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PDC Intelligent control-based theory for structure system dynamics

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Abstract. This paper deals with the problem of global stabilization for a class of nonlinear control systems. An effective approach is proposed for controlling the system interaction of structures through a combination of parallel distributed compensation (PDC) intelligent controllers and fuzzy observers. An efficient approximate inference algorithm using expectation propagation and a Bayesian additive model is developed which allows us to predict the total number of control systems, thereby contributing to a more adaptive trajectory for the closed-loop system and that of its corresponding model. The closed-loop fuzzy system can be made as close as desired, so that the behavior of the closed-loop system can be rigorously predicted by establishing that of the closed-loop fuzzy system.

Keywords: intelligent control function; Bayesian additive model; automated design

1. Introduction

Lately, fuzzy logic control (FLC) has been utilized in numerous fruitful useful control applications (Ying *et al*. 2019, Jeong *et al*. 2019, Battista and Varela 2019), “In spite of these achievements, it is obvious that several fundamental issues remain. In this study, a powerful Takagi-Sugeno (T-S) (1985) model is made by the application of these rules, to yield relations frameworks. This straight-forward and generally demonstratable technique endeavors to express each rule by a direct framework display which enables us to utilize straight criticism control strategies for input adjustment. The idea of parallel distributed compensation (PDC), as presented in Wang *et al*. (1996), is used to structure a controller to balance the model. The idea is to structure a compensator for each standard in the model. Each control rule is independently planned based on the comparison principle of the T-S model, so that the direct control structure strategies can be utilized to plan the PDC controller. A large controller, which is commonly nonlinear, is subsequently a mixing together all the individual direct controllers for each standard offered by equivalent settings in the model in the starting parts.

It has for some time been realized that the infusion of a high recurrence flag, known as a dither, into a nonlinear framework may enhance its execution. Better execution is seen as less bending in the framework yield, increased steadiness, and the extinguishing of limit cycles. A thorough investigation of steadiness in a general nonlinear framework with a dither control was given in Steinberg and Kadushin (1973), “Utilizing the causal technique, a casual model might be settled by suitably controlling the parameters of dither. In this way, dither of adequately high recurrence may result in yields of the casual model as close as wanted to that of the dithered framework (Mossaheb 1983), “It was demonstrated that the direction of a dithered framework can be anticipated by setting up that of its related model, the casual model (which relies upon the parameters of dither), given that the dither has a sufficiently high recurrence.

Additional insight can be obtained from well-known fields, which have received a lot of scientific consideration. Numerous algorithms have been derived from the tiny knowledge of the world of animals, which are incorporated into this field. When all is said in done, the swarm strategy requires transformative computational intelligence and is modeled on the specific practices or the ingrained instincts of animals. For example, cat swarm optimization (CSO) was propose based on demonstrated feline behaviors framed in mathematical terms (Chu *et al*. 2006); the improved artificial bee colony (IABC) algorithm was based on recreating the conduct of honey bees accumulating nectar (Tsai *et al*. 2009); and the evolved bat algorithm (EBA) was proposed dependent on the prey discovering procedure of bats (Tsai *et al*. 2012). These types of algorithms have been connected to tackle numerous issues in various fields.

2. Literature review

A few strategies for assessing solidness plans have been effectively connected, see Loria and Nesic (2003), Panteley and Loria (1998), Sontag and Wang (1995), and Sontag (1988). Computational insight methodologies, for example, neural systems and frameworks have additionally been

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utilized to show dynamics and applications in various regions. These apparatuses have turned out to be amazing and compelling. A few of the later works utilizing these methodologies can likewise be seen. Then again, swarm algorithms are likewise broadly used to build models of a framework or to find ideal solutions to issues in assembly, booking, and business coordination, background, and design. Panda *et al*. (2011) used CSO (see Chu and Tsai (2007)) to build a number of populace based learning rules for an interactive information retrieval (IIR) framework; Pardhan and Panda (2012) also used CSO to tackle numerous goal issues. In addition, Wang *et al*. (2012) utilized CSO to enhance data concealing outcomes. The IABC, which was proposed by Tsai *et al*. (2009), was effectively used to enhance the acknowledgment rate of the persistent confirmation framework (see Tsai *et al*. 2012) and to estimate the patterns outside the conversion scale (see Chang *et al*. 2014a); and EBA has been employed to provide the optimal recommended stock portfolio (see Chang *et al*. 2014b).

Although there have been many successful applications of intelligent algorithms, there are still some drawbacks to using them in any control scheme. A fuzzy Lyapunov method as well as an NN-fuzzy models have been used for dealing with the tension leg platform (TLP) stability problem; see Lam (2009), Lee *et al*. (2001), Liu and Zhang (2003), Park *et al*. (2003), Tanaka *et al*. (1996), Tanaka and Sano (1994), and Wang *et al*. (1996), "From this we study suitable mathematical modeling for the TLP system and discuss the interaction between a deformable floating structure and the surface wave motion by virtue of a partial differential equation as well as fuzzy logic theory.

Egresits *et al*. (1998) maintained that knowledge is firmly associated with picking up adjusting capacities, thus such abilities are considered as crucial to smart assembly frameworks. Various methodologies have been adapted to produce diverse machine learning strategies for assembly issues, beginning with standard enlistment in representative examples and area acknowledgment procedures in numerical, sub emblematic spaces. Artificial neural network (ANN) based learning strategies are currently the prevailing machine learning strategies utilized in assembly. They cannot exclusively be utilized for characterization and estimation, but can also be utilized for dependability investigation. For instance, the nonlinear Markov jump standard genetic regulatory network model can be built utilizing repetitive neural systems (see Zhu *et al*. 2013), "Be that as it may, essentially these arrangements have constrained modern acknowledgment on account of the 'discovery' idea of ANNs. The incorporation of neural and methods has been dealt with and previous arrangements investigated by Egresits *et al*. (1998). Narendra *et al*. (1998) depicted a clever current controller for the quick and adaptable control utilizing ANN and the logic worldview. Two strategies for altering the learning parameters are exhibited: a heuristic way to deal with and assess the learning rate as a polynomial of a vitality work is considered and learning parameters are examined. Fuzzy logic, genetic algorithms and neural systems are three prevalent man-made reasoning methods which are generally utilized (see Lian *et al*. 1998), "Due to their particular properties and favorable circumstances, they are being explored and coordinated to shape new models and methodologies in the regions of framework control right now.

Linear regression models provide an effective and attractively simple framework for understanding how each input variable relates with the observed target variables. However, they fail to capture non-linear dependencies between inputs and target variables, which are recurrent in the real-world. On the other hand, flexible models such as neural networks or Gaussian processes (GPs) lay on the opposite side of the spectrum, where the target variables are modeled as complex non-linear functions of all input variables simultaneously. Unfortunately, due to their black-box nature, the interpretativeness and the ability to understand how each input is contributing to the observed target are typically lost. Additive models (Ravikumar *et al*. 2009) contrast with these by specifying the target variable to be the result of a linear combination of non-linear functions of the individual inputs. Due to this structured form, additive models provide an interesting tradeoff between interpretability and flexibility.

3. Problem formulation

We consider models of the regression form as follows.

$$y = f_r(x^r) + \sum_{i=1}^{E} f_e(x^{e_i}) + \in, \tag{1}$$

where $\in \sim N(0, v)$ is the observation noise and E denotes the number of events that can affect the observed arrivals y. Hence, the number of events, E, varies between observations. However, if we assume the functions $f_r(x^r)$ and $f_e(x^{e_i})$ to be linear functions of their inputs, parameterized by a vector of coefficients $\boldsymbol{w}_r$ and $\boldsymbol{w}_e$ respectively, then we can write (1) as

$$y = (w_r)^T x^r + (w_e)^T \left(\sum_{i=1}^{E} x^{e_i} \right) + \in = w^T x + \in, \tag{2}$$

where we defined $\boldsymbol{w} \triangleq [\boldsymbol{w}_r; \boldsymbol{w}_e]$ and $\boldsymbol{x} \triangleq [\boldsymbol{x}^r; \sum_{i=1}^{E} x^{e_i}]$.

As we can see, in the case of linear functions, the feature vectors of all events can be aggregated by summation, which reduces the problem to a simple linear regression. However, that does not allow us to properly explore our domain knowledge.

4. Intelligent control based theory

4.1 Model description

The proposed Bayesian additive model builds on the assumption that there is a base routine component $y^r = f_r(x^r)$ and a variable number of event components $y^{e_i} = f_e(x^{e_i})$, whose contributions are summed up to obtain the total observed arrivals y in a given area. Since we wish to constrain the values of the individual components, y^r and $\{y^{e_i}\}_{i=1}^{E}$, to be non-negative, we define the latter to be oneside truncated Gaussians, which we denote as

$$y^r \sim \mathbb{I}\,(y^r\ > 0)\ \mathcal{N}\ (y^r|\, f_r(x^r),\ \beta_r), \tag{3}$$

$$y^{e_i} \sim \mathbb{I}\,(y^{e_i}\ > 0)\ \mathcal{N}\ (y^{e_i}|\, f_e(x^{e_i})\ \beta_e), \tag{4}$$

where $\mathbb{I}$ (a > 0) is an indicator function that takes the value 1 if and only if a > 0, β_r and β_e are the variances of routine and events components, respectively. Alternatively, we also consider a variant of the model that assumes the component values to be Poisson distributed with an exponential link function, such that

$$y^r \sim Poisson(\ y^r|\ e^{f_r(x^r)}), \tag{5}$$

$$y^{e_i} \sim Poisson(\ y^{e_i}|\ e^{f_e(x^{e_i})}), \tag{6}$$

In either case, the observed totals y are then defined as the sum of all components

$$\mathrm{y} = y^r + \sum_{i=1}^{E} y^{e_i} + \epsilon\,, \qquad \epsilon \sim\ \mathcal{N}\ (0, v). \tag{7}$$

Having specified the additive structure of the model, the next step is to specify how to model the functions f_r and f_e. Perhaps the simplest approach would be to assume $f_r(x^r)$ and $f_e(x^{e_i})$ to be linear functions of their inputs as in (2), "Although in this paper we focus on controlled systems, it is important to note that the intelligent algorithm described above is general enough to allow a large variety of models to be applied.

Letting the vectors $\mathbf{f}^r$ and $\mathbf{f}^e$ denote the functions $f_r(x^r)$ and $f_e(x^{e_i})$ evaluated for all feature vectors $\mathbf{x}^r$ and $\mathbf{x}^{e_i}$ respectively, Gaussian process modeling proceeds by placing a GP prior on $\mathbf{f}^r$ and $\mathbf{f}^e$, such that $\mathbf{f}^r \sim \mathcal{GP}\left(m_r(\mathbf{x}^r)\ \equiv 0, k_r\left(\mathbf{x}^r, \mathbf{x}^{r'}\right)\right)$ and $\mathbf{f}^e \sim \mathcal{GP}\left(m_e(\mathbf{x}^e)\ \equiv 0, k_e\left(\mathbf{x}^e, \mathbf{x}^{e'}\right)\right)$, where for the sake of simplicity (and without loss of generality, having our data centered) we assumed the GPs to have zero mean so that the GPs are completely defined in terms of the covariance functions k_r and k_e.

Fig. 1 shows a factor graph representation of the proposed model, which will be particularly useful in the following section for deriving a message passing algorithm to perform approximate Bayesian inference using expectation propagation (EP).

4.2 System analysis within structures

According to the factor graph in Fig. 1, the joint distribution of the proposed model with truncated Gaussian components is given by

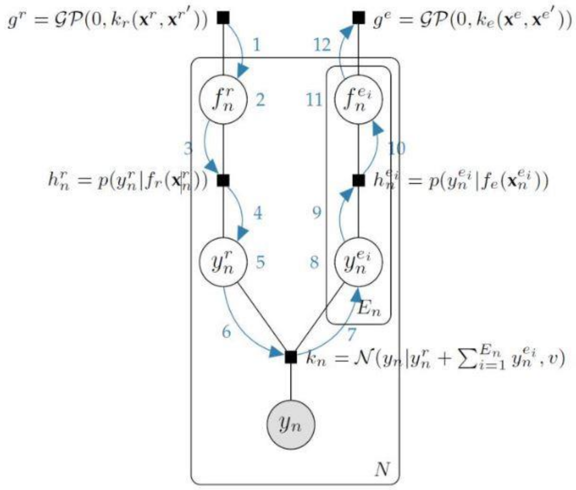

Fig. 1 Factor graph of the proposed Bayesian additive model with Gaussian process components. The blue arrows represent he message-passing algorithm for performing approximate Bayesian inference. The second flow of messages starting from the GP factor for the events component that goes in the opposite direction is not shown

$$\begin{aligned}
&p(\boldsymbol{f}^e, \boldsymbol{y}^r, \boldsymbol{Y}^e, y|\{\boldsymbol{x}_n^r, \boldsymbol{X}_n^e\}_{n=1}^N) \\
&= \mathcal{N}(\boldsymbol{f}^r|0, \boldsymbol{K}^r)\mathcal{N}(\boldsymbol{f}^e|0, \boldsymbol{K}^e)\prod_{n=1}^{N}\mathbb{I}(y_n^r > 0)\mathcal{N}(y_n^r|f_n^r, \beta_r) \\
&\times\left(\prod_{n=1}^{E_n}\mathbb{I}(y_n^{e_i} > 0)\mathcal{N}(y_n^{e_i}|f_n^{e_i}, \beta_e)\right)\mathcal{N}(y_n|y_n^r + \Sigma_{i=1}^{E_n} y_n^{e_i}, v)
\end{aligned} \tag{8}$$

where we defined $\boldsymbol{y} \triangleq \{y_n\}_{n=1}^N$, $\boldsymbol{y}^r \triangleq \{y_n^r\}_{n=1}^N$, and $\boldsymbol{Y}^e \triangleq \{y_n^e\}_{n=1}^N$, with $\boldsymbol{y}_n^e \triangleq \{y_n^{e_i}\}_{i=1}^{E_n}$. The covariance matrices $\boldsymbol{K}^r$ and $\boldsymbol{K}^e$ are obtained by evaluating the covariance functions $k_r(\boldsymbol{x}^r, \boldsymbol{x}^{r'})$ and $k_r(\boldsymbol{x}^e, \boldsymbol{x}^{e'})$ respectively between every pair of inputs.

The EP algorithm provides us with approximate posterior distributions for $\boldsymbol{f}^r$ and $\boldsymbol{f}^e$ given by $q(\boldsymbol{f}^r) = \mathcal{N}(\boldsymbol{f}^r|\mu^r,\ \Sigma^r)$ and $q(\boldsymbol{f}^e) = \mathcal{N}(\boldsymbol{f}^e|\mu^e,\ \Sigma^e)$. These estimates can be used to compute the predictive mean and variance of f_*^r and $\{f_n^{e_i}\}_{i=1}^{E_n}$, as in standard Gaussian process regression and classification. The predictive mean and variance for f_*^r are then given by

$$\begin{aligned}
&\mathbb{E}_q[f_*^r|\boldsymbol{f}^r, x_*^r, \{\boldsymbol{x}_n^r\}_{n=1}^N] = (\boldsymbol{K}_*^r)^T\left(\boldsymbol{K}^r + \tilde{\Sigma}^r\right)^{-1}\tilde{\mu}^r \\
&\mathbb{V}_q[f_*^r|\boldsymbol{f}^r, x_*^r, \{\boldsymbol{x}_n^r\}_{n=1}^N] \\
&= k_r(x_*^r, x_*^r) - (\boldsymbol{K}_*^r)^T\left(\boldsymbol{K}^r + \tilde{\Sigma}^r\right)^{-1}\boldsymbol{K}_*^r,
\end{aligned} \tag{9}$$

and similarly, for the events variables $\{f_*^{e_i}\}_{i=1}^{E_*}$.

The T-S fuzzy model of the model is established via the PDC scheme as follows.

Model Rule i:

$$\begin{aligned}
&\text{IF}\ \ x_{R1}(t)\ \text{is}\ M_{Ri1}(\alpha_m, \beta_m)\ \text{and}\ \ldots\ \text{and} \\
&\quad x_{Rk}(t)\ \text{is}\ \ M_{Rik}(\alpha_m, \beta_m)
\end{aligned} \tag{10}$$

THEN

$$\begin{cases}\dot{x}_R(t) = A_1(\alpha_m,\beta_m)x_R(t) + B_1(\alpha_m,\beta_m)u_R(t)\\ y_R(t) = D_i(\alpha_m,\beta_m)x_R(t),\end{cases} \quad i = 1,2,\dots,r. \tag{11}$$

Hence, the final state and final output of are

$$\dot{x}_R(t) = \frac{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)\begin{Bmatrix} A_i(\alpha_m,\beta_m)x_R(t)\\ +B_i(\alpha_m,\beta_m)u_R(t)\end{Bmatrix}}{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)}, \tag{12}$$

$$y_R(t) = \frac{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)D_i(\alpha_m,\beta_m)x_R(t)}{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)}. \tag{13}$$

Assume that the momentum equation can be characterized by the following differential equation

$$M\ddot{\bar{X}}(t) = -M\bar{r}\varphi(t), \tag{14}$$

where $\overline{X}(t) = [\bar{x}_1(t), \bar{x}_2(t) \cdots \bar{x}_n(t)] \in R^n$ is an n-vector; $\ddot{\bar{X}}(t)$, $\dot{\bar{X}}(t)$, $\bar{X}(t)$ are the acceleration, velocity, and displacement vectors, respectively. This is only a static model and M is the mass of the system; $M\bar{r}\varphi(t)$ is a wave-induced external force which can be expressed as follows

$$M\bar{r}\varphi(t) = F_{wx} - F_{TX}, \tag{15}$$

where F_{wx} is the horizontal wave force acting on the both sides of the structure; and F_{Tx} is the horizontal component of the static (or the pre-tensioned) tension applied by the tension legs. The static tension is given by $F_{Tx} = f\xi$.

IF $x_{R1}(t)$ is $M_{Ri1}(\alpha_m,\beta_m)$ and ... and $x_{Rk}(t)$ is $M_{Rik}(\alpha_m,\beta_m)$
THEN $u_R(t) = -K_i\hat{x}_R(t)$. (16)

Observer Rule *i*:

IF $x_{R1}(t)$ is $M_{Ri1}(\alpha_m,\beta_m)$ and ... and $x_{Rk}(t)$ is $M_{Rik}(\alpha_m,\beta_m)$
THEN $\dot{\hat{x}}_R(t) = A_i(\alpha_m,\beta_m)\hat{x}_R(t) + B_i(\alpha_m,\beta_m)u_R(t) + L_i\big(y_R(t) - \hat{y}_R(t)\big)$, (17)

where $y_R(t) = D_i(\alpha_m,\beta_m)x_R(t)$, $\hat{y}_R(t) = D_i(\alpha_m,\beta_m)\hat{x}_R(t)$ and $i = 1,2,\dots,r$.

Thus, the overall fuzzy controller and fuzzy observer can be written as

$$u_R(t) = -\frac{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)K_i\hat{x}_R(t)}{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)}, \tag{18}$$

$$\dot{\hat{x}}_R(t) = \frac{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)\begin{Bmatrix} A_i(\alpha_m,\beta_m)\hat{x}_R(t)\\ +B_i(\alpha_m,\beta_m)u_R(t)\\ +L_i\big(y_R(t)-\hat{y}_R(t)\big)\end{Bmatrix}}{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)}, \tag{19}$$

The premise variables depend on the state variables estimated by the fuzzy observer, which are unknown. Therefore, (20) is used instead of (18) as a fuzzy controller

$$u_R(t) = -\frac{\sum_{i=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)K_i\hat{x}_R(t)}{\sum_{i=1}^{r} w_i(\hat{x}_R(t),\alpha_m,\beta_m)}. \tag{20}$$

The closed-loop fuzzy relaxed system is rewritten as follows

$$\dot{\hat{x}}_R(t) = \frac{\begin{matrix}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)w_j(\hat{x}_R(t),\alpha_m,\beta_m)\\ \{A_i(\alpha_m,\beta_m) - B_i(\alpha_m,\beta_m)\hat{x}_R(t)\\ +L_iD_j(\alpha_m,\beta_m)\big(x_R(t)-\hat{x}_R(t)\big)\}\end{matrix}}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x_R(t),\alpha_m,\beta_m)w_j(\hat{x}_R(t),\alpha_m,\beta_m)}$$

From the discussion above, we can infer that if the dither has a sufficiently large frequency and a proper membership function is chosen, the trajectory of the closed-loop fuzzy relaxed system and that of the closed-loop dithered chaotic system would be made as close as desired. This enables a rigorous prediction of the stability of the closed-loop dithered chaotic system by establishing that of the closed-loop fuzzy relaxed system.

For controller design as proposed by Hammami (2001), Jankovic *et al*. (1996), Seibert and Suarez (1990), Sepulchre (2000), Sepulchre *et al*. (1997), Sontag (1989), and Sun *et al*. (2003), the standard first-order state equation is obtained from Eq. (21) assuming the equation of motion for a shear-type-building modeled by an n-degrees-of-freedom system controlled by actuators and subjected to an external force $\phi(t)$

$$\dot{X}(t) = AX(t) + E\varphi(t), \tag{21}$$

where $\Lambda^T(t) = [X^T(t) \quad U^T(t)]$, with $X^T(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_\delta(t)]$. We assume S layers and each layer has R^σ $(\sigma = 1, 2, \cdots, S)$ neurons, in which $x_1(t) \sim x_\delta(t)$ and $u_1(t) \sim u_m(t)$ are the input variables. The notation W^σ denotes the weight matrix of the σ^{th} $(\sigma = 1, 2, \cdots, S)$ layer. The transfer function vector of the σ^{th} layer is defined as $\Psi^\sigma(v) \equiv [T(v_1) \quad T(v_2) \quad \cdots \quad T(v_{R^\sigma})]^T$.

A neural-network-based model can be described as follows

$$\dot{X}(t) = \Psi^S\left(W^S\Psi^{S-1}\left(W^{S-1}\Psi^{S-2}\left(\cdots\cdot\Psi^2\left(W^2\Psi^1\left(W^1\Lambda(t)\right)\right)\cdots\cdots\right)\right)\right), \tag{22}$$

where $\Lambda^T(t) = [X^T(t) \quad U^T(t)]$, with $X^T(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_\delta(t)]$. We assume S layers and each layer has R^σ $(\sigma = 1, 2, \cdots, S)$ neurons, in which $x_1(t) \sim x_\delta(t)$ and $u_1(t) \sim u_m(t)$ are the input variables. The notation W^σ denotes the weight matrix of the σ^{th} $(\sigma = 1, 2, \cdots, S)$ layer. The transfer function vector of the σ^{th} layer is defined as $\Psi^\sigma(v) \equiv [T(v_1) \quad T(v_2) \quad \cdots \quad T(v_{R^\sigma})]^T$.

An LDI system can be described in the state-space representation (see Hu (2008) and Liu and Li (2010)) as follows

$$\dot{Y}(t) = A\big(a(t)\big)Y(t), \tag{23}$$

$$A(a(t)) = \sum_{i=1}^{r} h_i\big(a(t)\big)\bar{A}_i, \tag{23}$$

where r is a positive integer; $a(t)$ is a vector signifying the dependence of $h_i(\cdot)$ on its elements, i.e., $h_i\big(a(t)\big) \equiv h_i\big(a_1(t), a_2(t)\cdots, a_n(t)\big)$, $a(t) = [a_1(t), a_2(t), \cdots, a_n(t)]^T$. (In general, $a(t)$ coincides with the state vector $X(t)$); $\bar{A}_i$ ($i = 1, 2, \cdots, r$) are constant matrices; and $Y(t) = [y_1(t) \quad y_2(t) \quad \cdots \quad y_j(t)]^T$.

According to the interpolation method and Eq. (22), we can obtain

$$\begin{aligned}
&\dot{X}(t) \\
&= \left[\sum_{\varsigma^S=1}^{2} h_{\varsigma^S}(t) G_\varsigma^S (W^S[\cdot\cdot \right. \\
&\left. \cdot\,[\sum_{\varsigma^2=1}^{2} h_{\varsigma^2}(t) G_\varsigma^2 \left(W^2\left[\sum_{\varsigma^1=1}^{2} h_{\varsigma^1}(t) G_\varsigma^1 (W^1 \Lambda(t))]\big)\big]\cdots\right]\right)\right] \\
&= \sum_{\Omega^\sigma} h_{\Omega^\sigma}(t) E_{\Omega^\sigma} \Lambda(t)
\end{aligned} \tag{24}$$

Finally, based on Eq. (23), the dynamics of the NN model can be rewritten as the following LDI state-space representation

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(t) \bar{E}_i \, \Lambda(t), \tag{25}$$

also rearranged as follows

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(t) \{A_i X(t)\}, \tag{26}$$

where A_i is the partitions of E_i corresponding to the partition $\Lambda(t)$.

Based on the above modeling schemes for the NN-based approach, the nonlinear structural system (21) can be approximated as an LDI representation (26), "The LDI representation follows the same rules as the T-S fuzzy model, which combines the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools of a linear system theory into a unified framework. To ensure the stability of the TLP system, the T-S fuzzy model and the stability analysis are recalled. First, the ith rule of the T-S fuzzy model, representing the structural system, can be represented as follows:

Theorem 1: The augmented system (26) is asymptotically stable in the large if there exists a common positive definite matrix $\tilde{P}$, the controller gains K_i and observer gains L_i, $i = 1, 2, \ldots, r$ can be found to satisfy the following matrix inequalities

$$\begin{aligned}&\tilde{A}_{ii}^T = (\alpha_m, \beta_m)\tilde{P} + \tilde{P}\tilde{A}_{ii}(\alpha_m, \beta_m) < 0, \\ &i = 1, 2, \ldots, r\end{aligned} \tag{27}$$

$$\begin{aligned}&\left(\frac{\tilde{A}_{ij}(\alpha_m, \beta_m) + \tilde{A}_{ji}(\alpha_m, \beta_m)}{2}\right)^T \tilde{P} \\ &+\tilde{P}\left(\frac{\tilde{A}_{ij}(\alpha_m, \beta_m) + \tilde{A}_{ji}(\alpha_m, \beta_m)}{2}\right) < 0, \\ &i < j \le r.\end{aligned} \tag{28}$$

where

$$\begin{aligned}&\tilde{A}_{ij}(\alpha_m, \beta_m) \\ &= \begin{bmatrix} A_i(\alpha_m, \beta_m) - B_i(\alpha_m, \beta_m)K_j & B_i(\alpha_m, \beta_m)K_j \\ 0 & A_i(\alpha_m, \beta_m) - L_i D_j(\alpha_m, \beta_m)\end{bmatrix}.\end{aligned} \tag{29}$$

This proof is lengthy, so it is not repeated here. The above matrix inequalities can be also converted to the LMI form

$$\begin{aligned}
&QA_i^T(\alpha_m, \beta_m) + A_i(\alpha_m, \beta_m)Q \\
&\quad -Y_i^T B_i^T(\alpha_m, \beta_m) - B_i(\alpha_m, \beta_m)Y_i \quad < 0 \\
&A_i^T(\alpha_m, \beta_m)P_2 \ + P_2\ A_i(\alpha_m, \beta_m) \\
&\quad -D_i^T(\alpha_m, \beta_m)N_i^T - N_i D_i(\alpha_m, \beta_m) < 0 \\
&QA_i^T(\alpha_m, \beta_m) - Y_j^T B_i^T(\alpha_m, \beta_m) + QA_i^T(\alpha_m, \beta_m) \\
&\quad -Y_i^T B_i^T(\alpha_m, \beta_m) + A_i(\alpha_m, \beta_m)Q - B_i(\alpha_m, \beta_m)Y_j \\
&\quad +A_j(\alpha_m, \beta_m)Q - B_j(\alpha_m, \beta_m)Y_i \quad < 0, \\
&i < j
\end{aligned}$$

$$\begin{aligned}
&A_i^T(\alpha_m, \beta_m)P_2 \ - D_j^T(\alpha_m, \beta_m)N_i^T + P_2 A_i(\alpha_m, \beta_m) \\
&\quad -N_i D_j(\alpha_m, \beta_m) + A_j^T(\alpha_m, \beta_m)P_2 \ - D_i^T(\alpha_m, \beta_m)N_j^T \\
&\quad +P_2 A_j(\alpha_m, \beta_m) - N_j D_i(\alpha_m, \beta_m) < 0, \\
&i < j
\end{aligned}$$

where $Y_i = K_i Q$, $Y_j = K_j Q$, $N_i = P_2 L_i$, and $N_j = P_2 L_j$.

According to Theorem, we can appropriately regulate the parameters α_m and β_m of dither, to stabilize the closed-loop fuzzy relaxed system.

5. The experiment design and the simulation result

The proposed Bayesian additive model with Gaussian process components was implemented in the system modeled by Eq. (30) from the dynamics

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 9r \end{bmatrix} \tag{30}$$

Similar operations can be found in previous studies (see Liu and Lin (2012a, b, 2013)), "By combining the whole set of fuzzy rules, the approximation of the nonlinear system is completed. Thus, the fuzzy model approximated nonlinear system can be described as follows:

IF $x_1 \ge \frac{\pi}{3}$, then $\dot{x} = A_1\tilde{x} + B_1 u$,
RULE 2: IF $x_1 \approx \frac{\pi}{90}$, then $\dot{x} = A_2\tilde{x} + B_2 u$

According to **Theorem 1** described in section 3, it provides a useful criterion that ensures the system response is stable in the large. Base on **Theorem 1**, selecting the proper common positive definite matrix P and the control force K becomes the key problem to be dealt with. In this paper, we use Bayesian additive model with Gaussian process to discover the proper solutions. In this case, the obtained solutions can be classified into two categories: feasible and infeasible. It means that designing the fitness function in a binary operation form is a simpler way to answer to the need of this application. In this paper, the fitness function is designed based on the stability criterion derived from the LMI conditions via the Lyapunov function approach. The AND logical operation is employed in the fitness function for examining the solutions to produce the binary classification results on the discovered solutions. The fitness function is formulated as follows

$$F = \begin{cases} 1, & \text{if } \Theta < 0 \text{ and } P = P^T > 0 \\ 0, & \text{otherwise.} \end{cases},$$
$$\Theta = (A_i - B_i K)^T P + P(A_i - B_i K)$$

The matrix P is always constrained to be symmetric when using Bayesian additive model with Gaussian process to adjust the elements inside it. In addition, a boundary condition is used at the initialization process for both matrices P and K. The matrix P is kept influencing by the same range of boundary conditions for producing feasible solutions in a suitable range. All parameters used in our experiment for Bayesian additive model with Gaussian process are listed in Table 1.

The number of run listed in Table 1 aims to provide a series of experimental results for examining by statistical methods. In this paper, we choose a fixed iteration number to be the termination criterion. A larger population size requires more memory resource and computation power. Hence, we set the population size to be 26 in the experiment. The number of feasible solutions obtained by Bayesian additive model with Gaussian process in different runs are shown in Fig. 2.

The statistical analysis of the results obtained by Bayesian additive model with Gaussian process over 40

Table 1 Parameters for Bayesian additive model with Gaussian process

Boundary condition for matrix P and K	[-5, 5]
Medium material	Air
Number of run	40
Population size	26
Number of iteration	600

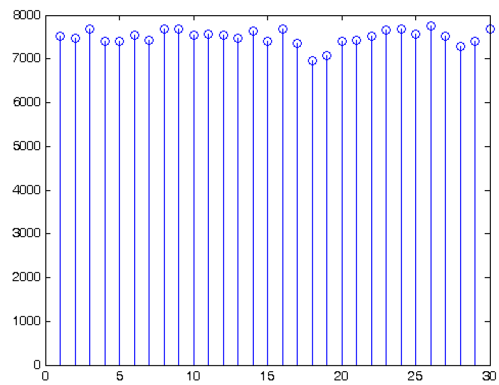

Fig. 2 Number of feasible solutions obtained in 30 runs

Table 2 Statistical analysis of the obtained feasible solutions

Mean	7502
Minimum	6950
Maximum	7758
Mode	7404
Standard Deviation (STD)	180.7580

runs is given in Table 2. Although the STD of the obtained feasible solution in every run is a bit large, the number of found feasible solutions is still much more than enough to decide the system parameters in the application.

According to the experimental results, Bayesian additive model with Gaussian process produces 7,502 feasible solutions in average. Assuming that every artificial agent allocates a feasible solution successfully in all iterations, the maximum number of feasible solutions that can be found in one run is 8,000. This implies that the success rate for utilizing the Bayesian additive model with Gaussian process to find feasible solutions is 93.77% in average. The solutions found by the Bayesian additive model with Gaussian process are determined as feasible if the eigen values are all negative, because the negative eigen values result in the control system staying stable in the large.

6. Conclusions

We proposed BAM-GP: A Bayesian additive model (BAM) with Gaussian process (GP) components that allows for an observed variable to be modeled as a sum of a variable number of non-linear functions on subsets of the input variables. We developed an efficient approximate inference algorithm using expectation propagation (EP), which allows us to both make predictions about the unobserved totals and to estimate the marginal distributions of the additive components. The proposed model is then capable of being flexible, while retaining its interpretability characteristics. Then, the fuzzy controller, the fuzzy observer and the dither signal are simultaneously introduced to transfer the chaotic motions to the origin. Finally, we believe that the presented methodology is quite general and that it can be easily adapted beyond the fuzzy controller and the fuzzy observer.

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