzed bending behaviour of S-FGM plate

resting on elastic foundation under hygro-thermo-mechanical environment by using four variable trigonometry plate theory. Zohra *et al.* (2016) performed buckling behaviour analysis of FGM sandwich structure with clamped boundary condition by adopting four unknown variables. Van Tung (2015) analyzed the buckling/post-buckling and bending behaviour of FGM sandwich structure under the combined thermal as well as mechanical loading in association with von-Karman nonlinearity in the framework of the FSDT. El-Haina *et al.* (2017) investigated the thermal buckling behaviour of thick FG-sandwich by implementing both the sinusoidal SDT and stress function. Menasria *et al.* (2017) studied thermal stability of FG sandwich plate by implementing new displacement field function which is undetermined integral terms. Ozdemir *et al.* (2018) applied 6-DOFs mesh free modeling to linear buckling analysis of stiffened plates with curvilinear surfaces. Boudierba *et al.* (2013) analyzed FGM plate with elastic foundation and implemented the refined plate theory to examine the static behaviour. Natarajan and Manickam (2012) used finite element method and various type plate theories with shear consideration, for development of mathematical model of FG sandwich plates. Kiani and Eslami (2012) compute the buckling response and post-buckling response of FGM sandwich plate using the FSDT kinematic model and von-Karman type geometrical nonlinearity utilizing pure metal core whereas the graded face and bottom part. Fouad *et al.* (2018) developed model of FGM plate by using novel kinematic and considering undetermined integral for analysis of thermo-mechanical bending. Dash *et al.* (2019) investigated flexural strength of FG curved sandwich shell structure by using FEM and higher order polynomial shear deformation kinematics. Ghannadpour and Mehrparvar (2020) investigated post buckling response of thick FG-plate with oblique elliptical cut-out by using plate assembly technique. Mehar and Panda (2017) by using FEM analyzed vibration response without considering thermal effect of FG-carbon nano tube reinforced curved panel with sandwich arrangement. Wang and Shen (2011) implemented two-step perturbation method to solve the governing equation of temperature dependent FGM sandwich structure. Katariya *et al.* (2017) analyzed buckling responses of shape memory alloy with sandwich arrangement of shell panel and included the Green Lagrange strain-displacement in geometrical nonlinearity analysis. Subsequently they used FEM model with HSDT for above analysis. Panda and Singh (2013) evaluated nonlinear fundamental natural frequency of spherical panel composed of shape memory alloy with layered composite by using Green-Lagrange strain-displacement, HSDT and the FEM with direct iterative method. El Meiche *et al.* (2011) implemented five number of unknowns in hyperbolic SDT whereas other plate theories contain four unknowns to analyze the buckling response and fundamental frequency response of laminated sandwich plates. Kettaf *et al.* (2013) analyzed buckling response of FG-sandwich by considering constant, linear and nonlinear temperature distributions across the thickness functionally graded sandwich plates.

The review of above articles is indicating two major

research gaps i.e., i) mathematical model of graded sandwich structure derived using lower-order kinematics (FSDT) instead of HSDT and ii) the geometrical distortion included using von-Karman type nonlinearity. According to the published data on the graded sandwich structure, no study has been reported on the buckling behaviour using the HSDT kinematics and Green-Lagrange type of nonlinear geometrical distortion. Hence, the present article aims to numerically predict the thermal buckling load parameter using displacement controlled isoparametric FE model under the influence of variable kinds of temperature loading (uniform, linear and nonlinear). The convergence of critical buckling temperature (λ_{cr}) and the validity has been confirmed by solving different numerical examples as same as the published articles. Lastly, the model is utilized to show the influences of various structural parameters (curvature ratio, thickness ratio, power-law index (k), support conditions and the FG sandwich type) on the buckling characteristics of the graded sandwich curved panel structures. The obtained solutions and their relevance to the engineering analysis have been highlighted.

2. Mathematical modelling

A general mathematical formulation is prepared for the doubly curved functionally graded sandwich shell panel. The core is considered to be purely ceramic whereas the face sheets have been graded (from ceramic to metal) functionally along the thickness direction using the power-law formulae. The material properties (Young’s modulus, density and Poisson’s ratio) vary as per the following relation

$$P(z) = P_m + (P_c - P_m)V_f^{(n)} \tag{1}$$

where P_m : Properties of metal and P_c : Properties of ceramic. Additionally, $V_f^{(n)}$: Volume fraction ($n = 1, 2, 3$).

The volume fraction of the ceramic varies through the thickness as per the following power-law

$$\begin{aligned} V_f^{(1)} &= \left(\frac{z - h_0}{h_1 - h_0}\right)^{n_z}, & z \in [h_0, h_1] \\ V_f^{(2)} &= 1, & z \in [h_1, h_2] \\ V_f^{(3)} &= \left(\frac{z - h_3}{h_2 - h_3}\right)^{n_z}, & z \in [h_2, h_3] \end{aligned} \tag{2}$$

where n_z is the power law index and the thickness coordinate (z) levels h_0, h_1, h_2 and h_3 are defined in Fig. 1 which illustrates the geometry of the shell panels based on the principal radii of curvature along the longitudinal and transverse directions.

2.1 Displacement field

The curved panels (Fig. 1) with adequate geometrical dimensions ($a \times b \times h$) m^3 to construct a rectangular base (projection of the curved panel would be a rectangle) panel model has been considered. The thickness of the core is

denoted as h_c whereas the thickness of the bottom and top face sheets are denoted as h_{f1} and h_{f2} , respectively such that $h = h_c + h_{f1} + h_{f2}$. The R_1 is radius of curvature in x direction and R_2 is in y direction. The curved panel geometrical configurations are defined as: Cylindrical ($R_1 = R, R_2 = \infty$), spherical ($R_1 = R, R_2 = R$), elliptical ($R_1 = R, R_2 = 2R$), hyperboloid ($R_1 = R, R_2 = -R$) and flat ($R_1 = R_2 = \infty$) on the basis of curvature, where R is a constant. The displacement model based on HSDT mid-plane kinematics (Kant and Swaminathan 2002) is utilized to model the FG shell panels in the present work.

The nine unknown parameters $p_0, q_0, r_0, p_1, q_1, r_1, p_3, q_3, r_3$ global displacement p, q and r are the polynomial functions. At any point of sandwich shell panel the global displacement can be evaluated as

$$\begin{aligned} p &= p_0 + zp_1 + z^2p_2 + z^3p_3 \\ q &= q_0 + zq_1 + z^2q_2 + z^3q_3 \\ r &= r_0 \end{aligned} \tag{3}$$

where p_0, q_0 and r_0 are displacement components of any point in x, y and z -directions respectively. Now, some parameters in the equation act for the rotation of normal (p_1 and q_1) in y and x axes respectively. The terms p_1, q_1, p_3 , and q_3 represented the expansion of Taylor’s series of composite structure at any point on the mid-plane at $z = 0$. The unknown parameters are defined at the mid-plane are given as

$$\begin{aligned} p_0 &= p, & q_0 &= q, & r_0 &= r \\ p_1 &= \frac{\partial p}{\partial z}, & q_1 &= \frac{\partial q}{\partial z}, & p_2 &= \frac{1}{2} \frac{\partial^2 p}{\partial z^2}, & q_2 &= \frac{1}{2} \frac{\partial^2 q}{\partial z^2} \\ p_3 &= \frac{1}{6} \left(\frac{\partial^3 p}{\partial z^3}\right), & q_3 &= \frac{1}{6} \left(\frac{\partial^3 q}{\partial z^3}\right) \end{aligned}$$

Now, the above Eq. (3) in matrix form

$$\{\lambda\} = [H_1]\{\lambda_0\} \tag{4}$$

in which $\{\lambda\} = \{p \ q \ r\}^T$ and $\{\lambda_0\} = \{p_0 \ q_0 \ r_0 \ p_1 \ q_1 \ p_2 \ q_2 \ p_3 \ q_3\}^T$.

$[H_1]$ is the matrix of thickness co-ordinate, so

$$[H_1] = \begin{bmatrix} 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\frac{\partial p}{\partial x} + \frac{r}{R_1}\right) \\ \left(\frac{\partial q}{\partial y} + \frac{r}{R_2}\right) \\ \left(\frac{\partial p}{\partial y} + \frac{\partial q}{\partial x} + \frac{2r}{R_{12}}\right) \\ \left(\frac{\partial p}{\partial z} + \frac{\partial r}{\partial x} - \frac{p}{R_1}\right) \\ \left(\frac{\partial q}{\partial z} + \frac{\partial r}{\partial y} - \frac{q}{R_2}\right) \end{Bmatrix} \tag{5}$$

By substituting Eq. (3) in Eq. (4), the strain vector can further be given as

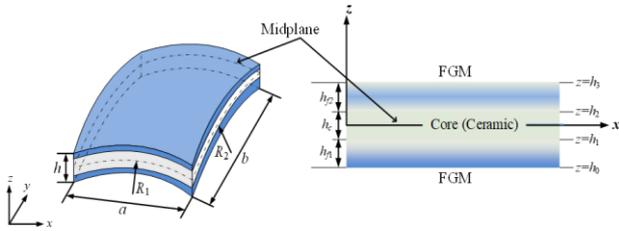


Fig. 1 Geometry of FGM sandwich panels and material variation in with ceramic core

$$\begin{aligned} \varepsilon = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} &= \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^1 \\ k_y^1 \\ k_{xy}^1 \\ k_{xz}^1 \\ k_{yz}^1 \end{Bmatrix} \\ &+ z^2 \begin{Bmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \\ k_{xz}^2 \\ k_{yz}^2 \end{Bmatrix} + z^3 \begin{Bmatrix} k_x^3 \\ k_y^3 \\ k_{xy}^3 \\ k_{xz}^3 \\ k_{yz}^3 \end{Bmatrix} \end{aligned} \quad (6)$$

in which

$$\{\bar{\varepsilon}\} = \{\varepsilon_x^0 \quad \varepsilon_y^0 \quad \varepsilon_{xy}^0 \quad \varepsilon_{xz}^0 \quad \varepsilon_{yz}^0 \quad k_x^1 \quad k_y^1 \quad k_{xy}^1 \quad k_{xz}^1 \quad k_{yz}^1 \quad k_x^2 \quad k_y^2 \quad k_{xy}^2 \quad k_{xz}^2 \quad k_{yz}^2 \quad k_x^3 \quad k_y^3 \quad k_{xy}^3 \quad k_{xz}^3 \quad k_{yz}^3\}$$

$[H_2]_{5 \times 20} = [I_1 \quad zI_1 \quad z^2I_1 \quad z^3I_1]$ and $[I_1]$ is a unit matrix having dimension 5×5 .

The strain-stress relationship of FGM sandwich structure can be expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{zx} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} - \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta t \right) \quad (7)$$

or $\{\sigma\} = [Q]\{\varepsilon - \alpha\Delta t\}$

where $Q_{11} = E_1/(1 - (\mu_{21}\mu_{12}))$, $Q_{22} = E_2/(1 - (\mu_{21}\mu_{12}))$, $Q_{12} = \nu_{12}E_2/(1 - (\mu_{21}\mu_{12}))$, $Q_{44} = G_{13}$, $Q_{55} = G_{23}$ and $Q_{66} = G_{12}$. Further, the values of shear modulus are taken as $G_{13} = G_{12}$ and $G_{23} = 1.2 \times G_{12}$.

Δt : represents temperature rise.

The total strain energy (U) of the sandwich structure can be written as

$$\begin{aligned} U &= \frac{1}{2} \left(\iint \left(\int_{-h/2}^{h/2} \{\varepsilon\}^T \{\sigma\} dz \right) dx dy \right) \\ \text{or } U &= \frac{1}{2} \left(\iint \left(\int_{-h/2}^{h/2} \{\bar{\varepsilon}\}^T [H_2]^T \{Q\} [H_2] \{\bar{\varepsilon}\} dz \right) dx dy \right) \end{aligned} \quad (8)$$

$$\text{or } U = \frac{1}{2} \iint \left(\int_{-h/2}^{h/2} [B]^T \{\lambda_0\}^T [H_2]^T \{Q\} [H_2] \{\lambda_0\} dz \right) dx dy$$

$$\text{or } U = \frac{1}{2} \iint \{\lambda_0\}^T [B]^T \{Q\} [B] dx dy$$

where

$$[D] = \int_{-h/2}^{h/2} [H_2]^T \{Q\} [H_2] dz$$

Now FGM sandwich panel is under thermal load then, it can be evaluated as

$$\begin{aligned} \{f_T\} &= \{M_{xT} \quad M_{yT} \quad M_{xyT} \quad 0 \quad 0\}^T \\ &= \int_{-h/2}^{h/2} [Q] \{\alpha_1 \quad \alpha_2 \quad 0 \quad 0 \quad 0\}^T \Delta t dz \end{aligned} \quad (9)$$

The work done can be calculated due to temperature rise as

$$W_T = \int_V \left(\begin{aligned} &\frac{1}{2} \{ (p_x)^2 + (q_x)^2 + (w_x)^2 \} M_{xT} \\ &+ \frac{1}{2} \{ (p_y)^2 + (q_y)^2 + (w_y)^2 \} M_{yT} \\ &+ \{ p_x p_y + q_x q_y + w_x w_y \} M_{xyT} \end{aligned} \right) dv \quad (10)$$

$$W_T = \int_V \begin{Bmatrix} p_x \\ p_y \\ q_x \\ q_y \\ r_x \\ r_y \end{Bmatrix}^T \begin{bmatrix} M_{xT} & M_{xyT} & 0 & 0 & 0 & 0 \\ M_{xyT} & M_{yT} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{xT} & M_{xyT} & 0 & 0 \\ 0 & 0 & M_{xyT} & M_{yT} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{xT} & M_{xyT} \\ 0 & 0 & 0 & 0 & M_{xyT} & M_{yT} \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ q_x \\ q_y \\ r_x \\ r_y \end{Bmatrix} dv \quad (11)$$

$$\text{or } W_T = \int_q \{\varepsilon_G\}^T [S_T] \{\varepsilon_G\} dq$$

$$\text{or } W_T = \int_q \{\bar{\varepsilon}_G\}^T [H_G]^T [S_T] [H_G] \{\bar{\varepsilon}_G\} dq$$

or $W_T = \iint \{\bar{\varepsilon}_G\}^T [D_G] \{\bar{\varepsilon}_G\} dx dy$ where $\{\bar{\varepsilon}_G\} = \{p_x \quad p_y \quad q_x \quad q_y \quad r_x \quad r_y\}^T$ is the mid-plane geometric strain vector and $[H_G]_{6 \times 24} = [I_2 \quad zI_2 \quad z^2I_2 \quad z^3I_2]$ is thickness coordinate of geometrical matrix. $[I_2]$ is a 6×6 unit matrix. $[D_G]_{24 \times 24}$ is a matrix of material properties which may be defined as $[D_G] = \int_{-h/2}^{h/2} [H_G]^T [S_T] [H_G] dz$.

In this analysis, FEM is employed to compute the buckling temperature of FGM sandwich structure. To obtain the elemental equation of the sandwich shell structure an isoparametric element having nine nodes and nine degrees of freedom (DOF) per node is implemented.

The mid-plane displacement vector is further expressed in terms of interpolation function $\{M_i\}$ and nodal displacement vector $\{\lambda_{0i}\}$ as same as the source (Cook *et*

Table 1 Different support condition combinations

A	SSSS	Each side having simply supported
B	CCCC	All sides clamped
C	SFSF	Two opposite sides simply supported and two other sides are free
D	SCSC	Two sides clamped and other two sides are simply supported
E	CFCF	Two opposite sides free and two other sides are clamped
F	CFFF	Cantilever (one side clamped, others free)

Table 2 Coordinates of different sandwich structure

Sandwich type	h_0	h_1	h_2	h_3
1-0-1	$-h/2$	0	0	$h/2$
1-1-1	$-h/2$	$-h/6$	$h/6$	$h/2$
1-2-1	$-h/2$	$-h/4$	$h/4$	$h/2$
2-1-2	$-h/2$	$-h/10$	$h/10$	$h/2$
2-2-1	$-h/2$	$-h/10$	$h/10$	$h/2$

al. 2009)

$$\{\lambda_0\} = \sum_{i=1}^9 M_i \{\lambda_{0i}\} \tag{12}$$

where

$$\{\lambda_{0i}\} = [p_{0i} \ q_{0i} \ r_{0i} \ p_{1i} \ q_{1i} \ p_{2i} \ q_{2i} \ p_{3i} \ q_{3i}]^T.$$

The strain vectors can be expressed in terms of nodal displacement vector as

$$\{\bar{\varepsilon}\} = \{\lambda_{0i}\}[B] \tag{13}$$

$$\{\bar{\varepsilon}_G\} = [B_G]\{\lambda_{0i}\} \tag{14}$$

where $[B_G]$ and $[B]$ represent the product of differential operators and interpolation functions. Their matrix details are available in Mehar *et al.* (2017).

The variational principle applied to the FG sandwich curved panel for derive governing equation of buckling analysis.

$$\delta \Pi = \delta(U - W_T) = 0 \tag{15}$$

The final steady state equation of FG shell panel for the buckling analysis in terms of eigenvalue and eigenvectors is present in Eq. (16)

$$([K_L] + \lambda_{cr}[K_G])\{\Delta\} = 0 \tag{16}$$

where $[K_L] = \iint [B]^T [D] [B] dx dy$ and $[K_G] = \iint [B]^T [D_G] [B] dx dy$. The linear and geometrical stiffness matrices are $[K_L]$ and $[K_G]$, respectively. Also, λ_{cr} is critical buckling temperature load factor.

For numerical analysis different edge conditions are considered which are given below (Table 1):

I. Simply-supported edge conditions (S)

$$x = 0, a; \ q_0 = r_0 = q_1 = q_2 = p_3 = q_3 = 0$$

$$y = 0, b; \ p_0 = r_0 = p_1 = p_2 = p_3 = 0$$

II. Clamped edge conditions (C)

$$x = 0, a; \ y = 0, b$$

$$p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 = 0$$

III. Free edge conditions (F)

$$x = 0, a; \ y = 0, b$$

$$p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 \neq 0$$

In the present study, various kinds of symmetry of the FGM sandwich plate are used and denoted as: 1-1-1, 1-2-1, 2-1-2, 1-0-1 and 2-2-1. Considering the thickness of plate as

“ h ” and the plane of symmetry to be at the mid-surface of plate, the arrangements of the core and the face sheets for each of the sandwich symmetries are as follows the Table 2.

3. Results and discussions

In the current investigation, the proposed HSDT type FE model has been utilised to compute the thermal buckling response of a FG sandwich structure. The study reveals the convergence test followed by the validation of the model. Then, a series of numerical examples are taken up to solve various structural parametric effect on the buckling temperature values of graded sandwich structure including the geometrical configuration, temperature loading and sandwich type (symmetric and unsymmetric). Now, the geometrical parameters and the material dependent components are defined in a tabular fashion, which are utilized for the numerical analysis purpose.

In the FGM sandwich curved shell panel, temperature of the bottom surface is T_B and temperature of top surface is T_T .

The material properties are adopted for the current analysis are: Titanium-alloy/Zirconia [$Ti \ 6Al \ 4V/ZrO_2$] FG sandwich properties (Titanium alloy: $E = 66.2 \text{ GPa}$, $\alpha = 10.3 \times 10^{-6}$, $\nu = 0.3$; Zr: $E = 244.27 \text{ GPa}$, $\alpha = 12.766 \times 10^{-6}$, $\nu = 0.3$).

The symmetry of FG panels is defined in terms of the ratio of face and core thickness and represented as $h_{f1} - h_c - h_{f2}$. The core-face thickness ratio (CFR) is defined as the ratio of thickness of core to the face ($CFR = h_c/h_f$, where, $h_{f1} = h_{f2} = h_f$). The panels are assumed to have the following geometrical and material properties throughout, unless specified otherwise. The panel is square cross-section $a/b = 1$, $a/h = 10$ and power-law-index ($n_z = 2$). Also, the Titanium-alloy/Zirconia [$Ti \ 6Al \ 4V/ZrO_2$] properties are utilized for throughout the current analysis if not stated explicitly.

The buckling response of FGM sandwich panel focused on the temperature distribution through thickness as the following.

Uniformly temperature distribution through thickness:

In this case the initial temperature of the FG sandwich plate is assumed to be T_i which subsequently increases to T_f . Then the change in temperature is given by

$$\Delta T = T_f - T_i$$

Graded temperature distribution across thickness:

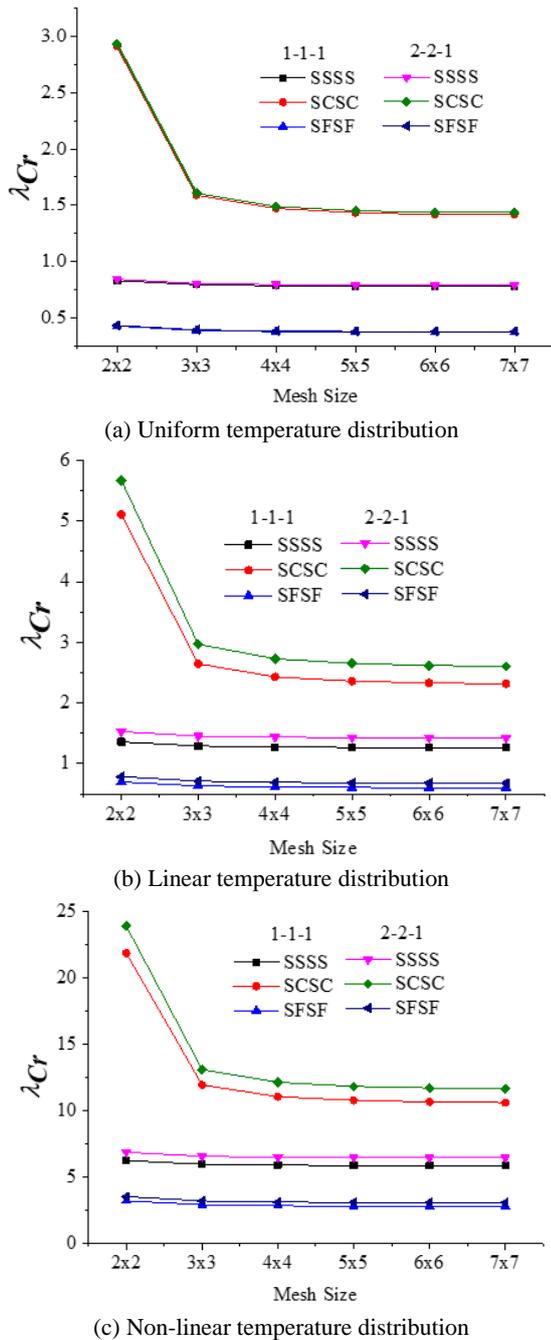


Fig. 2 Convergence of buckling response of cylindrical panels (a) uniform temperature distribution; (b) linear temperature distribution; (c) non-linear temperature distribution

It is assumed that the temperature variation across the thickness of FG sandwich panel is regulated through power law variation given as (Zenkour and Sobhy 2010)

$$T(z) = \Delta t \left(\frac{z}{h} + \frac{1}{2} \right)^\gamma + T_T$$

where T_T and T_B are the top and bottom surface temperatures respectively. γ is the temperature exponent having values $0 < \gamma < \infty$. The temperature difference is $\Delta t = T_B - T_T$. For $\gamma = 1$, the temperature distribution will

be linear across the thickness and for other values of γ , it will be nonlinear.

3.1 Convergence of critical buckling temperature

At the very beginning, the convergence test of the present structural model has been performed. The first critical buckling temperature difference (λ_{Cr}) is obtained for different mesh sizes by considering the material properties scheme 1-1-1 and 2-2-1 cylindrical shell panels subjected to [SSSS], [SFSF] and [SCSC] support conditions. The physical properties for the *Ti-alloy/ZrO₂* panels is: $a/h = 10$, square cross-section, $R/a = 10$, and $n_z = 2$.

Figs. 2(a)-(c) are illustrating the sensitivity behaviour of critical buckling temperature (λ_{Cr}) for variable mesh divisions. The responses are presented for the uniform, linear and nonlinear temperature distributions in Figs. 2(a), (b) and (c), respectively considering the earlier defined material properties. The results are following the necessary convergence with mesh refinement for each type of temperature distribution. It can also be concluded that a (6 × 6) mesh is good enough for the numerical computation of the buckling results.

3.2 Validation of thermal buckling response

The numerically calculated critical buckling temperature (λ_{Cr}) using current HSDT based approach is compared with the analytical values (of λ_{Cr}) reported by Zenkour and Sobhy (2010). The validation is confirmed as the present values are matching well with the values from Zenkour and Sobhy (2010) and presented in Tables 3-5, for different cases of temperature distributions across the FG sandwich panel thickness. The critical buckling temperature (λ_{Cr}) of FG flat sandwich shell panels ($a/b = 1$, $a/h = 10$), as considered by Zenkour and Sobhy (2010), corresponding to power law index 0.5 and 2 for the symmetries 1-0-1 and 2-1-2 are obtained using the present scheme and listed in Tables 3-5 along with the reference values. λ_{Cr} corresponding to uniform temperature distribution, linear temperature distribution and nonlinear temperature distribution is presented in Tables 3-5 respectively. It may be noted that the λ_{Cr} values in the above Tables comprise of analytical solutions of the reference and present HSDT based numerical solutions. Moreover, the present approach utilizes a nine-noded isoparametric element and each node having 9 DOF which is used for discretizing the shell panel domain. It is clearly observed that the present λ_{Cr} values are in good agreement with that of the reference. The new value is hardly lesser compared to the higher-order theory based solution reported by the reference. Therefore, it is safe to conclude that the current model yields correct and valid results.

3.3 Effect of curvature ratio (R/a) on buckling response

Firstly, the influence of R/a on the buckling response of FGM sandwich curved panels with [SCSC] support conditions is investigated considering four different types of panel geometry (cylindrical, spherical, hyperboloid, and

Table 3 Validation of critical buckling temperature (λ_{Cr}) under uniform temperature distribution

Sandwich type	n_z	Source	$a/h = 5$	$a/h = 10$	$a/h = 15$	$a/h = 25$
1-0-1	0.5	SPT (Zenkour and Sobhy 2010)	2.87276	0.80328	0.36504	0.13294
		HPT (Zenkour and Sobhy 2010)	2.87073	0.80313	0.36501	0.13294
		FPT (Zenkour and Sobhy 2010)	2.83506	0.80036	0.36444	0.13286
		CPT (Zenkour and Sobhy 2010)	3.34559	0.83639	0.37173	0.13382
		Present	2.8421	0.79451	0.36102	0.13148
	2	SPT (Zenkour and Sobhy 2010)	2.63459	0.71815	0.32462	0.11789
		HPT (Zenkour and Sobhy 2010)	2.63018	0.71783	0.32455	0.11788
		FPT (Zenkour and Sobhy 2010)	2.57355	0.71357	0.32368	0.11776
		CPT (Zenkour and Sobhy 2010)	2.962	0.7405	0.32911	0.11848
		Present	2.611	0.71573	0.32389	0.1177
2-1-2	0.5	SPT (Zenkour and Sobhy 2010)	2.83194	0.79232	0.3601	0.13116
		HPT (Zenkour and Sobhy 2010)	2.83029	0.7922	0.36007	0.13115
		FPT (Zenkour and Sobhy 2010)	2.79675	0.78959	0.35954	0.13108
		CPT (Zenkour and Sobhy 2010)	3.30065	0.82516	0.36673	0.13202
		Present	2.801	0.78323	0.35591	0.12962
	2	SPT (Zenkour and Sobhy 2010)	2.39953	0.65098	0.29396	0.10671
		HPT (Zenkour and Sobhy 2010)	2.39637	0.65075	0.29392	0.1067
		FPT (Zenkour and Sobhy 2010)	2.34733	0.6471	0.29317	0.1066
		CPT (Zenkour and Sobhy 2010)	2.68016	0.67004	0.29779	0.1072
		Present	2.3807	0.64827	0.29296	0.10638

Table 4 Under linear temperature distribution validation of critical buckling temperature (λ_{Cr})

Sandwich type	n_z	Source	$a/h = 5$	$a/h = 10$	$a/h = 15$	$a/h = 25$
1-0-1	0.5	SPT (Zenkour and Sobhy 2010)	5.69553	1.55657	0.68008	0.21589
		HPT (Zenkour and Sobhy 2010)	5.69147	1.55627	0.68002	0.21588
		FPT (Zenkour and Sobhy 2010)	5.62013	1.55073	0.67888	0.21573
		CPT (Zenkour and Sobhy 2010)	6.64118	1.62279	0.69346	0.21764
		Present	5.458963	1.581754	0.726454	0.266733
	2	SPT (Zenkour and Sobhy 2010)	5.21919	1.38631	0.59924	0.18578
		HPT (Zenkour and Sobhy 2010)	5.21036	1.38566	0.59911	0.18576
		FPT (Zenkour and Sobhy 2010)	5.0971	1.37714	0.59736	0.18553
		CPT (Zenkour and Sobhy 2010)	5.874	1.431	0.60822	0.18696
		Present	4.975096	1.413244	0.646227	0.236681
2-1-2	0.5	SPT (Zenkour and Sobhy 2010)	5.61388	1.53464	0.6702	0.21231
		HPT (Zenkour and Sobhy 2010)	5.61059	1.5344	0.67015	0.21231
		FPT (Zenkour and Sobhy 2010)	5.5435	1.52919	0.66908	0.21216
		CPT (Zenkour and Sobhy 2010)	6.55131	1.60032	0.68347	0.21405
		Present	5.382616	1.560241	0.716634	0.26314
	2	SPT (Zenkour and Sobhy 2010)	4.74906	1.25196	0.53793	0.16341
		HPT (Zenkour and Sobhy 2010)	4.74274	1.2515	0.53784	0.1634
		FPT (Zenkour and Sobhy 2010)	4.64467	1.2442	0.53635	0.1632
		CPT (Zenkour and Sobhy 2010)	5.31032	1.29008	0.54559	0.16441
		Present	4.531798	1.2815	0.585399	0.214275

Table 5 Under non linear temperature distribution validation of critical buckling temperature (λ_{Cr})

Sandwich type	n_z	Source	$a/h = 5$	$a/h = 10$	$a/h = 15$	$a/h = 25$
1-0-1	0.5	SPT (Zenkour and Sobhy 2010)	21.62877	5.91108	2.58262	0.81985
		HPT (Zenkour and Sobhy 2010)	21.61337	5.90995	2.58239	0.81982
		FPT (Zenkour and Sobhy 2010)	21.34245	5.8889	2.57804	0.81924
		CPT (Zenkour and Sobhy 2010)	25.21986	6.16255	2.63342	0.82651
		Present	20.7304	6.006704	2.758705	1.012918
	2	SPT (Zenkour and Sobhy 2010)	23.0683	6.12734	2.64858	0.82115
		HPT (Zenkour and Sobhy 2010)	23.02926	6.12449	2.648	0.82107
		FPT (Zenkour and Sobhy 2010)	22.52869	6.08684	2.64029	0.82005
		CPT (Zenkour and Sobhy 2010)	25.96247	6.32487	2.68827	0.82634
		Present	21.9894	6.246391	2.856255	1.046104
2-1-2	0.5	SPT (Zenkour and Sobhy 2010)	21.35073	5.83656	2.54893	0.80746
		HPT (Zenkour and Sobhy 2010)	21.33821	5.83566	2.54875	0.80744
		FPT (Zenkour and Sobhy 2010)	21.08306	5.81584	2.54466	0.80689
		CPT (Zenkour and Sobhy 2010)	24.91597	6.08637	2.59941	0.81408
		Present	20.47118	5.933912	2.725505	1.000775
	2	SPT (Zenkour and Sobhy 2010)	22.38252	5.90053	2.53532	0.77017
		HPT (Zenkour and Sobhy 2010)	22.35275	5.89838	2.53488	0.77011
		FPT (Zenkour and Sobhy 2010)	21.89054	5.86398	2.52785	0.76918
		CPT (Zenkour and Sobhy 2010)	25.02775	6.08019	2.57139	0.77488
		Present	21.03511	5.956555	2.721782	0.996428

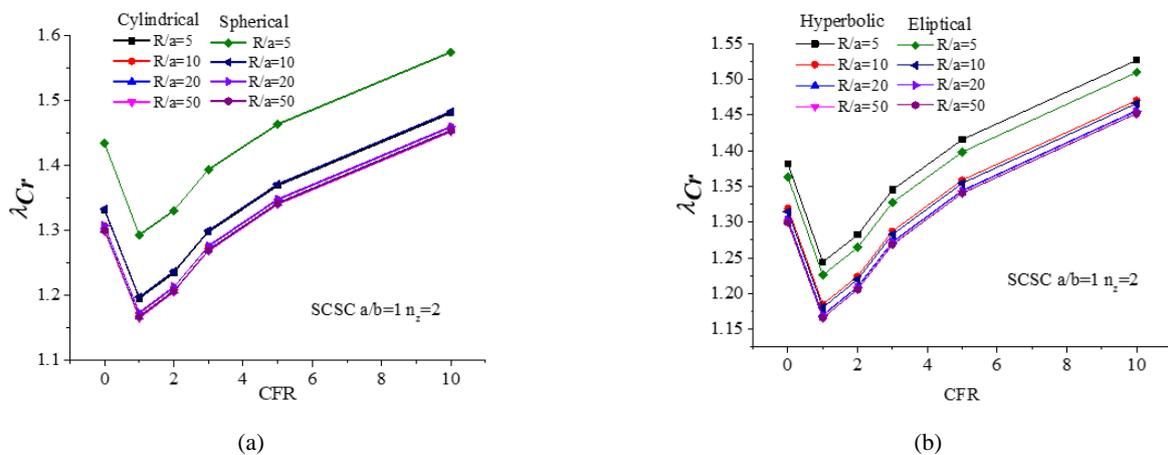


Fig. 3 Thermal buckling temperature load variation for different curvature ratio (R/a) under uniform temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

elliptical) subjected to three types of temperature distributions (uniform, linear and nonlinear) across the thickness. The results are displayed in Figs. 3-5 taking λ_{Cr} as abscissa, CFR as ordinate and keeping other parameters fixed like $n_z = 2$, $a/b = 1$ and $a/h = 10$. The effect of variation of curvature ratio on λ_{Cr} is carried out by taking $R/a = 5, 10, 20$ and 50 . The λ_{Cr} is computed for increasing CFR under uniform temperature distribution and illustrated in Fig. 3(a) and (b) for cylindrical, spherical and hyperboloid, elliptical geometry, respectively. In this case of temperature distribution, with the increase of CFR value, the λ_{Cr} value initially decreases, attains a minimum value

corresponding to $CFR = 1$ and then increases with further increase in CFR. The same trend is observed in the case of linear temperature distribution shown in Figs. 4(a) and (b). However, unlike two previous cases, the critical buckling temperature monotonically decreases with increase in CFR value in case of nonlinear temperature distribution as illustrated in Figs. 5(a) and (b) for all of the geometries considered.

It is significant to mention that for a fixed (R/a) ratio, T_{Cr} values for the spherical shell panels are higher than the corresponding values for cylindrical panels and the values for the hyperboloid shell panels are higher than the values

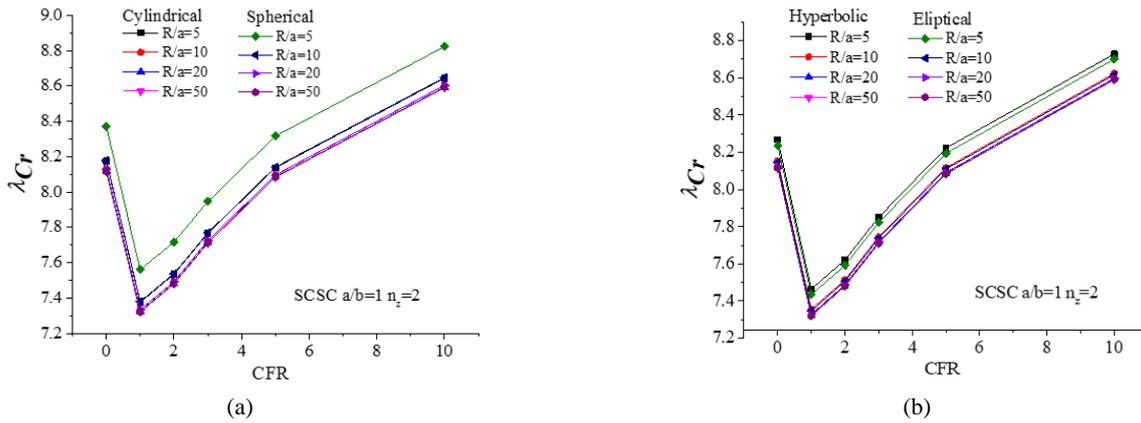


Fig. 4 Thermal buckling temperature load variation for different curvature ratio (R/a) under linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

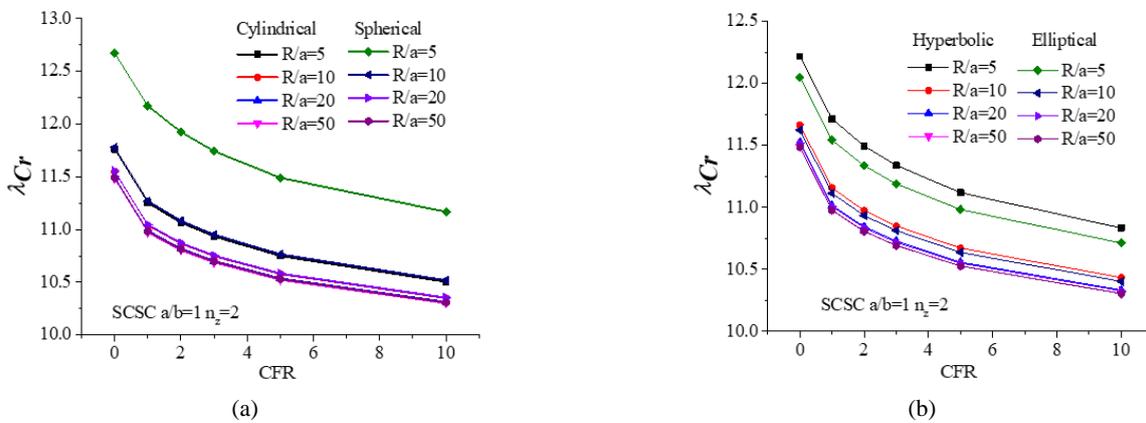


Fig. 5 Thermal buckling temperature load variation for different curvature ratio (R/a) under non linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

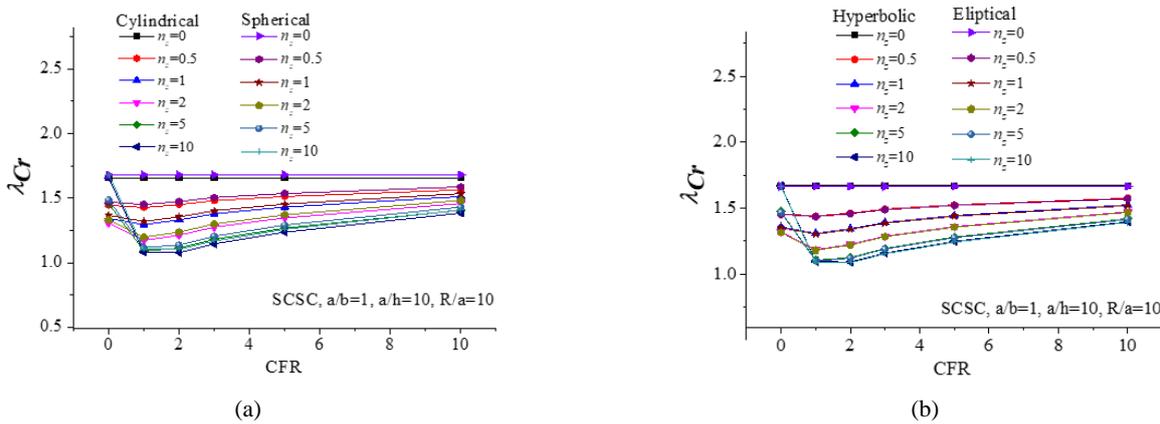


Fig. 6 Thermal buckling temperature load variation for different n_z values under uniform temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

of elliptical panels for all temperature distributions.

3.4 Effect of power law index on buckling response

The influence of power law index on the buckling response of FGM sandwich curved panels with $a/h = 10$, $R/a = 10$ and [SCSC] support conditions, is investigated

considering four different types of panel geometry (cylindrical, spherical, hyperboloid, and elliptical) under three types of temperature distributions (uniform, linear and nonlinear) throughout thickness. The results are displayed in Figs. 6-8 with $n_z = 0, 0.5, 1, 2, 5$ and 10 . The λ_{Cr} is computed for increasing CFR under uniform temperature distribution and illustrated in Fig. 6(a) and (b) for cylind-

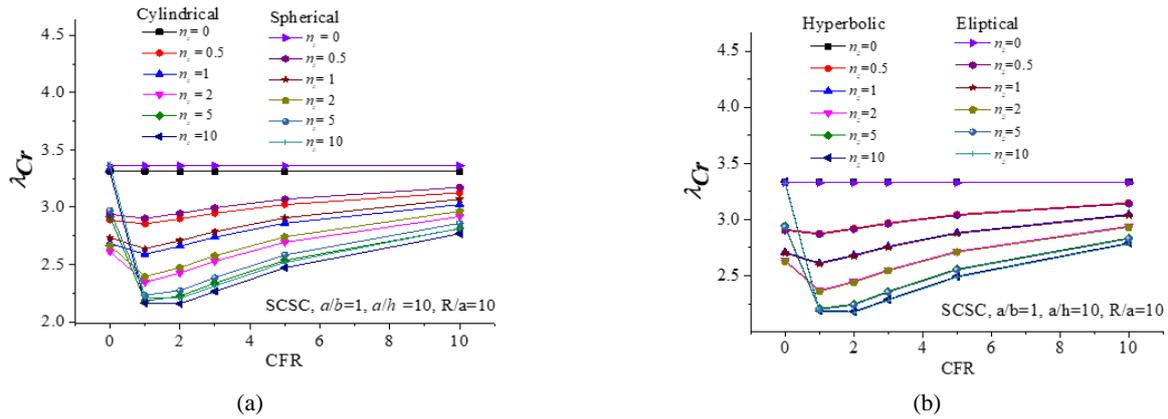


Fig. 7 Thermal buckling temperature load variation for different n_z values under linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

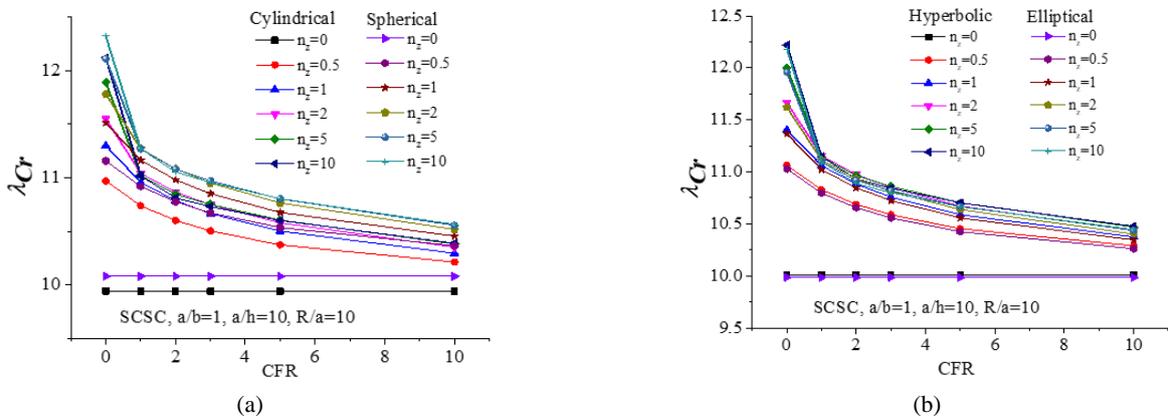


Fig. 8 Thermal buckling temperature load variation for different n_z values under non linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

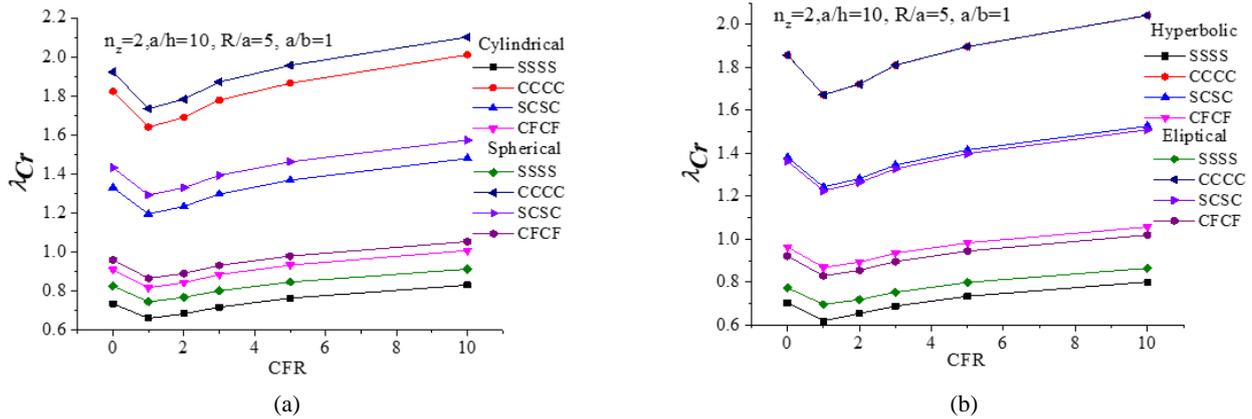


Fig. 9 Thermal buckling temperature load variation for different support condition under uniform temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

rical, spherical and hyperboloid, elliptical geometry, respectively. In this case of temperature distribution, with the increase of k value, the λ_{Cr} value decreases for a fixed CFR value (other than CFR = 0). It may be observed that for all geometric panels, the λ_{Cr} initially decreases then increases for all values of k but however the change is more pronounced in case of $n_z = 5$ and 10. The similar behav-

our is observed in case of linear temperature distribution as shown in Figs. 7(a) and (b). Unlike uniform and linear temperature distribution, in case of nonlinear temperature distribution, the critical buckling temperature decreases monotonically with increase of CFR but for a fixed value of CFR, the λ_{Cr} value increases with increase of k value as displayed in Figs. 8(a) and (b) for all of the geometries

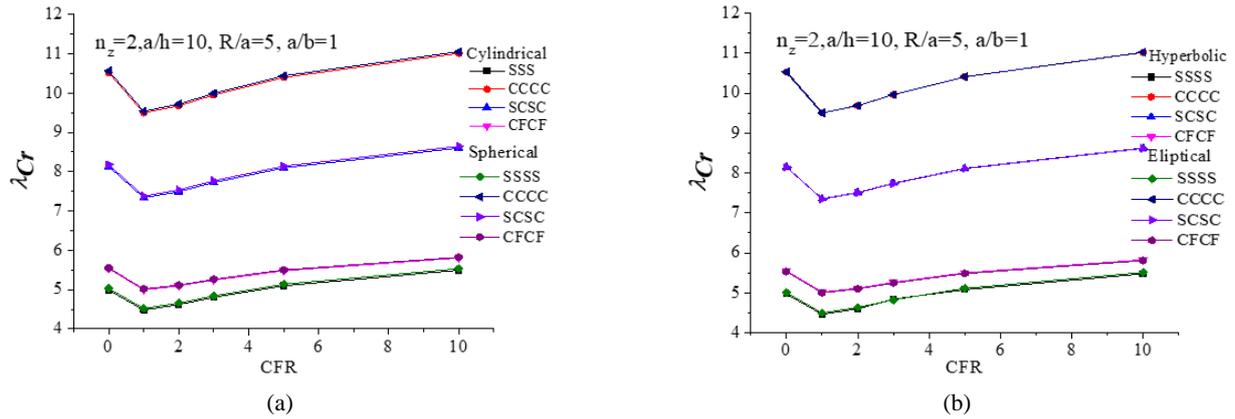


Fig. 10 Thermal buckling temperature load variation for different support condition under linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

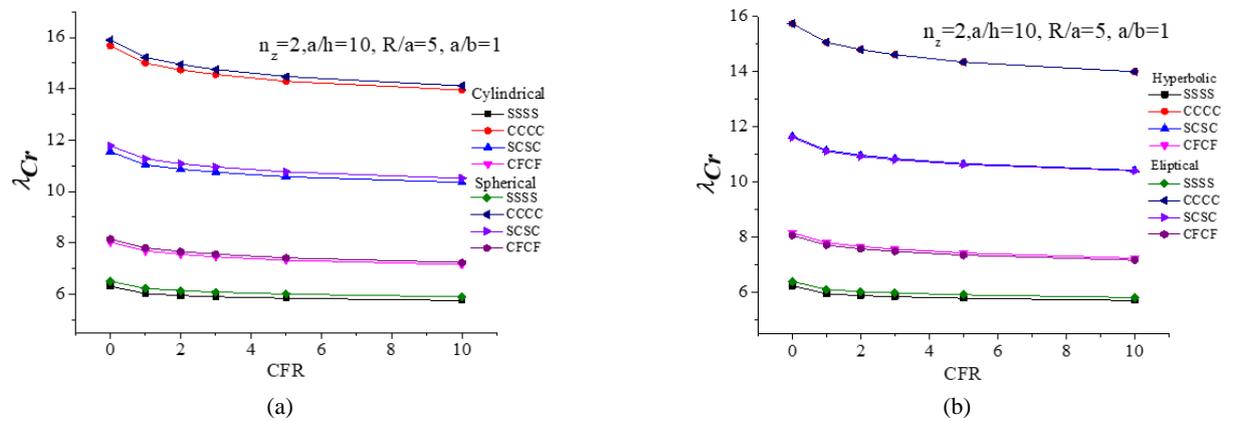


Fig. 11 Thermal buckling temperature load variation for different support condition under non linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

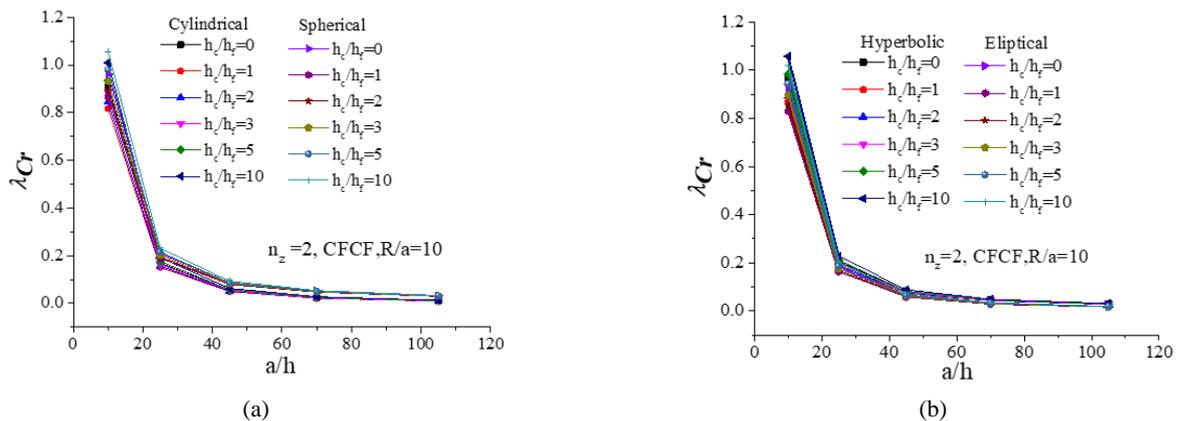


Fig. 12 Thermal buckling temperature load variation for core face thickness ratio (h_c/h_f) under uniform temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

considered. It may be noted here that for $n_z = 0$, the T_{Cr} becomes independent of variation of CFR for all temperature distributions and all geometric panels.

3.5 Effect of support condition on buckling response

The influence of support conditions on the buckling

response of FGM sandwich curved panels with $a/h = 10$, $R/a = 5$, $n_z = 2$, $a/b = 1$ is investigated considering four different types of panel geometry subjected to various temperature distributions as mentioned in the preceding subsections (3.3 and 3.4).

The results are displayed in Figs. 9-11. The sandwich panel subjected to [SSSS], [CCCC], [SCSC] and [CFCF]

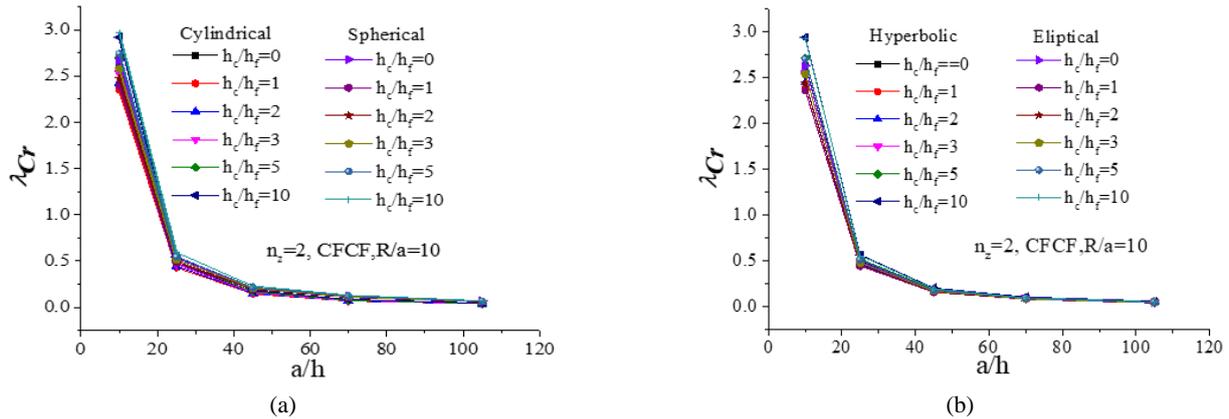


Fig. 13 Thermal buckling temperature load variation for core face thickness ratio (h_c/h_f) under linear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

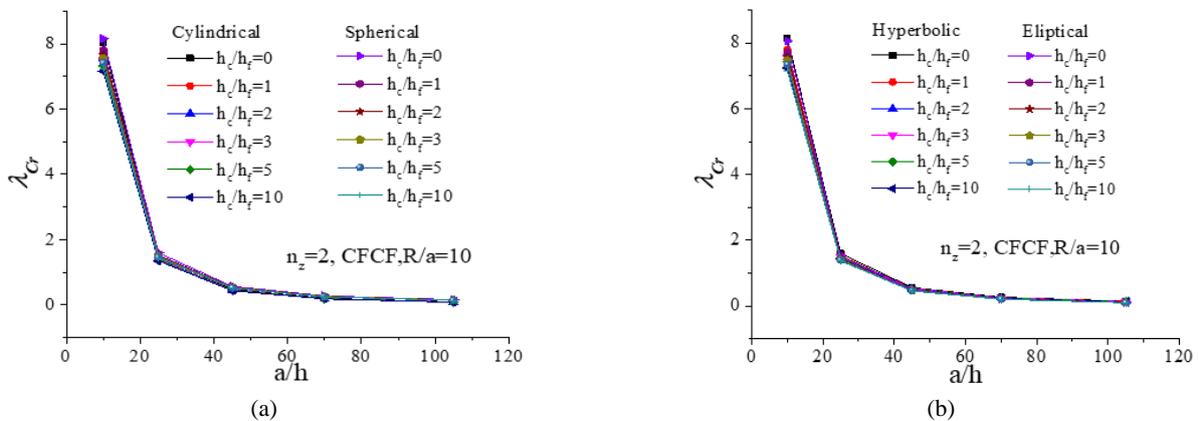


Fig. 14 Thermal buckling temperature load variation for core face thickness ratio (h_c/h_f) under nonlinear temperature distribution (a) cylindrical and spherical; (b) hyperboloid and elliptical FG sandwich shell panels

support conditions. It has been observed from Figs. 9-11 that the maximum and minimum values of critical buckling temperature (λ_{Cr}) occurs for [CCCC] and [SSSS] support conditions respectively in all four geometries. This is attributed that the increasing number of constraints has an effect of increasing stiffness of the shell panel. For uniform and linear temperature distributions, the λ_{Cr} increases monotonically with increase in CFR beyond CFR = 1 for all geometries and support conditions corresponding to Figs. 9-10. However in case of nonlinear temperature distribution Fig. 11, the value of λ_{Cr} monotonically decreases with increase in CFR for all geometries.

It is significant to mention that the λ_{Cr} values are higher for spherical geometry compared to the corresponding values of cylindrical geometry for all support conditions and uniform (Fig. 9(a)) and nonlinear (Fig. 11(a)) temperature distributions. Moreover for linear temperature, their T_{Cr} values are nearly identical (Fig. 10(a)).

3.6 Effect of CFR (core to face thickness ratio) on buckling response

The influence of CFR on the buckling response of FGM sandwich curved panels with $R/a = 10$, $n_z = 2$, $a/b = 1$ is investigated considering four different type of panel

geometries with three types of temperature distributions throughout thickness as mentioned earlier. Both core (h_c) and face thickness (h_f) are varied but the total thickness (h) of the panel is constant.

The results are displayed in Figs. 12-14. The CFR is varied to have the values 0, 1, 2, 3, 5 and 10. The λ_{Cr} is computed for increasing thickness ratio (a/h) under uniform temperature distribution and illustrated for cylindrical and spherical shells in Fig. 12(a) and hyperboloid and elliptical shells in Fig. 12(b). In this case, with increase of a/h ratio, the λ_{Cr} value decreases. The similar behaviour is observed in the case of linear and nonlinear temperature distributions as shown in Figs. 13(a) and (b) and Figs. 14(a) and (b) respectively. For a fixed (a/h) ratio, the λ_{Cr} value increases with increase in CFR value in uniform and linear temperature distributions and for all geometries but surprisingly the λ_{Cr} value falls with rise in CFR value in case of nonlinear temperature distribution as depicted in Figs. 14(a) and (b). spherical shells in Fig. 12(a) and hyperboloid and elliptical shells in Fig. 12(b). In this case, with increase of a/h ratio, the λ_{Cr} value decreases. The similar behaviour is observed in the case of linear and nonlinear temperature distributions as shown in Figs. 13(a) and (b) and Figs. 14(a) and (b) respectively. For a fixed (a/h) ratio, the λ_{Cr} value increases with increase in CFR

value in uniform and linear temperature.

3.7 Effect of sandwich symmetry type

The buckling responses of *Ti-alloy/ZrO₂* FGM sandwich spherical shell panels is studied accounting the influence of sandwich symmetry type with $a/h = 10$, $R/a = 10$ and clamped boundary condition [CCCC]. The schemes such as 1-1-1, 2-2-1, 1-1-3, 2-1-2, 2-1-3, 1-5-1 and 3-1-4 are considered here. The symmetry types are 1-1-1, 1-5-1, 2-1-2 and non-symmetry ones are 2-2-1, 1-1-3, 2-1-3, 3-1-4. The value of λ_{Cr} is plotted with increasing (n_z) value and depicted in Fig. 15. In case of uniform and linear temperature distribution all schemes have identical behaviour excluding 1-1-3 which gives higher λ_{Cr} value than other scheme in linear temperature distribution. But in case of nonlinear temperature distribution all schemes are λ_{Cr} value increases with increasing n_z value and 1-1-3 has highest λ_{Cr} value. The non-linear temperature distribution exhibits higher values of λ_{Cr} corresponding to all of the power law index values. Finally, 1-1-3 scheme gives the highest λ_{Cr} value than other unsymmetrical sandwich with respect to all temperature distribution. Similarly, 1-5-1 gives highest λ_{Cr} value than other symmetrical sandwich in case of uniform and linear temperature distribution but in case of non-linear distribution 2-1-2 scheme gives highest λ_{Cr} value.

4. Conclusions

Thermal buckling load of FGM sandwich curved shell panels are predicted numerically using Green-Lagrange type large distortion in the framework of the HSDT. The computational responses are evaluated using in-house higher-order FE MATLAB code with the help of the current mathematical model. The solution validity as well as the convergence are checked initially using the similar kind of results available in the published domain. Finally, a set of numerical examples are solved to bring out the influences of curvature ratio, CFR (h_c/h_f), power-law index (k), support conditions and the FG sandwich including the types of sandwich i.e., symmetrical and unsymmetrical on buckling temperature. Moreover, the results are obtained for the four different geometrical shapes viz. cylindrical, spherical, hyperboloid, and elliptical including three kinds of temperature loading (uniform, linear and nonlinear) through the panel thickness. The following concluding remarks are listed in the following lines to understand the capability of the proposed model to investigate the sandwich structure made from graded material.

- The critical buckling temperature (λ_{Cr}) is increasing while the curvature ratio increases under the uniform and linear temperature distributions whereas following a reverse path for the nonlinear temperature distribution.

- The buckling load parameters are decreasing while the power-law indices (n_z) increase under the uniform temperature distribution irrespective of all kind of geometries adopted in this analysis whereas the values follow the reverse line for nonlinear temperature loading.

- For all four geometrical shapes, as well as the temperature distributions, the λ_{Cr} values are decreasing when the thickness ratio (a/h) increases. However, for a particular thickness ratio, the λ_{Cr} values follow an increasing line when CFR value increases under the uniform and linear temperature distributions. However, the values follow a similar downline type for the nonlinear temperature distribution.

- The clamped graded structure is showing the stiffest configuration while compared to all other kinds of end boundaries.

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