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Structural damage identification of truss structures using self-controlled multi-stage particle swarm optimization

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Abstract. The present work proposes a self-controlled multi-stage optimization method for damage identification of structures utilizing standard particle swarm optimization (PSO) algorithm. Damage identification problem is formulated as an inverse optimization problem where damage severity in each element of the structure is considered as optimization variables. An efficient objective function is formed using the first few frequencies and mode shapes of the structure. This objective function is minimized by a self-controlled multi-stage strategy to identify and quantify the damage extent of the structural members. In the first stage, standard PSO is utilized to get an initial solution to the problem. Subsequently, the algorithm identifies the most damage-prone elements of the structure using an adaptable threshold value of damage severity. These identified elements are included in the search space of the standard PSO at the next stage. Thus, the algorithm reduces the dimension of the search space and subsequently increases the accuracy of damage prediction with a considerable reduction in computational cost. The efficiency of the proposed method is investigated and compared with available results through three numerical examples considering both with and without noise. The obtained results demonstrate the accuracy of the present method can accurately estimate the location and severity of multi-damage cases in the structural systems with less computational cost.

Keywords: damage identification; self-controlled multi-stage optimization; particle swarm optimization; mode shape and natural frequency; inverse problem

1. Introduction

Structural damage detection is an important aspect of structural health monitoring (SHM). Damage detection can be broadly classified into local damage detection and global damage detection. Vibration-based damage detection (VBDD) is one of the branches of the global damage detection method using non-destructive testing (NDT). The main idea behind the VBDD is that the changes in physical parameters (stiffness, damping, and mass), which will affect the vibration parameters (frequency, frequency response function, mode shape) of the structure. Two basic types of data are used in the vibration methods: time domain and frequency domain. Among them, natural frequencies and mode shapes are used widely as it can be measured easily. For this, so many different types of damage detection methods and indices are being developed using modal and frequency data. Some are discussed in the literature such as change in natural frequencies (Cawley and Adams 1979, Kim *et al.* 2003), changes in mode shapes (Kim *et al.* 2003), mode shape curvature (Pandey *et al.* 1991), changes in flexibility (Pandey and Biswas 1994).

Recently there is enormous development in soft computing tools, mathematics, and optimization techniques, which are being used for damage detection problems. In optimization technique, structural damage detection problem can be defined as an inverse problem where an error function is defined using the vibration data from the structure. The error function is subsequently minimized with proper optimization methods to identify damage location and severity of damage.

Mares and Surace (1996) presented a method to locate and quantify the damage with genetic algorithm (GA) by using a residual force method. Perera and Torres (2006) conducted a numerical as well as experimental study to detect the damage by using GA from the change of natural frequencies and mode shapes. A multi-chromosome genetic algorithm (GA) was introduced by Villalba and Laier (2012) for damage detection. Yu and Wan (2008) used an improved particle swarm optimization (IPSO) to detect the structural damage of a plane frame structure. Mohan *et al.* (2013) used particle swarm optimization (PSO) to detect the damage of structures from frequency response function (FRF) data. Miguel *et al.* (2012) used harmony search method to detect damage of the structure, under ambient vibration. Beside these ant colony optimization (Majumdar *et al.* 2012), cuckoo search method (Hosseinzadeh *et al.* 2014), magnetic charged system search (Kaveh and Maniat 2015), improved charged system search (Kaveh and Zolghadr 2015), artificial bee colony (Ding *et al.* 2015), Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO) (Kaveh and Mahdavi 2016), echolocation search algorithm (Nobahari *et*

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al. 2017), Cyclical Parthenogenesis Algorithm (Kaveh and Zolghadr 2017) were used in damage detection of structures. Kaveh *et al*. (2014) proposed a mixed particle swarm-ray optimization together with harmony search (HRPSO) is applied to detect damage in the structure. Kaveh and Maniat (2014) applied Charged System Search (CSS) algorithm for damage detection in skeletal structure using incomplete modal data. Pan *et al*. (2016) used hybrid self-adaptive Firefly-Nelder-Mead algorithm for structural damage detection. Pholdee and Bureerat (2016) performed a comparative study to identify the damage of three truss structures using different meta-heuristics. Meta-heuristic methods which they studied includes differential evolution (DE) artificial bee colony algorithm (ABC), real-code ant colony optimisation (ACOR), league championship algorithm (LCA), simulated annealing (SA), particle swarm optimisation (PSO), evolution strategies (ES), teaching-learning-based optimisation (TLBO), adaptive differential evolution (ADE), evolution strategy with covariance matrix adaptation (CMAES), success-history based adaptive differential evolution (SHADE) and SHADE with linear population size reduction (L-SHADE). Chun and Yu (2017) proposed a PSO-based algorithm for structural damage detection using Bayesian multi-sample objective function. Big Bang and Big Crunch (BB-BC) optimization algorithm was used by Haung and Lu (2017) to detect the damage of beam, plate and European Space Agency Structures utilizing acceleration responses (AR) of the structure as objective function. Kaveh and Dadras (2018) developed an enhanced thermal exchange optimization (ETEO) algorithm and applied it to a wide range of structures to identify the damage. Ghannadi and Kourehli (2019) used moth-flame optimization (MFO) to detect damage in truss and shear frame structures. A comparison of results for identifying the damaged members of truss structures using Vibrating Particle system (VPS) and Enhanced Vibrating Particle system (EVPS) is performed by Kaveh *et al*. (2019).

Optimization methods mentioned above show high accuracy in damage detection of structures when the number of design variables (structural elements) is small. It was found that the computational cost is very high to determine the damage location and quantify the damage simultaneously. The accuracy of the solution decreased with the increment of the design variables (Seyedpoor 2011). Hence, many researchers are interested to find a way to decrease the dimension of the optimization problem. This is implemented by excluding the elements of the structure which are less susceptible to damage. As a result, the algorithm locates the damaged element and its extent accurately with less computational time. The first approach is a hybrid technique, consists of two different methods which are used in two successive steps. Friswell *et al.* (1998) adopted a hybrid approach incorporating GA to locate the damage position, and standard eigensensitivity method was used to optimize the damage extent. Au *et al.* (2003) used elemental energy quotient difference in the first stage to locate the damaged domain approximately. Then the micro-genetic algorithm was incorporated in the second stage to quantify the damage using incomplete and noisy modal data. He and Hwang (2007) introduced a hybrid method, firstly a grey relation analysis was performed, and the most potentially damaged elements were detected. In the next step, the real parameter genetic algorithm with simulated annealing and an adaptive mechanism was used to estimate the damage of the pre-identified locations. Guo and Li (2009) described a two-stage method to locate and quantify the structural damage. The evidence theory was used in the first stage, to locate multiple damage sites. In the next stage, a micro search algorithm is used to detect the damage extent. Naser *et al.* (2010) developed a strategy based on design variable elimination using continuous genetic algorithm combined with sensitivity analysis and micro search for damage detection. A two-stage damage detection method was described by Fallahian and Seyedpoor (2010). A new type of objective function was introduced by Kaveh *et al*. (2018) where the computational burden of single-stage is reduced by a two-phase technique. It was used to identify the damage of skeletal structures. In the first phase, only the natural frequencies were used to compute the objective function. If the results are satisfactory, then only the second phase of the damage identification algorithm is performed using less number of mode shapes. Apart from these, other hybrid methods for damage detection utilizing optimization methods can be found in the reference (Baghmisheh *et al*. 2012, Rao *et al*. 2012, Seyedpoor 2012, Xiang and Liang 2012, Fadel *et al*. 2013, Hosseinzadeh *et al*. 2013, Xiang *et al*. 2013, Wang *et al*. 2014, Seyedpoor and Maryam 2016, Dinh-Cong *et al*. 2018b). These hybrid methods work well if the locations of the damage are identified accurately in the first stage. It is reported that there is a chance to exclude some actual damage elements in the first stage in some damage scenarios (Kaveh and Zolghadr 2017).

Another design variable reduction technique is the multistage technique, where a single algorithm is used in multiple stages. Damage variables are reduced simultaneously in the successive stages and lead to damage identification. The benefit of this approach is that one can use only a single technique which gives better control over the problem. Seyedpoor (2011) proposed a multistage PSO (MSPSO) algorithm to detect the damage. The main idea was to eliminate the design variables with zero damage value, from the search space of the next stage. Thus, search space of the design variables reduced at the successive stages and predicted the damage accurately. An adaptive multi-stage optimization method utilizing a modified particle swarm optimization (MPSO) was adopted to identify the multiple damage cases of the structural systems by Nouri Shirazi *et al*. (2014). The healthy elements were successively eliminated during the stages. The healthy elements were identified by a parameter, which changes with the increment of the stages. The algorithm required an upper and lower limit of the parameter. A specified range of the upper and lower limit of the parameter would help to identify the elements with the damaged severity in between that range, but a greater range would increase the computational cost, and the benefits of the damage variable reduction could not be fully utilized. Dinh-Cong *et al*. (2017) presented an efficient multi-stage optimization approach for damage detection in a plate structure. A modified differential evolution (MDE) algorithm was used to minimize the objective function established via flexibility changes of structure. The main idea was to eliminate the low damage variables in each stage, to reduce the search space.

There are three main limitations of these multi-stage approaches. Firstly, the above methods are based on a multi-stage strategy, and the criteria to shift the stages are based on maximum numbers iterations on each stage. These heuristic algorithms are stochastic in nature; sometimes, it may require lesser numbers of iteration than the maximum number of iterations to converge. In that case, an unnecessary extra computational effort will be required. On the contrary, if the algorithm does not reach to desire accuracy within the maximum numbers of iterations, then there is a chance of losing actual damaged members in the next stage. Secondly, former approaches need a predefined threshold value of damage index to identify the healthy elements. This threshold value had to be set by using previous experience. If it is kept too small, then the computational cost eventually will increase. On the other way, if the threshold value is more, it can neglect actual damage members with less damage index. Lastly, in the former approaches, the identified healthy elements were removed from the search space of the next stage. Generally, the numbers of healthy elements are more than damaged elements. So, elimination of healthy elements in successive stages will increase the number of required stages to converge, as well as will increase the computational cost.

A multi-stage optimization technique is a simple and efficient technique to detect damage precisely, but the role of the stopping criteria of each stage and the threshold value of the damage index plays a significant part. A novel multi-stage algorithm is adopted here to reduce the number of damage variables, and a standard PSO is used as an optimizer. Any heuristic based optimization technique can be adopted for the proposed multi-stage damage detection algorithm. But, PSO has been selected in the present study to compare with the previous results of multistage methods based on PSO (Nouri Shirazi *et al.* 2014). Here, proposed multi-stage method will overcome aforesaid limitations and has threefold advantages. Firstly, in this method stopping criteria of different stages depends on the tolerance value of the objective function instead of the number of iterations per stage. Thus, the algorithm becomes independent of the optimization technique used as well as takes care of the stochastic nature of the algorithms, which will eventually save computational effort and enhance the accuracy of the prediction. Secondly, the algorithm does not need a predefined threshold value of damage index for identifying the damaged elements. This algorithm adopts a self-controlled strategy which automatically decides this threshold value of the damage index, no prior knowledge is needed. The algorithm sets a different threshold value for different problems. This value has been updated for each stage even for the same problem and automatically include all the damaged members in search space. Finally, it is observed that in a structure the number of healthy elements is greater than the number of damaged elements. In this multi-stage technique only the most susceptible damaged elements are included in successive stages, instead of excluding only healthy elements. Therefore, lesser numbers of design variables are included in search space, which will produce a higher accuracy and lower computational cost than existing single-stage and multi-stage techniques. For numerical experimentation of the proposed methodology, four examples are considered with and without noise addition to demonstrate the effectiveness of this method.

2. Vibration-based structural damage detection

In the present study, the structural damage is identified utilizing vibration data.

2.1 Theory and background

The equation of motion of an n-degree of freedom system without damping can be expressed as

$$[M]\{\ddot{X}\} + [K]\{X\} = \{f\} \tag{1}$$

[M] and [K] are the (n x n) global mass, and stiffness matrices of the structure respectively. $\{f\}$, $\{X\}$ and $\{\ddot{X}\}$ are the vector externally applied load, displacement vector and acceleration vector respectively. The eigenvalue equation associated with Eq. (1) is given by

$$([K] - \omega_i^2[M])\{\varphi_i\} = 0; \qquad i = 1,2,\dots.n \tag{2}$$

There exists *n* numbers of solutions of Eq. (2) $\{\omega_i\}$ and $\{\varphi_i\}$ represents the natural frequency and corresponding vibration mode shape.

2.2 Damage modeling

Damage to the structure is considered as the reduction of the stiffness and mass. In the present study, it is assumed that damage will affect only the stiffness of the structure, but not the mass of the structure. Damage can be modeled by reducing one of the element's stiffness parameters, such as the moment of inertia, cross-sectional area or elastic modulus. For the present study, the damage is modeled by reducing the elasticity modulus. A scalar parameter 'damage index'(x) is introduced here for reduction of the modulus of elasticity. The value of x varies from 0 to 1, where zero value indicates the no damage case and a value near to one corresponds to complete damage of the corresponding member. The x is defined as follows

$$x_i = \frac{E_i^u - E_i^d}{E_i^u} \tag{3}$$

where $[E]_i^u$ and $[E]_i^d$ are the elastic moduli of the i^{th} member of the structure in the undamaged and damaged state respectively and x_i is the damage index for the corresponding member. Then the stiffness matrix of the i^{th} member can be expressed as

$$[k]_{ei}^d = (1 - x_i)[k]_{ei} \tag{4}$$

$[k]_{ei}$ and $[k]_{ei}^{d}$ are the initial and updated damaged stiffness matrix of the i^{th} member of the structure, respectively. This element stiffness matrix of damaged and undamaged members is assembled to create the total stiffness matrix of the structural system.

The change of mass due to the presence of damage is neglected in the present study (Majumdar *et al*. 2012, Nanda *et al.* 2012). So, the eigenvalue equation of the damaged system is expressed as

$$([K]_e^d - (\omega_i^d)^2[M])\{\varphi_i^d\} = 0; \qquad i = 1,2,\dots.n \quad (5)$$

where $[K]_e^d$ is the global stiffness matrix of the damaged system, and $[M]$ is the global mass matrix. ω_i^d and φ_i^d represents the natural frequency, and corresponding vibration mode shape for the damaged structure respectively.

2.3 Modelling of noise

Noise because of human errors are always present in the results of experimental modal testing. So, the result predicted by the mathematical model differs from the experimental result. It is essential to check the sensitivity of the proposed method with the uncertainty of these measurements errors. So, the simulated natural frequencies and mode shapes are polluted with uniformly distributed random noise. The frequencies and mode shape data including noise can be obtained using the following equations

$$\omega_i^{noise} = \omega_i\left(1 + 2 \times \frac{N_L}{100}(rand - 0.5)\right) \quad (6)$$

$$\varphi_{ji}^{noise} = \varphi_{ji}\left(1 + 2 \times \frac{N_L}{100}(rand - 0.5)\right) \quad (7)$$

where, ω_i^{noise} is the i^{th} natural frequency contaminated by noise and φ_i^{noise} is the j^{th} component of the i^{th} mode shape vector polluted by noise; N_L is the noise level in percentage (e.g., 3.0 relates to 3.0% noise level) and '*rand*' is the random number uniformly distributed in the range [0,1]. It is known that the natural frequencies are less contaminated by noise in comparison with mode shapes. As reported by Dinh-Cong *et al*. (2017) the frequencies and mode shapes are contaminated with a standard error of $\pm 0.15\%$ and $\pm 3.0\%$ respectively.

2.4 Objective function based on modal parameter

The presence of damage will change the modal parameters (frequencies and mode shapes) of the structure. A correlation between the measured and predicted data will lead to the location and severity of the damage. For the present problem, this correlation is achieved by an objective function, which is defined as the error between the measured and numerical modal data. So, the damage detection problem reduced to a minimizing optimization problem. Frequency can be measured very easily and less affected by noise, but it is not possible to locate the damage using only frequency. For this inverse problem, a combination of frequency and mode shapes is selected as the objective function,

The error in frequency

$$F1(X) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\{\omega_{N,i}(X) - \omega_{M,i}\}^2} \quad (8)$$

The correlation between two sets of equal order vectors $\{\varphi_A\}$ and $\{\varphi_x\}$, can be expressed by the Modal Assurance Criteria (Pastor *et al*. 2012). The MAC is defined as

$$MAC(r,q) = \frac{\left|\{\varphi_A\}_r^T\{\varphi_x\}_q\right|^2}{(\{\varphi_A\}_r^T\{\varphi_A\}_r)(\{\varphi_x\}_q^T\{\varphi_x\}_q)} \quad (9)$$

Where $\{\varphi_A\}_r$ represents r^{th} mode shape. A MAC value of 0 indicates no correlation, whereas a MAC value 1 indicates two correlated modes.

The error in the MAC value

$$F2(X) = \sum_{i=1}^{n}\left(1 - diag\left(MAC(\{\varphi_{N,i}(X)\},\{\varphi_{M,i}\})\right)\right)^2 \quad (10)$$

In the Eqs. (8) and (10) subscripts *N* and *M* refer to numerical and measured value, respectively. ω_i is the i^{th} natural frequency and φ_i is the i^{th} mode shape.

The error combination of frequency and The MAC value

$$F(X) = \gamma_1 \times F1(X) + \gamma_2 \times F2(X) \quad (11)$$

where *F(X)* represents a scalar value of error that depends on a vector $X^T = \{x_1, x_2, \dots\dots\dots\dots, x_t\}$, which represents the damage variable containing the damage severity (x_i, i = 1, 2,...t) for all t structural elements of the numerical model. γ_1 and γ_2 are the two weighting factors ensures a well-conditioned objective function (Harrison and Butler 2001). For the present study, the values of γ_1 and γ_2 were taken as 0.01 and 100 respectively. Different numbers of frequencies and mode shape data are used in different examples. So, damage detection problem can be solved by using an optimization algorithm by finding a set of damage variables minimizing *F(X).* This optimization problem is defined as

$$\begin{array}{ll} \text{Find:} & X^T = \{x_1, x_2, \dots\dots\dots\dots, x_t\} \\ \text{Minimize:} & F(X) \\ \text{Subjected to:} & x^l \le x_i \le x^u; \quad i = 1,2\dots\dots t \end{array} \quad (12)$$

where x^l and x^u are the lower and upper bound of the damage index. For the present problem damage index (x) ranges from 0 to 0.9.

The quality of the damage detection technique depends on the sparsity of the measured mode shape data. Mode shape data combining all degrees of freedom (dof) generally leads to better results. But in real life, it is not possible to get the data from all the dofs. So, it is essential to study optimum sensor placement (OSP) to collect the optimum data from the structure for structural damage detection. Guo *et al.* (2004) used improved genetic algorithm to identify the optimal locations of sensors for structural health monitoring. Dinh-Cong *et al.* (2018a) used limited numbers of sensors to detect damage in laminated composite structure using iterated improved reduced system (IIRS) method and Jaya algorithm. Dinh-Cong *et al.* (2018b) applied Neumann series expansion-based second-order model reduction method (NSEMR-II) to condense the structural physical properties due to a limited number of sensors placed on the structure. Next, TLBO is used to identify the structural damage in the truss structures. But, in the present study, it is considered that the sensors are placed at all the nodes of the structures. So, the vibration data from all degrees of freedom are available and used the in the present numerical experimentation. The former approaches can be adopted in case of incomplete or sparse data.

3. The proposed self-controlled multi-stage PSO algorithm

The selection of the optimization algorithm is the most important part of the optimization based damage detection problem. The optimization algorithms often fall into the local optima trap; this is the main drawback of this type of damage detection methods. Local optima trap leads to false detection of damage. To avoid this problem and detect the damage properly an efficient algorithm is prescribed, which is eventually reduces the computational cost as well. In this study, a self-controlled multi-stage optimization method is proposed using PSO. The standard PSO is described at first and then proposed methodology is discussed in detail.

3.1 PSO algorithm

Particle swarm optimization is a population-based optimization technique which is proposed by Kennedy and Eberhart (1995). This is a stochastic optimization method, which is based on the social behavior of animals such as a swarm of birds and fish schooling. A swarm of particles is generated with random position and velocity. The position of each particle is a potential solution to the problem. The position of a swarm particle which is the closest to the destination is considered as the optimum solution of the problem. The procedure is described as follows:

(1) The position of a particle in space will represent the value of design variables. Initially, the position of a particle is randomly generated with the upper and lower limit as mentioned in the optimization problem. Damage indices (x), which are given by Eq. (3) are treated as the design variables in this problem. The position of a particle is defined as $\{x_1, x_2 \ldots x_t\}$, where t is the number of variables of the problem, which is the number of structural members for the present problem.

(2) A swarm of P number of particles is generated with different and random positions. Each of the particles is a potential candidate for the solution. So, the position of the total population is represented as

$$[X]_S = \{x_1, x_2, \ldots\ldots, x_t\}_S : S \in \{1,2,\ldots\ldots,P\} \tag{13}$$

The objective function F which is given by the Eq. (12) is evaluated as F_S for each particle location. These particles travel in the search space and simultaneously upgrade its position to reach the destination, which is the optimum solution of the problem. The lowest value of the objective function F_S indicates the best solution, which is the nearest position of the particle to the destination and let the best position of the best particle is $[X]^{best}$. Let $[X]_S^{best} = [X]_S$

(3) A loop is initiated for a user-defined number of iterations N. Each particle position is updated with the help of Eqs. (14a) and (14b)

$$[X]_S^{i+1} = [X]_S^i + [V]_S^{i+1} : i \in \{1,2,\ldots.,N\}, \quad \forall S \in \{1,2,\ldots\ldots,P\} \tag{14a}$$

here [V] is the velocity of the particle, it updates the particle position.

$$[V]_S^{i+1} = w^i [V]_S^i + C_1 r_1 \left([X]_S^{best} - [X]_S^i\right) + C_2 r_2 \left([X]^{best} - [X]_S^i\right) \tag{14b}$$

The objective function $F_S^{i+1}\left([X]_S^{i+1}\right)$ is evaluated for the new particle position using Eq. (12).

$$\begin{aligned} F_S^{i+1} &< F([X]^{best}), \quad \text{Then let} \quad [X]^{best} = [X]_S^{i+1} \\ F_S^{i+1} &< F([X]_S^{best}), \quad \text{Then let} \quad [X]_S^{best} = [X]_S^{i+1} \end{aligned} \tag{15}$$

(4) End of the loop

(5) After completion of all iterations particle position $[X]^{best}$ is considered as the best solution.

Where $[X]_S^i = \{x_1, x_2, \ldots\ldots, x_t\}_S : S \in \{1,2,\ldots\ldots,P\}$ Represents position vector of the S th particle in the ith iteration for the t variables.

$[V]_S^{i+1}$ is the velocity vector of the Sth particle in the $(i+1)$th iteration,

$[X]_S^{best}$ is the best position of the Sth particle between first and ith iteration,

$[X]^{best}$is the best position achieved by the whole swarm of a particle between first and i th iteration.

r_1 and r_2 are random numbers uniformly distributed within [0,1]. C_1 and C_2 are the cognitive and social scaling parameter respectively. The term w^i denotes the inertial weight, which controls the balance between the global-local searches. The inertia weight w^i is defined by a linearly decreasing function from a maximum value w_{max} to a minimum value w_{min}. More information about the PSO in the application of structural damage detection can be found in the works of Saada *et al.* (2013), Nanda *et al.* (2012). A study on the PSO parameters is performed to get an optimum result with minimum computational cost efficiency. Thus PSO parameters for the present problem is selected which are reported in Table 1.

Table 1 PSO parameters for all examples

Parameters	Description	Value
P	The number of particles	75
C_1	Cognitive parameter	2.05
C_2	Social parameter	2.05
w_{max}	Maximum of inertia weight	1.10
w_{min}	Minimum of inertia weight	0.10

3.2 Self controlled multistage optimization method

A self-controlled multi-stage PSO (SCMSPSO) algorithm is proposed to accurately detect the damage location and severity with less computational cost. In general, only a few members of a structure are damaged, and the other members remain undamaged. But in a single stage method sometimes undamaged members are shown as damaged members with lesser severity than actual damaged members. As a result, the accuracy of the method is affected, and the original damage location and severity are not properly detected. In the present study, a self-controlled multi-stage strategy is proposed to minimize the objective function. In previous multi-stage studies stage, the termination criterion was based on the iteration number, which does not guarantee accuracy and cost efficiency simultaneously. In this method, the first stage termination criterion is based on the desired tolerance of the objective function. This will ensure that the original solution will remain in the search space. After satisfying the convergence criteria of the first stage, a threshold value of damage index is defined by the algorithm. This damage index threshold value will be updated in each stage by the algorithm until the problem converges. Previously this threshold value was taken arbitrarily and needed previous expertise. The design variables more than that value are considered as unhealthy elements, which are the most likely elements of the structure to be damaged. Only these damage prone elements are included in the next stage which results in the reduction in the dimension of the search space. This method includes all the damaged elements in a successive way, and the accuracy of the PSO algorithm increases subsequently. Thus, the algorithm achieves to exact localization and quantification of damage. Different steps involved in this method are enumerated as follows:

(1) Set the parameters of PSO, and the number of initial design variables is t, which is the original total number of structural members. The values of other parameters (β) of the algorithm are set.
(2) Randomly generate initial position and velocity of the P particles in the t dimensional space with specified limits

$$X^l \leq X_i \leq X^u \quad \text{and} \quad V^l \leq V_i \leq V^u$$
$$i = 1, 2, \dots P$$

(3) Find an initial solution, $X_{pso}^T = \{x_1, x_2, \dots\dots, x_t\}$ by using standard PSO and achieve the first stage convergence criteria.
(4) Find the maximum value of damage index (x_{max}) from the last solution. The threshold value of damage index to identify most damage prone elements are defined by a parameter μ. The value of μ is set by (x_{max}/β).
(5) The members having damage index more than value μ is considered as an unhealthy element. And find the total numbers of such members.
(6) Only unhealthy members (v) are kept in the set of design variables of the search space of the next optimization. So, the dimension of the optimization is reduced to v from t. Thus the dimension reduction of search space is (t-v). The damage indexes of healthy elements are set as zero.
(7) Solve the problem using standard PSO and get the new solution, i.e., $X_{pso}^T = \{x_1, x_2, \dots\dots, x_v\}$
(8) Check the convergence criteria. If satisfy go to step 9. Otherwise, go to step 4 with an increment of β.
(9) Save the final optimal solution and stop the optimization process.

From step 1 to 9 is described by a flowchart which is shown in Fig. 1. This algorithm works for a high number of design variables as well as low numbers of design variables, which is shown in examples considered in the study. A

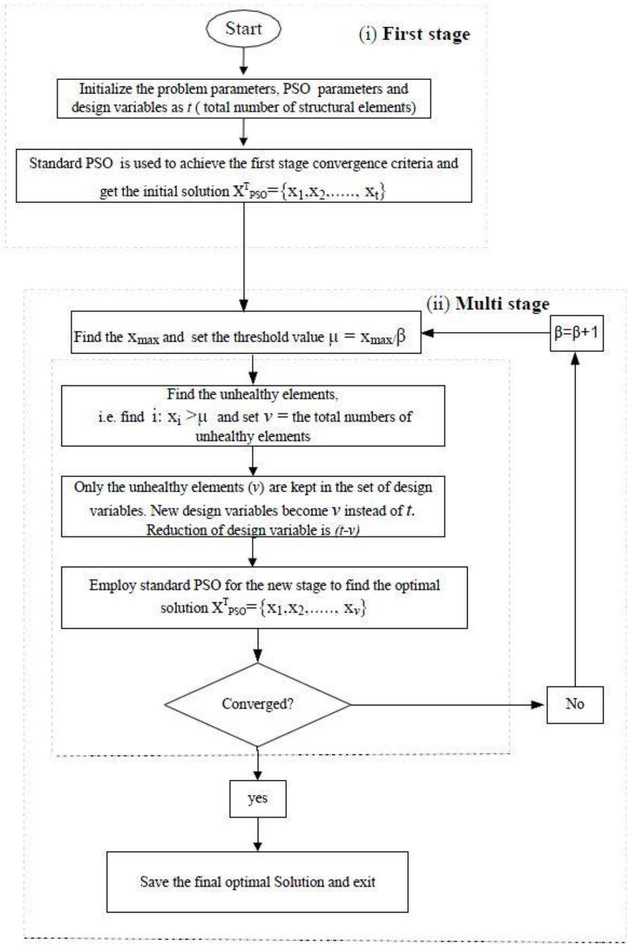

Fig. 1 The flowchart of the self-controlled multi-stage optimization algorithm

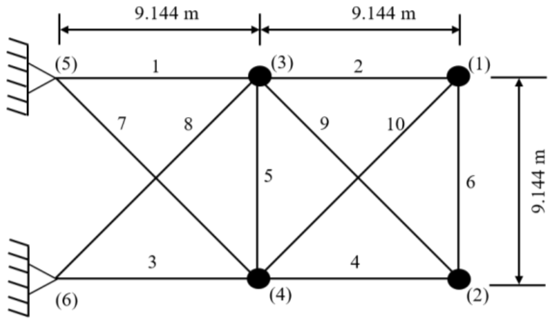

Fig. 2 10-bar planar truss (Kaveh and Mahdavi 2016)

Table 2 Two damage scenarios induced in 10-bar planar truss

Damage scenario I		Damage case scenario II	
Element number	Damage percentage	Element number	Damage percentage
1	5.0%	2	5.0%
-	-	4	10.0%

higher number of design variables takes a higher number of iterations to converge. In the present study, the initial value of β is taken as 2. The value of β gives the stage number (i.e., the 2th stage will have the value of β = 2). Stopping criteria for the first stage of the algorithm is considered as 1e-2. And, the final stopping criteria for the present algorithm either 1e-7 (Dinh-Cong *et al.* 2018b) or no relative changse in the objective function for the next 50 iterations.

4. Numerical examples

In order to show the efficiency of the proposed method for structural damage detection, four illustrative test examples are considered (Majumdar *et al.* 2012, Nouri Shirazi *et al*. 2014, Kaveh and Mahdavi 2016). The first example is a 10-bar planar truss, the second one is a 31-bar planar truss, the third example is a 47-bar planar truss and the last example is a 25- bar spatial truss. All the examples are studied in two circumstances of noise-free and noisy measurement data. In first three examples, frequencies and mode shapes are contaminated with 0.15% and 3.0% noise respectively. In the last example 0.5% and 5.0% noise is added to pollute the frequencies and mode shapes respectively to check the efficiency of the algorithm at higher level of noise. This will prove that noise addition will not affect the solution at all. Ten independent runs are made for each damage scenario to take care of the stochastic nature of the optimization algorithm.

4.1 10 -bar planar truss

A planar truss consists of ten members shown in Fig. 2 is selected from Kaveh and Mahdavi (2016). The modulus of elasticity, material density, and cross-sectional area is considered as 2770 kg/m^3, 69.8 GPa and 0.0025 m^2. Two independent damage scenarios which are considered in this example are described in Table 2. Kaveh and Mahadavi utilized the first eight natural frequencies, and corresponding mode shapes to detect the damage in this structure. So, in the present study also the first eight frequencies are used to detect damage and results are compared with Kaveh and Mahdavi (2016).

The performance of the proposed method for damage detection of the 10 bar planar truss is compared with the results obtained by Enhanced Colliding Bodies Optimization (ECBO) (Kaveh and Mahdavi 2016) and PSO. ECBO is selected to compare the results with SCMSPSO because it is one of the most recent algorithms. The performance of single stage ECBO is excellent in damage detection problem reported by Kaveh and Mahdavi (2016). Only average value of predicted damage without noise is available for ECBO. Table 3 shows the average value of damage index, standard deviation, and an average number of structural analyses of the ECBO, PSO, and SCMSPSO for the scenarios I and II respectively. All three methods can detect the damage location accurately with a very small

Table 3 The statistical results of damage assessment in 10- bar planar truss obtained by ECBO, PSO and SCMSPSO for each scenario with and without noise

Scenario	Actual location	Without noise									With noise					
		ECBO (Kaveh and Mahdavi 2016)			PSO			SCMSPSO			PSO			SCMSPSO		
		Avg. Value	Std. Dev	Avg. NSA	Avg. value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. value	Std. Dev	Avg. NSA
1	α^1	0.04977	_	_	0.0500	0.0	4700	0.05	0.0	1970	0.0387	7.43e-03	14300	0.0523	6.00e-03	6740
2	α^2	0.1000	_		0.9994	1.68e-04	25890	0.10	5.14e-06	8925	0.1002	2.54e-03	17650	0.1005	5.50e-03	9425
	α^4	0.0499	_	_	0.05005	1.65e-04		0.05	8.44e-06		0.0487	2.31e-03		0.0505	5.71e-03	

*Avg. Value = average value of damage index with respect to *F*; Std Dev = standard deviation with respect to *F*;
Avg. NSA= an average number of structural analyses

Table 4 Damage variable identified in the different stages of SCMSPSO and PSO for the scenario I of the 10-bar planar truss with noise free and noisy data

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
1	0.05	**<u>0.0541</u>**	0.05	0.05	**0.0157**	0.0485	0.0466
2	0.0	0	-	0	0.0009	-	0.0029
3	0.0	0	-	0	**<u>0.0280</u>**	0.0	0
4	0.0	0	-	0	0	-	0
5	0.0	0	-	0	0	-	0
6	0.0	0	-	0	0	-	0
7	0.0	0	-	0	0	-	0
8	0.0	0	-	0	0	-	0
9	0.0	0	-	0	0.0117	-	0.0045
10	0.0	0	-	0	0	-	0.0036
TVDI ($\boldsymbol{\mu}$)	-	0.0207	-	-	0.0140	-	-
RSA		450	300	5025	825	5555	20550
TRSA		-	750	5025	-	6380	20550
Design variables		10	1	10	10	2	10
Elimination		0	9	-	0	8	-

*TVDI = threshold value of damage index; RSA = required structural analyses; TRSA = total required structural analyses

error in quantifying the severity of damage. Particularly, for the scenario I without noise, the error in the quantification of damage index is 0.46%, 0.0% and 0.0% for ECBO, PSO, and SCMSPSO respectively. The cost efficiency of the algorithms can be measured by comparing the average number of structural analyses (NSA). SCMSPSO takes 58.08% less computational effort (NSA) than PSO. For the scenario I with noise (Table 3), the errors in damage index quantification are 22.6% and 4.6% for PSO and SCMSPSO respectively. The accuracy of SCMSPSO is better than PSO, as SCMSPSO successfully avoids false damage detection. Moreover, SCMSPSO takes 52.87% less NSA than PSO. Similarly, for the scenario II SCMSPSO performed better than ECBO and PSO, both without noise and with measurement noise. SCMSPSO shows better accuracy than the other two algorithms, in terms of damage index quantification with less computational cost.

A typical run of the scenario I obtained by SCMSPSO and PSO without noise and with noise measurement is shown in Table 4. The problem converges in the second stage for SCMSPSO for both noisy as well as noise-free measurements. The required number of structural analyses in each stage, the total number of required structural analyses, design variables in each stage and elimination of damage variables in successive stages are described in Table 4. In case of without noise-free measurements, the first stage of SCMSPSO the number of variables is ten, which is equal to the total number of structural members. The optimization algorithm runs until the objective function value reaches to stopping criteria of the first stage. Then the problem enters into the next stage, and only the members with damage index more than the threshold value is considered here. In the second stage of the present run, the design variable number reduces to one from ten. Thus, the numbers of the damage variables decrease with the increment of stage, and consequently the accuracy of damage prediction increases. Whereas, PSO requires much more average numbers of structural analyses (NSA) than SCMSPSO, to achieve the same level of damage prediction accuracy. The damage prediction with noise by SCMSPSO successfully predicts only the damage members, whereas PSO predicts some other members with very less severity damage. Thus, SCMSPSO produces more accurate results than PSO and successfully avoids false damage prediction with considerably less computational cost. The damage identification results for the two damage scenarios are shown in Figs. 3 and 4 with noise and without noise. These figures show that this method accurately finds the damage location and severity without any false identification. Both the scenarios converge at the second stage of SCMSPSO.

4.2 31 -bar planar truss

The 31-bar planar truss shown in Fig. 5 is selected from Nouri Shirazi *et al.* (2014). The elasticity modulus and density of the material are 70 Gpa and 2770 kg/m^3. In the present problem, three different damage scenarios are selected from Nouri Shirazi *et al.* (2014). These three damage scenarios are shown in Table 5. According to Nouri Shirazi *et al.* (2014), the first ten natural frequencies and corresponding mode shapes are utilized to predict the damage.

The performance of the proposed method compared with MPSO (Nouri Shirazi *et al.* 2014) and PSO. The

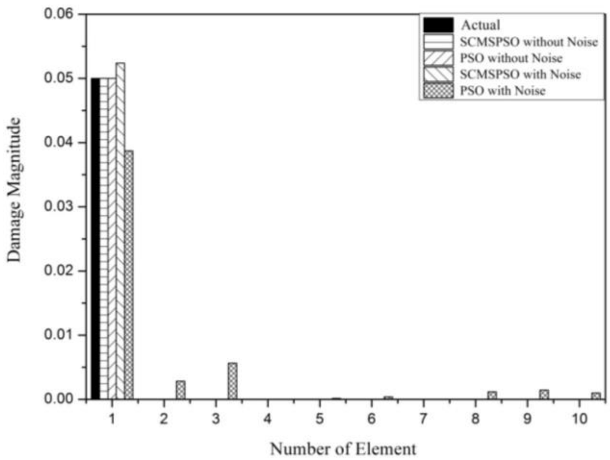

Fig. 3 The obtained results of damage prediction for 10-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario I

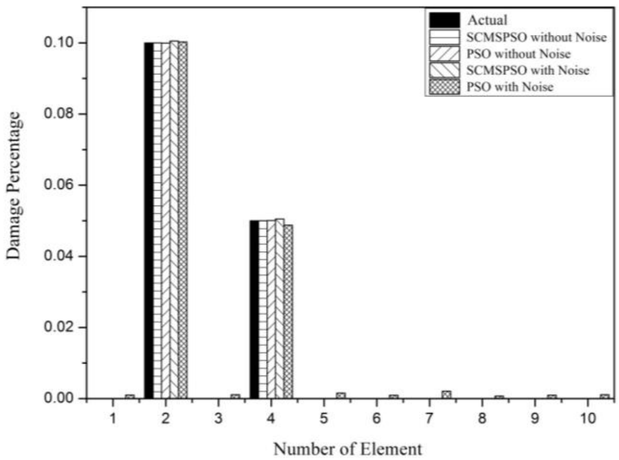

Fig. 4 The obtained results of damage prediction for 10-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario II

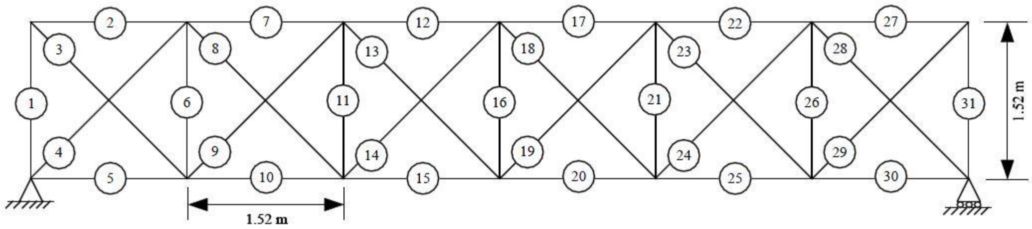

Fig. 5 31-bar planar truss (Nouri Shirazi *et al*. 2014)

Table 5 Three damage scenarios induced in 31-bar planar truss

Damage scenario I		Damage scenario II		Damage scenario III	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
11	0.25	16	0.30	1	0.30
25	0.15	-	-	2	0.20

Table 6 The statistical results of damage assessment in 31- bar planar truss obtained by MPSO, PSO and SCMSPSO for each scenario with and without noise

Scenario	Actual location	Without noise									With noise					
		MPSO (Nouri Shirazi *et al.* 2014)			PSO			SCMSPSO			PSO			SCMSPSO		
		Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA
I	α^{11}	0.2495	1.38e-3	35700	0.25	0.0	42900	0.25	0.0	6685	0.1959	8.64e-02	108645	0.2633	3.16e-02	12135
	α^{25}	0.1496	9.24e-4		0.15	0.0		0.15	0.0		0.1192	2.39e-02		0.1447	1.41e-02	
II	α^{16}	0.3000	0.0	14100	0.30	0.0	16660	0.30	0.0	7780	0.2484	7.36e-02	53430	0.3214	2.84e-02	15840
III	α^{1}	0.3000	0.0	33900	0.30	0.0	48540	0.30	0.0	8260	0.2859	1.03e-02	112575	0.2988	4.83e-03	11750
	α^{2}	0.2000	0.0		0.20	0.0		0.20	0.0		0.1518	6.67e-02		0.2024	8.78e-03	

*Avg. Value = average value of damage index with respect to *F*; Std Dev = standard deviation with respect to *F*;
Avg. NSA= an average number of structural analyses.

Table 7 Damage variable identified in the different stages of SCMSPSO and PSO for the scenario III of the 31-bar planar truss with noise free and noisy data

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
1	0.0	0	-	0.0	0.0	-	0.0
2	0.0	0.0021	-	0.0	0.0277	-	0.0547
3	0.0	0.0	-	0.0	0.0068	-	0.0044
4	0.0	0.0	-	0.0	0.0	-	0.0
5	0.0	0.0	-	0.0	0.0001	-	0.0
6	0.0	0.0057	-	0.0	0.0	-	0.0
7	0.0	0.0	-	0.0	0.0010	-	0.0018
8	0.0	0.0	-	0.0	0.0469	-	0.0
9	0.0	0.0	-	0.0	0.0	-	0.0
10	0.0	0.0	-	0.0	0.0328	-	0.0
11	0.25	**0.2201**	0.25	0.25	**0.1039**	0.2596	0.1451
12	0.0	0.0	-	0.0	0.0	-	0.0
13	0.0	0.0	-	0.0	0.0	-	0.0
14	0.0	0.0	-	0.0	0.0	-	0.0
15	0.0	0.0	-	0.0	0.0	-	0.0
16	0.0	0.0	-	0.0	0.0	-	0.0
17	0.0	0.0	-	0.0	0.0	-	0.0
18	0.0	0.0	-	0.0	0.0	-	0.0
19	0.0	0.0	-	0.0	0.0	-	0.0
20	0.0	0.0	-	0.0	0.0	-	0.0
21	0.0	**0.1453**	0.0	0.0	0.0	-	0.0521
22	0.0	0.0	-	0.0	0.0	-	0.0074
23	0.0	0.0008	-	0.0	0.0	-	0.0
24	0.0	0.0031	-	0.0	0.0	-	0.0
25	0.15	**0.1240**	0.15	0.15	**0.1198**	0.1473	0.1250
26	0.0	0.0026	-	0.0	0.0220	-	0.0520
27	0.0	0.0	-	0.0	0.0	-	0.0
28	0.0	0.0	-	0.0	0.0	-	0.0
29	0.0	0.0	-	0.0	0.0	-	0.0043
30	0.0	0.0064	-	0.0	0.0	-	0.0
31	0.0	0	-	0.0	0.0	-	0.0
TVDI ($\boldsymbol{\mu}$)		0.110	-	-	0.0599	-	-
RSA		4350	2625	25500	7400	5225	112575
TRSA		-	6975	25500	-	12625	112575
Design variables		31	3	31	31	2	31
Elimination		0	28	-	0	29	-

*TVDI = threshold value of damage index; RSA = required structural analyses; TRSA = total required structural analyses

average solutions, standard deviation and an average number of structural analyses of the MPSO, PSO, and SCMSPSO for all the scenarios of this problem are shown in Table 6. It can be observed from Table 6; all the algorithms can predict damage location as well as severity with an acceptable precision without noise. However, the difference can be noticed in terms of computational cost. SCMSPSO proves to be the most cost-efficient among the algorithms.

Apart from this, in the presence of noise, the error will increase in all the scenarios. In the case of noisy measurement data, only PSO results are compared with the

SCPSPSO results due to unavailability of the results of MPSO. It can be observed from Table 6 that SCMSPSO predicts the damage severity more accurately with a lesser average number of structural analyses than PSO. More specifically, in the scenario I without noise all the algorithms predict damage severity and location accurately. Whereas MPSO requires lesser computational cost than PSO, but SCMSPSO takes even lesser computational cost among than PSO. SCMSPSO requires 81.27% and 84.41% less computational time (NSA) than MPSO and PSO respectively. As seen from Table 6, for the scenario I SCMSPSO predicts the damage severity with an error of 5.32% and 3.53% for the elements 11 and 25 respectively. While the error between the obtained averages damage index and actual damage index by PSO is 21.64% and 20.53% for the elements 11 and 25 respectively. Similarly, in scenario II and III, all the algorithm performs excellently in regards of damage severity prediction without noise. But the computational cost is the lowest for the SCMSPSO. In the case of measurements with noise (Table 6) SCMSPSO predicts the damage extent with a mean error of 7.13% and 0.8% for scenario II and III respectively while those errors for PSO is 17.2% and 14.4% respectively. The accuracy of the damage severity prediction of actual damaged elements is improved for SCMSPSO as it successfully avoids false damage prediction. Moreover, with noisy measurements, SCMSPSO performs far better than PSO in terms of computational cost.

Table 7 shows a typical run of scenario III obtained by SCMSPSO and PSO without noise and with noise measurement data. It is observed from Table 7, in the first stage of SCMSPSO with noise-free measurements; the variable number is the same as total numbers of truss elements which are thirty-one. In the first stage, SCMSPSO predicts the actual damaged elements and some false damaged elements in stage one, with few numbers of structural analyses. But, in the second stage, the algorithm keeps only three design variables in the search space. As a result, the problem converges in the second stage with a small number of structural analyses. Whereas, PSO requires

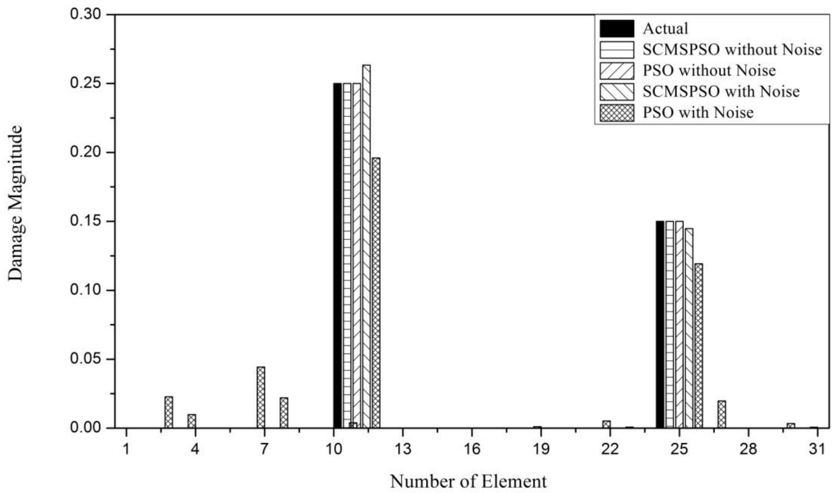

Fig. 6 The obtained results of damage prediction for 31-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario I

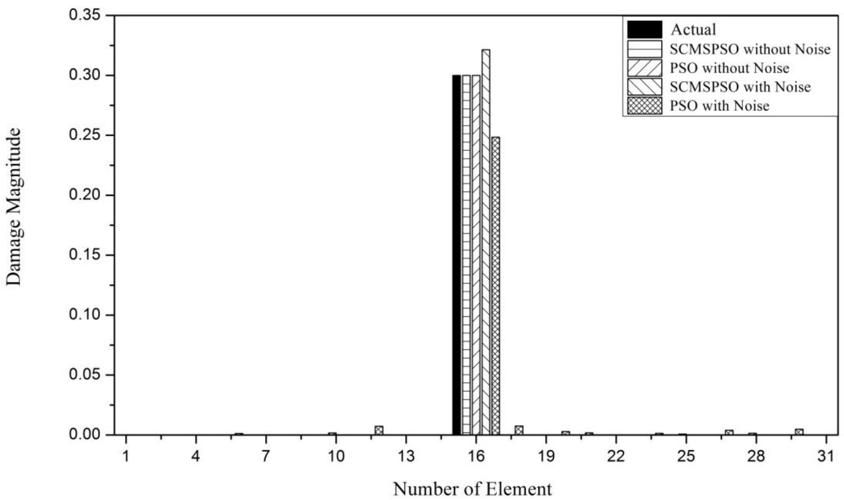

Fig. 7 The obtained results of damage prediction for 31-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario II

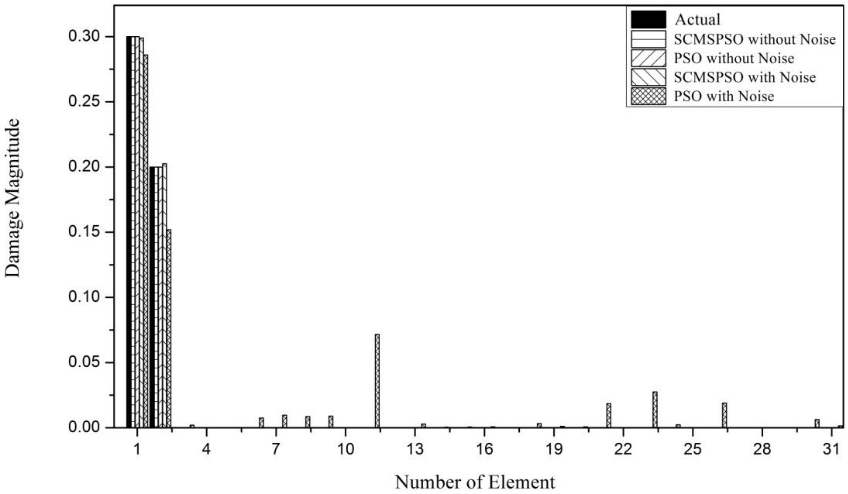

Fig. 8 The obtained results of damage prediction for 31-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario III

much more structural analyses to achieve the same accuracy of the solution. For noisy data, SCMSPSO shows less accurate results at the first stage as the number of variables is thirty-one. But, in the second stage, the number of design variables reduced to two, and it gives better accuracy than single stage PSO, as it successfully eliminates members without damage. SCMSPSO requires lesser structural analyses than PSO for damage detection with noise. Fig. 6, Figs. 7 and 8 show the three damage scenarios identified by SCMSPSO and PSO with and without noise. It can be observed from figures that SCMSPSO predict the damage location and damage extent with acceptable accuracy with and without noise. All the scenarios converge at the second stage of the algorithm.

4.3 47 -bar planar truss

A two-dimensional 47- bar power line tower shown in Fig. 9 is selected from Nouri Shirazi *et al.* (2014). The material density and elastic modulus for the members of the problem are 0.3 lb/in^3 and 30,000 ksi respectively. For the present problem, four different damage cases are selected from Nouri Shirazi *et al.* (2014) , which are shown in Table 8. The first ten natural frequencies and corresponding mode shapes are utilized to predict the damage.

As seen in Table 9 without noise measurements PSO and SCMSPSO both can identify and quantify damage with zero error, but SCMSPSO takes much less computational time than PSO. More specifically SCMSPSO requires 54.84%, 61.99%, 75.95% and 71.56% less structural analyses than PSO for damage detection of scenario I, II, III and IV respectively. While with noise measurements, SCMSPSO can identify damage and quantify it with minor error and computational time than PSO. The mean error between obtained average damage indices and actual damage indices by SCMSPSO are 0.63%, 1.03%, 0.95% and 0.625% for scenarios I, II, III and IV respectively. While those for PSO are 2.9%, 6.4%, 4.03% and 13.71% respectively. It should be emphasized that the SCMSPSO requires much less number of average structural analyses than PSO in noise measurements also. Thus, self-controlled multistage strategy helps the PSO algorithm to increase the accuracy and decrease the computational cost significantly.

The damage identification by MPSO is represented by Nouri Shirazi *et al.* (2014) differently. The comparison of results between SCMSPSO and MPSO is shown in Tables 10 and 11, which follows the representation of Nouri Shirazi *et al.* (2014). In Tables 10 and 11, the sign ‘+’ represents the identification of the exact location and magnitude of the damage. The sign ‘x’ represents the identification of the exact location without proper identification of damage extent. The sign ‘-’ represents the failure to identify the location and extent of the damage. Tables 10 and 11 show SCMSPSO identifies the damage with better accuracy and lesser computational cost than MPSO in all scenarios and both noisy and noise-free measurements. A typical run of scenario III obtained by SCMSPSO and PSO without noise and with noise measurement data is shown in Table 12. SCMSPSO utilizes the total number of structural members as initial design variables, and in the second stage, the design variables are reduced by the algorithm. As a result, SCMSPSO achieves

Table 8 Four damage scenarios induced in 47-bar planar truss

Damage scenario I		Damage scenario II		Damage scenario III		Damage scenario IV	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
10	0.30	30	0.30	10	0.30	40	0.30
-	-	-	-	30	0.30	41	0.20

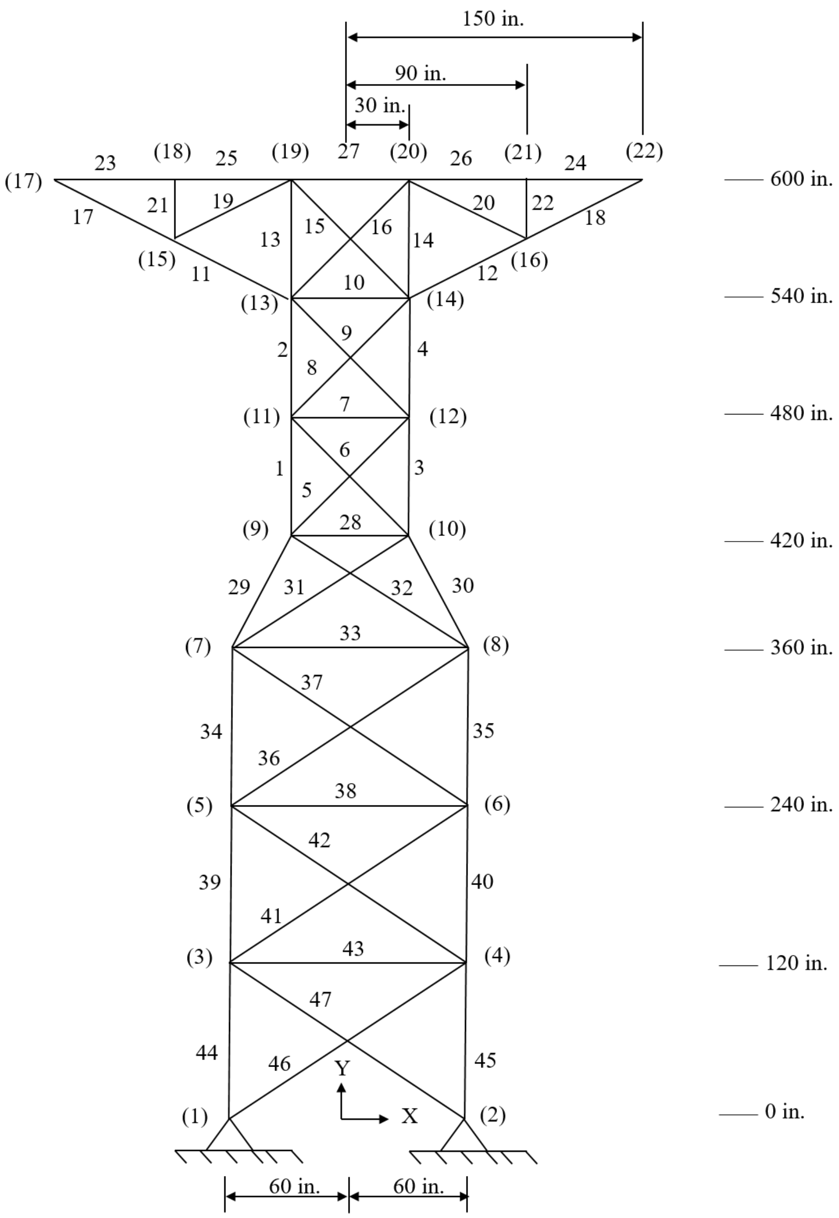

Fig. 9 47-bar planar truss (Nouri Shirazi *et al*. 2014)

Table 9 The statistical results of damage assessment in 47- bar planar truss obtained by PSO and SCMSPSO for each scenario with and without noise

Scenario	Actual location	Without noise						With noise					
		PSO			SCMSPSO			PSO			SCMSPSO		
		Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA
I	α^{10}	0.30	0.0	17685	0.30	0.0	7985	0.2913	8.21e-03	67640	0.3019	7.60e-03	12915
II	α^{30}	0.30	0.0	23285	0.30	0.0	8850	0.2808	1.14e-02	57320	0.2969	1.38e-02	15120
III	α^{10}	0.30	0.0	44925	0.30	0.0	10800	0.2914	9.24e-03	6980	0.2991	7.54e-03	20708
	$\alpha30$	0.30	0.0		0.30	0.0		0.2844	9.29e-03		0.2952	1.29e-02	
IV	α^{40}	0.30	0.0	46840	0.30	0.0	13320	0.2696	2.47e-02	86790	0.2967	1.00e-02	20340
	α^{41}	0.20	0.0		0.20	0.0		0.1654	6.35e-02		0.2003	5.81e-03	

*Avg. Value = average value of damage index with respect to F; Std Dev = standard deviation with respect to F;
Avg. NSA = an average number of structural analyses

Table 10 The damage identification results of 47-bar planar truss for scenario I to IV using MPSO and SCMSPSO without noise

Sample no.	Damage identification result							
	MPSO (Nouri Shirazi *et al.* 2014)				SCMSPSO			
	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario I	Scenario II	Scenario III	Scenario IV
1	+	+	x	+	+	+	+	+
2	-	+	+	+	+	+	+	+
3	+	+	x	+	+	+	+	+
4	+	+	+	+	+	+	+	+
5	-	+	x	-	+	+	+	+
6	+	+	+	+	+	+	+	+
7	+	x	+	+	+	+	+	+
8	+	+	+	+	+	+	+	+
9	+	+	+	+	+	+	+	+
10	+	+	+	x	+	+	+	+
Proper identification (%)	80	90	70	90	100	100	100	100
Avg. NSA	40800	17340	34170	28662	12915	8850	10800	13320

*Avg. NSA = an average number of structural analyses

Table 11 The damage identification results of 47-bar planar truss for scenario I to IV using MPSO and SCMSPSO with noise

Sample no.	Damage identification result							
	MPSO (Nouri Shirazi *et al.* 2014)				SCMSPSO			
	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario I	Scenario II	Scenario III	Scenario IV
1	x	x	x	-	x	x	x	x
2	-	x	-	x	x	x	x	x
3	-	x	x	x	x	x	x	x
4	-	-	x	x	x	x	x	x
5	x	x	x	x	x	x	x	x
6	x	x	x	x	x	x	x	x
7	x	x	x	x	x	x	x	x
8	x	-	x	-	x	x	x	x
9	x	x	x	x	x	x	x	x
10	x	-	-	x	x	x	x	x
Proper identification (%)	70	70	80	80	100	100	100	100
Avg. NSA	180790	156550	165640	265630	7985	15120	20708	20340

*Avg. NSA = an average number of structural analyses

better accuracy with less computational cost than PSO for both with and without noise.

The final damage prediction of this 47-bar planar truss for the scenario I and scenario II without noise and with noise is also performed by Nobahari *et al.* (2017) utilizing genetic algorithm (GA) and echolocation search algorithm (ESA). The results of GA and ESA are also compared with SCMSPSO only for these two scenarios with and without noise. The final damage prediction of scenario I with and without noise is shown in Figs. 10 and 11 respectively. Fig. 10 shows that SCMSPSO performs better than PSO, ESA, and GA. Fig. 11 shows that SCMSPSO predicts damage severity better than other algorithms as it avoids false damage detection where the other algorithms predict false damaged elements. Similarly, Figs. 12 and 15 show structural damage prediction with and without noise measurements by SCMSPSO and PSO for case II and IV respectively. Damage identification by SCMSPSO, PSO, GA and ESA for case III with and without noise is shown in Figs. 13 and 14 respectively. As seen from Figs. 10 to 15, SCMSPSO performs the best among the algorithms and successfully avoids false damage detection for both noisy and noise-free measurements.

Table 12 Damage variable identified in the different stages of SCMSPSO and PSO for the scenario III of the 47-bar planar truss with noise free and noisy data

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
1	0.0	0.0178	-	0.0	0.0002	-	0.0
2	0.0	0.0	-	0.0	0.0	-	0.0
3	0.0	0.0	-	0.0	0.0	-	0.0
4	0.0	0.0	-	0.0	0.0	-	0.0
5	0.0	0.0	-	0.0	0.0	-	0.0
6	0.0	0.0	-	0.0	0.0165	-	0.0
7	0.0	0.0	-	0.0	0.0	-	0.0
8	0.0	0.0	-	0.0	0.0	-	0.0
9	0.0	0	-	0.0	0.0	-	0.0
10	0.30	<u>**0.2963**</u>	0.30	0.30	**0.2767**	0.3014	0.29234
11	0.0	0.0	-	0.0	0.0	-	0.0
12	0.0	0.0002	-	0.0	0.0070	-	0.0
13	0.0	0.0019	-	0.0	0.0021	-	0.0
14	0.0	0.0	-	0.0	0.0	-	0.0
15	0.0	0.0380	-	0.0	0.0	-	0.0
16	0.0	0.0	-	0.0	0.0	-	0.0
17	0.0	0.0	-	0.0	0.0	-	0.0
18	0.0	0.0	-	0.0	0.0039	-	0.0
19	0.0	0.0063	-	0.0	0.0	-	0.0
20	0.0	0.0	-	0.0	0.0	-	0.0
21	0.0	0.0238	-	0.0	**0.1257**	0.0	0.0
22	0.0	**0.1697**	0.0	0.0	0.0109	-	0.0
23	0.0	0.0	-	0.0	0.00863	-	0.0
24	0.0	0.0	-	0.0	0.0	-	0.0
25	0.0	0.0	-	0.0	0.0	-	0.0
26	0.0	0.0	-	0.0	0.0	-	0.0
27	0.0	0.0002	-	0.0	0.0007	-	0.0
28	0.0	0.0	-	0.0	0.0239	-	0.0
29	0.0	0.0007	-	0.0	0.0	-	0.0
30	0.30	**0.2798**	0.30	0.30	<u>**0.2932**</u>	0.2802	0.29172
31	0.0	0.0006	-	0.0	0.02857	-	0.0
32	0.0	0.0052	-	0.0	0.0	-	0.0
33	0.0	0.0	-	0.0	0.0021	-	0.0
34	0.0	0.0	-	0.0	0.0	-	0.0
35	0.0	0.0	-	0.0	0.0	-	0.0
36	0.0	0.0006	-	0.0	0.0	-	0.0
37	0.0	0.0	-	0.0	0.0	-	0.0
38	0.0	0.0001	-	0.0	0.0	-	0.0
39	0.0	0.0001	-	0.0	0.0006	-	0.0059
40	0.0	0.0006	-	0.0	0.0001	-	0.0
41	0.0	0.0005	-	0.0	0.0	-	0.0
42	0.0	0.0	-	0.0	0.0	-	0.0
43	0.0	0.0	-	0.0	0.0	-	0.0
44	0.0	0.0026	-	0.0	0.0	-	0.0

Table 12 Continued

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
45	0.0	0.0	-	0.0	0.0001	-	0.0
46	0.0	0.0	-	0.0	0.0	-	0.0027
47	0.0	0.0117	-	0.0	0.0	-	0.0
TVDI (μ)		0.1481	-	-	0.1466		
RSA		6075	4125	52050	6000	10575	95700
TRSA		-	10200	52050	-	16575	95700
Design variables		47	3	47	47	3	47
Elimination		0	44	-	0	44	-

*TVDI = threshold value of damage index; RSA = required structural analyses; TRSA = total required structural analyses

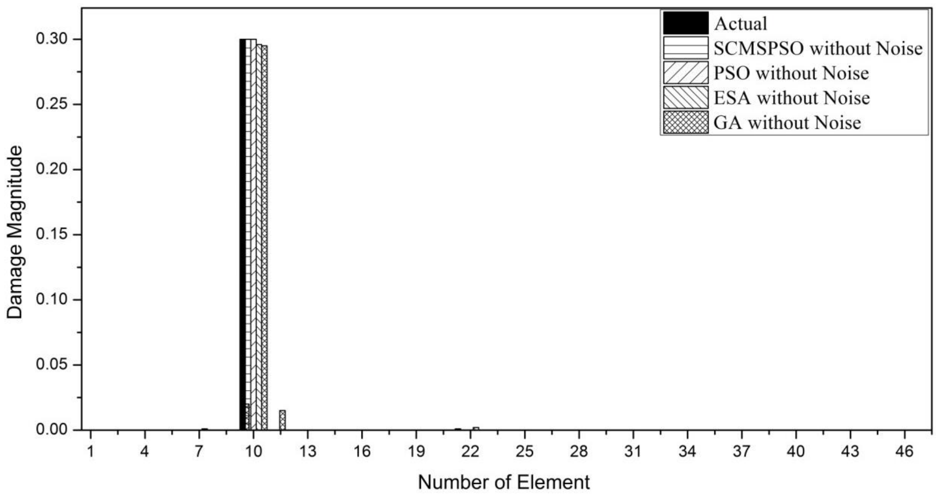

Fig. 10 The obtained results of damage prediction for 47-bar planar truss using GA, PSO, ESA and SCMSPSO considering noise free data for damage scenario I

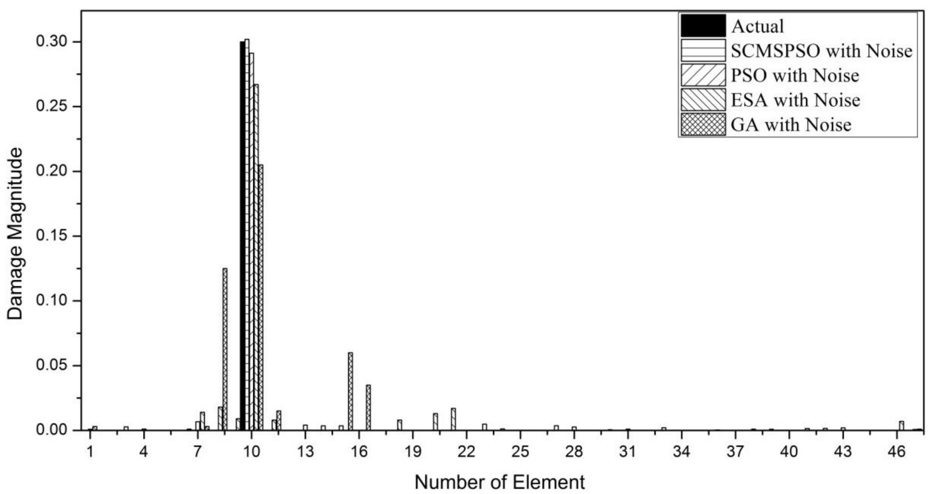

Fig. 11 The obtained results of damage prediction for 47-bar planar truss using GA, PSO, ESA and SCMSPSO considering with noise data for damage scenario I

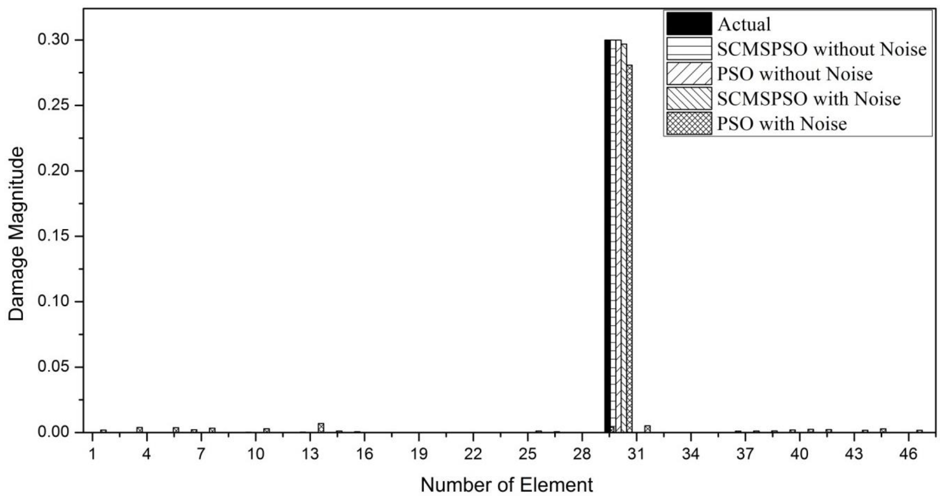

Fig. 12 The obtained results of damage prediction for 47-bar planar truss using PSO and SCMSPSO considering noise free and with noise data for damage scenario II

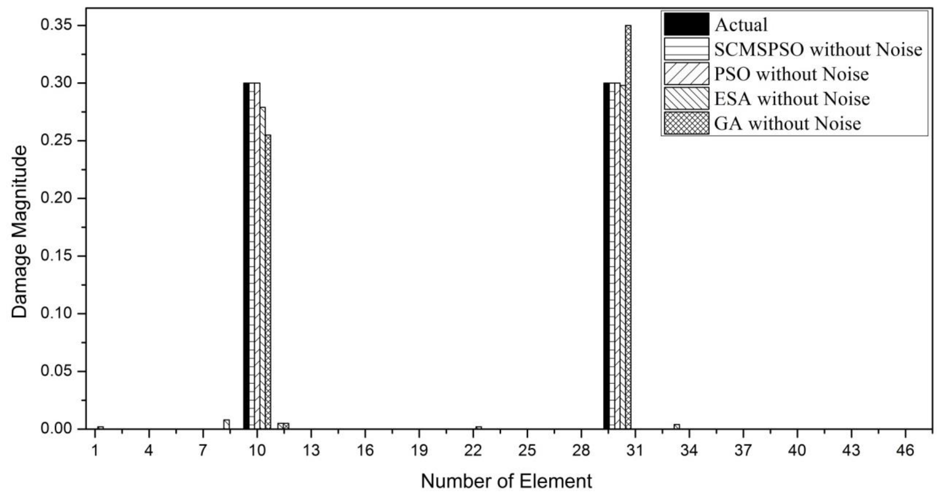

Fig. 13 The obtained results of damage prediction for 47-bar planar truss using GA, PSO, ESA and SCMSPSO considering noise free data for damage scenario III

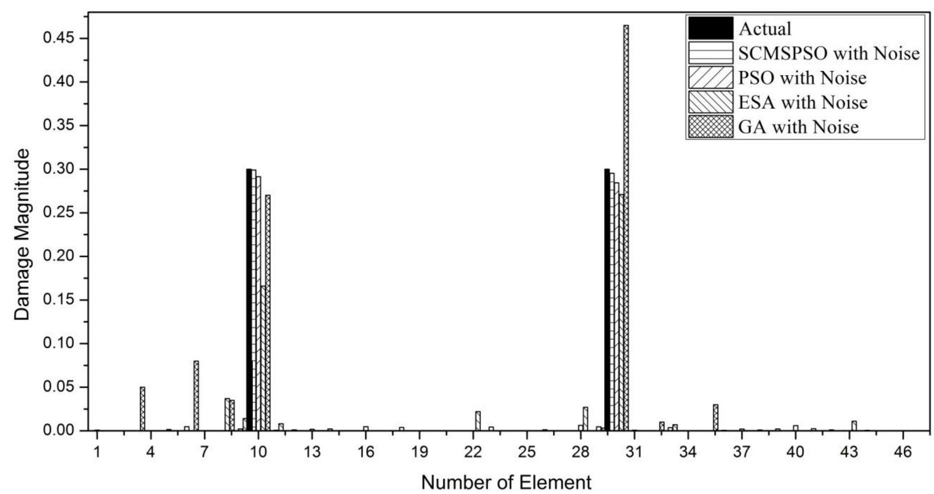

Fig. 14 The obtained results of damage prediction for 47-bar planar truss using GA, PSO, ESA and SCMSPSO considering with noise data for damage scenario III

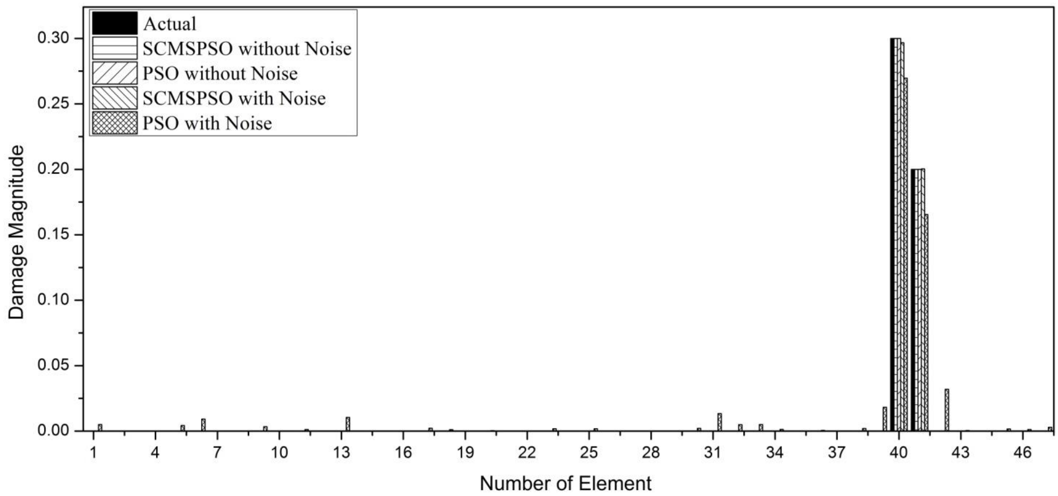

Fig. 15 The obtained results of damage prediction for 47-bar planar truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario IV

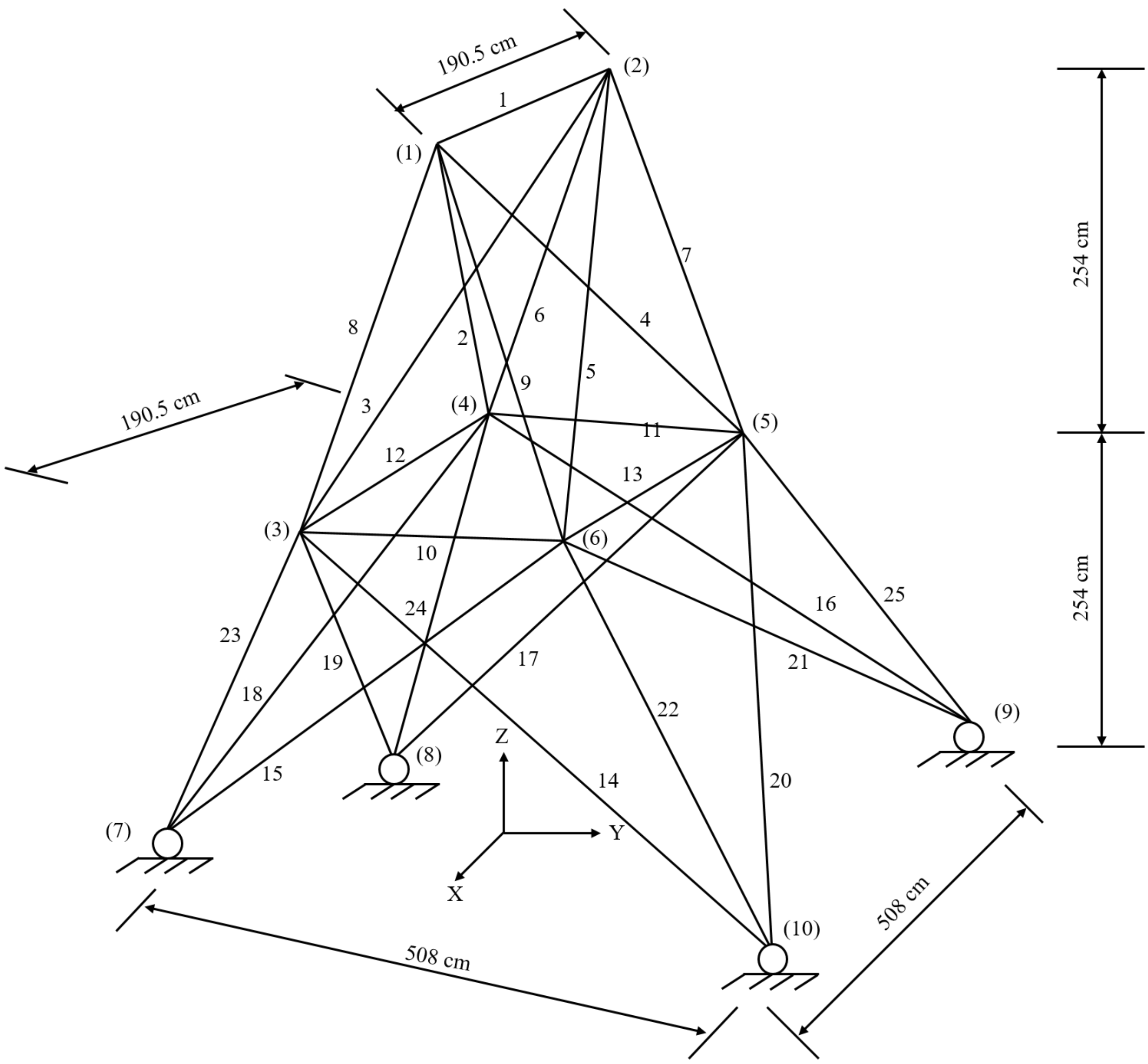

Fig. 16 25-bar spatial truss (Majumdar *et al*. 2012)

Table 13 Two damage scenarios induced in 25-bar spatial truss

Damage scenario I		Damage case scenario II	
Element number	Damage percentage	Element number	Damage percentage
14	0.25	7	0.30
-	-	16	0.30

4.4 25 -bar spatial truss

A 25- bar spatial truss considered by Majumdar *et al.* (2012) is considered as the last example for the present study to show the robustness of the proposed algorithm, is shown in Fig. 16. The elastic modulus, material density, and cross-sectional area of all members are 200 GPa, 7850 kg/m^3, and 6.4165 mm^2 respectively. For the present study, one single member damage and two members damage scenarios are considered, which are shown in Table 13. The first five natural frequencies and corresponding mode shapes are considered to identify the damages.

The performance of the algorithm is compared with single-stage PSO, considering both noise and without noise data. In the present example, natural frequencies and mode shapes are polluted with 0.5% and 5.0% noise respectively to check the efficiency of the algorithm in case of a higher level of noise.

Results of the current problem are shown in Table 14. It is observed from results that PSO and SCMSPSO performed well in scenario I for without noise. SCMSPSO takes less 73.91% less computational effort than PSO. In scenario II without noise, PSO shows an average error of 12.11% to quantify the damage and two false damage locations also identified. But SCMSPSO accurately identifies the damage and quantify the same with 94.05% less computational cost than PSO. In the case of scenario I with noisy data, PSO identifies damage with an error of 2.31%. But it identifies three false damage locations. Whereas, SCMSPSO detects the damage without any false damage, and takes 78.31% less computational time with an error of only 0.34%. Similarly, in scenario II, with noisy data PSO and SCMSPSO identify the damage location with an average error of 17.90% and 0.73%, respectively. PSO identifies six false damage locations, whereas SCMSPSO identifies one false damage location with only 1.58% of damage. In this scenario, SCMSPSO takes 69.58% less

Table 14 The statistical results of damage assessment in 25- bar spatial truss obtained by PSO and SCMSPSO for each scenario with and without noise

Scenario	Actual location	Without noise						With noise					
		PSO			SCMSPSO			PSO			SCMSPSO		
		Avg. value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. Value	Std. Dev	Avg. NSA	Avg. value	Std. Dev	Avg. NSA
1	α^{14}	0.25	.0	18975	0.25	0.0	4950	0.2442	0.0122	59550	0.2508	7.60e-03	12915
2	α^{7}	0.2325	3.46e-04	117300	0.30	0.0	6975	0.2061	0.0675	105300	0.2970	0.0397	32025
	α^{16}	0.2947	7.07e-06		0.30	0.0		0.2863	0.0138		0.3014	0.0099	

*Avg. Value = average value of damage index with respect to *F*; Std Dev = standard deviation with respect to *F*; Avg. NSA = an average number of structural analyses

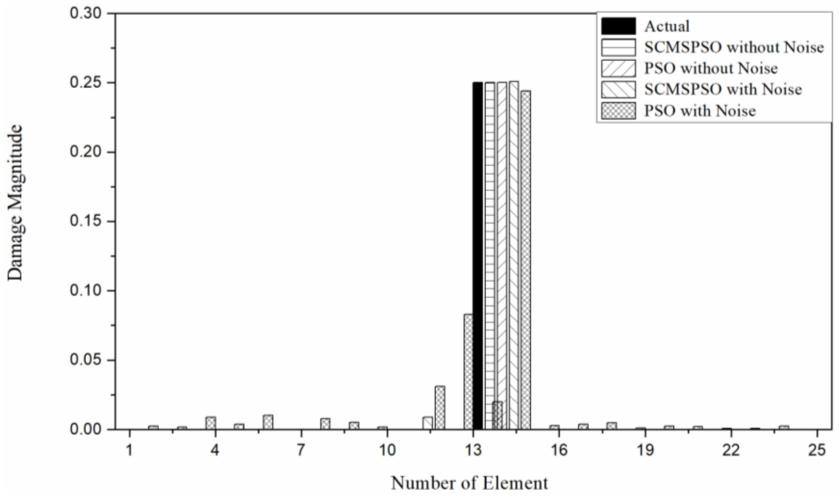

Fig. 17 The obtained results of damage prediction for 25-bar spatial truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario I

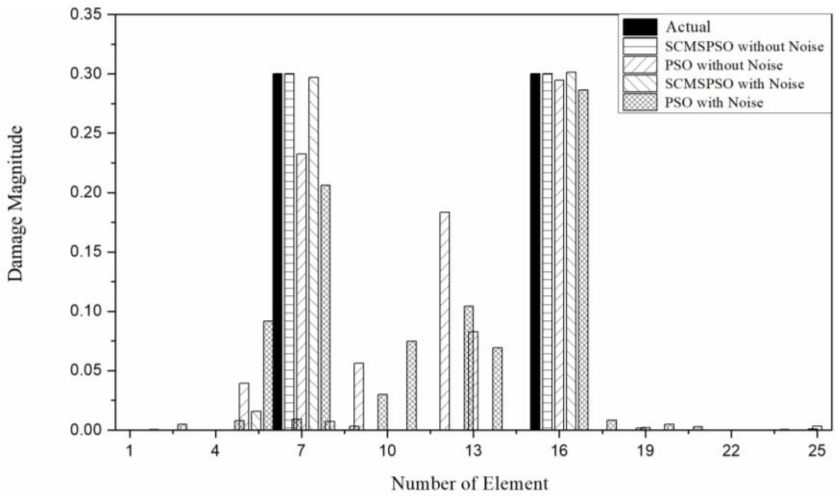

Fig. 18 The obtained results of damage prediction for 25-bar spatial truss using PSO and SCMSPSO considering noise free and noisy data for damage scenario II

Table 15 Damage variable identified in the different stages of SCMSPSO and PSO for the scenario II of the 25-bar planar truss with noise free and noisy data

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
1	0	0	-	0.0	0.0002	-	0.0
2	0	0	-	0.0	0.0241	-	0.0326
3	0	0	-	0.0	0.0	-	0.0
4	0	0	-	0.0	0.04432	-	0.0465
5	0	0.009	-	0.0390	0.0	0	0.0
6	0	0.0	-	0.0	0.0	-	0.0
7	0.3	**0.211**	0.30	0.2328	0.1880	0.2821	0.1825
8	0	0.0	-	0.0075	0.0	-	0.0
9	0	0.0961	-	0.0558	0.0	-	0.0
10	0	0.0070	-	0.0	0.0	-	0.0
11	0	0.0	-	0.0	0.0	-	0.0
12	0	**0.3697**	0.0	0.1835	0.2708	0.0	0.2986
13	0	**0.3547**	0.0	0.0825	0.1408	-	0.1516
14	0	0.0	-	0.0	0.0	-	0.0
15	0	0.0	-	0.0	0.0	-	0.0
16	0.3	**0.2876**	0.30	0.2947	0.3060	0.31	0.3051
17	0	0.0003	-	0.0	0.0	-	0.0
18	0	0.0012	-	0.0	0.0	-	0.0
19	0	0.0141	-	0.0019	0.0110	-	0.0112
20	0	0.0	-	0.0	0.0	-	0.0
21	0	0.0	-	0.0	0.0	-	0.0
22	0	0.0	-	0.0	0.0	-	0.0
23	0	0.0002	-	0.0	0.0	-	0.0
24	0	0.0	-	0.0	0.0	-	0.0
25	0	0.0010	0.1473	0.0031	0.1198	0.1473	0.0570

Table 15 Continued

Element no.	Actual damage magnitude	Without noise			With noise		
		SCMSPSO		PSO	SCMSPSO		PSO
		Stage 1	Stage 2		Stage 1	Stage 2	
TVDI ($\boldsymbol{\mu}$)		0.1848	-	-	0.1525	-	-
RSA		6975	8175	187575	40425	8250	112575
TRSA		-	15150	187575	-	48675	112575
Design variables		25	4	31	31	4	25
Elimination		0	21	-	0	21	-

*TVDI = threshold value of damage index; RSA = required structural analyses;
TRSA = total required structural analyses

computational effort than PSO. In all scenarios, the standard deviation of results in the case of PSO is higher than SCMSPSO, which indicates that the consistency of SCMSPSO is better than PSO to identify and quantify the damage.

A typical run of scenario II obtained by SCMSPSO and PSO without noise and with noise measurement data is shown in Table 15. SCMSPSO utilizes the total number of structural members as initial design variables, and in the second stage, the design variables are reduced by the algorithm. SCMSPSO successfully avoids false damage cases for both noise free and noisy measurements, which improves the accuracy of the results with less computational cost compared to single-stage PSO. The final damage identification results of scenario I, and scenario II are shown in Figs. 17 and 18 respectively.

5. Conclusions

Structural damage detection problem is formulated as an unconstrained optimization problem with an objective function based on natural frequencies and mode shapes. A self-controlled multi-stage strategy using particle swarm optimization is successfully implemented to estimate the damage location and severity in structures by minimizing the objective function. The methodology involves achieving an initial solution in the first stage when a small value of the objective function is reached, which has been selected as the first stage stopping criterion of the problem. Choosing a stopping criterion based on a small value of objective function instead of a fixed maximum number of iterations makes it independent of the optimization technique chosen. Moreover, this enables the algorithm to retain the actual solution in the search space. The algorithm also provides an automatic way of choosing an adaptable threshold value of the damage index to decide the most damage-prone elements which have to be kept in the search space of the next stage. The threshold value of the damage index chosen here adapts itself based on the peak value of damage index after the end of each stage.

The performance of the proposed algorithm has been demonstrated by performing damage identification in four different example problems. It was found that the algorithm outperforms some of the well-established existing optimization based damage identification methods. More specifically, the comparison presented in this paper showed that there is a significant increase in the accuracy of damage identification in the case of noisy data with a considerable reduction in computational cost. On the other hand, in the case of noise-free data, the accuracy of all the algorithms considered in this work are found to be almost equal, but the proposed algorithm is significantly efficient computationally. These improvements can be attributed to the contributions made towards choosing the first stage stopping criteria and an adaptable threshold value of damage index.

Although the self-controlled multi-stage strategy proposed here is used with PSO in this work, it has the potential to be combined with other optimization algorithms. Application of this strategy can lead to an effective and efficient identification of damage, which is evident from the results obtained in this paper.

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Appendix I. Notation

The following symbols are used in this paper:

C_1	=	cognitive parameter
C_2	=	social parameter
E^d	=	elastic modulus of damage member;
E^u	=	elastic modulus of undamage member;
$F1(X)$	=	objective function based on error in the frequency
$F2(X)$	=	objective function based on error in the MAC value
$F(X)$	=	objective function based on linear combination of error in frequency and MAC
{f}	=	load vector;
[K]	=	global stiffness matrix;
$[K]_e^d$	=	global stiffnessmatrix of damage system;
$[k]_{ei}$	=	undamage stiffnessmatrix of i^{th} member;
$[k]_{ei}^d$	=	damage stiffnessmatrix of i^{th} member;
[M]	=	global mass matrix;
N_L	=	noise level in percentage;
P	=	number of particles;
r_1, r_2	=	random number uniformly distributed within [0,1];
$rand$	=	uniformly distributed number in the range [0,1]
t	=	initial total number of structural member;
v	=	total number of unhealthy elements;
{X}	=	displacement vector;
$\{\ddot{X}\}$	=	acceleration vector;
x_i	=	damage index of i^{th} member;
x_{max}	=	maximum damage index;
w_{max}	=	maximum inertia weight;
w_{min}	=	minimum inertia weight;
β	=	stage indicator of multi-stage;
μ	=	threshold value of damage index;
γ_1, γ_2	=	weighting factors;
φ_i	=	i^{th} mode shape of undamaged structure;
φ_i^d	=	i^{th} mode shape of the damage structure;
φ_{ji}^{noise}	=	j^{th} component of i^{th} mode shape of the damage structure polluted with noise;
ω_i	=	i^{th} natural frequency;
ω_i^d	=	i^{th} natural frequency of the damage structure;
$\omega_{N,i}(X)$	=	i^{th} natural frequency of numerical model;
$\omega_{M,i}$	=	i^{th} natural frequency of the experimental model;

Appendix II. Acronyms

The following acronyms are used in this paper:

Avg. Value	=	average value of damage index;
CBO	=	colliding body optimization;
ECBO	=	enhanced colliding body optimization;
ESA	=	echolocation search algorithm;
FRF	=	frequency response function;
GA	=	genetic algorithm;
IPSO	=	improved particle swarm optimization;
MAC	=	modal assurance criteria;
MDE	=	modified differential evolution;
MPSO	=	modified particle swarm optimization;
MSPSO	=	multi-stage particle swarm optimization;
NDT	=	non-destructive testing;
NSA	=	an average number of structural analyses;
PSO	=	particle swarm optimization;
RSA	=	required structural analyses;
SCPSO	=	self-controlled multi-stage particle swarm optimization;
Std. Dev	=	standard deviation of damage index;
TVDI	=	threshold value of damage index;
TRSA	=	Total required structural analyses;
VBDD	=	vibration-based damage detection;