Free vibration analysis of magneto-rheological smart annular three-layered plates subjected to magnetic field in viscoelastic medium

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Abstract. Magneto-rheological fluids and magneto-strictive materials are of the well-known smart materials which are used to control and reduce the vibrations of the structures. Vibration analysis of a smart annular three-layered plate is provided in this work. MR fluids are used as the core's material type and the face sheets are made from MS materials and is assumed they are fully bonded to each other. The structure is rested on visco-Pasternak foundation and also is subjected to a transverse magnetic field. The governing motion equations are derived based on CPT and employing Hamilton's principle and are solved via GDQ as a numerical method for various boundary conditions. Effect of different parameters on the results are considered and discussed in detail. One of the salient features of this work is the consideration of MR fluids as the core, MS materials as the faces, and all of them under magnetic field. The outcomes of this study may be led to design and create smart structures such as sensors, actuators and also dampers.

Keywords: smart materials; magneto-rheological fluids; magneto-strictive materials; vibration analysis; annular plate; visco-pasternak foundation

1. Introduction

Nowadays scientists are trying to use new materials to improve the mechanical responses of the structures. Smart materials can be named of the best materials which can adopt the mechanical behaviors of the structures in accordance with the desired ones. Using smart materials leads to achieving the desired action from the engineering structures. Magneto-strictive (MS) materials are from one of the well-known smart materials. They deform when are subjected to a magnetic field. Cobalt, iron, nickel, and ferrite are known as MS materials. These materials are appropriate to provide enormous forces, strains, high energy densities, noise, and vibration control and can be used as sensors and actuators (Squire 1999, Duc et al. 2015a, 2016a, Tabbakh and Nasihatgozar 2018, Duc 2018, Duc and Cong 2018, Zucca et al. 2015). The MS strain originates from the rotation of the atomic magnetic moment in a magnetic field without changing crystallographic orientation or structure (Ghorbanpour Arani et al. 2017b). MS materials are taken into consideration by the researchers, recently. The analysis of a curved beam with MS layers was provided by Bayat et al. (2015). They used Terfenol-D as the MS material. Ebrahimi and Dabbagh (2018b) studied about MS sandwich composite nanoplates. They used classical plates theory (CPT) to describe the

*Corresponding author, Assistant Professor, E-mail: samir@kashanu.ac.ir; saeid_amir27111@yahoo.com displacements of the structure and modified strain gradient theory to take into account the small scale effect and also solved the equations analytically. Ghorbanpour Arani and Khoddami Maraghi (2016) employed sinusoidal shear deformation theory to consider the vibrational behavior of an MS plate. They concluded MS materials help to control vibrations of the structures. Suman et al. (2017) presented bending and strength analyses of the laminated plate with MS layers. They used ANSYS to analyze their model. The results for the effect of thermo-magnetic fields on the behavior of three-layered MS plates which its top and bottom layers were made of ceramic presented by Ebrahimi and Dabbagh (2018a). Their study was on small scale and considered different variants effects. Trigonometric higher order shear deformation theory used by Ghorbanpour Arani et al. (2017b) to control vibrations of the MS plate which subjected multi-physical was to loads Mohammadrezazadeh and Jafari (2019) used classical shells theory to consider vibration control of a laminated truncated conical shell which was integrated by MS layers. They obtained the results via Galerkin method and compared them with those of finite element software. Also, transient response of an FG nanobeam which two MS layers was bonded to its top and bottom surfaces presented by Ghorbanpour Arani and Abdollahian (2017). They used modified couple stress theory to account the small scale effect.

Rheological fluids are a well-known branch of the smart materials which are attracted the scientist attention recently that includes two types, namely magneto-rheological (MR) and electro-rheological (ER) which are sensitive to magnetic and electric fields, respectively. MR fluids refer to

the colloidal suspensions which are made from ferrous particles in the low permeability oil. By exposure to a magnetic field, ferrous particles attached to each other quickly, form chain-like in the applied field direction and MR fluids are converted to quasi-solid materials from the liquid phase. The most important features of MR fluids that encouraged the scientist to use them in the smart devices and structures are their high capacity for vibration damping, quick response, good reversibility, and controllable performance (Duan et al. 2019, Huang et al. 2019, Tzou et al. 2004, Zhang et al. 2019). An experimentally vibrations test of MR cantilever sandwich beams was provided by Lara-Prieto et al. (2010). They considered both partial and full activation effects of the MR beams and changes in natural frequencies shows the controllability of the structure's vibration. Sandwich rectangular plates vibrations with MR elastomer damping presented by Yeh (2013). He found that the MR elastomer has a noteworthy effect on the vibrational response of the rectangular plate and obtained the results for different values of magnetic field intensity. Manoharan et al. (2014) carried out a dynamic analysis of laminated composite MR fluids plate and the motion equations were presented in FE formulation form and considered variations of frequencies and loss factor by changing the other parameters such as a magnetic field. The vibration of a partially treated laminated composite MR fluid sandwich rectangular plate considered by Ramamoorthy et al. (2016). They presented the results for both free and forced vibration cases. Eshaghi et al. (2015) presented both experimental and theoretical analyses about the effect of MR fluid on the vibration of sandwich plates. They considered two different sandwich plates with polyethylene terephthalate face layers. It is noticeable that they employed classical plate theory (CPT) to extract the motion equations. Babu and Vasudevan (2016) carried out a dynamic analysis of tapered laminated sandwich plate which they used MR elastomer as the core and composite laminates as the face sheets. They derived governing equations based on CPT and solved them numerically and presented the results for featured parameters such as applied field intensity. Naji et al. (2016) used layerwise theory to obtain more accurate results about the dynamic characterization of beam structures with MR layers and solved the equations via finite element method (FEM). They provided an experimental set-up to validate FE model. They investigated vibrational behavior of laminated composite beams with MR layers in another study (Naji et al. 2018). They used experimental tests to validate their analytical obtained results. MalekzadehFard et al. (2017) discussed mechanical behavior cylindrical sandwich panels with MR fluid core. They analyzed vibration and buckling behaviors of the mentioned structure for a simply supported cylinder based on higher order panel theory and validated their results with that of simulated with ABAQUS.

Different displacement fields have been used by the researchers to analyze the structures' behaviors. For example, CPT neglects from the shear deformation effect, while the higher-order ones take the shear deformation effect into account. Bui and Nguyen (2011) used a novel

meshfree method for free vibration analysis of classical Kirchhoff's plates. In another study, Bui et al. (2011a) analyzed buckling of plates subjected to uniformly uniaxial, biaxial in-plane compression and pure shear loads using an efficient novel meshfree method based on Reissner-Mindlin plate theory. Minh and Duc (2019) discussed the effect of cracks on the stability of the functionally graded (FG) plates higher-order variable-thickness using with shear deformation theory (HSDT). Buckling isogeometric analysis of FG plates under combined thermal and mechanical loads provided by Yu et al. (2017). Thom et al. (2017) analyzed bi-directional FG plates by FEM and a new third-order shear deformation plate theory. Nonlinear dynamic analysis of Sigmoid FG circular cylindrical shells on elastic foundations using the third-order shear deformation theory in thermal environments provided by Duc et al. (2015b). Bui et al. (2016) presented new numerical results of mechanical behaviors of FG plates in high temperatures. They developed a FEM based on a new third-order shear deformation plate theory. An effective isogeometric analysis for modeling laminated composite plates with cutouts carried out by Yu et al. (2016). They described cutouts by the level set method. An efficient meshfree method for vibration analysis of laminated composite plates presented by Bui et al. (2011b). Their formulation was based on the CPT while the moving Kriging interpolation satisfied the delta property was employed to construct the shape functions. In another study, Bui et al. (2009) used a similar interpolation-based meshless method for numerical simulation of Kirchhoff's plate problems. Moreover, in series of paper, the influence of different structures' theories on dynamic response and also their structural behavior in different thermo-mechanical environments considered by various authors (Duc 2014, Duc et al. 2015c, Anh et al. 2015, Chan et al. 2019, Minh et al. 2018).

The aim of this work is to analyze the vibrational behavior of a smart annular sandwich plate which is made from an MR fluid core and two MS face sheets. The structure is located on visco-Pasternak medium and is subjected to a magnetic field. To the best author's knowledge, there is no study about such a structure in the literature. Three different MR fluid types are considered as the core's type. The kinematics relations are on the basis of CPT which neglects shear deformation effects and also the relation between strains and displacements are investigated according to the Von-Karman assumptions. Using energy method and Hamilton's principle the motion equations are extracted and are solved for various boundary conditions by employing GDQ as an accurate and rapid-convergence numerical method. Influences of the most prominent parameters of the structure such as the geometrical size of it and also different MR fluid types, applied magnetic field, velocity feedback gain and visco-Pasternak medium are considered in detail. The outcomes of this work may be useful to design more accurate smart structures and devices such as sensors and actuators.



Fig. 1 Schematic of the under consideration smart annular sandwich plate

2. Mathematical formulations

One MR smart annular plate which is located between two MS layers is taken under investigation as can be seen in Fig. 1. The origin of the coordinate system is located at the center of the mid-plane of the core. The inner radius of the plate is shown by *a* and the outer one is *b*. also, h_i (i = t, c, b) represents the thickness of each layer. To consider the effect of the medium, a viscoelastic type foundation is selected and the structure is rested on.

The formulations are presented based on the following assumptions:

- The core and face layers are fully bonded to each other and their transverse displacement is the same.
- The Young's elasticity modulus of the MR fluid is lower than MS face sheets. Therefore, the normal stresses of the MR fluids vanish.
- The shear strains of the core are much greater than the face layers. So, those of the face layers are relinquished.

CPT is employed to describe the displacement components of the faces. Based on the CPT, the effect of shear deformations is not taken into consideration and the displacements are presented as follows (Arshid and Khorshidvand 2018)

$$U_{1i}(r,\theta,z,t) = u_i(r,\theta,t) - z \frac{\partial w(r,\theta,t)}{\partial r},$$

$$U_{2i}(r,\theta,z,t) = v_i(r,\theta,t) - z \frac{\partial w(r,\theta,t)}{r\partial \theta},$$

$$U_3(r,\theta,z,t) = w(r,\theta,t)$$
(1)

in which U_{1i} , U_{2i} (i = t, b), and U_3 are displacements of an

arbitrary point of the faces in the radial, tangential and transverse directions, respectively and u_i , v_i and w are those of their mid-plane.

Also, the strain-displacement relations are expressed based on the Von-Karman assumptions which indicate (Brush *et al.* 1975)

$$\varepsilon_{ij} = \frac{1}{2} [\nabla U_k + (\nabla U_k)^T], \qquad k = 1,2,3$$
(2)

where ε_{ij} (*i*, *j* = *r*, θ , *z*) are the strain components.

The constitutive relations for each of the face sheets may be demonstrated as (Ghorbanpour Arani and Khoddami Maraghi 2016)

$$\begin{cases} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{r\theta} \end{cases} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{44} \end{bmatrix} \begin{cases} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \gamma_{r\theta} \end{cases} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ H_z \end{pmatrix}$$
(3)

where σ_{ij} and ε_{ij} are stress and strain tensors, respectively. e_{ij} and H_z also are the MS coupling moduli and external transverse magnetic field, respectively. Furthermore, c_{ij} denotes the MS stiffness components which are expressed as

$$c_{11} = \frac{E_{MS}}{1 - v_{MS}^2}, \quad c_{12} = v_{MS}c_{11}, \quad c_{44} = \frac{E_{MS}}{2(1 + v_{MS})}$$
(4)

in which E_{MS} denotes Young's elasticity modulus of MS layers and v_{MS} represents their Poisson's ratio.

The generated transverse magnetic field may be defined as (Ebrahimi and Dabbagh 2018a)

$$H_z = K_c I(r, \theta, z, t) \tag{5}$$

where coil constant is shown by K_c and depends on the coil's turns (n_c) , width (b_c) and radius (r_c) which can be determined using $K_c = \frac{n_c}{\sqrt{b_c^2 + 4r_c^2}}$. Also, the coil current $(I(r,\theta,z,t))$ is defined as $C(t) \frac{\partial w(r,\theta,z,t)}{\partial w(r,\theta,z,t)}$ where C(t) is the

 $(I(r,\theta,z,t))$ is defined as $C(t) \frac{\partial w(r,\theta,z,t)}{\partial t}$ where C(t) is the constant control gain. By introducing the velocity feedback gain (K_{vcf}) as the multiple of coil constant and control gain, the transverse magnetic field can be rewritten as follow

$$H_z = K_{vcf} \frac{\partial U_3(r, \theta, z, t)}{\partial t}$$
(6)

To determine MS coupling moduli, the following relations can be employed

$$e_{31} = e_{31MS} \cos^2 \psi + e_{32MS} \sin^2 \psi,
e_{32} = e_{32MS} \cos^2 \psi + e_{31MS} \sin^2 \psi,
e_{34} = (e_{31MS} - e_{32MS}) \cos \psi \sin \psi$$
(7)

where the angle of induced magnetic anisotropy is shown by ψ .

As stated before, the core' type is from MR fluids. The shear properties of MR fluids are presented by numerous researchers. Pre- and post-yield are the regions that both shear stress and strain of MR fluids are characterized in. Also, should be noted that according to experimental tests, the magnetic field has a significant effect on their behavior, especially their shear properties. In the pre-yield region, the shear stress and strain are related to each other on the basis of linear viscoelastic theory as below (Ghorbanpour Arani *et al.* 2017a)

$$\{\tau_{rz}, \quad \tau_{\theta z}\} = G_c\{\gamma_{rz}, \quad \gamma_{\theta z}\}$$
(8)

The complex shear modulus of MR fluid is presented by G_c which introduced the viscoelastic behavior in the preyield regime

$$G_c = G' + iG'' \tag{9}$$

where G' is the storage modulus which depicts the average energy stored of MR fluid during a cycle of deformation and G'' is the loss modulus which is related to average energy dissipated during a cycle. The MR fluid behavior in the post-yield regime can be investigated like to the Bingham plastic model, approximately

$$\tau = \tau_y + \mu \left(\frac{\partial \gamma}{\partial t}\right) \tag{10}$$

Here, τ_y , μ and $\frac{\partial \gamma}{\partial t}$ are respectively the induced dynamic yield stress magnetic field, the plastic viscosity, and rate of shear strain.

For the MR fluid core, the shear strains can be defined as follows (Ghorbanpour Arani *et al.* 2017b)

$$\gamma_{rz}^c = \frac{\partial U_3}{\partial r} + \frac{\partial u_c}{\partial z},\tag{11}$$

$$\gamma_{\theta z}^{c} = \frac{\partial U_{3}}{r \partial \theta} + \frac{\partial v_{c}}{\partial z}$$
(12)

Based on the geometric relationship between the displacements as shown in Fig. 2 for before and after deformation (Chen and Hansen 2005), those of the MR fluid core can be described as follow

$$\frac{\partial u_c}{\partial z} = \frac{\left[\left(\frac{(h_t + h_b)}{2}\right)\left(\frac{\partial U_3}{\partial r}\right) + (u_t - u_b)\right]}{h_c},$$
(13)

$$\frac{\partial v_c}{\partial z} = \frac{\left[\left(\frac{(h_t + h_b)}{2}\right)\left(\frac{\partial U_3}{r\partial \theta}\right) + (v_t - v_b)\right]}{h_c}$$
(14)



Replacing Eqs. (13)-(14) into Eqs. (11)-(12), the following relations for the MR fluid core may be achieved

$$\gamma_{rz}^{c} = \left[\frac{\left(\frac{h_{t}}{2} + h_{c} + \frac{h_{b}}{2}\right)}{h_{c}}\right]\frac{\partial U_{3}}{\partial r} + \frac{(u_{t} - u_{b})}{h_{c}},\qquad(15)$$

$$\gamma_{\theta z}^{c} = \left[\frac{\left(\frac{h_{t}}{2} + h_{c} + \frac{h_{b}}{2}\right)}{h_{c}}\right] \frac{\partial U_{3}}{r \partial \theta} + \frac{(v_{t} - v_{b})}{h_{c}}, \quad (16)$$

To consider the effect of different types of MR fluids, in the present study three types of them are investigated. The first type was presented by Yeh (2014) and has the following specifications

$$G' = -3.3691B^2 + 4997.5B + 873000,$$

$$G'' = -0.9B^2 + 812.4B + 185500,$$
 (17)

$$\rho = 3500$$

The second type of MR fluid was presented by Ramamoorthy *et al.* (2016) as follow

$$G' = -0.05035B^{2} + 428.455B + 858.8,$$

$$G'' = -0.057B^{2} + 452.105B + 848.35,$$
 (18)

$$\rho = 2812$$

And Aguib *et al.* (2014) presented the third type of MR fluid with the following specifications

$$G' = -3.238 \times 10^{-6}B^3 + 0.02733B^2 +38.29B + 1600000, G'' = -6.889 \times 10^{-6}B^3 + 0.02122B^2 +70.11B + 330000, \rho = 1100$$
(19)

Noted that the above-mentioned specifications are obtained based on experimental tests and B denotes the magnetic field intensity in Gauss.

3. Governing equations

Energy method and Hamilton's principle are used in this work to extract the governing motion equations and



Fig. 2 Geometrics of MR fluids before and after deformation

boundary conditions of the smart annular plate as following (Ghorbanpour Arani *et al.* 2019, Karami and Shahsavari 2019)

$$\int_0^t \delta[U - T - V] dt = 0$$
 (20)

in which U, T, and V are strain energy, kinetic energy, and works of external loads. The total strain energy of the smart plate may be expressed as follow (Arshid *et al.* 2019c, Sidhoum *et al.* 2018)

$$U = 0.5 \int_{faces} \int_{r} \int_{\theta} (\sigma_{ij} \varepsilon_{ij}) r \, dr \, d\theta \, dz + 0.5 \int_{core} \int_{r} \int_{\theta} (\tau_{kz}^{c} \gamma_{kz}^{c}) r \, dr \, d\theta \, dz$$
(21)

in which ij = rr, $\theta\theta$ and $r\theta$ and k = r, θ .

Also, the total kinetic energy of the structure can be obtained using the below relation (Amir *et al.* 2019a, Amir 2019)

$$T = 0.5 \int_{faces} \int_{r} \int_{\theta} \rho_{t,b} \left[\left(\frac{\partial U_{1}}{\partial t} \right)^{2} + \left(\partial \frac{U_{2}}{\partial t} \right)^{2} + \left(\frac{\partial U_{3}}{\partial t} \right)^{2} \right] r \, dr \, d\theta \, dz$$
$$+ 0.5 \int_{core} \int_{r} \int_{\theta} \rho_{c} \left[\left(\frac{\partial U_{3}}{\partial t} \right)^{2} + z^{2} \left(\left(\frac{\partial \gamma_{rz}^{c}}{\partial t} \right)^{2} + \left(\frac{\partial \gamma_{\theta z}^{c}}{\partial t} \right)^{2} \right) \right] r \, dr \, d\theta \, dz$$
(22)

Noted that the kinetic energy of the MR fluid core is due to the transverse motion and the rotational deformation of the core as can be seen in the above relation.

Also, the smart plate is rested on visco-Pasternak elastic foundation. The force due to this type of elastic foundation can be demonstrated as (Arshid *et al.* 2019a, Mohammadimehr *et al.* 2019, Duc *et al.* 2017)

$$F = K_1 w(r, \theta, t) - K_2 \nabla^2 w(r, \theta, t) + D\left(\frac{\partial w(r, \theta, t)}{\partial t}\right)$$
(23)

where K_1 , K_2 , and D are the springs, shear layer, and dampers constants, respectively and ∇^2 denotes Laplacian operator. Consequently, the work of the foundation can be stated as (Amir *et al.* 2018, Guerroudj *et al.* 2018, Duc *et al.* 2016b)

$$V = 0.5 \int_{r} \int_{\theta} (F.U_3) \ r \, \mathrm{d}r \, d\theta \tag{24}$$

Rewriting the strains in terms of displacements and inserting in the strain and kinetic energies and also external work relations, using the variational formulation and by mathematical manipulations, the governing motion equations in the axial-symmetric case will be obtained. It should be noted that in axial-symmetric case, the derivatives respect to θ will be removed

$$\delta u_{t}: \quad rA_{110}u_{t}^{"} + (A_{110})u_{t}^{'} - \left(\frac{rG_{1}}{h_{c}^{2}} + \frac{A_{220}}{r}\right)u_{t} \\ + \left(\frac{rG_{1}}{h_{c}^{2}}\right)u_{b} - rA_{111}w^{"} - (A_{111})w^{"} \\ - \left(\frac{rG_{1}d}{h_{c}^{2}} - \frac{A_{221}}{r}\right)w^{'} - rP_{310}\left(\frac{\partial w^{'}}{\partial t}\right)$$
(25)

$$-(P_{310} - P_{320})\left(\frac{\partial w}{\partial t}\right) - \left(I_0 + \frac{J_2}{h_c^2}\right)r\left(\frac{\partial^2 u_t}{\partial t^2}\right) + \left(\frac{rJ_2}{h_c^2}\right)\left(\frac{\partial^2 u_b}{\partial t^2}\right) - \left(\frac{J_2 d}{h_c^2} - I_1\right)r\left(\frac{\partial^2 w}{\partial t^2}\right) = 0$$

$$rG_1 \qquad (25)$$

$$\begin{split} \delta u_{b} : \quad \frac{rG_{1}}{h_{c}^{2}} u_{t} + rB_{110}u_{b}^{"} + (B_{110b})u_{b}^{'} \\ &- \left(\frac{B_{220}}{r} + \frac{rG_{1}}{h_{c}^{2}}\right)u_{b} - rB_{111}w^{"} - (B_{111})w^{"} \\ &+ \left(\frac{B_{221}}{r} + \frac{rG_{1}d}{h_{c}^{2}}\right)w^{'} - rT_{310}\left(\frac{\partial w^{'}}{\partial t}\right) \\ &+ (T_{320} - T_{310})\left(\frac{\partial w}{\partial t}\right) + \left(\frac{rJ_{2}}{h_{c}^{2}}\right)\left(\frac{\partial^{2}u_{t}}{\partial t^{2}}\right) \\ &- \left(\frac{J_{2}}{h_{c}^{2}} + Y_{0}\right)r\left(\frac{\partial^{2}u_{b}}{\partial t^{2}}\right) + \left(\frac{J_{2}rd}{h_{c}^{2}} + Y_{1}r\right)\left(\frac{\partial^{2}w^{'}}{\partial t^{2}}\right) \\ &= 0 \end{split}$$
(26)

$$\begin{split} \delta w: \quad rA_{111}u_{t}^{"} + (2A_{111})u_{t}^{"} + \left(\frac{drG_{1}}{h_{c}^{-2}} - \frac{A_{221}}{r}\right)u_{t}^{'} \\ &+ \left(\frac{A_{221}}{r^{2}} + \frac{dG_{1}}{h_{c}^{-2}}\right)u_{t} + rB_{111}u_{b}^{"} + (2B_{111})u_{b}^{"} \\ &- \left(\frac{B_{221}}{r} + \frac{drG_{1}}{h_{c}^{-2}}\right)u_{b}^{'} + \left(\frac{B_{221}}{r^{2}} - \frac{dG_{1}}{h_{c}^{-2}}\right)u_{b} \\ &- (A_{112} + B_{112})rw^{(4)} - 2(A_{112} + B_{112})w^{"} \\ &+ \left(\frac{A_{222}}{r^{2}} + \frac{B_{222}}{r} + \frac{d^{2}rG_{1}}{h_{c}^{-2}} + rK_{2}\right)w^{"} \\ &- \left(\frac{A_{222}}{r^{2}} + \frac{B_{222}}{r^{2}} - \frac{d^{2}G_{1}}{h_{c}^{-2}} - K_{2}\right)w^{'} - rK_{1}w \\ &- (P_{311} + T_{311})r\left(\frac{\partial w^{"}}{\partial t}\right) \\ &- (2T_{311} - P_{321} - T_{321} + 2P_{311})\left(\frac{\partial w^{'}}{\partial t^{2}}\right) \\ &- rD\left(\frac{\partial w}{\partial t}\right) - \left(I_{1} - \frac{J_{2}d}{h_{c}^{-2}}\right)r\left(\frac{\partial^{2}u_{t}^{'}}{\partial t^{2}}\right) \\ &- \left(I_{1} - \frac{J_{2}d}{h_{c}^{-2}}\right)\left(\frac{\partial^{2}u_{t}}{\partial t^{2}}\right) - \left(Y_{1} + \frac{J_{2}d}{h_{c}^{-2}}\right)r\left(\frac{\partial^{2}u_{b}^{'}}{\partial t^{2}}\right) \\ &- \left(\frac{J_{2}d}{h_{c}^{-2}} + Y_{1}\right)\left(\frac{\partial^{2}u_{b}}{\partial t^{2}}\right) \\ &+ \left(I_{2} + Y_{2} + \frac{J_{2}d^{2}}{h_{c}^{-2}}\right)r\left(\frac{\partial^{2}w^{"}}{\partial t^{2}}\right) \\ &+ \left(I_{2} + Y_{2} + \frac{J_{2}d^{2}}{h_{c}^{-2}}\right)\left(\frac{\partial^{2}w}{\partial t^{2}}\right) \\ &- (I_{0} + Y_{0} + J_{0})r\left(\frac{\partial^{2}w}{\partial t^{2}}\right) = 0 \end{split}$$

Noted that the prime sign denotes derivative with respect to *r*. Also

$$A_{ijk} = \int_{top} c_{ijk} z^k dz, \quad i, j = 1, 2, \quad k = 0, 1, 2$$

$$B_{ijk} = \int_{bottom} c_{ijk} z^k dz, \quad i, j = 1, 2, \quad k = 0, 1, 2 \quad (28)$$

$$P_{ijk} = \int_{top} e_{ijk} K_{vcf} z_t^k dz, \quad ij = 31, 32, \quad k = 0, 1$$

$$T_{ijk} = \int_{bottom} e_{ijk} K_{vcf} z_b^k dz, \quad ij = 31,32, \quad k = 0,1$$

$$G_1 = \int_{core} G_c dz,$$

$$\{J_i, \quad I_i, \quad Y_i\} = \left\{ \int_{core} \rho_c, \quad \int_{top} \rho_t, \quad \int_{bottom} \rho_b \right\} z^i dz i$$

$$= 0,1,2$$

$$d = \frac{h_t}{2} + h_c + \frac{h_b}{2}$$
(28)

where

$$z_t = z - \frac{h_c}{2} - \frac{h_t}{2}, \qquad z_b = z + \frac{h_c}{2} + \frac{h_b}{2}$$
 (29)

The conditions of both inner and outer edges of the plate may be one of simply supported or clamped. For the simply supported edges conditions, the following relation should be ruled in inner or outer edges

$$u_{t} = 0, \quad u_{b} = 0, \quad w = 0, -rA_{111}u'_{t} - rB_{111}u'_{b} + (B_{112} + A_{112})rw'' + (B_{122} + A_{122})w' + (T_{311} + P_{311})r\left(\frac{\partial w}{\partial t}\right) = 0$$
(30)

And for the clamped edges annular plate the boundary conditions are as follows

$$u_t = 0, \quad u_b = 0, \quad w = 0, \quad w' = 0$$
 (31)

4. Results and discussion

4.1 Solution procedure

In this study, GDQM is used as the numerical solution method to solve the governing motion equations. Based on the GDQ basis, the differential equations are discretized and are converted to the algebraic ones based on the following relation (Arshid *et al.* 2019b, Shu 2012)

$$f^{(m)}(x_i) = \sum_{j=1}^{N} C_{ij}^{(m)} f(x_j), \quad i = 1, 2, \dots, N$$
(32)

in which $C_{ij}^{(m)}$ is the weighting coefficient of the GDQM and *N* refers to the number of grid points.

Also, the grid points are distributed radially and nonuniformly according to the *Chebyshev* pattern as follow (Shokravi 2018, Tohidi *et al.* 2018)

$$x_i = a + \frac{(b-a)}{2} \left\{ 1 - \cos\frac{(i-1)\pi}{(N-1)} \right\}, \quad i = 1, 2, \dots, N$$
(33)

Using the GDQ relations, with regard to the harmonic manner for the displacements, and by converting the motion and boundary conditions equations to the following matrix form, the frequencies of the structure can be achieved by solving it (Amir *et al.* 2019b, c)

$$([K] + i\omega[C] - \omega^2[M])\{X\} = 0$$
(34)

in which [K], [C] and [M] are the stiffness, damping and mass matrices, respectively, and displacements vector is shown by $\{X\}$ which is defined as follow

$$\{X\} = \{u_t, \ u_b, \ w\}^T \tag{35}$$

4.2 Validation and convergence of the results

In order to examine the reliability of the results, they should be compared to the other studies in the literature. Since there is no study about such a plate, the results are obtained for the simpler state and are compared with those of previous studies. To this aim, a single-layer homogenous annular plate is taken under consideration in the absence of elastic foundation and magnetic field. The results for different boundary conditions are presented in Table 1. Noted that in this table, the Poisson's ratio is equal to 1/3and the inner to outer radius ratio is 0.4. Also, the dimensionless frequency for this table is reported as $\Omega =$ $\omega a \sqrt{\rho h/A_{110}}$. As can be seen in this table, the results are in good coincident with the previous studies' ones (Chakraverty et al. 2001, Zhou et al. 2011) and the little difference is raised from different solution methods that the mentioned works used, namely Rayleigh-Ritz and variable seperation methods. Therefore, the reliability of the results is ensured.

Furthermore, the convergence of the results for the under consideration plate is studied. In the numerical

Table 1 Comparison of the present results with previously published studies

	Ω1					
Boundary conditions	Present	Chakraverty et al. (2001)	Percentage error	Zhou <i>et al.</i> (2011)	Percentage error	
C-C	61.881	61.88	0.001%	61.872	0.015%	
S-C	44.928	44.93	0.004%	44.932	0.009%	
C-S	41.273	41.27	0.007%	41.261	0.030%	
S-S	28.121	28.08	0.145%	28.184	0.224%	



Fig. 3 Considering the convergence rate of the results

methods by increasing the number of grid points, the results should converge to their final values. Rate of convergence and moreover, using the lowest possible number of nodes to converge, show the power of the solution method. Here, for example, the convergence of the natural frequencies for various boundary conditions is shown in Fig. 3. The second type of MR fluids which was presented by Ramamoorthy *et al.* (2016) is selected for the core of the structure and the inner radius of the plate is considered to be 5 cm. Also, the outer to inner radii ratio which is shown by λ is equal to 5.

It is seen that for all the boundary conditions the results are converged with approximately a few numbers of nodes, about 11. Therefore, the convergence of the results is also ensured and all of the following results, are extracted with 15 number of grid points.

4.3 Case study

The motion equations of the smart sandwich structure which its faces were subjected to a uniform magnetic field, were extracted using Hamilton's principle and solved numerically via the GDQ method. Three types of MR fluids are considered for the core's material type and the Terfenol-D is selected as the face sheets with E = 30 GPa, $\rho = 9250$ kg/m³, v = 0.25, and $e_{31} = e_{32} = 442.55$ N/m.A (Ghorbanpour Arani *et al.* 2017b). Now, the effect of different geometric and physical parameters such as radius and aspect ratios of the annular plate, magnetic field intensity, velocity feedback gain, types of MR fluids, viscoelastic medium and also boundary conditions will be carried out.

Fig. 4 shows the influence of different types of MR fluids on the results. By considering this figure, can be found that enhancing the magnetic field intensity, causes an increase in the natural frequencies values due to increasing the rigidity of the structure. As stated before, when the MR fluids are subjected to a magnetic field, they will be converted to a quasi-solid phase from liquid one. So, it leads to increasing the rigidity of the plate. Also, by changing the magnetic field intensity in a logical range, the MR fluid type 2 will be most affected in comparison to two



Fig. 4 Effect of magnetic field intensity on the three types of MR fluids

other types. By reviewing the Eqs. (17)-(19) which denote the MR fluids types specifications, it can be expected that since the third type of MR fluid is affected by the cube of magnetic field intensity, and also due to its very small factor, therefore the magnetic field intensity has the least effect on this type. Vice versa, for the second type, according to its specifications, the magnetic field intensity has the maximum effect among the mentioned types of MR fluids. Generally, the shear complex modulus affects the stiffness of the structure and it's increasing, leads the natural frequency of the plate to enhance.

Tables 2 and 3 illustrate the effect of MR fluid type on the results. Table 2 shows this effect on the four first modes and for both edges clamped annular plate which can be seen that the third type of MR fluids that was presented by Aguib *et al.* (2014) has the maximum values of the results and the second type (Ramamoorthy *et al.* 2016) has the minimum one.

Table 2 MR fluids types influence on the first four frequencies of the C-C annular plate

	ω (×10 ⁴)				
MR fluid	First mode	Second mode	Third mode	Fourth mode	
Type 1	1.3407	3.0744	3.4724	5.8467	
Type 2	1.3055	3.0578	3.3882	5.8132	
Type 3	1.4466	3.1341	3.7181	5.9416	

Table 3 Effect of MR fluids types on the natural frequencies for various boundary conditions

MR Fluid	$\omega_1 (\times 10^4)$				
	C-C	C-S	S-C	S-S	
Type 1	1.3407	0.8724	1.0437	0.6548	
Type 2	1.3055	0.8497	1.0164	0.6379	
Type 3	1.4466	0.9441	1.1278	0.7109	



Fig. 5 Face sheets thickness variations effect on the fundamental frequency of the plate



Fig. 6 Radii ratio effect on the natural frequency of the structure for a) various boundary conditions; b) two first modes

Table 3 presents a similar effect but for the first mode and different boundary conditions. Since the rigidity of the structure for both edges clamped plate is more than the other conditions, so its results have the most values, too. And vice versa about both simply supported edges plate which has the least values of the frequencies.

The effect of MS face sheets thickness on the results is shown in Fig. 5. The intensity of the magnetic field for this figure is 100 Gauss. It is found that due to more stiffness of the face sheets of the structure than its core, as the face layers become thicker, so the stiffness of the structure will be enhanced and following it, the natural frequency increases. Also, again the effect of three mentioned types of MR fluids is seen in this figure which confirms the previous findings.

Influence of radii ratio is presented in Fig. 6. Fig. 6(a) depicts that by increasing the outer radius more than inner ones, the stiffness of the structure will be reduced and the natural frequency of the structure will be enhanced. Fig. 6(b) shows this effect but for the two first modes.

Fig. 7 presents the effect of thicknesses variations simultaneously. As can be expected, increasing both of the

core's and face's thicknesses will enhance the natural frequency, but the effect of that of faces is more than the core's due to its more rigidity.

Figs. 8 and 9 show the viscoelastic foundation and also velocity feedback gain effects on the results. It can be understood that by increasing the K_{vcf} , the natural frequencies will be reduced. These two figures are plotted for the second type of MR fluid core. Generally, the effect of velocity feedback gain is not significant and a large value of it is needed to show the effect clear. Different models of elastic foundation and their effect on the vibrational behavior of the plate are compared in Fig. 8. It is noteworthy that the medium of a structure can be much effective on its vibrations. Although the elastic medium is simulated by an external load, its effect enters to stiffness matrix and increasing the stiffness of the elastic medium, leads to higher values of the stiffness matrix. Noted that the stiffness of the elastic medium should be coordinated with the structure's application and always increasing the foundation stiffness is not good. For example, to reduce the vibrations of an engineering structure without changing its geometrical size and material properties due to specific



Fig. 7 Thicknesses influence on the results



Fig. 8 Velocity feedback gain and foundation types effect on the results



Fig. 9 Viscoelastic foundation effect on the first frequency of the C-C plate

limitations, adding elastic foundation can help to achieve this aim. Nowadays different types of elastic medium in various fields such as civil, mechanical or industrial engineering are used widely. With regard to Fig. 8, it is found when the elastic foundation is neglected, the frequencies have the least values. But when springs are added to the structures, in other words, the foundation is simulated by the Winkler model, the frequencies are increased for increasing the rigidity of the structure. Also, by enhancing the spring constant, the frequencies are raising, too. Adding the shear layer converts the foundation to Pasternak model and leads the frequency to rise.

But adding the dampers to the foundation as is shown in Fig. 9, which causes the foundation converts to viscoelastic one, reduces the frequency. Increasing the damping constant reduces the rigidity and consequently, frequencies reduce.

5. Conclusions

Free vibration analysis of a smart annular three-layered plate provided in this work. MR fluids are used as the core's material type and the face sheets are made from MS materials and it is assumed that they are fully bonded to each other. The sandwich plate is rested on a visco-Pasternak foundation and also is subjected to a magnetic field. The kinematic relations are provided based on the CPT which neglects the shear deformations effect which is more common for thin structures. Three different types of MR fluids are considered as core's material. The differential motion equations and associated boundary conditions are extracted by employing Hamilton's principle and are solved via GDQ as an accurate numerical method.

Different parameters effect on the results was discussed in details and the following items can be concluded:

- Enhancing the magnetic field intensity leads the natural frequencies to increase.
- The applied magnetic field affects the second type of MR fluids more the two other types.
- Comparing three types of MR fluids, it can be found that the third type leads to the most and the second

type leads to the least values of the natural frequencies.

- As both core and faces become thicker, the natural frequencies of the plate increase. Noted that the effect of face sheets' thickness variations is more due to its more rigidity.
- Increasing the velocity feedback gain values leads to reduce the natural frequency.
- The frequencies of both edges clamped plate are more than other types of boundary conditions.
- By neglecting the elastic foundation, the results are in their minimum values and by adding the spring and shear layer, they lead to increase.
- Adding the dampers to the foundation reduces the frequencies of the structure.

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