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Influence of the distribution shape of porosity on the bending of FGM beam using a new higher order shear deformation model

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Abstract. In this paper, a new higher order shear deformation model is developed for static analysis of functionally graded beams with considering porosities that may possibly occur inside the functionally graded materials (FGMs) during their fabrication. The model account for higher-order variation of transverse shear strain through the depth of the beam and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. The present work aims to study the effect of the distribution forms of porosity on the bending of simply supported FG beam. Based on the present higher-order shear deformation model, the equations of motion are derived by the principle of virtual works. Navier type solution method was used to obtain displacement and stresses, and the numerical results are compared with those available in the literature. A comprehensive parametric study is carried out to assess the effects of volume fraction index, porosity fraction index, and geometry on the bending of imperfect FG beams. It can be concluded that the proposed model is simple and precise for the resolution of the behavior of flexural FGM beams while taking into account the shape of distribution of the porosity.

Keywords: functionally graded materials; bending; volume fraction of porosity; Navier's solution; shear deformation theory

1. Introduction

Nowadays, with the fast progress of technological advancement especially in various engineering industries such as nuclear, mechanical engineering, aerospace structures, civil engineering, etc., the need for the use of materials that ensure a better performance along with durability, high resistance and stiffness against the surrounding environment has become a necessary thing for a progress with permanent creativity. Due to these critical points, the usage of advanced composites has great benefits, which permit to prolong the active lifespan of such structures with a reduced exploitation cost and less pollution to the environment.

Functionally graded materials (FGMs) are known as new sort of inhomogeneous composite materials that are introduced to be thermal barrier components for aerospace structural utilizations and fusion reactors (Bayat *et al.* 2009). FGMs are advanced composite materials made of two or more constituent phases. The concept of FGM was first considered in Japan in 1984 during a space plane project.

Nowadays, functionally graded beams and plates structures are widely used in many industries including nuclear engineering. Therefore, accurate structural analysis of FG beams is required to predict their correct bending, and vibration behavior. It is grasped that in the process of FGM manufacturing, microvoids (known as porosities) can take place within the materials during the sintering action (Zhu *et al.* 2001). This phenomenon is related to the large difference in solidification temperatures between material components (Wang *et al.* 2017). Due to the importance of this subject, several studies have been carried out to explore the porosity effects. For example, Ait Atmane *et al*. (2015) studied the free vibration of microbeams made of porous graded materials using a higher order shear deformation theory. Saadatfar and Aghaie-Khafri (2015) investigated the Electro magneto thermo elastic behavior of a rotating imperfect hybrid functionally graded hollow cylinder. Şeref Doğuşcan Akbaş (2015) studied the forced vibration analysis of functionally graded porous deep beams. Gupta and Talha (2017) examined the effect of porosity on the frequency response of FG plates in the presence of a thermal effect by using a non-polynomial higher-order shear and normal deformation theory. Benferahat *et al*. (2016a, b, c) studied the effect of porosity on the bending and free vibration response of functionally graded plates resting on Winkler-Pasternak foundations by introducing in the mathematical formulation a volume fraction of porosity. Mirza *et al*. (2018) used various theories to analysis of Laminated and FGM Beams. Zghal *et al*. (2017) analyze the static of functionally graded carbon nanotube-reinforced plate and shell structures. Draiche *et al*. (2019) studied the static of laminated reinforced composite plates using a simple first-order shear deformation theory. Belbachir *et al*. (2019) analyze the bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and

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mechanical loadings. Sahla *et al*. (2019) developed the free vibration analysis of angle-ply laminated composite and soft-core sandwich plates. Chaabane *et al*. (2019) investigated the analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation. Draoui *et al*. (2019) used the FSDT for the static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT). Adda Bedia *et al*. (2019) developed a new hyperbolic two-unknown beam model for Bending and buckling analysis of a nonlocal strain gradient nanobeams. Meksi *et al*. (2019) used an analytical solution for bending, buckling and vibration responses of FGM sandwich plates. Hellal *et al*. (2019) analyze the dynamic and stability of functionally graded material sandwich plates in hygro-thermal environment using a simple higher shear deformation theory. Frikha *et al*. (2016) used a new higher order C^0 mixed beam element for FGM beams analysis. Mallek *et al*. (2019a) analyze dynamic of functionally graded carbon nanotube–reinforced shell structures with piezoelectric layers under dynamic loads. Mallek *et al*. (2019b) investigated the piezoelastic response of smart functionally graded structure with integrated piezoelectric layers using discrete double directors shell element. Mallek *et al*. (2019c) analyzed the geometrically non-linear of FG-CNTRC shell structures with surface-bonded piezoelectric layers. Hajlaoui *et al*. (2019) analyzed the geometrically nonlinear analysis of FGM shells using solid shell element with parabolic shear strain distribution. Mellouli *et al*. (2019) investigated the meshfree implementation of the double director shell model for FGM shell structures analysis. Zghal *et al*. (2018) studied the non-linear bending analysis of nanocomposites reinforced by graphene-nanotubes with finite shell element and membrane enhancement. Recently, Addou *et al*. (2019) used a quasi 3D HSDT for studied the influences of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation. Medani *et al*. (2019) analyzed the static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate. Alimirzaei *et al*. (2019) analyzed the nonlinear of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions. Berghouti *et al*. (2019) analyzed the vibration of nonlocal porous nanobeams made of functionally graded material. Bourada *et al*. (2019) investigated the dynamic of porous functionally graded beam using a sinusoidal shear deformation theory. Batou *et al*. (2019) studied the wave dispersion properties in imperfect sigmoid plates using various HSDTs. Ebrahimi *et al*. (2019) analyzed the vibration of porous metal foam shells rested on an elastic substrate.

In addition, in recent years, many researchers have dealt the effect of stretching the thickness on FGM structures (Khiloun *et al*. 2019, Addou *et al*. 2019, Boutaleb *et al*. 2019, Zarga *et al*. 2019, Boulefrakh *et al*. 2019, Boukhlif *et al*. 2019, Mahmoudi *et al*. 2019, Zaoui *et al*. 2019). Also, many literature reviews on FGM and other types of materials has published recently, for example we can cite the research paper of Karami *et al*. (2019a), Hussain *et al*. (2019), Semmah *et al*. (2019), Tlidji *et al*. (2019), Karami *et al*. (2019b, c, d, e).

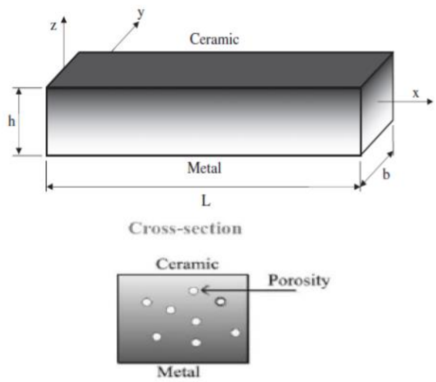


Fig. 1 Geometry and coordinate of a FG beam

The objective of this work is to use a new higher order shear deformation model to study the effect of the distribution form of porosity on static behavior of FGM beams. The effect due to porosity is included using a modified mixture law covering the porosity phases proposed by Wattanasakulpong (2014), Demirhan (2019) and Younsi (2018). The properties of the material of the FGM beam are supposed to vary according to a power law distribution of the volume fraction of the constituents. The equation of motion for FGM beams is obtained by the principle of virtual works. The effects of power index, pore volume fraction, geometry ratio, and thickness ratio on FGM beam deflection are also studied.

2. Geometric configuration and material properties

Consider a functionally graded beam with length L and rectangular cross section $b \times h$, with b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction.

In this study, we consider an imperfect FGM beam with a volume fraction of porosity β $(\beta \prec\prec 1)$ with different form of distribution between the metal and the ceramic. The modified mixture rule proposed by Wattanasakulpong (2014) is

$$P(z) = P_m\left(V_m - \frac{\beta}{2}\right) + P_c\left(V_c - \frac{\beta}{2}\right) \tag{1}$$

The power law of the volume fraction of the ceramic is assumed as

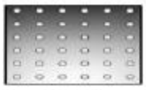
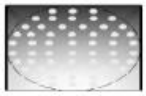
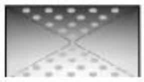
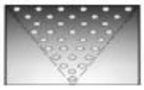
$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{2}$$

The modified mixture rule becomes

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m - (P_c + P_m)\frac{\beta}{2} \tag{3}$$

where, k is the power law index that takes values greater than or equals to zero. The FGM beam becomes a fully ceramic beam when k is set to zero and fully metal for

Table 1 Different distribution forms of porosity

Porosity distribution form	Elastic modulus expression	Schema
Homogeneous shape	$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\beta}{2}$	
Form "O" shape	$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\beta}{2}\left(1-2\frac{\lvert z\rvert}{h}\right)$	
Form "X" shape	$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\beta}{2}\left(2\frac{z}{h}\right)$	
Form "V" shape	$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\beta}{2}\left(\frac{1}{2}+\frac{z}{h}\right)$	

large value of k.

The Young's modulus E of the imperfect FG can be written as a functions of thickness coordinate Z (middle surface), as follows (Ait Atmane *et al*. 2015, Hassaine Daouadji *et al*. 2016, Hadji *et al*. 2015).

$$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\beta}{2} \quad (4)$$

The material properties of a perfect FGM beam can be obtained when the volume fraction of porosity β is set to zero. Due to the small variations of the Poisson ratio ν, it is assumed to be constant. Several porosity distributions have been studied in the present work, such as "O", "V" and X" (Table 1).

3. Kinematic, strain and stress relations

The displacement field of the present higher order shear deformation model is given by the following expression (Hadji *et al*. 2016).

$$u_1(x,z) = u(x) - z\frac{\partial w_0}{\partial x} + \frac{1}{\lambda_x}\left[\frac{3}{2}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right]Q_x \quad (5a)$$

$$u_2(x,z) = 0 \quad (5b)$$

$$u_3(x,z) = w_0(x) \quad (5c)$$

Where u is displacements at any point $(x,0)$ on the reference plane in the x direction. Q_x is the transverse shear stress resultant with λ_x being the unknown constants. The constant λ_x can be determined by considering the definition of Q_x

$$Q_x = \int_{-h/2}^{h/2} \tau_{xz} dz \quad (6)$$

The strains associated with the displacements in Eq. (5) are

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{\lambda_x}\left[\frac{3}{2}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right]\frac{\partial Q_x}{\partial x} \quad (7a)$$

$$\gamma_{xz} = \frac{1}{\lambda_x}\left[\frac{3}{2h} - \frac{6z^2}{h^3}\right]Q_x \quad (7b)$$

The stress–strain relationship at any point in the beam is given by the one-dimensional Hooke's law as follows

$$\sigma_x = Q_{11}(z)\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z)\,\gamma_{xz} \quad (8)$$

where

$$Q_{11}(z) = E(z) \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \quad (9)$$

Using Eq. (7b) in the constitutive relations for τ_{xz} and then substituting in the Eq. (7) λ_x may be obtained as follow

$$\lambda_x = \frac{E}{2(1+\nu)} \quad (10)$$

3.1 Equations of motion

The equations of motion are obtained using the principle of variational energy and virtual work.

$$\frac{\partial N_x}{\partial x} = 0 \quad (11a)$$

$$\frac{\partial^2 M_x}{\partial x^2} = -q \quad (11b)$$

$$\frac{\partial}{\partial x}\left[M_x - \frac{4}{3h^2}P_x\right] - \left[Q_x - \frac{4}{h^2}R_x\right] = 0 \quad (11c)$$

The stress resultants and moment resultants are given by

$$(N_x, M_x, P_x) = \int_{-h/2}^{h/2} \sigma_x(1, z, z^3) dz \quad (12a)$$

$$R_x = \int_{-h/2}^{h/2} z^2 \tau_{xz} dz \quad (12b)$$

The various stiffness parameters are defined as follows

$$(A_{11}, B_{11}, D_{11}, E_{11}, F_{11}, H_{11}) = \int_{-h/2}^{h/2} Q_{11}(z)\,(1, z, z^2, z^3, z^4, z^6) dz \quad (13a)$$

$$(A_{55}, D_{55}, F_{55}) = \int_{-h/2}^{h/2} Q_{55}(z)\,(1, z^2, z^4)dz \tag{13b}$$

4. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_0, Q_x can be written by assuming the following variations.

$$u_0 = \sum_{m=1}^{\infty} U_m \cos\frac{m\pi x}{x} \tag{14a}$$

$$w_0 = \sum_{m=1}^{\infty} W_m \sin\frac{m\pi x}{L} \tag{14b}$$

$$Q_x = \sum_{m=1}^{\infty} Q_{xm} \cos\frac{m\pi x}{L} \tag{14c}$$

The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda\, x) \tag{15}$$

where Q_m is the load amplitude calculated from

Table 2 Materials properties

Material	Properties	
	$E\ (GPa)$	ν
Ceramic (Alumina, Al_2O_3)	380	0.3
Ceramic (Zirconia, ZrO_2)	151	0.3
Metal (Aluminum, Al)	70	0.3

$$Q_m = \frac{2}{L}\int_0^L q(x)\sin(\lambda\, x)dx \tag{16}$$

The coefficients Q_m are given below for some typical loads.

$$Q_m = q_0 \quad \text{Simusoidal load} \tag{17a}$$

$$Q_m = \frac{4q_0}{m\pi} \quad \text{Uniform load} \tag{17b}$$

Eqs. (14) and (15) reduce the governing equations to the following form.

For flexural analysis

$$\begin{aligned} &[C]\{\Delta\} = \{f\} \\ &\{\Delta\}^T = \{U_m, W_m, Q_{xm}\} \quad \text{and} \quad \{f\}^T = \{0, Q_m, 0\} \end{aligned} \tag{18}$$

where $[C]$ refer to the flexural stiffness.

5. Results and discussion

In this study, the bending analysis of FGM beams by the new higher order shear deformation model is suggested for investigation, the effect of the distribution form of porosity is also studied; the Poisson's ratio is fixed at $\nu = 0.3$. Comparisons are made with the solutions available in the literature in order to verify the accuracy of this analysis. The properties of the materials used in this analysis are presented in Table 2.

For a sake of simplicity the displacements and stresses, are presented in a non-dimensional form

$$\begin{aligned} &\overline{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right), \quad \overline{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2}, \frac{h}{2}\right), \\ &\overline{\tau}_{xz} = \frac{h}{q_0 L}\tau_{xz}(0,0) \end{aligned} \tag{19}$$

Table 3 Comparison of the maximum vertical displacement $\bar{w}$ of FG beams

L/h	Theory	β	$k = 0$	$k = 1$	$k = 2$	$k = 5$	$k = 10$
5	Li *et al.* (2010)	$\beta = 0$	3.1657	6.2599	8.0602	9.7802	10.8979
	CBT*	$\beta = 0$	2.8783	5.7746	7.4003	8.7508	9.6072
	FBT*	$\beta = 0$	3.1657	6.2599	8.0303	9.6483	10.7194
	TBT*	$\beta = 0$	3.1654	6.2594	8.0677	9.8281	10.9381
	Present	$\beta = 0$	3.1654	6.2594	8.0677	9.8281	10.9381
		$\beta = 0.1$	3.3646	7.2508	10.0092	12.9184	14.5261
		$\beta = 0.2$	3.5906	8.6775	13.5422	20.0935	23.1357
20	Li *et al.* (2010)	$\beta = 0$	2.8962	5.8049	7.4415	8.8151	9.6879
	CBT*	$\beta = 0$	2.8783	5.7746	7.4003	8.7508	9.6072
	FBT*	$\beta = 0$	2.8962	5.8049	7.4397	8.8069	9.6767
	TBT*	$\beta = 0$	2.8962	5.8049	7.4421	8.8182	9.6905
	Present	$\beta = 0$	2.8962	5.8049	7.4421	8.8182	9.6905
		$\beta = 0.1$	3.0785	6.7458	9.2856	11.6159	12.7847
		$\beta = 0.2$	3.2853	8.1093	12.6873	18.2673	20.2309

* Reference of Vo *et al.* (2015)

Table 4 Comparison of the axial stress $\bar{\sigma}_x$ of FG beams

L/h	Theory	α	$k=0$	$k=1$	$k=2$	$k=5$	$k=10$
5	Li *et al*. (2010)	$\beta=0$	3.8020	5.8837	6.8812	8.1030	9.7063
	FBT*	$\beta=0$	3.7500	5.7959	6.7676	7.9428	9.5228
	TBT*	$\beta=0$	3.8020	5.8836	6.8826	8.1106	9.7122
	Present	$\beta=0$	3.8019	5.8835	6.8824	8.1104	9.7119
		$\beta=0.1$	3.8019	6.2196	7.5679	9.0712	10.9462
		$\beta=0.2$	3.8019	6.7062	8.8019	11.0614	13.1949
20	Li *et al*. (2010)	$\beta=0$	15.0130	23.2054	27.0989	31.8112	38.1372
	FBT*	$\beta=0$	15.0000	23.1834	27.0704	31.7711	38.0913
	TBT*	$\beta=0$	15.0129	23.2053	27.0991	31.8130	38.1385
	Present	$\beta=0$	15.0129	23.2051	27.0989	31.8127	38.1382
		$\beta=0.1$	15.0129	24.5347	29.8064	35.5387	42.8965
		$\beta=0.2$	15.0129	26.4623	34.6992	43.2984	51.4893

* Reference of Vo *et al*. (2015)

Table 5 Comparison of the shear stress $\bar{\tau}_{xz}$ of FG beams

L/h	Theory	α	$k=0$	$k=1$	$k=2$	$k=5$	$k=10$
5	Li *et al*. (2010)	$\beta=0$	0.7500	0.7500	0.6787	0.5790	0.6436
	FBT*	$\beta=0$	0.5976	0.5976	0.5085	0.3914	0.4279
	TBT*	$\beta=0$	0.7332	0.7332	0.6706	0.5905	0.6467
	Present	$\beta=0$	0.7329	0.7329	0.6704	0.5904	0.6465
		$\beta=0.1$	0.7329	0.7329	0.6595	0.5495	0.6169
		$\beta=0.2$	0.7329	0.7329	0.6432	0.4733	0.5497
20	Li *et al*. (2010)	$\beta=0$	0.7500	0.7500	0.6787	0.5790	0.6436
	FBT*	$\beta=0$	0.5976	0.5976	0.5085	0.3914	0.4279
	TBT*	$\beta=0$	0.7451	0.7451	0.6824	0.6023	0.6596
	Present	$\beta=0$	0.7437	0.7437	0.6812	0.6013	0.6586
		$\beta=0.1$	0.7437	0.7437	0.6702	0.5601	0.6290
		$\beta=0.2$	0.7437	0.7437	0.6538	0.4831	0.5617

* Reference of Vo *et al*. (2015)

As the first example, FG beams under am uniformly distributed load are considered. The maximum displacements and stresses obtained from the different theories are given in Tables 3-5 along with the results from previous studies using CBT and TBT (Vo *et al*. 2015, Li *et al*. 2010).

As we can see on this table, close agreements were obtained between the results of the present model and those of literature (when $\beta=0$, perfect beam). It is observed from Table 3 that the deflections increase by increasing the span-to-depth ratio L/h. By introducing the volume fraction of porosity (β), it can be noted that the increase of this factor induces an increase in dimensionless deflections of FGM beams. This is expected because the imperfect FG beam $(\beta \neq 0)$ is the one with the lowest stiffness and the perfect FG beam is the one with the highest stiffness. Due to ignoring the shear deformation effect, CBT underestimates deflection of moderately deep beams.

Displacements and stresses of FGM beams under sinusoidal load obtained by using the present model, parabolic beam theory (PSDBT) of Reddy (1984) and first order beam theory (FSDBT) of Timoshenko (1921), and the inverse hyperbolic theory of Sayyad and Ghugal (2018) are presented in Table 6. These results are obtained for $L/h=5$ and 20 and various values of power law index k and the volume fraction of porosity β. It is seen that the displacements and stresses obtained from the present theory are in excellent agreement with those obtained from PSDBT. The FSDBT underestimates the displacements and stresses. Furthermore, it is observed from the Table 6 that the displacements are increased with the increase in power-law index whereas stresses are identical when beam is made of fully ceramic $(k=0)$ or fully metal $(k=\infty)$. This is due to the fact that an increase of the power-law index makes FG beams more flexible i.e., reduces the stiffness. The same remarks for the effect of the porosity are noticed in the case of a sinusoidal loading.

In Figs. 2 and 3, we present the effect of the distribution

Table 6 Non-dimensional displacements and stresses of functionally graded beam subjected to sinusoidal load

k	Theory	β	$L/h = 5$			$L/h = 20$		
			$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0	Reddy (1984)	$\beta = 0$	2.5020	3.0916	0.4769	2.2838	12.171	0.4774
	Timoshenko (1921)	$\beta = 0$	2.0523	3.0396	0.2653	2.2839	12.158	0.2653
	Sayyad and Ghugal (2018)	$\beta = 0$	2.5019	3.0922	0.4800	2.2839	12.171	0.4806
	Present	$\beta = 0$	2.5019	3.0916	0.4769	2.2839	12.171	0.4774
		$\beta = 0.1$	2.6594	3.0916	0.4769	2.4276	12.171	0.4774
		$\beta = 0.2$	2.8381	3.0916	0.4769	2.5907	12.171	0.4774
1	Reddy (1984)	$\beta = 0$	4.9458	4.7857	0.5243	4.5773	18.813	0.5249
	Timoshenko (1921)	$\beta = 0$	4.8807	4.6979	0.5376	4.5734	18.792	0.5376
	Sayyad and Ghugal (2018)	$\beta = 0$	4.9441	4.7867	0.5248	4.5774	18.814	0.5245
	Present	$\beta = 0$	4.9458	4.7857	0.4769	4.5774	18.814	0.4774
		$\beta = 0.1$	5.7286	5.0589	0.4769	5.3193	19.892	0.4774
		$\beta = 0.2$	6.8549	5.4542	0.4769	6.3944	21.454	0.4774
5	Reddy (1984)	$\beta = 0$	7.7723	6.6057	0.5314	6.9540	25.794	0.5323
	Timoshenko (1921)	$\beta = 0$	7.5056	6.4382	0.9942	6.9373	25.752	0.9942
	Sayyad and Ghugal (2018)	$\beta = 0$	7.7739	6.6079	0.5274	6.9541	25.795	0.5313
	Present	$\beta = 0$	7.7723	6.6057	0.3856	6.9540	25.794	0.3863
		$\beta = 0.1$	10.2156	7.3904	0.3593	9.1603	28.816	0.3599
		$\beta = 0.2$	15.8848	9.0136	0.3099	14.4051	35.109	0.3105
10	Reddy (1984)	$\beta = 0$	8.6530	7.9080	0.4226	7.6421	30.999	0.4233
	Timoshenko (1921)	$\beta = 0$	8.3259	7.7189	1.2320	7.6215	30.875	1.2320
	Sayyad and Ghugal (2018)	$\beta = 0$	8.6539	7.9102	0.4237	7.6422	30.923	0.4263
	Present	$\beta = 0$	8.6530	7.9080	0.4224	7.6421	30.923	0.4231
		$\beta = 0.1$	11.4934	8.9173	0.4035	10.0824	34.782	0.4042
		$\beta = 0.2$	18.3084	10.7602	0.3603	15.9549	41.753	0.3611
∞	Reddy (1984)	$\beta = 0$	13.582	3.0916	0.4769	12.398	12.171	0.4774
	Timoshenko (1921)	$\beta = 0$	12.552	3.0396	0.3183	12.398	12.158	0.3183
	Sayyad and Ghugal (2018)	$\beta = 0$	13.582	3.0922	0.4800	12.329	12.171	0.4806
	Present	$\beta = 0$	13.582	3.0916	0.4769	12.398	12.171	0.4774
		$\beta = 0.1$	20.016	3.0916	0.4769	18.271	12.171	0.4774
		$\beta = 0.2$	38.029	3.0916	0.4769	34.715	12.171	0.4774

shape of porosity on the dimensionless deflections of FG beam for different span-to-depth ratio L/h, made with Al/Al_2O_3 and Al/ZrO_2 respectively. As we can see on Figs. 2 and 3, the dimensionless deflections decrease in increasing the span-to-depth ratio L/h.

It can also be noted that the distribution shape of porosity slightly influences the variation of the dimensionless deflections as a function of the geometry ratio. The highest values of dimensionless were obtained for the "H" shape of porosity distribution while the lowest ones correspond to the "X" shape of porosity distribution. Comparing the two FGM beams, it can be noted that the different curves are spaced for the beam with Al/Al_2O_3 than for that made of Al/ZrO_2. It can also be observed that the different curves respect the same order for different distribution shape of porosity except the two O and V shape of porosity distribution.

The effect of the distribution shape of porosity on the axial stress $\overline{\sigma}_x$, through the depth of an Al/Al_2O_3 FGM beam is presented in Fig. 4. The stresses are tensile at the top surface and compressive at the bottom surface. Also, it is clear that the axial stress is maximum for H distribution shape of porosity and it is minimal for "X" shape of porosity distribution.

Fig. 5 show the distribution of the shear stresses $\overline{\tau}_{xz}$ through the thickness of an Al/Al_2O_3 FG beam. It's clear that the distributions are not parabolic. Also, the shear stress is maximum for H distribution shape of porosity and it is minimal for "X" shape of porosity distribution. It can also be noted that the distribution shape of porosity slightly influences the variation of the shear stress.

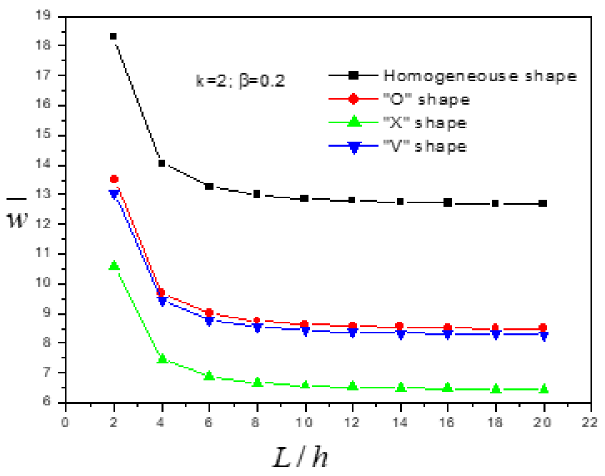


Fig. 2 Effect of the shape of porosity distribution on the non-dimensional deflections $\overline{w}$ versus length-to-thickness ratio L/h of an Al/Al_2O_3 FGM beam ($k = 2$ and $\beta = 0.2$)

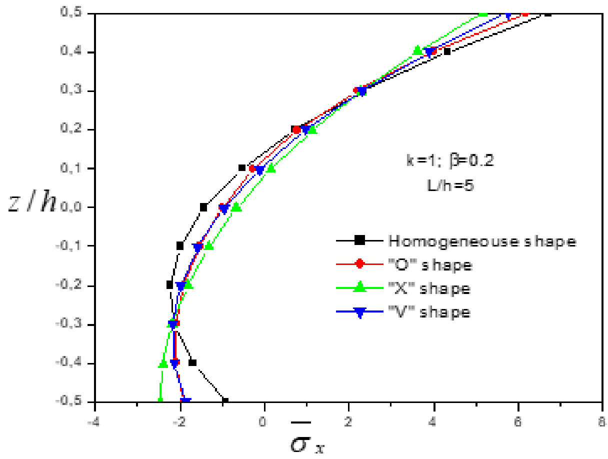


Fig. 4 Effect of the shape of porosity distribution on the axial stress $\overline{\sigma}_x$ through-the-thickness of an Al/Al_2O_3 FGM beam ($k = 1$ and $\beta = 0.2$)

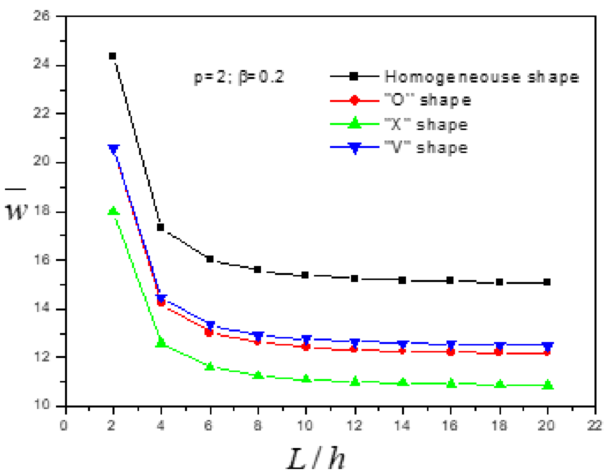


Fig. 3 Effect of the shape of porosity distribution on the non-dimensional deflections $\overline{w}$ versus length-to-thickness ratio L/h of an Al/ZrO_2 FGM beam ($k = 2$ and $\beta = 0.2$)

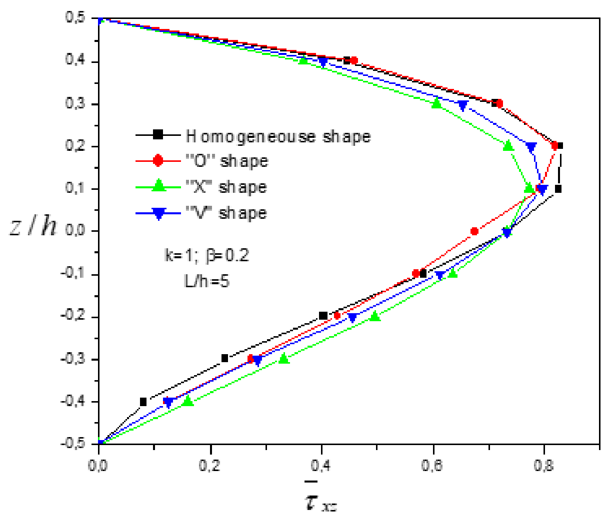


Fig. 5 Effect of the shape of porosity distribution on the shear stress $\overline{\tau}_{xz}$ through-the-thickness of an Al/Al_2O_3 FGM beam ($k = 1$ and $\beta = 0.2$)

6. Conclusions

The study was focused on the effect of the distribution shape of porosity on bending of FGM beam. The mathematical formulation is based on the use of new higher order shear deformation model. The properties of the material are assumed to vary according to the thickness direction of the beam and the rule of the mixture that has been reformulated to evaluate the characteristics of the materials with different distribution shape of porosity. The equation of motion for FGM beams is obtained by the principle of virtual works. The Navier method is used for analytical solutions of the FGM beam with simply supported boundary conditions. A parametric study was conducted, including volume fraction indices, thickness ratios and volume fraction of porosity. According to the typical results, it can be concluded that distribution shape of porosity has a significant effect on the deflections of FGM beams as well as on the axial stress and the shear stress developed in the beam.

Finally, in view of this research, it is very important to study the effect of other parameter such as the effect of boundary conditions and other type of porosity distribution, and to see how these parameters can affect the stability of this type of porous beam.

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