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A novel multistage approach for structural model updating based on sensitivity ranking

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Abstract. A novel multistage approach is developed for structural model updating based on sensitivity ranking of the selected updating parameters. Modal energy-based sensitivities are formulated, and maximum-normalized indices are designed for sensitivity ranking. Based on the ranking strategy, a multistage approach is proposed, where these parameters to be corrected with similar sensitivity levels are updated simultaneously at the same stage, and the complete procedure continues sequentially at several stages, from large to small, according to the predefined levels of the updating parameters. At every single stage, a previously developed cross model cross mode (CMCM) method is used for structural model updating. The effectiveness and robustness of the multistage approach are investigated by implementing it on an offshore structure, and the performances are compared with non-multistage approach using numerical and experimental vibration information. These results demonstrate that the multistage approach is more effective for structural model updating of offshore platform structures even with limited information and measured noise. These findings serve as a preliminary strategy for structural model updating of an offshore platform in service.

Keywords: multistage; sensitivity ranking; model updating; CMCM

1. Introduction

The increasing depth of water and the harsh marine environment in offshore oil exploitation make it hard to perform the visual inspection of structural damage. The problem has resulted in the fast development of more straightforward monitoring techniques for damage identification by inspecting changes in modal characteristics. The primary thought is being developed since the early 1970s (Viero and Roitman 1999). Of late, a large body of research has been undertaken in the structural health monitoring using the modal information, and many methods have been proposed (Cheng *et al.* 2009, Doebling *et al.* 1998, Fan and Qiao 2011, Liang *et al.* 2019, Perez *et al.* 2017, Oliveira *et al.* 2018). An accurate model, e.g., the finite element (FE) model, is a need as the baseline model to detect the modal characteristics changes of the actual structure for most methods. Any uncertainty associated with the model would cause unfavorable locations and extents of damage. Thus the baseline model invariably has to be verified and, if necessary, updated for further applications like damage detection, structural control, evaluation, and assessment. In essence, the model updating itself is a damage detection procedure with updating parameters as the presence, location and extent of the damage (Li *et al.* 2008), before which, the baseline model updated to eliminate the modeling errors is likewise indispensable.

Structural model updating is to correct the baseline model using test/measured data to produce a target model that could better predict the dynamic behavior of the actual structure. A sea of papers has been written (Bayraktar *et al.* 2010, Chung *et al.* 2012, Ni and Ye 2019, Song *et al.* 2017, Wang and Wu 2014), and a complete book (Friswell and Mottershead 1995) has been spent on this subject. Traditionally, the existing methods can be broadly classified into two groups (Hu and Li 2007): (i) direct matrix methods, and (ii) indirect physical property change methods. The first one is generally of non-iterative, which is inspired by computing changes on mass and stiffness matrices. Such changes may have succeeded in generating a target model, but these models cannot be interpreted sensibly. On the contrary, the method in the second group tries to find correction factors for each element or design parameter, which is closer to physically realizable quantity. However, it is iterative and requires more significant computation effort. Taking an entirely different view, Hu *et al.* (2007) developed the cross model cross mode (CMCM) method for simultaneous updating stiffness, mass, and damping matrices. It is non-iterative and very cost-effective and holds the merits of preserving the baseline model configuration and physical connectivity of the target one. Applying model reduction/modal expansion strategy, the CMCM method can be utilized with incomplete measured data. It has been proved to be valid with many numerical and experimental studies (Li *et al.* 2008, Liu *et al.* 2018, Wang 2014, Wang *et al.* 2015). The problem of model

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updating can be transformed into one of solving the system of equations, e.g., $\mathbf{A}\boldsymbol{x} = \mathbf{F}$.

Many problems needed further investigation are the selection of updating parameters and solution of updating equations in the presence of polluted noise and/or spatially incompleteness of measurements. As a critical step, the parameters selection could affect the condition of the system of equations. Moreover, the sensitivity analysis is dependent on it. Further, these parameters should be chosen with the aim of correcting recognized uncertainty in the model, and the measured data should be sensitive to them (Friswell and Mottershead 1995, Yuan *et al.* 2019). Generally, these parameters with higher sensitivity have a more accurate estimation. Otherwise, the errors are greater. Huge differences in the sensitivity could result in an ill-conditioned problem of a system of equations (Banan and Huelmstad 1994). A small perturbation on **F** and/or **A** would lead to a large deviation on actual solution $\boldsymbol{x}$ if the system of equations is ill-conditioned. For the CMCM-based method, modal parameters with measured noise and uncertainty usually lead to considerable perturbation. The estimation of the system of equations is divergent due to ill-conditioned problem. Accordingly, a novel approach is indispensable to overcome the ill-conditioned problem and acquire accurate/stable estimation for structural model updating.

In this study, a novel multistage model updating approach is presented based on sensitivity ranking. Modal energy-based sensitivities are formulated, and maximum-normalized sensitivity indices are developed for sensitivity ranking. These updating parameters with the same sensitivity level are updated simultaneously in one single stage, and the whole updating procedure continues sequentially in several stages. Further, the cross model cross mode method is utilized in each stage for structural model updating.

The study is organized as follows. In Section 2, the CMCM method and updating parameter sensitivity are briefly summarized as the theoretical background. After that, the modal energy-based sensitivity index and sensitivity ranking of updating parameters are formulated, and a detail introduction of the multistage approach is found in Section 3. Then numerical study and experimental validation of a jacket platform structure are studied to illustrate the effectiveness of the multistage approach in Section 4 and 5. Finally, Section 6 gives a conclusion with some concluding remarks.

2. Theoretical background

2.1 CMCM method

For the *i*-th mode of the baseline model and *j*-th mode of the measured structure, a CMCM updating equation was formed

$$C_{ij}^{\dagger} + \sum_{n=1}^{N_K} \alpha_n C_{n,ij}^{\dagger} = \lambda_j^*(D_{ij}^{\dagger} + \sum_{n=1}^{N_M} \beta_n D_{n,ij}^{\dagger}) \tag{1}$$

Where

$$\begin{aligned} C_{ij}^{\dagger} &= \boldsymbol{\phi}_i^T \mathbf{K} \boldsymbol{\phi}_j^*; \qquad C_{n,ij}^{\dagger} = \boldsymbol{\phi}_i^T \mathbf{K}_n \boldsymbol{\phi}_j^*; \\ D_{ij}^{\dagger} &= \boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_j^*; \qquad D_{n,ij}^{\dagger} = \boldsymbol{\phi}_i^T \mathbf{M}_n \boldsymbol{\phi}_j^* \end{aligned} \tag{2}$$

Note that $\boldsymbol{\phi}_i$, $\mathbf{K}$ and $\mathbf{M}$, $\mathbf{K}_n$ and $\mathbf{M}_n$ are the *i*-th mode shape, system stiffness and mass matrices, *n*-th element stiffness mass matrices of the baseline model; λ_j^* and $\boldsymbol{\phi}_j^*$ are the *j*-th natural frequency and mode shape of the measured structure; α_n and β_n are the *n*-th stiffness and mass correction coefficients; N_K and N_M are the numbers of the corresponding coefficients to be corrected, respectively.

Using a new index '*m*' to replace '*ij*' and rearranging Eq. (1), one obtains

$$\sum_{n=1}^{N_K} \alpha_n C_{n,m}^{\dagger} + \sum_{n=1}^{N_M} \beta_n E_{n,m}^{\dagger} = F_m^{\dagger} \tag{3}$$

Where

$$E_{n,m}^{\dagger} = -\lambda_j^* D_{n,m}^{\dagger}; F_m^{\dagger} = -C_m^{\dagger} + \lambda_j^* D_m^{\dagger} \tag{4}$$

When N_i modes of the baseline model and N_j modes of the measured structure are considered, totally $N_m = N_i \times N_j$ CMCM equations were formed and arranged in a matrix form as follows

$$\mathbf{A}\boldsymbol{x} = \mathbf{F}, \textit{where } \mathbf{A} = [\mathbf{C}, \mathbf{E}] \textit{ and } \boldsymbol{x} = [\boldsymbol{\alpha}; \boldsymbol{\beta}] \tag{5}$$

Note that **C** and **E** are the $N_m \times N_K$ and $N_m \times N_M$ matrices; $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and **F** are the column vectors of size N_K, N_M, and N_m, respectively.

These coefficients to be corrected $\boldsymbol{x}$ were calculated by solving Eq. (5) with the singular value decomposition (SVD) algorithm.

$$\tilde{\boldsymbol{x}} = \mathbf{A}^{\#}\mathbf{F} \tag{6}$$

Where $\mathbf{A}^{\#}$ is the generalized inverse of **A**.

Substituting the estimation $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{\alpha}}; \tilde{\boldsymbol{\beta}}]$ into Eq. (7) yields the corrected stiffness and mass matrices of the target model of the measured structure.

$$\begin{aligned} \mathbf{K}^* &= \mathbf{K} + \Delta\mathbf{K} = \mathbf{K} + \sum_{n=1}^{N_K} \tilde{\alpha}_n \mathbf{K}_n; \\ \mathbf{M}^* &= \mathbf{M} + \Delta\mathbf{M} = \mathbf{M} + \sum_{n=1}^{N_M} \tilde{\beta}_n \mathbf{M}_n \end{aligned} \tag{7}$$

2.2 Updating parameter sensitivity

To establish a sensitivity ranking strategy, the sensitivities of eigenvalue and eigenvector on updating parameters should be firstly formulated. These parameters are the correction coefficients α_n and β_n associated with each the elementary mass and stiffness matrices. The eigenvalue sensitivity was determined analytically by differentiation of the un-damped eigenvalue equation, and the eigenvector sensitivity was also determined due to its simplicity of implementation based on expanding the gradients into a weighted sum of the eigenvectors (Fox and Kapoor 1968).

$$\frac{\partial \lambda_i}{\partial \alpha_n} = \boldsymbol{\phi}_i^T \mathbf{K}_n \boldsymbol{\phi}_i; \qquad \frac{\partial \lambda_i}{\partial \beta_n} = -\lambda_i \boldsymbol{\phi}_i^T \mathbf{M}_n \boldsymbol{\phi}_i \tag{8}$$

$$\frac{\partial \boldsymbol{\phi}_i}{\partial \alpha_n} = \sum_{r=1}^{N} b_{inr}^{\alpha} \boldsymbol{\phi}_r; \qquad \frac{\partial \boldsymbol{\phi}_i}{\partial \beta_n} = \sum_{r=1}^{N} b_{inr}^{\beta} \boldsymbol{\phi}_r \tag{9}$$

with the factors b_{inr}^{α} and b_{inr}^{β} as follows:

$$b_{inr}^{\alpha} = \begin{cases} \dfrac{\boldsymbol{\phi}_r^T \mathbf{K}_n \boldsymbol{\phi}_i}{\lambda_i - \lambda_r}, (r \neq i) \\ 0, (r = i) \end{cases}; \qquad b_{inr}^{\beta} = \begin{cases} -\dfrac{\lambda_i \boldsymbol{\phi}_r^T \mathbf{M}_n \boldsymbol{\phi}_i}{\lambda_i - \lambda_r}, (r \neq i) \\ -\dfrac{1}{2} \boldsymbol{\phi}_r^T \mathbf{M}_n \boldsymbol{\phi}_i, (r = i) \end{cases} \tag{10}$$

Note that $\boldsymbol{\phi}_i^T \mathbf{K}_n \boldsymbol{\phi}_i$ and $\lambda_i \boldsymbol{\phi}_i^T \mathbf{M}_n \boldsymbol{\phi}_i$ denote the *i*-th MSE and modal kinetic energy (MKE) for the eigenvalue sensitivity, respectively. Further the eigenvector sensitivity can also be represented by MSE and MKE.

3. Novel multistage methodology

3.1 Modal energy-based sensitivity index

As mentioned above, the eigenvalue and eigenvector sensitivities on correction coefficients can both be represented by MSE and MKE, and the modal energy-based sensitivity indices are defined as follows

$$\begin{cases} SI_{ni}^{K} = \boldsymbol{\phi}_i^T \mathbf{K}_n \boldsymbol{\phi}_i, n = 1, \dots, N_K \\ SI_{ni}^{M} = \lambda_i \boldsymbol{\phi}_i^T \mathbf{M}_n \boldsymbol{\phi}_i, n = 1, \dots, N_M \end{cases} \tag{11}$$

where SI_{ni}^{K} and SI_{ni}^{M} represents the *i*-th mode sensitivity indices to the correction coefficients α_n and β_n, respectively.

To take N_j modes into consideration, one can take the average of sensitivity indices as the new ones

$$\begin{cases} SI_n^K = \dfrac{1}{N_j} \sum_{i=1}^{N_j} \boldsymbol{\phi}_i^T \mathbf{K}_n \boldsymbol{\phi}_i, i = 1,2, \dots, N_j \\ SI_n^M = \dfrac{1}{N_j} \sum_{i=1}^{N_j} \lambda_i \boldsymbol{\phi}_i^T \mathbf{M}_n \boldsymbol{\phi}_i, i = 1,2, \dots, N_j \end{cases} \tag{12}$$

For the sake of brevity and convenience, the sensitivity index is re-defined as follows

$$\begin{cases} SI_n = SI_n^K, \; n = 1, \dots, N_K \\ SI_{n+N_K} = SI_n^M, \; n = 1, \dots, N_M \end{cases} \tag{13}$$

The merits of the presented sensitivity definition include that the computation is very simple and sensitivity has an obvious physical meaning.

3.2 Sensitivity ranking

Numerical studies indicate that the sensitivity indices to different correction coefficients have magnitude differences. Thus a natural logarithmic operation is firstly applied to these sensitivity indices, which are normalized to the maximum value. After all the manipulation, one obtains a maximum-normalized sensitivity index MSI_n as follows

$$MSI_n = \frac{ln(SI_n)}{max\{ln(SI_1), ln(SI_2), \dots, ln(SI_{N_K+N_M})\}} \tag{14}$$

Where SI_n can be interpreted as the sensitivity index to the correction coefficient α_n or β_n, and MSI_n is between 0 and 1.

In practice, these correction coefficients with higher sensitivities have more accurate estimations. Otherwise, the estimations are wrong. The huge difference in the sensitivity indices usually results in ill-conditioned problems (Banan and Huelmstad 1994). Thus a novel approach to classify these coefficients into multilevel is necessary to overcome this shortcoming. There is no specific pattern for sensitivity ranking, and either equal or non-equal spacing division can be adopted. For example, if n_s levels are determined by either engineering experiences, the sensitivity ranking with the same interval $\Delta S = 1/n_s$ can be conducted automatically as listed in Table 1. If the bar map has an evident gradient distribution, these coefficients located in the same stair are classified into one same level. Thus another option is to classify non-equally or artificially with the help of a bar map.

3.3 Multistage approach

Based on the ranking strategy, a multistage approach is proposed, where these correction coefficients with the same sensitivity level are updated simultaneously in one single stage. The complete procedure continues sequentially in several stages, and CMCM method is used in each stage. A procedure of the multistage approach is showed as follows.

- **Step 1**: Computation and ranking of the sensitivity indices

The modal energy-based sensitivity indices (Eq. (13)) of the $N = N_K + N_M$ correction coefficients are firstly calculated. These coefficients are classified into *K* levels ranked from *I* to *K* in descending order based on the maximum-normalized sensitivity indices (Eq. (14)) and ranking strategy.

$$\boldsymbol{x} = [\boldsymbol{x}_I; \boldsymbol{x}_{II}; \dots; \boldsymbol{x}_K] \tag{15}$$

- **Step 2**: Model updating of stage *I*

Table 1 The n_s levels sensitivity ranking with equal interval

Levels	1	2	k	n_s
Range	$[n_s - 1 \sim n_s]\Delta S$	$[n_s - 2 \sim n_s - 1]\Delta S$	$[n_s - k \sim n_s - k + 1]\Delta S$	$[0 \sim 1]\Delta S$

In the first stage, the N_m CMCM equations are formulated by considering all the correction coefficients $[\boldsymbol{x}_I; \boldsymbol{x}_{II}; \dots; \boldsymbol{x}_K]$. These unknown values are estimated with the SVD algorithm and the result is given as follows:

$$\widetilde{\boldsymbol{x}}^{(1)} = [\widetilde{\boldsymbol{x}}_I^{(1)}; \widetilde{\boldsymbol{x}}_{II}^{(1)}; \dots; \widetilde{\boldsymbol{x}}_K^{(1)}] \tag{16}$$

Where the superscript '(k)' denotes the k-th stage. Obviously $\widetilde{\boldsymbol{x}}_I^{(1)}$ has the minimum estimated errors with the highest sensitivity level. Thus only $\widetilde{\boldsymbol{x}}_I^{(1)}$ is retained and the other estimations are set to be zero. The final estimation in the first stage is arranged as

$$\widetilde{\boldsymbol{x}}^{(1)} = [\widetilde{\boldsymbol{x}}_I^{(1)}; \boldsymbol{0}; \dots; \boldsymbol{0}] \tag{17}$$

Updating the baseline model with $\widetilde{\boldsymbol{x}}^{(1)}$ using Eq. (7), the updated model at the first stage (UM-I for short) can be obtained easily.

- **Step 3**: Model updating of stage *II*

Taking UM-I as the baseline model, N_m CMCM equations are also reconstructed to identify the remaining coefficients $[\boldsymbol{x}_{II}; \boldsymbol{x}_{III}; \dots; \boldsymbol{x}_K]$. The estimation in the second stage is given by

$$\widetilde{\boldsymbol{x}}^{(2)} = [\widetilde{\boldsymbol{x}}_{II}^{(2)}; \widetilde{\boldsymbol{x}}_{III}^{(2)}; \dots; \widetilde{\boldsymbol{x}}_K^{(2)}] \tag{18}$$

Likewise, only the estimations with the highest sensitivity level *II* are kept and the new estimation in stage *II* is arranged as

$$\widetilde{\boldsymbol{x}}^{(2)} = [\widetilde{\boldsymbol{x}}_{II}^{(2)}; \boldsymbol{0}; \dots; \boldsymbol{0}] \tag{19}$$

Similarly, a new updated model (UM-II) is obtained by updating UM-I with $\widetilde{\boldsymbol{x}}^{(2)}$.

- **Step 4**: Repeat the above process unless the completion of *K* stages

The process continues until the last sensitivity level (Level *K*). After *K* stages model updating, the final estimation of correction coefficients is arranged

$$\widetilde{\boldsymbol{x}} = [\widetilde{\boldsymbol{x}}_I^{(1)}; \widetilde{\boldsymbol{x}}_{II}^{(2)}; \dots; \widetilde{\boldsymbol{x}}_K^{(k)}] \tag{20}$$

UM-K can be regarded as the target model of the measured structure.

4. Numerical study

4.1 Baseline model

A baseline model of a four-leg offshore platform structure is established. It comprises of 48 nodal points and 77 elements, including 20 leg brace (LB), 24 horizontal brace (HB), and 12 horizontal diagonal brace (HDB), 16 diagonal brace (DB) in vertical planes and 5 deck plate (DP) elements as shown in Fig. 1. Listed below in Table 2 is the corresponding properties. The plane dimensions are 0.52×0.365 and 0.77×0.54 m at top and bottom elevations,

Table 2 The properties of elements

Types	Section shapes	Outer diameters (mm)	Thicknesses (mm)	
LB	tubular	20	2	
HB/HDB/DB	tubular	10	2	
DP	rectangular	700 (length)	545 (width)	10 (height)

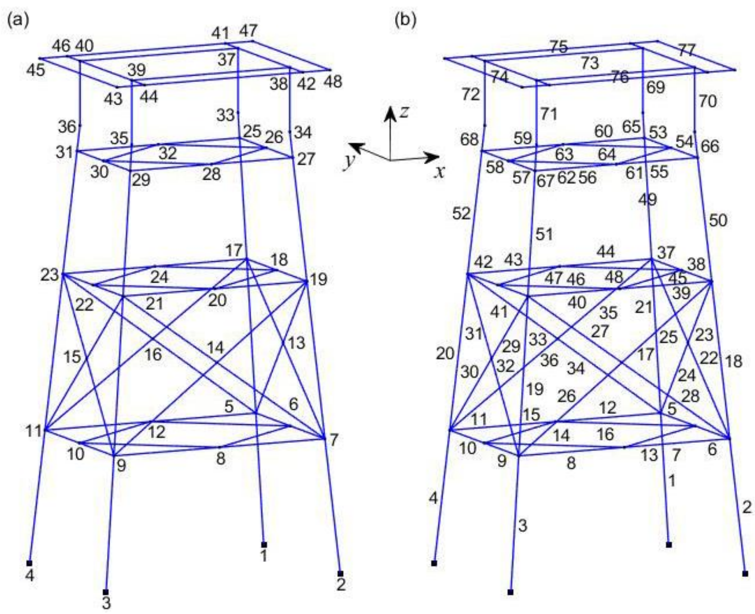

Fig. 1 The sketch of the offshore platform structure: (a) Nodal points; (b) Element numbers

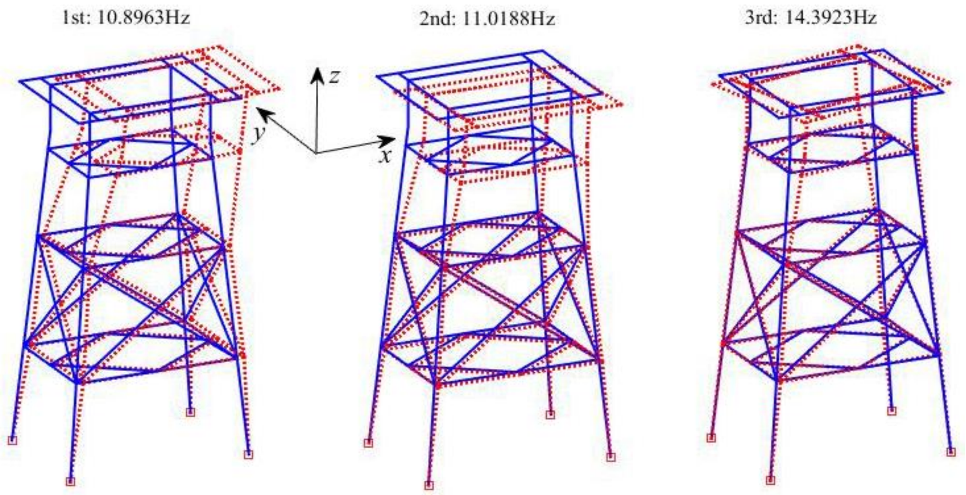

Fig. 2 The first three modal parameters of the baseline model

which are +0.51 and -1.02 m with reference to the still water level, respectively. The platform is fixed on the ground, and the elevations of three stories are +0.23, -0.14, and -0.62 m. The essential material properties are: Young's modulus E = 2.06×10^{11} Pa, mass density ρ = 7850 kg/m^3, and Poisson ratio v = 0.3.

Modal analysis was performed by developing a program in Matlab environment to acquire the modal parameters of the baseline model. The first three natural frequencies (*FREs*) and corresponding mode shapes are exhibited in Fig. 2. The first and second modes are vibrated dominantly in the x (long-span) and y (short-span)-directions, respectively, and the third mode is a torsion mode around z axis.

Table 3 *FREs*, *REFs* and *MACs* of the baseline model and measured structure

Models	Factors	1st	2nd	3rd
Measured	*FREs* (Hz)	11.5864	11.7159	15.1260
Baseline	*FREs* (Hz)	10.8963	11.0188	14.3923
Difference	*REFs* (%)	5.96	5.95	4.85
	MACs (%)	97.88	98.62	99.85

4.2 Measured structure

The measured structure was simulated by the baseline model with some modeling errors because the offshore platform is not readily measured in service. The stiffness and mass correction coefficients of partial (not all) elements are selected and preset as special values to illustrate the effectiveness of the multistage approach. For example, the stiffness coefficients of two LB 18/51, two HB 58/60, two DB 25/30 elements, and mass coefficients of three DP elements 73/74/75 are considered. Note that these correction coefficients are expected to have the same or similar values to avoid ill-conditioned problems. Therefore, α_{18}, α_{25}, α_{30}, α_{51}, α_{58}, α_{60}, β_{73}, β_{74}, and β_{75} are preset as the same values -0.20.

Performing the modal analysis again, one obtains the modal parameters of the measured structure. Shown in Table 3 is the *FREs*, relative errors of frequencies (*REFs*), and modal assurance criteria (*MACs*) (Ewins 2000) between mode shapes derived from the baseline model and measured structures. It is noteworthy that the *FREs* are 10.8963, 11.0188, and 14.3923 Hz with the relative error 5.96, 5.95, and 4.85% to that of the measured structure, respectively. The minimum value of *MACs* is 97.88% in the first mode.

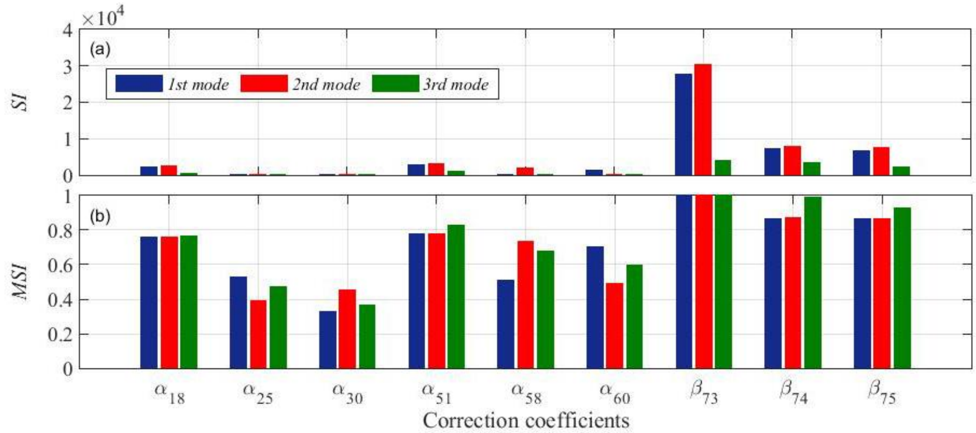

Fig. 3 Sensitivity indices for the first three modes

It is unavoidable that the identified modal parameters always contain errors, which are not only due to the measurement noise but also to the uncertainties in the modal parameter identification. Therefore, the error distribution in modal parameters has a lot of uncertainties. However, the errors in modal parameters are usually simulated by adding a series of uncorrelated random numbers on the theoretically calculated modal parameters in numerical studies (Li *et al.* 2007, 2016, Modak *et al.* 2002) to study the error propagation from these modal parameters to the estimation of other parameters. The primary goal is to understand how sensitive the noise to the estimation of the correction coefficients in the study. Thus the measurement of the *i*-th polluted mode displacement at the *v*-th degrees of freedom (DoFs), denoted by $\hat{\phi}_{iv}^{*}$, is simulated by adding a Gaussian random error to the corresponding actual value.

$$\hat{\phi}_{iv}^{*} = \phi_{iv}^{*}(1 + n_{\phi}\gamma_{\phi}) \tag{21}$$

Where γ_{ϕ} is the Gaussian random number with zero mean and unit standard deviation; n_{ϕ} denotes the noise level; ϕ_{iv}^{*} extracted from $\boldsymbol{\phi}_{i}^{*}$ is the *i*-th measured mode displacement at the *v*-th DoFs. Compared to the noise in mode shapes, the noise in natural frequencies is usually negligible (Hu *et al.* 2006, Messina *et al.* 1998). Thus the proportional noise for the mode shapes is introduced to investigate the noise robustness of the multistage approach.

4.3 Sensitivity analysis and ranking

Sensitivity analysis of these correction coefficients is conducted, and the sensitivity indices are illustrated in Fig. 3, where the top panel is the modal energy-based sensitivity index defined by Eq. (20), and the bottom panel corresponds to the maximum-normalized sensitivity index. It is noteworthy that the sensitivity indices of different correction coefficients vary significantly from each other, as shown in Fig. 3.

Based on the bar map (see Fig. 4) and ranking strategy, nine coefficients are classified into five levels $\boldsymbol{x} = [\boldsymbol{x}_I; \boldsymbol{x}_{II}; \boldsymbol{x}_{III}; \boldsymbol{x}_{IV}; \boldsymbol{x}_V]$, where $\boldsymbol{x}_I = \beta_{73}$, $\boldsymbol{x}_{II} = \{\beta_{74}; \beta_{75}\}$, $\boldsymbol{x}_{III} = \{\alpha_{51}; \alpha_{18}\}$, $\boldsymbol{x}_{IV} = \{\alpha_{58}; \alpha_{60}\}$, and $\boldsymbol{x}_V = \{\alpha_{25}; \alpha_{30}\}$. Note that these DP elements 73/74/75 sensitivities to mass coefficients are higher than LB, HB, and DB elements 18/51, 58/60, and 25/30 sensitivities to stiffness coefficients.

4.4 Model updating results

To demonstrate the multistage approach and verify its effectiveness, the first three measured modes are used, and the corresponding mode shapes are polluted by the small noise n_{ϕ} = 0.5% and large noise n_{ϕ} = 5.0% for these two scenarios. Structural model updating is conducted and compared by the CMCM and multistage approach.

4.4.1 Small measured noise scenario

First, the CMCM method is conducted by using the small noise polluted data for model updating. Given the first three measured modes together with baseline modes, totally nine CMCM equations are constructed for estimating the nine correction coefficients. The estimation sorted by sensitivity ranking $\tilde{\boldsymbol{x}}$ = [β_{73}; β_{74}; β_{75}; α_{51}; α_{18}; α_{58}; α_{60}; α_{25}; α_{30}] is [-0.1977; -0.1943; -0.2070; -0.1845; -0.2086; -0.1822; -0.1892; -0.1687; -0.0610] with relative errors [1.15%; 2.85%; 3.50%; 7.75%; 4.30%; 8.90%; 5.40%; 15.65%; 69.50%] as shown in Fig. 5. It is concluded that these coefficients except α_{30} preferably match the preset value.

Substituting the estimation $\tilde{\boldsymbol{x}}$ into Eq. (7), one obtains the corrected stiffness and mass matrices of the target model. The *FREs* are 11.5855, 11.7187, and 15.1329 Hz, respectively, which are close to the measured ones. Compared to the baseline model, the *REFs* are negligible (e.g., the maximum value 0.05%) and *MACs* are greatly improved in all three modes, as shown in Table 4. Thus, the target model can be accurately obtained, which exactly matches the measured structure by using the CMCM method.

For clarity, the procedure of the multistage approach is showed as follows.

- **Step 1**: Computation and ranking of the sensitivity indices

When the first three modes are chosen as the target modes, maximum-normalized sensitivity indices are computed, as shown in Fig. 4. The nine coefficients are classified into five levels based on ranking strategy, and the following updating procedure is conducted in five stages.

- **Step 2**: Model updating of stage *I*

With the first three measured modes and baseline modes, nine CMCM equations are formed again. Using the SVD algorithm, one obtains the estimation $\tilde{\boldsymbol{x}}^{(1)} =$

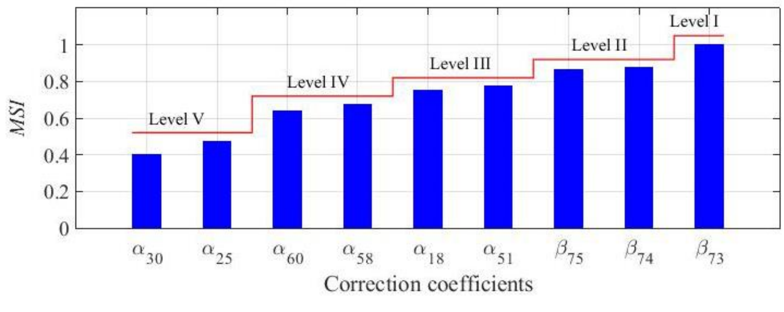

Fig. 4 Maximum-normalized sensitivity index and ranking for the first three modes

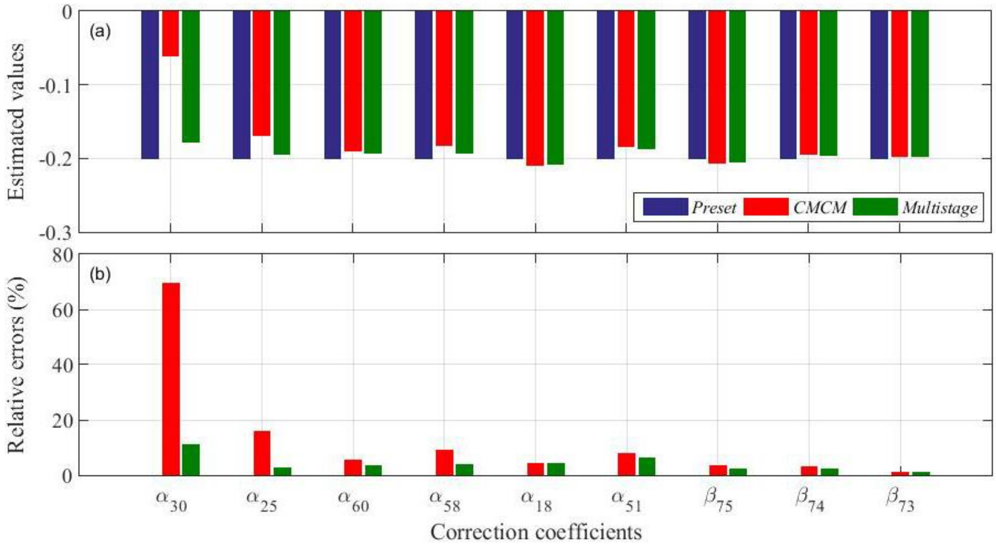

Fig. 5 The estimated values and relative errors in the small noise scenario

Table 4 *FREs*, *REFs* and *MACs* before and after model updating in the small noise scenario

Factors	Modes	Baseline	CMCM	Multistage				
				UM-I	UM-II	UM-III	UM-IV	UM-V
FREs (Hz)	1st	10.8963	11.5855	11.4334	11.7361	11.7361	11.5894	11.5864
	2nd	11.0188	11.7187	11.5613	11.8629	11.7542	11.7193	11.7159
	3rd	14.3923	15.1329	14.7380	15.3238	15.1789	15.1333	15.1260
REFs (%)	1st	5.96	0.01	1.32	1.29	0.29	0.02	0.00
	2nd	5.95	0.02	1.32	1.26	0.33	0.03	0.00
	3rd	4.85	0.05	2.56	1.31	0.35	0.05	0.00
MACs (%)	1st	97.88	100.00	97.89	99.80	99.94	100.00	100.00
	2nd	98.62	100.00	98.63	99.87	99.93	100.00	100.00
	3rd	99.85	100.00	99.85	99.97	99.99	100.00	100.00

$[\tilde{\boldsymbol{x}}_I^{(1)};\tilde{\boldsymbol{x}}_{II}^{(1)};\tilde{\boldsymbol{x}}_{III}^{(1)};\tilde{\boldsymbol{x}}_{IV}^{(1)};\tilde{\boldsymbol{x}}_V^{(1)}]$ (same the estimation $\tilde{\boldsymbol{x}}$ by using CMCM method) with errors [1.15%; {2.85%; 3.50%}; {7.75%; 4.30%}; {8.90%; 5.40%}; {15.65%; 69.50%}]. Note that the correction coefficients with higher sensitivities have the smaller relative errors. Thus only $\tilde{\boldsymbol{x}}_I^{(1)}$={$\beta_{73}$=-0.1977} is preserved and other estimations are set to be zero vectors. Finally the estimation at the first stage is $\tilde{\boldsymbol{x}}^{(1)} = [\tilde{\boldsymbol{x}}_I^{(1)}; \mathbf{0}; \mathbf{0}; \mathbf{0}; \mathbf{0}]$.

Updating the baseline model with $\tilde{\boldsymbol{x}}^{(1)}$, UM-I is obtained and the corresponding results are listed in Table 4. The *FREs* are closer to the measured ones than that of the baseline model. The *REFs* of UM-I are all reduced whereas *MACs* have no significantly improved. The condition number of the coefficient matrix is equal to 1.1437×10^{6}.

- **Step 3**: Model updating of stage *II*

In the second stage, UM-I is regarded as the baseline model. Similarly the nine CMCM equations to identify the remaining correction coefficients can be formed by using the first three measured modes and baseline modes of UM-I. The coefficient matrix is of full column rank and its condition number is 1.0728×10^{5}, which is smaller than that of stage I. The SVD algorithm is also used to solve the over-determined CMCM equations.

The eight coefficients are estimated $\tilde{\boldsymbol{x}}^{(2)} = [\tilde{\boldsymbol{x}}_{II}^{(2)};\tilde{\boldsymbol{x}}_{III}^{(2)};\tilde{\boldsymbol{x}}_{IV}^{(2)};\tilde{\boldsymbol{x}}_V^{(2)}]$ = [{-0.1957; -0.2045}; {-0.1896; -0.1929}; {-0.1825; -0.2086}; {-0.1601; -0.0748}] with the errors [{2.15%; 2.25%}; {5.20%; 3.55%}; {8.75%; 4.30%}; {19.95%; 62.60%}]. Only $\tilde{\boldsymbol{x}}_{II}^{(2)}$= {$\beta_{74}$=-0.1957; β_{75} = -0.2045} is kept and the final estimation is $\tilde{\boldsymbol{x}}^{(2)} = [\tilde{\boldsymbol{x}}_{II}^{(2)}; \mathbf{0}; \mathbf{0}; \mathbf{0}]$. After updating UM-I with $\tilde{\boldsymbol{x}}^{(2)}$, UM-II are also obtained, and the *MACs* of UM-II are improved compared to that of UM-I but *REFs* have no significant improvement.

- **Step 4**: Model updating of stage *III* and *IV*

Similarly, the estimations are $\tilde{\boldsymbol{x}}^{(3)}$ = [α_{51} = -0.1875; α_{18} = -0.2081};**0**;**0**]$ and $\tilde{\boldsymbol{x}}^{(4)}$ = [{α_{58} = -0.1920; α_{60} = -0.1933};**0**] in the stage *III* and *IV*, respectively. Note that the *REFs* are gradually reduced and *MACs* are improved.

- **Step 5**: Model updating of stage *V*

Finally with UM-IV as the baseline model, nine CMCM equations for the last two coefficients are formed using the first three measured modes and baseline modes of UM-IV. The coefficient matrix is full rank and its condition number is only 3. The estimations $\widetilde{\boldsymbol{x}}^{(5)} = \widetilde{\boldsymbol{x}}_V^{(5)}$ = {α_{25} = -0.1946; α_{30} = -0.1783} can be gained easily. Compared to the coefficients ({α_{25} = -0.1687; α_{30} = -0.0610}) from the CMCM method, these coefficients associated with the lowest sensitivity levels are greatly improved.

The *FREs* are 11.5864, 11.7159, and 15.1260 Hz without errors to the measured ones, respectively, and *MACs* are all equal to true values. Thus the baseline model can be updated very well. The finally estimations is $\widetilde{\boldsymbol{x}}^{(1)} = [\widetilde{\boldsymbol{x}}_I^{(1)}; \widetilde{\boldsymbol{x}}_{II}^{(2)}; \widetilde{\boldsymbol{x}}_{III}^{(3)}; \widetilde{\boldsymbol{x}}_{IV}^{(4)}; \widetilde{\boldsymbol{x}}_V^{(5)}]$ with the errors [1.15%; {2.15%; 2.25%}; {6.25%; 4.05%}; {4.00%; 3.35%}; {2.70%; 10.85%}] as shown in Fig. 5. The largest relative error has been greatly reduced to 10.85% compared to the value 69.50% of the CMCM method. It is also concluded that the correction coefficients are estimated accurately by using the multistage approach. Obviously, it has a great downward trend from 1.1437×10^6 to 3 for the condition numbers, which indicates that the ill-condition is greatly improved stage by stage.

Four factors, including the mean of *REFs* (*mREF*), mean of *MACs* (*mMAC*), mean and standard deviation of relative errors of correction coefficients *mREC* and *stdREC*, are utilized and listed in Table 5 for comparison. The *mREF* and *mMAC* are all about the target values. Thus the CMCM and multistage approach have excellent performances on structural model updating in the small noise scenario. Moreover, the *mREC* and *stdREC* are 4.08% and 2.93, which are much less than 13.22% and 21.54 by using CMCM method, respectively. In short, the multistage approach obtains more accurate and stable estimations. Due to the ill-conditioned problem, the small level noise 0.5% could result in a significant change on the estimation, especially for the lower sensitivity coefficients (e.g., α_{30} with 69.50% error) by using CMCM method.

Table 5 Four factors using the CMCM and multistage approach in the small noise scenario

Factors	*mREF* (%)	*mMAC* (%)	*mREC* (%)	*stdREC*
CMCM	0.03	100.00	13.22	21.54
Multistage	0.00	100.00	4.08	2.93

4.4.2 Large measured noise scenario

Similarly, using the first three corrupted measured modes with large noise and baseline modes, the estimated result, relative errors, and other factors are shown in Fig. 6 and Table 6. One can see that the *REFs* are all about 0.30%, and *MACs* are greater than 99.93%. The CMCM method could give a good agreement between the target model and the measured structure, even in the large noise scenario. However, these correction coefficients are deviated far away from their counterparts, especially for those with lower sensitivity. These coefficients with higher sensitivity have more accurate estimations (e.g., β_{73} and β_{74}), and contrarily the errors are larger (e.g., α_{25} and α_{30} with errors 90.30 and 200.80%). Further, α_{30} is equal to 0.2016, which is very different from the preset value. The main reason for this is the large condition number 4.1076×10^8 caused by huge sensitivity differences between these coefficients.

As shown in Fig. 6 and Table 6, the multistage approach could give better agreement and obtain more accurate and stable estimation than that of CMCM method in the large noise scenario. Although the estimation α_{25} is improved, they still have the biggest relative error than others. Conditioning of the updating equations is not the main reason, because the condition number is less than 10 in stage *V*. Much of this is due to errors caused by the first four stages accumulate. Thus the estimation in the last level usually has poor reliability.

4.5 Discussions

4.5.1 Effects of sensitivity rankings

Classified by artificially, the other type of sensitivity ranking is considered here. Totally nine parameters can be classified into three levels $\boldsymbol{x} = [\boldsymbol{x}_I; \boldsymbol{x}_{II}; \boldsymbol{x}_{III}]$, where $\boldsymbol{x}_I$ = {β_{73}; β_{74}; β_{75} }, $\boldsymbol{x}_{II}$ = {α_{51}; α_{18}; α_{58}; α_{60}}, and $\boldsymbol{x}_{III}$ = {α_{25}; α_{30}} using the same interval ΔS = 0.2. As shown in Fig. 7 and Table 7, the three stage approach can also give a better agreement and estimation (except α_{25}). Note that the *mREC* and *stdREC* of five stages approach are smaller than that of three stages, and it has best performance. The error of coefficient in Level *II* α_{60} of three stages is considerably larger than that of five stages. One of reasons is that

Table 6 *FREs*, *REFs* and *MACs* before and after model updating in the large noise scenario

Factors	Modes	Baseline	CMCM	Multistage
FREs (Hz)	1st	10.8963	11.5462	11.5826
	2nd	11.0188	11.6761	11.7152
	3rd	14.3923	15.0873	15.1483
REFs (%)	1st	5.96	0.35	0.03
	2nd	5.95	0.34	0.01
	3rd	4.85	0.26	0.15
MACs (%)	1st	97.84	99.93	99.97
	2nd	98.63	99.94	99.97
	3rd	99.84	99.97	99.99

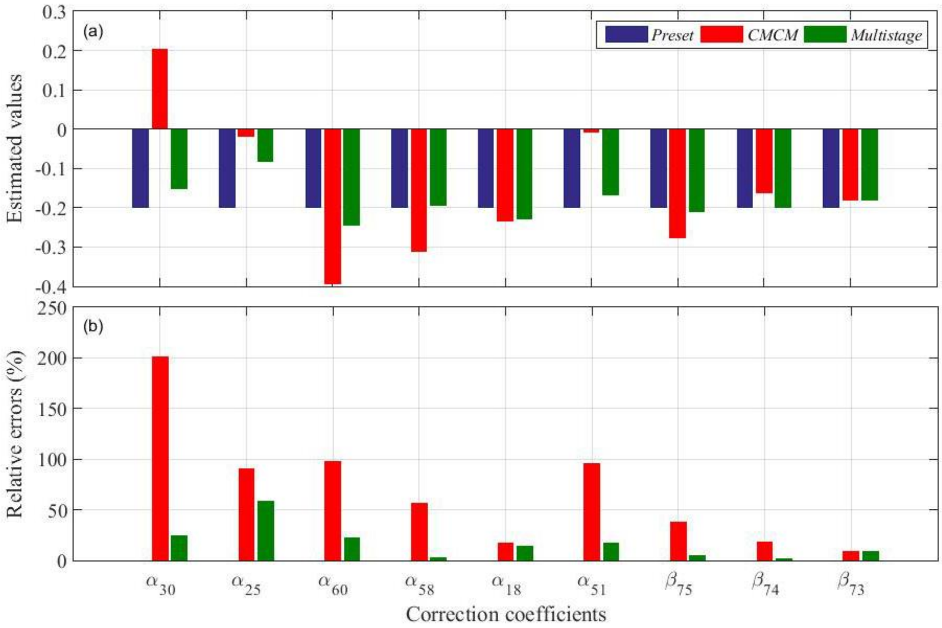

Fig. 6 The estimated values and relative errors in the large noise scenario

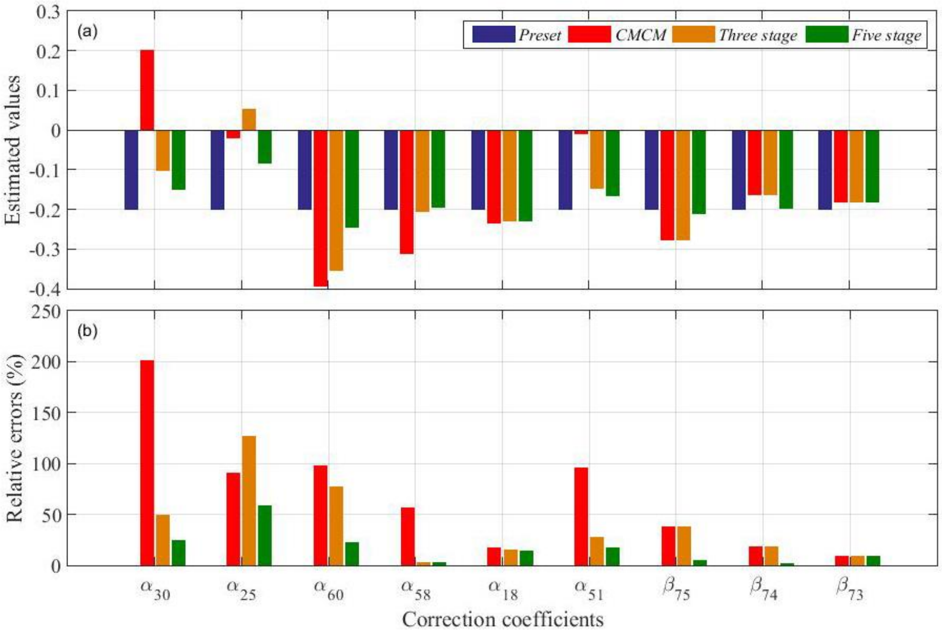

Fig. 7 The estimated values and relative errors using the CMCM and three/five stages approach

Table 7 Four factors using different sensitivity rankings

Factors	*mREF* (%)	*mMAC* (%)	*mREC* (%)	*stdREC*
CMCM	0.32	99.95	69.19	60.50
Three stage	0.09	99.97	40.35	39.65
Five stage	0.06	99.98	17.06	17.63

condition numbers are still very large (than 10^5), which results in a bigger error for α_{60} and simultaneously gives a larger accumulated error to stage *III* (α_{25}). Through the comparative research, we think that automatic ranking with equal intervals may be not suitable in some instances and human judgment according to the distribution of the bar map is indispensable.

4.5.2 Effects of mode combinations

The number of the measured mode N_j is set to be 3 due to the high-order modal parameters are not available in practice. Three combinations N_i = 3, 5, and 9 & N_j = 3 are investigated under large measured noise. The performances of the four factors turn better with the increase of the baseline modes, as shown in Table 8. Maximum values of *mREC* and *stdREC* are 17.06% and

Table 8 Four factors using different mode combinations

Combinations	Methods	*mREF* (%)	*mMAC* (%)	*mREC* (%)	*stdREC*
N_i = 3 & N_j = 3	CMCM	0.32	99.95	69.19	60.50
	Multistage	0.06	99.98	17.06	17.63
N_i = 5 & N_j = 3	CMCM	0.20	99.98	61.06	52.76
	Multistage	0.05	99.99	16.01	16.36
N_i = 9 & N_j = 3	CMCM	0.17	99.98	60.89	50.13
	Multistage	0.02	99.99	15.46	16.16

17.63, which are greatly improved than that of CMCM method even with N_i = 3 baseline modes, respectively. It is indicated that the multistage approach guarantees superb performances on structural model updating with a few of modal parameters.

5. Experimental validation

5.1 Experiment setup

Model updating was carried out for the prototype of the simulated numerical structure given in Section 4. The experimental structure (see Fig. 8) was with a height of 1.53 m and was fixed to the ground via bolts. A steel plate was installed at the structure top to simulate the topside mass. The elevations of four stories of the structure were 1.53, 1.25, 0.88, and 0.40 m, from top to bottom, respectively.

Fig. 8 The measured structure of offshore platform

Vibration responses were excited via a hammer by beating on the corner of the top plate. Totally 26 accelerometers were installed on the joints of the structure to collect the vibration signals in 62 DoFs, of which 18 three-component acceleration sensors (Model 4803A-0002) were installed at nodal points 13-16/17/19/21/23/25-32/37/39}, and the other 8 one-component acceleration sensors (Model 2220-002) were installed at nodal points 5/7/9/11 in the x or y direction. These sensors were connected to a CRONOS PL 64-DCB8 dynamic data acquisition system. Fig. 9 shows two typical acceleration signals with the 500 Hz sampling rate with a duration of 10 s. The Eigen-system realization algorithm (Wang and Liu 2010) was used for identifying the modal parameters. Because only partial DoFs were measured, an interpolation modal expansion technique based on the optimal fitting method (Zhang and Wei 1999) was used to obtain spatially-completeness mode shapes. The identified *FREs* are 10.9026, 11.0321, and 14.7827 Hz, respectively. Note that the baseline model of the measured structure was established as given in Section 4. The third frequency of the baseline model is 14.3923 Hz, with a relative difference 2.64% to that of the measured structure. The *MACs* are 93.13, 91.81, and 99.99% between the mode shapes of the baseline model and those of the measured structure.

5.2 Sensitivity analysis and ranking

The special components (e.g., the flange replacements, as shown in Fig. 8) that would change the dynamic features of the structure were not considered into the baseline model. Thus the baseline model has to be updated to eliminate the modeling errors. In addition, the plate was divided into five elements with different masses. For simplicity, the lumped mass matrix was used for these elements in the modeling, which could also cause modeling errors unavoidable, especially for the third torsional mode. Thus the mass of the top plate is also under an obligation to be updated. In this study, these correction coefficients α_{25}, α_{51}, α_{60}, β_{73}, β_{74}, β_{75}, β_{76}, and β_{77} are selected in the experimental validation.

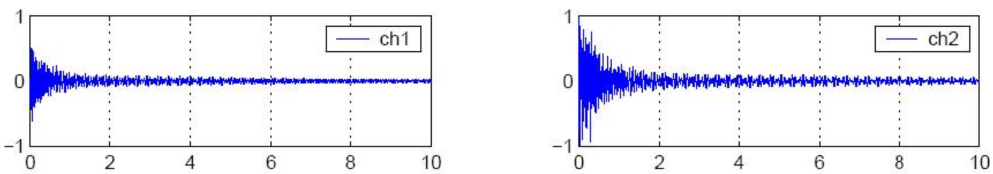

Fig. 9 The typical acceleration signals of measured structure

Table 9 *MSI* and sensitivity ranking in experimental validation

Correction coefficients	α_{25}	α_{60}	α_{51}	β_{75}	β_{76}	β_{74}	β_{77}	β_{73}
MSI	0.419	0.590	0.743	0.862	0.862	0.867	0.867	1.000
Ranking A	III	III	II	I	I	I	I	I
Ranking B	V	IV	III	II	II	II	II	I

Table 10 *FREs*, *REFs* and *MACs* before and after model updating in experimental validation

Factors	Modes	Baseline	Three stage	Five stage
FREs (Hz)	1st	10.8963	10.9090	10.9030
	2nd	11.0188	11.0290	11.0310
	3rd	14.3923	14.7778	14.7831
REFs (%)	1st	0.06	0.06	0.00
	2nd	0.12	0.03	0.01
	3rd	2.64	0.03	0.00
MACs (%)	1st	93.13	93.57	94.74
	2nd	91.81	92.45	93.72
	3rd	99.99	99.99	100.00

Sensitivity analysis and ranking strategy were conducted and showed in Table 9. These coefficients were divided into three and five levels with the same interval ΔS = 0.2 and gradient distribution, respectively.

5.3 Model updating results

Given the first three measured modes together with five baseline modes, fifteen CMCM equations were constructed. The estimations are $\tilde{\boldsymbol{x}}$ = [β_{73}; β_{77}; β_{74}; β_{76}; β_{75}; α_{51}; α_{60}; α_{25}] = [0.0252; -0.1817; -0.1776; 0.0748; -0.0057; 0.1512; -0.6840; -1.4043]. Note that the correction coefficient α_{25} has no physical meaning because these coefficients are generally not less than -1 in practice, and thus be discarded. The estimation using the multistage approach is $\tilde{\boldsymbol{x}}$ = [0.0252; {-0.2288; -0.3021; 0.2473; 0.1896}; 0.0018; -0.0007; -0.0086]. The *REFs* are negligible and *MACs* are larger than that of the baseline model as shown in Table 10. It is concluded that the multistage approach can achieve better agreement between the target model and the measured structure. The correction coefficients has definite physical meaning. Same conclusion was drawn from the three stage approach.

6. Conclusions

A novel multistage approach is developed based on sensitivity ranking for structural model updating. Modal energy-based sensitivities are formulated, and maximum-normalized sensitivity indices are developed for sensitivity ranking. Based on the ranking strategy, a multistage model updating strategy is proposed, where updating parameters with the same sensitivity level are estimated simultaneously in one single stage, and the whole updating procedure continues sequentially in several stages. In each single stage, the previously developed cross model cross mode (CMCM) method is used for model updating. Numerical study and experimental validation are examined to verify the effectiveness of the proposed multistage approach. Compared to CMCM method, three main conclusions are drawn as follows.

- The multistage approach guarantees more stable and accurate estimations for structural model updating in the presence of noise and a few of the lower order modal parameters in the numerical study. Better agreements are also achieved between the target model and measured structure in the numerical study and experimental validation.
- The precision and accuracy of correction coefficients are greatly improved by using the multistage approach.
- Ill-conditioned problems of CMCM equations with polluted noise are significantly improved by means of the multistage approach.

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