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Long term structural health monitoring for old deteriorated bridges: a copula-ARMA approach

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Abstract. Long term structural health monitoring has gained wide attention among civil engineers in recent years due to the scale and severity of infrastructure deterioration. Establishing effective damage indicators and proposing enhanced monitoring methods are of great interests to the engineering practices. In the case of bridge health monitoring, long term structural vibration measurement has been acknowledged to be quite useful and utilized in the planning of maintenance works. Previous researches are majorly concentrated on linear time series models for the measurement, whereas nonlinear dependences among the measurement are not carefully considered. In this paper, a new bridge health monitoring method is proposed based on the use of long term vibration measurement. A combination of the fundamental ARMA model and copula theory is investigated for the first time in detecting bridge structural damages. The concept is applied to a real engineering practice in Japan. The efficiency and accuracy of the copula based damage indicator is analyzed and compared in different window sizes. The performance of the copula based indicator is discussed based on the damage detection rate between the intact structural condition and the damaged structural condition.

Keywords: structural health monitoring; copula; long term assessment; bridge structure; ARMA model

1. Introduction

Gradual deterioration in bridges due to aging has become one of the critical issues in civil engineering practices in recent years. It is now a very common problem encountered by the developed countries, and for sure will be in the developing countries as well in the near future. Taking Japan as an example, the bridges in Japan which are in service for more than fifty years will reach 67% in 2033, see Fig. 1. There is indeed great need of a reliable and accurate bridge health monitoring system for evaluating the potential risks contained in the structures. Particularly, it is more important for short-span and mid-span bridges which take up a considerably high proportion among the all bridges. In accordance with theory, damage in bridge structures is supposed to be diagnosed from the change in bridge modal parameters as these are direct reflections of the structural conditions.

An effective structural monitoring system needs to be established based on proper measurement. Among all the developed approaches, structural health monitoring (SHM) using vibration measurement data showed the most promising performances. In this approach, the vibration data from the bridge is collected and analyzed to estimate the modal properties of structures which will then be assessed in the risk analysis (Doebling *et al*. 1996, Salawu 1997, Ko and Ni 2005, Wang *et al*. 2016, Xu *et al*. 2019a, b). The fundamental concept of this technology is that modal parameters can be directly related to structures' physical properties. Therefore, changes in physical properties, such as reductions in stiffness, will mean a decrease of structural strength. However, the sensitivity of bridge modal parameters to the damages is usually not very obvious (Spiridonako *et al*. 2016). It is of great importance to establish a method in observing the structural conditions based on more sensitive features. Meanwhile, it also has to be adequate enough to cope with the damage detection algorithm, e.g., window size, in order to provide reliable results.

In recent times, many techniques have been developed and devoted to tackle this issue. It aims to identify the hidden information of structural conditions from the structural vibration data (Deraemaeker *et al*. 2007, Dilena and Morassi 2011, Kim *et al*. 2012, Zhang and Lam 2015a, Zhang *et al*. 2015, 2019). From the acquired vibration data, one aims to extract damage-sensitive features and then to discriminate among features from the damaged and undamaged bridges quantitatively (Worden *et al*. 2000, Gul and Catbas 2009, Cho *et al*. 2010, Dohler *et al*. 2012). These two procedures can also be named as feature

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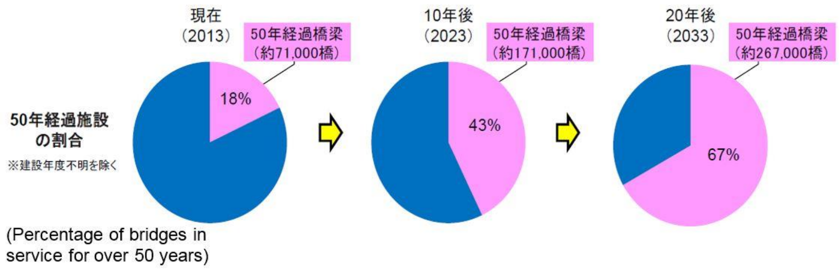

Fig. 1 Percentage of bridges in serves for over 50 years (Zhang *et al*. 2017a)

extraction and feature discrimination. Plenty research works have been done on the development of these procedures for vibration measurement data based structural health monitoring. The fundamental concept is established based on the use of structural dynamic properties, such as frequency, damping constant and mode shapes (Xia and Ni 2016). Magalhaes *et al*. (2010) had applied a strategy to identify the structural damages based on the bridge vibration data after removing of environmental and operational effects. An output-only based structural health monitoring approach is proposed for the bridge health monitoring by Reynders *et al*. (2013). Spiridonakos and Chatzi (2014) had proposed a stochastic structural identification method for observing the old bridges based on vibration and environmental data. Until recently, Spiridonakos *et al*. (2016) had adopted polynomial chaos expansion models for modeling the operational variability in the monitoring of structures. Some researches also tend to use artificial neural network in the damage identifications (Gopal *et al*. 2019, Garg *et al*. 2019). In general, from the recent advances in structural health monitoring, it is now widely recognized that the time series record of the structural modal parameters can be employed as an indicator.

In analyzing the time series data, linear regression models focusing on individual statistical analysis are commonly applied. However, the use of nonlinear dependency between different time series data in the structural damage detection has not been considered yet. Usually, the nonlinear dependences in time series data is either ignored or directly considered in linear manner. However, as a matter of fact, nonlinear dependences are quite common among sensor measurement (Huynh *et al*. 2018). It should be taken into account when analyzing the time series data. Under such circumstances, a copula based method is proposed in this study for handling the problem. Copula is a promising tool in modeling the dependences among different variables. It is very accurate in capturing the dependency changes between different time series data. Many former references have studied and investigated the feature of this approach (Zhang and Lam 2016, Yang *et al*. 2017, Zhang *et al*. 2018a, b). Therefore, it would be valuable if a copula based damage identification method can be formulated for structural health monitoring.

Realizing these initiatives, this paper attempts to propose a new structural damage identification method by combining the copula theory with time series model. The main objective is to utilize the concept of copula in modeling the dependences between the output data, and try to use ARMA model to characterize the statistical properties among the time series output. The copula would serve as a tool to identify the anomalous data among the multivariate. Section 2 will first introduce the fundamental concepts in vibration data based structural health monitoring method. The copula based structural identification method is then proposed and discussed in Section 3. A real bridge located in Japan is then selected as a case study for demonstrating the proposed approach in Section 4. Section 5 provides a further discussion on the results from the structural damage identification analysis. Finally, the findings and conclusions are summarized in Section 6.

2. Framework of long term bridge health monitoring

The most convenient structural health monitoring approach for detecting damages in bridge structures is through the use of a linear time series model such as the autoregressive (AR) model for the modal parameters. Several precedent studies on the use of time series vibration data are summarized as follows.

2.1 AR model

In the monitoring of a bridge structure, the data of acceleration are measured from the bridge through the sensor system. A linear time-series model, Autoregressive (AR) model can be constructed to characterize the system output. The coefficients of the optimal AR model can be adopted to represent the system for bridge diagnosis. The AR model can be formulated as following

$$y(k) = \sum_{i=1}^{p} a_i y\,(k-i) + e(k) \tag{1}$$

where $y(k)$ is the output of the structural dynamic system at time k, a_i indicates the AR coefficient of i-th order and $e(k)$ represents the error term.

Based on the first three AR coefficients, the parameter known as Damage Sensitive Feature (DSF) or Damage Indicator (DI) can be defined by Eq. (2)

$$DI = \frac{|a_1|}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \tag{2}$$

where a_1, a_2 and a_3 indicate the first, second and third AR coefficients respectively. Nair *et al*. (2006) showed that the first three AR coefficients are the significant ones among all the AR coefficients that can characterize the modal properties of a structural system. This is also agreed by Kim *et al*. (2013) who discovered that the first three order AR coefficients is a promising choice in bridge health monitoring.

2.2 Mahalanobis distance

The DI could be utilized as a reference in observing the structural condition. If there is a large deviation with the observations of structure at its intact condition, the structural condition is believed to have some damages. Such indicator based structural health monitoring method is very convenient and widely utilized in engineering applications. However, many researchers also critique this concept for its inefficiency in data usage. The construction of a time series model usually requires too much information which usually requires a very long observation time period. Moreover, the stability of the damage index in single time series is sometimes inadequate enough in structural safety assessments. Therefore, many studies attempt to develop structural health monitoring method by utilizing multiple measurement sources. Among these, the Mahalanobis Distance (MD) is one of the widely adopted approaches (Chang and Kim 2016).

MD is a multivariate statistical distance that quantitatively shows how different the current condition is from the intact condition with the following equation.

$$MD = \left(\frac{1}{k}\right) Z_i^T C^{-1} Z_i \tag{3}$$

where Z_i is the normalized vector obtained by normalized values of X_i (i = 1,…, k) and $Z_i = \frac{X_i - m_i}{s_i}$; X_i is the value of i-th characteristic, m_i is the mean of i-th characteristic, s_i is the standard deviation of i-th characteristic, k_i indicates number of characteristic/variables and C^{-1} represents the inverse of the correlation matrix.

Although MD is a simple concept that can be efficiently applied, the drawbacks are also obvious. It is recognized MD might not offer an accurate measure of structural damage if the multiple measurement are nonlinearly dependent. Therefore, with the aim of advancing the field of structural health monitoring, there is a strong need of finding a more robust damage identification approach which could handle multivariate data sources with nonlinear dependences.

3. Copula-ARMA damage detection method

To arrive at a more efficient multivariate damage detection method, the concept of copula is introduced in the time series data analysis. Copula is a highly applicable statistical tool for modeling multivariate data. It has been widely applied in various engineering fields including offshore engineering (Zhang *et al*. 2017b), reliability theory (Zhang and Lam 2015b) as well as in hydrology and environmental sciences (Salvadori and De Michele 2007). This section discusses a new copula based damage detection methodology proposed this study, the copula-ARMA approach. The detailed explanations will start from introducing ARMA and copula theory first, and then followed by the theoretical combination of these two.

3.1 ARMA model

Autoregressive-moving-average (ARMA) model is acknowledged as one of the most useful statistical tool for modeling and predicting future values in the time series. Theoretically, it is composed of an autoregressive part and a moving average part. The general fundamental properties of ARMA model can be found in any mathematical text books, see (Liebscher 2008).

An ARMA model is denoted as ARMA(p, q) where p refers to the number of autoregressive terms and q refers to the number of moving-average terms. A general formulation of ARMA(p, q) model can be given as below.

$$X_t = \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{4}$$

where X_t is the time series value at time t, φ_i (i = 1, 2,…, p) represent the auto-regressive parameters, θ_j (j = 1, 2,…, q) are the moving average parameters and $\{\varepsilon_t\}$ denotes a normal white noise process.

The prediction of X_t can be calculated based on previous observations X_{t-i} and its associated error terms ε_{t-j}. The AR(p) part is corresponding to the regression terms associate the present value and its past values. The MA(q) part is mainly referring to the modeling error terms, which determines the dependences between different observations.

In order to judge the best fit model orders, and Akaike Information Criterion (AIC) can be employed. The AIC is a measure for the relative quality of statistical models. It serves as a useful tool for model selection as it calculates relative quality of one model compared to the others (Akaike 1974). The formulation of AIC is given as below.

$$AIC = 2\mathrm{k} - 2\ln(\hat{L}) \tag{5}$$

where k is the number of parameters in the statistical model. In ARMA model, if the model under consideration is a linear regression, k is referring to the number of regressors, including the intercept. $\hat{L}$ represents the maximized likelihood function value of the assessed model M, which can be computed from following.

$$\hat{L} = p(x \;/\; \hat{\theta}, M) \tag{6}$$

where $\hat{\theta}$ indicates the parameter values that maximize the likelihood function and x refers to the observed data. The AIC consists of two terms; the first term is the log-likelihood function and the second term is a penalty function for the number of AR order.

3.2 Copula

Copula is a concept in statistics for modeling multivariate data by using only uniform marginal distributed variables. It is very powerful in characterizing the dependences between random variables. Copula has been widely applied in quantitative financial analysis and risk management. Nowadays, it also becomes quite popular in engineering practices. Especially in reliability engineering field, it is recognized as one of the most powerful tools for modeling and simulating random variables in multivariate analysis (Zhang *et al*. 2018a, b).

A copula function is a joint distribution function of multiple random variables U_1, U_2, … ,U_p, each of which is marginally uniformly distributed from 0 to 1, e.g., $U(0, 1)$. The construction of a copula model originates from the joint multivariate model. For random variables $X_1, X_2, \ldots, X_p$ with a joint cumulative distribution function, it can be formulated as following.

$$F(x_1, x_2, \ldots, x_p) = P\ (X_1 \leq x_1,\ X_2 \leq x_2, \ldots, X_p \leq x_p) \tag{7}$$

By substituting the marginal cumulative distribution function as following

$$F_j(x) = P(X_j \leq x), \qquad j = 1, 2, \ldots, p \tag{8}$$

A copula function would then be generated by the below expression

$$F\ (x_1, x_2, \ldots, x_p) = C\ [F_1\ (x_1), F_2\ (x_2), \ldots, F_p\ (x_p)] \tag{9}$$

If each $F_j\ (x)$, if it is continuous, the copula function$C$ would be unique. In other words, the joint distribution of $X_1, X_2, \ldots, X_p$can be described by the marginal distributions $F_j\ (x)$ and the copula C. This is also known as Sklar's theorem (Nelsen 2006).The copula links the marginal distributions together to form the joint distribution. From a modeling perspective, Sklar's theorem allows separating the modeling of the marginal distributions $F_j\ (x)$ from the dependence structure, which provides high flexibility in copula function.

In real practices, there are two well-known families of copulas, Gaussian copula and Archimedean copulas. In this study, the Gaussian copula is selected for the structural health monitoring as it is the most general and fundamental copula model. A Gaussian copula is constructed from a multivariate normal distribution by using the probability integral transform. The Gaussian copula can be written as following equation.

$$C(u_1, \ldots, u_N; \rho) = \Phi_\rho(\Phi^{-1}(u_1) + \cdots + \Phi^{-1}(u_N)) \tag{10}$$

where ρ is the correlation coefficient matrix, $\Phi_\rho(\cdot,\ldots,\cdot)$ stands for the standard multivariate normal distribution function, $\Phi^{-1}(\cdot)$ represents the inverse function of standard normal distribution function.

In the case of two variables, the Gaussian copula has only the single parameter ρ. It conveniently incorporates the correlation into a function that combines each of the marginal distributions to produce a bivariate cumulative distribution function. In terms of bivariate dependence, there are some dependence measures like Pearson correlation coefficient, Spearman's rho and Kendall's tau which worth for a mention before the copula approach is applied.

The Pearson correlation coefficient is also referred to as Pearson product-moment correlation coefficient, equaling to the covariance of two variables divided by the product of their standard deviations, is formulated as below.

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sigma_x \sigma_y} \tag{11}$$

where σ_x, σ_y are standard deviations of variables x and y respectively, $Cov(X, Y)$ is the covariance, which can be expressed by the mean $\mu_x \mu_y$ and expectation function E as following.

$$Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)] \tag{12}$$

Therefore, the Pearson correlation coefficient can be simplified as

$$\rho(X, Y) = \frac{E[XY] - E[X]E[Y]}{\sqrt{E[X^2] - [E[X]]^2}\sqrt{E[Y^2] - [E[Y]]^2}} \tag{13}$$

In addition to Pearson's correlation coefficient, rank based correlation coefficients such as Spearman's rho and Kendall's tau are believed to be more accurate. They measure a different type of dependence, which is the association among the random variables rather than population correlation coefficient. The Spearman's rho is equivalent to the Pearson correlation coefficient between the ranked variables. The Pearson's correlation assesses linear relationships, while Spearman's correlation assesses monotonic relationships no matter it is linear or not. The standard formula for calculating Spearman's rho is given below.

$$\rho_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \tag{14}$$

where d_i indicates the difference between the ranks of corresponding values X_i, Y_i, n is the number of observations. Another well-known rank based dependence measure is Kendall's tau. The Kendall's tau is a non-parametric measure for concordance between random variables. A general formulation is given as following.

$$\tau = \frac{N_c - N_d}{\frac{1}{2} n(n - 1)} \tag{15}$$

where N_c is the number of concordant, N_d is number of discordant and n stands for the number of observations.

In addition, Spearman's rho and Kendall's tau can also be expressed based on copula function parameters (Nelson 2006). For example, the following equations can be used to relate the dependence measures with the copula functions

$$\rho_s(X,Y) = 12 \iint_{[0,1]^2} C(u,v)dudv - 3 \tag{16}$$

$$\tau(X,Y) = 4 \iint_{[0,1]^2} C(u,v)dC(u,v) - 1 \tag{17}$$

3.3 Copula-ARMA Model

The combination of copula and ARMA model can be utilized in the time series data analysis. Copula-ARMA model is based on data characterization by both ARMA model and copula model. ARMA models are firstly created separate time series data, and then the dependences between different time series data can be captured by the copula function. The detailed procedures are elaborated in this section.

In constructing the copula-ARMA model, the error terms of ARMA model should be calculated at the initial stage. Here, the error terms are simply estimated by the difference between the observed data y and its theoretical values $\hat{y}$.

$$\text{Error term (y)} = \text{y} - \hat{y} \tag{18}$$

Copula models can then be applied to model the error terms from different time series data, e.g., C (Error term (x), Error term (y); θ). Based on the selected copula, the corresponding copula parameter could then be utilized as an indicator for the statistical properties of the time series data. Moreover, as a reference, the stability of values in Kendall's tau and Spearman's rho between different time series data can also be employed as an indicator of dependences in the time series data. The changes in dependences could then be detected in the time series data through the use of these copula based dependence measure concepts. As for bridge structural health monitoring, the copula parameter computed from the residuals of ARMA model is supposed to show certain tendencies in the long term measurement as a result of bridge deterioration.

3.4 Framework of Copula-ARMA damage detection method

Following the given concept, the flowchart of copula-ARMA approach for long-term measurement based bridge health monitoring is illustrated in Fig. 2. Generally speaking, it includes two major steps.

<u>*Step 1: Data collection and structural modal analysis*</u>

The initial step in the structural health monitoring is to extract the key structural information from the observed dynamic data. In this first step of the proposed framework, the long term measurement of the bridge including the

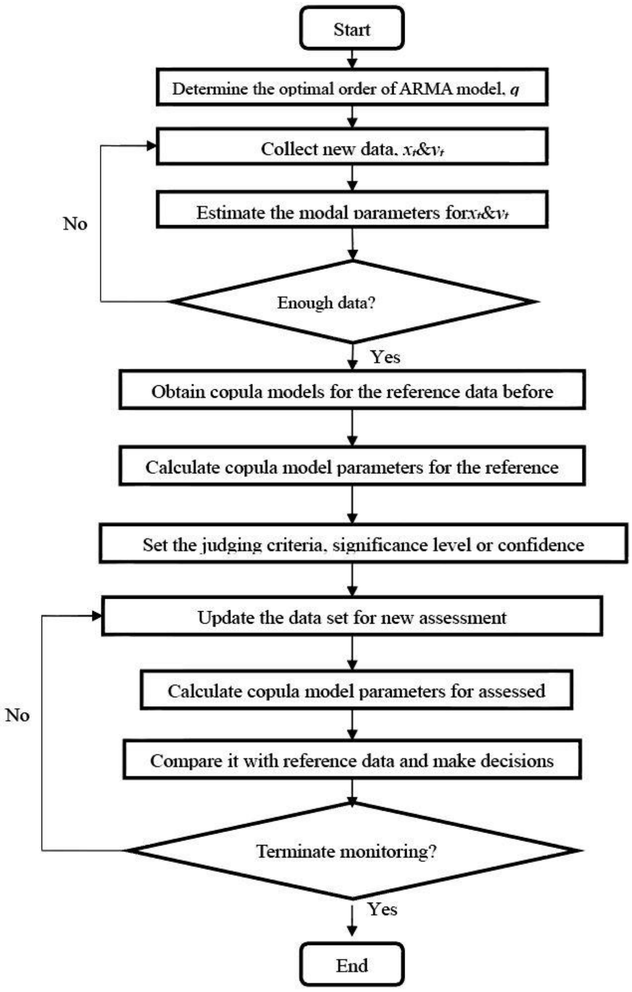

Fig. 2 Flowchart of the copula-ARMA approach for long-term bridge health monitoring

accelerations and other types of dynamic data are gathered. This kind of data will serve as reference data for structural health monitoring in a future evaluation. Following the structural modal analysis provided in Section 2.1, the frequency and damping ratio of the bridge are estimated. Based on the results, the time series data for modal parameter from different observations (sensors) can be obtained.

<u>*Step 2: Calculate the copula model parameter and evaluate the structural health condition*</u>

The next step is to calculate the copula model parameter between different time series data. During this step, the copula model is fitted to the estimated residuals from each theoretical model as introduced in Section 3.3. Based on the data, a pair of modal parameter time series data can be utilized to compute the copula parameter. And its stability in the timeline would be considered as an indicator for the structural health monitoring. The value of copula parameters will be updated when new data is withdrawn from the measurement.

In the present study, the first mode frequency and damping ratio is chosen for calculating the copula model

Fig. 3 The monitored girder bridge

parameter. The reason is because many former works showed that these two modal parameters are the most important factors indicating the structural health conditions. Once the result of copula model parameter is estimated, it can be compared with the reference value to judge whether the structural condition is changed or not. In this sense, the value obtained in the reference data step serves as a comparison and the new updated data would represent the current structural condition. To demonstrate the proposed approach, a case study on a real bridge health monitoring is provided in the following sections.

4. Case study – Short span girder bridge

A seven-span plate-girder bridge with Gerber-system in Himeji constructed in 1960 is selected for the investigation in this study. It has a total span of 187 m and it experiences high traffic volume for more than 50 years. Fig. 3 shows a site picture of the target bridge and the monitored span. The bridge information is summarized in Table 1. The monitored bridge span is 16 m long, with sensors on specified locations as shown in Fig. 4. Four accelerometers (UA1, UA2, DA1 and DA2) and two thermometers (T5 and T6) measuring the ambient vibration and temperature are placed in the middle and the end of the bridge. The acceleration is recorded hourly and the sampling frequency is 200 Hz. The thermometer measures the temperature every half an hour. The whole monitoring system was started in August 2008 and keeps working until now. However, it suffers discontinuity of measurement in the past eight years which is resulted from hardware problems and other operational mistakes.

In this study, the bridge dominant frequency and damping ratio are identified from original ambient vibration data measured at each sensor. In order to have an efficient analysis in the acceleration data, the size of moving time windows is set as 40.96 s as shown in Fig. 5. Average values of frequency and corresponding damping ratio within each hour are considered as the output and used to calculate the modal parameters. The calculated modal parameter values as shown in Fig. 6 shows that dominant frequency is identified around 4 Hz and thus is used for the damage detection. Therefore, the frequency around 4 Hz will be estimated from each sensor based on the proposed model. In order to make things clearer, the results of modal parameters are labeled based on the sensor numbers. For

Table 1 Information of the girder bridge

Construction year			March 1960
Bridge length			L = 187.00 m
Span length	Hanging girder		L_1 = 16.00 m
	Anchoring girder		L_2 = 6.2 m + 28.4 m + 6.2 m
Width			W = 8.00 m
Structural type	Superstructure		Steel girder-plate bridge with 7 spans
	Substructure	Abutment	Wall type abutment
		Pier	Wall type pier
	Foundation		Caisson
Design load			TL-20

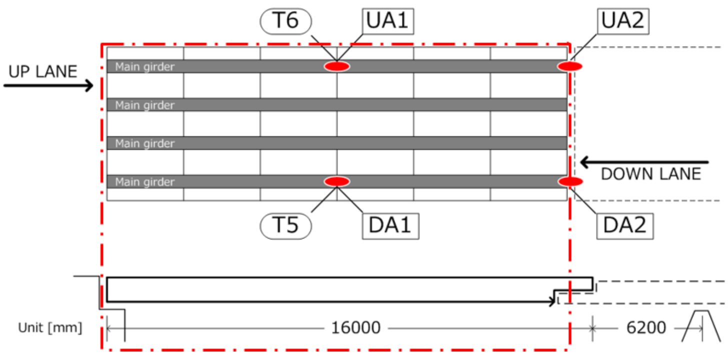

Fig. 4 Sensor locations in the monitored bridge span

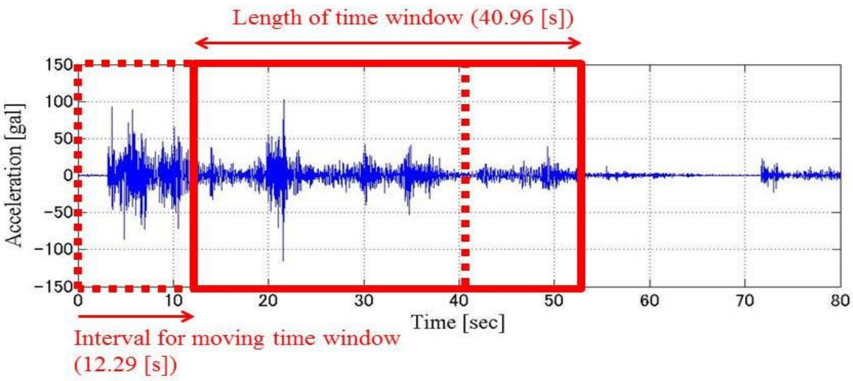

Fig. 5 Time windows for identifying modal parameters

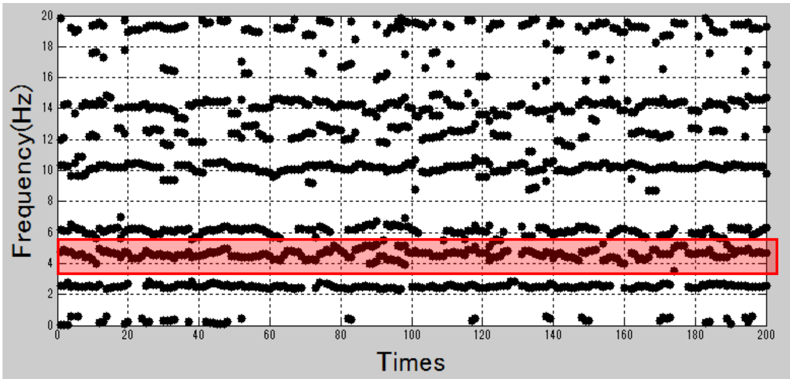

Fig. 6 A sample of the output of one hour of acceleration data and dominant frequency

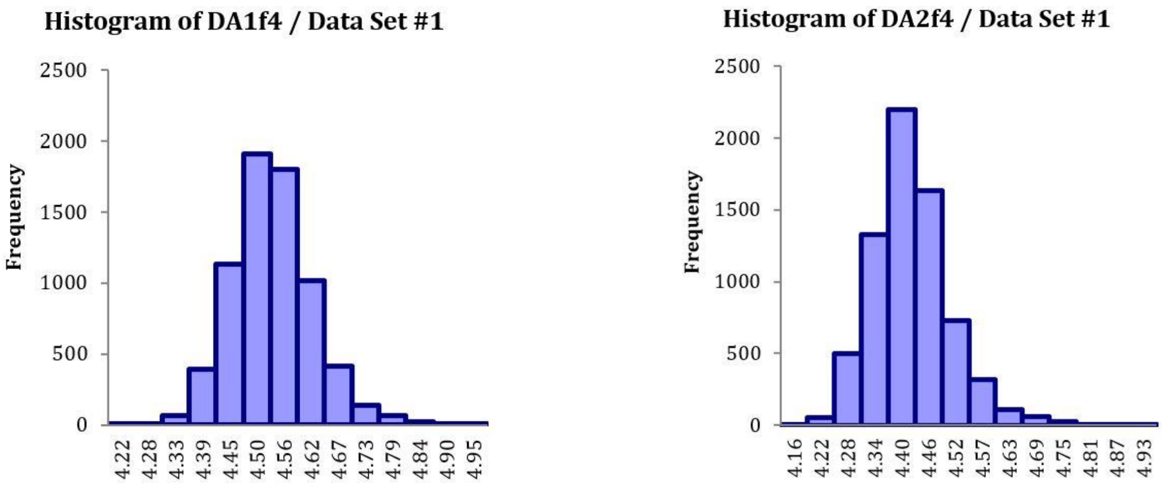

Fig. 7 Histograms of DA1f4 and DA2f4

instance, for the sensor DA1, frequency around 4 Hz and 10 Hz, and its associated damping ratios are denoted as DA1f4, DA1f10, DA1d4 and DA1d10 respectively.

Based on the long term record of measurement data, three time periods of data are extracted for the analysis. These include the follows:

- Dataset #1 - from 2008.8.5 to 2009.5.22 (290 days),
- Dataset #2 - from 2014.1.1 to 2014.7.23 (204 days), and
- Dataset #3 - from 2014.9.4 to 2015.4.8 (217 days).

5. Data analysis

5.1 Basic statistical properties of reference data

The statistical characteristics of the first year data are examined and served as the reference data. To demonstrate the use of copula approach in structural health monitoring for bivariate data, in this study, only sensor DA1 and DA2 are been analyzed. The histograms of calculated frequencies based on these two sensors are shown in Fig. 7. These would serve as the fundamental references. Any

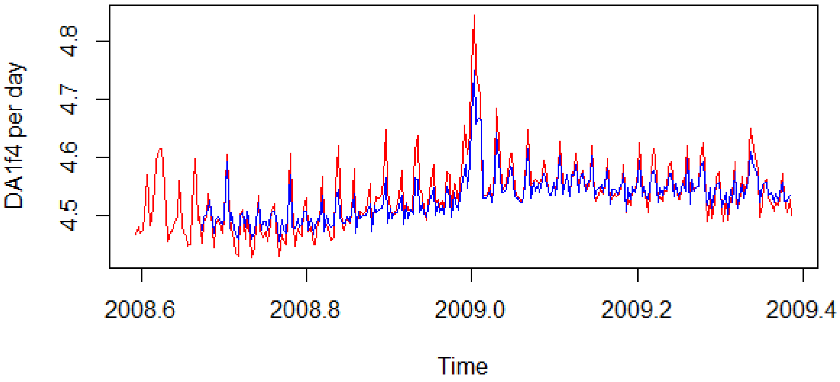

Fig. 8 ARMA model based forecast for DA1f4

observations having deviations from these are believed to indicate some damages in the bridge structure.

5.2 Time series analysis using ARMA model

Based on the first year data (dataset #1), basic time series analysis including forecasting is conducted by the use of ARMA model. This is aimed to check the stationarity in the time series. If the prediction cannot be done very, it indicates a nonstationary time series. And perhaps, the adopted reference dataset should be changed. In this prediction, the expected value of a forecast point is obtained by taking every data observed before that time point. After the prediction, the forecast point moves on to the next, and the prediction process repeats until the end of time period. Taking the starting point of forecasting as an example, the 31st analytical value is predicted by the ARMA model based on the first 30 data, and then the process moves on to forecast the 32nd point and so on. The plot of comparison between real data (red line) and forecast (blue line) is shown in Fig. 8.

It is obvious that the forecasted data is comparable with the real data, which means the ARMA model does show good performance in time series analysis. The time series data is also considered to be stationary. However, when it comes to peak values, the error between the forecasted result and real data is quite large. This is as a consequence of the limitations due to the assumed linear dependences in ARMA model. Therefore, some improvements to the statistical model should be considered. Therefore, it is necessary to combine copula with the existing ARMA model in the structural damage detections.

5.3 Copula-ARMA: detecting changes in modal parameter time series

Before constructing a copula-ARMA model, the seasonal effect has to be removed. Without the time varying modeling, the value of copula parameter is expected to change with the time. However, due to data scarceness, the copula approach is difficult to be conducted. And in fact, the correlation between the modal parameters and temperature are quite small. For instance, the dependence between frequency DA1f4 and temperature is quite minimal. In this case, the Spearman's rho equals to 0.077, while the Kendall's tau equals to 0.054 which indicates a very weak dependency.

Therefore, we consider to use regression method to remove the temperature influence to the reference data. A linear regression model between model parameter and temperature can be established as following.

$$f = \alpha T + \beta \tag{19}$$

where f is the modal parameter, T is the temperature, α is the regression coefficient and β is a constant.

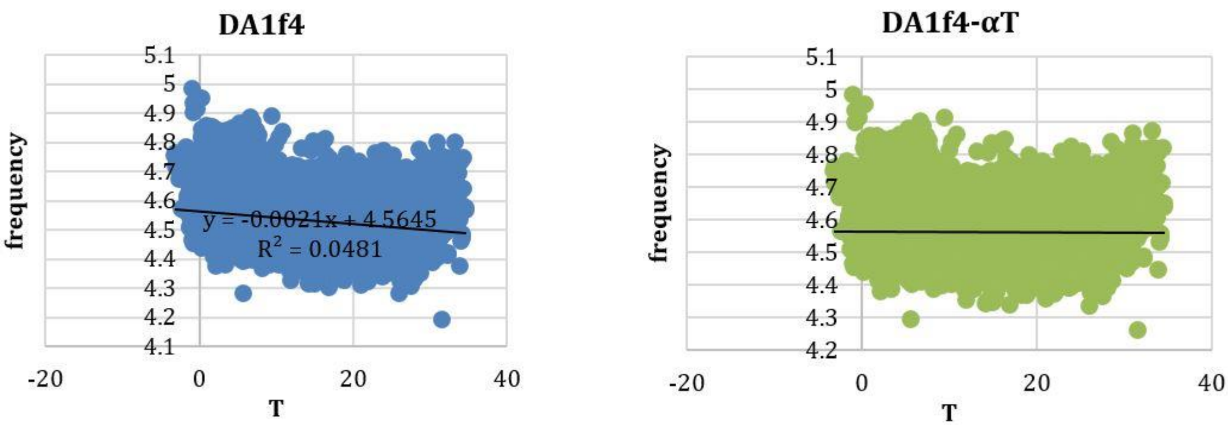

Fig. 9 Elimination of temperature influence for DA1

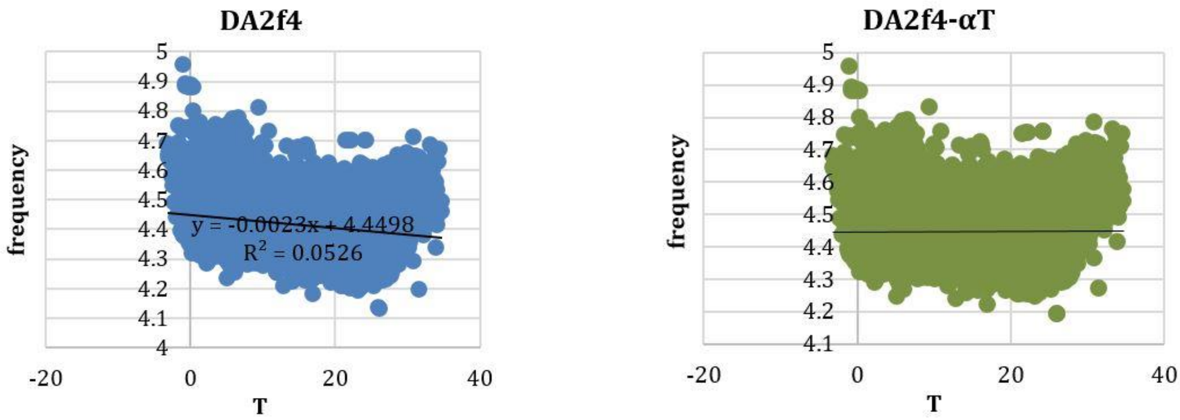

Fig. 10 Elimination of temperature influence for DA2

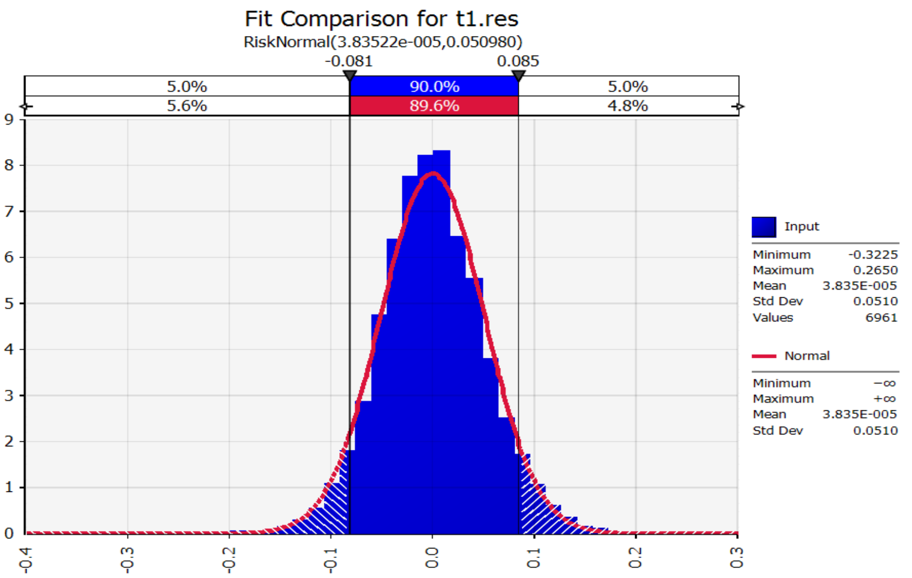

Fig. 11 Distribution fitting to DA1f4 residuals

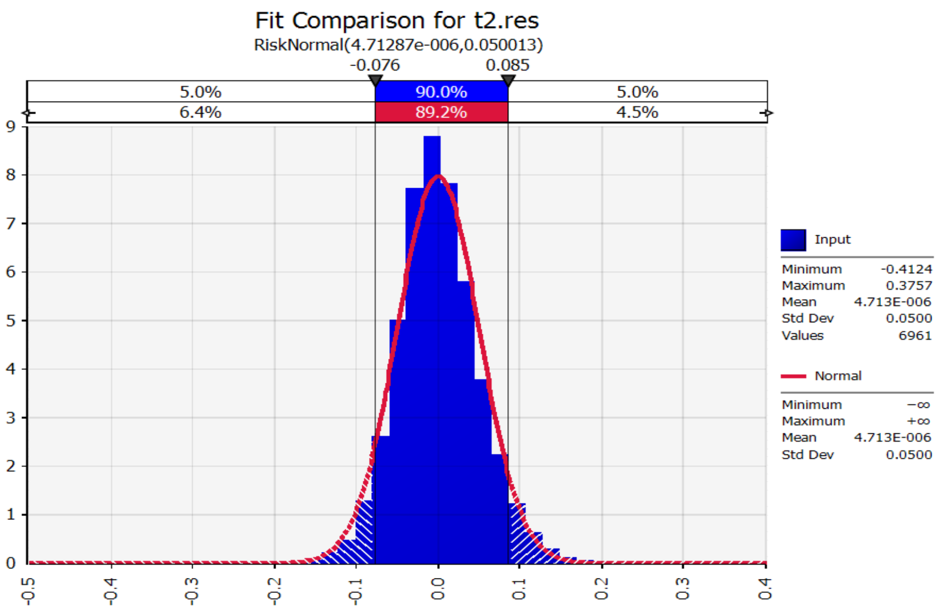

Fig. 12 Distribution fitting to DA2f4 residuals

After subtracting temperature effect αT from modal parameters, the output becomes a temperature-independent variable. For DA1f4 and DA2f4, the frequency data before and after removing temperature influence are depicted in Figs. 9 and 10.

Based on the modified modal parameter time series data, the best fit ARMA models for two sensors are then obtained. The two ARMA models are given as follows.

ARMA(5,2) is the best model for DA1f4 as shown in Eq. (20).

$$\begin{aligned} X_t &= 2.3473X_{t-1} - 1.5994X_{t-2} + 0.2104X_{t-3} \\ &\quad -0.1593X_{t-4} + 0.1877X_{t-5} + \varepsilon_t \\ &\quad -1.9264\varepsilon_{t-1} + 0.9932\varepsilon_{t-2} \end{aligned} \tag{20}$$

ARMA(2,3) is the best model for DA2f4 as shown in Eq. (21).

$$\begin{aligned} X_t &= 1.5334X_{t-1} - 0.5903X_{t-2} + \varepsilon_t \\ &\quad -0.9816\varepsilon_{t-1} + 0.2365\varepsilon_{t-2} + 0.0689\varepsilon_{t-3} \end{aligned} \tag{21}$$

To check the quality of two models, residuals between these ARMA models and the original data are computed and plotted in Figs. 11 and 12. It can be seen the errors are quite small which implies an adequate fitting in the model. After the residuals are calculated, the copula model is applied for the residuals. In this study, the Gaussian copula is selected as the candidate model. The next step would then be investigating the variations of copula parameters with respect to time. And compare it with the reference data to see whether there is a statistical change or not. While calculating copula parameters, the window size is initially

set as 20 hours. In order to compare all the statistical properties, the output from the continuous windows would include Spearman's rho, Kendall's tau, Pearson correlation

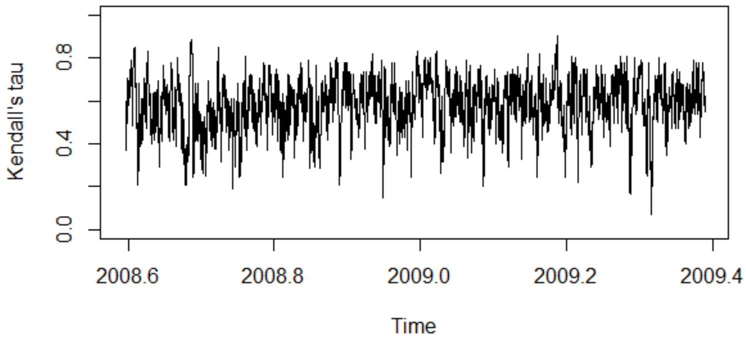

Fig. 13 Kendall's tau in assessed time series data

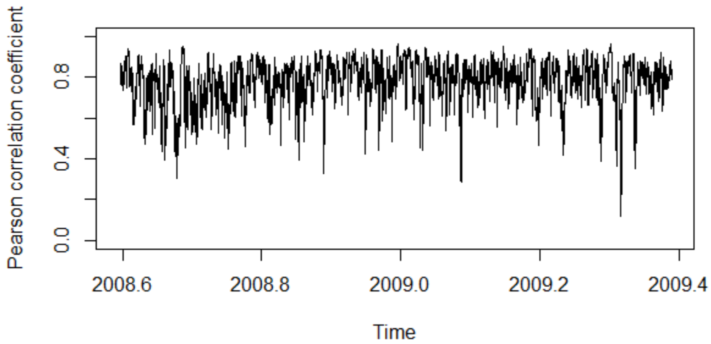

Fig. 14 Pearson correlation coefficient in assessed time series data

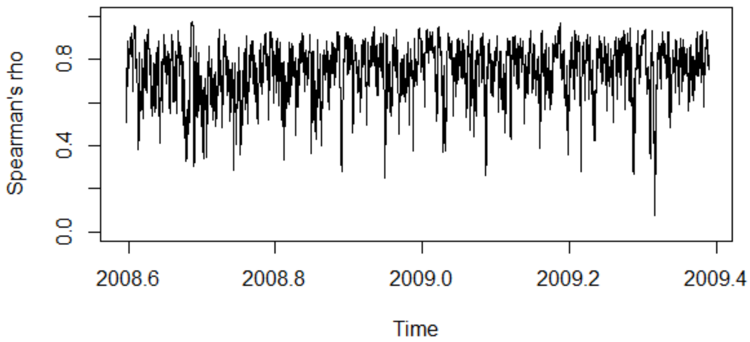

Fig. 15 Spearman's rho in assessed time series data

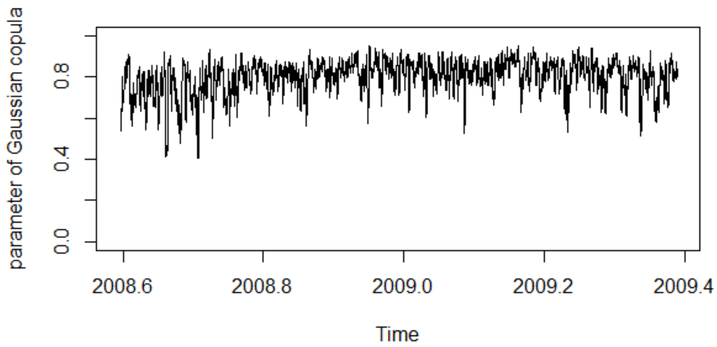

Fig. 16 Parameter of Gaussian copula hourly in assessed time series data

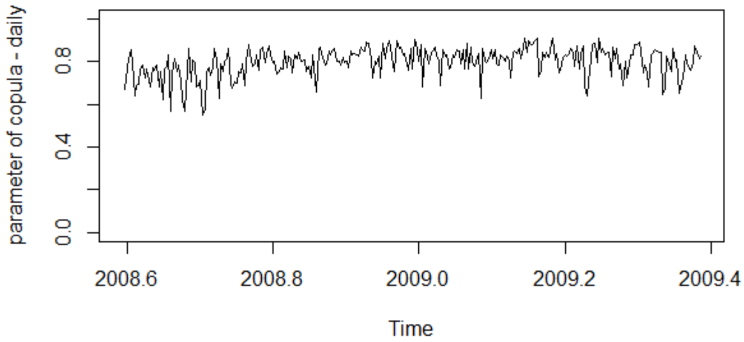

Fig. 17 Daily change of Gaussian copula parameter

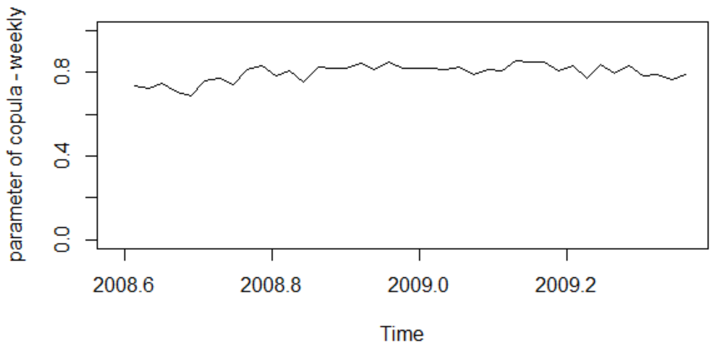

Fig. 18 Weekly changes of Gaussian copula parameter

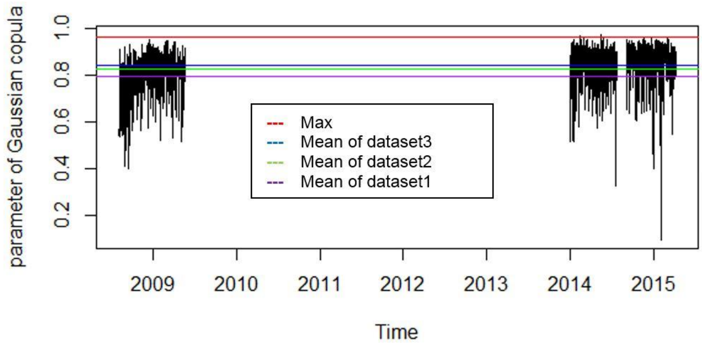

Fig. 19 Copula parameter fluctuation with time

coefficient, and parameter of Gaussian copula. These are shown in Figs. 13 to 16. For a better representation, the plots of copula parameter changes are aggregated into daily and weekly and shown in Figs. 17 and 18. The Gaussian copula parameter shows increasing tendency, although the amount is small.

Both dataset #2 and dataset #3 are investigated by the same ARMA model constructed for dataset #1. The copula

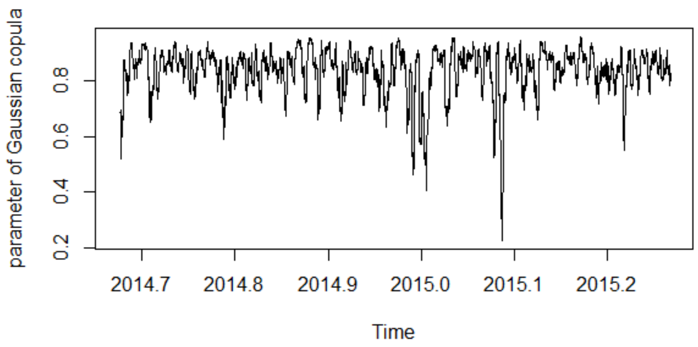

Fig. 20 Copula parameter change with window size of 24 hours

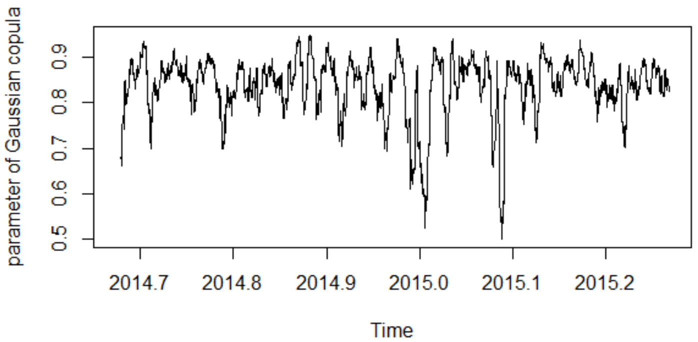

Fig. 21 Copula parameter change with window size of 48 hours

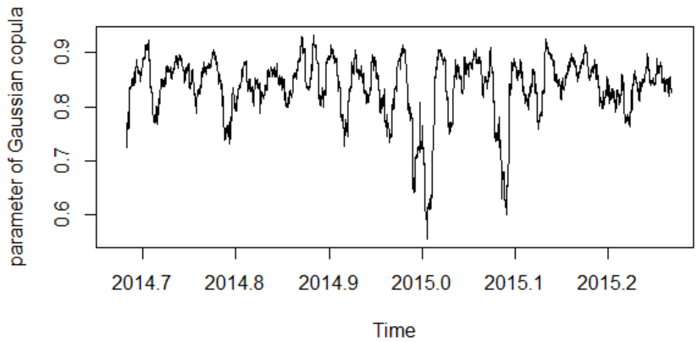

Fig. 22 Copula parameter change with window size of 72 hours

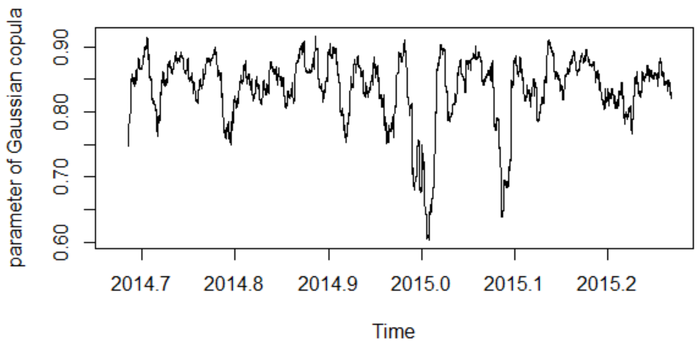

Fig. 23 Copula parameter change with window size of 96 hours

parameter for these periods are all estimated and plotted in Fig. 19. It is not obvious to tell the exact tendency by referring to timeline. The change in the parameter of copula is too little, although the parameter of copula does show some increasing tendency.

However, there is an interesting founding in dataset #3, where an obvious peak value in modal parameter was detected. Therefore, to have a more detailed study over this

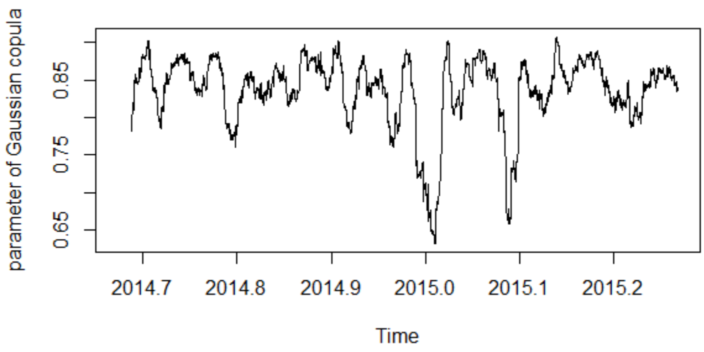

Fig. 24 Copula parameter change with window size of 120 hours

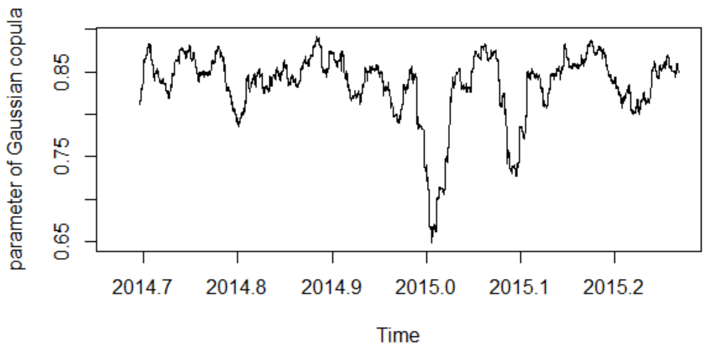

Fig. 25 Copula parameter change with window size of 192 hours

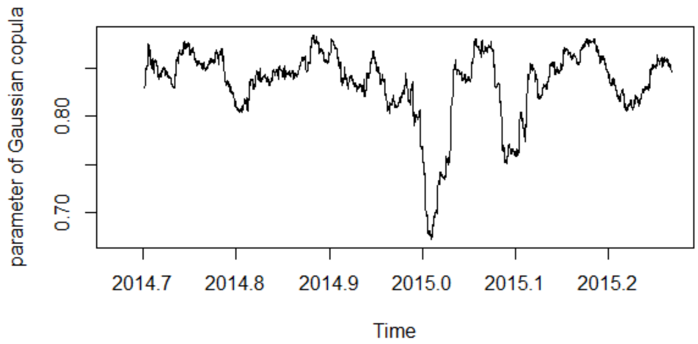

Fig. 26 Copula parameter change with window size of 240 hours

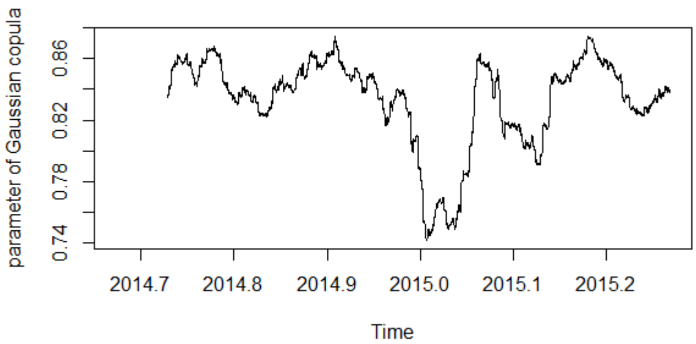

Fig. 27 Copula parameter change with window size of 480 hours

period, we changed the window size to 24, 48, 72, 96, 120, 192, 240, 480 hours for a comparison study. The results are shown in Figs. 20 to 27. It can be seen the shorter the window size is, the more sensitive the parameter is with the peak value. On the other hand, when the window size becomes too large, the sensitivity of the copula parameter decreases. For dataset #3, a particular peak during New Year holidays is observed in the original frequency values.

This is the consequence of the sudden drop of traffic weight. Meanwhile, in the plot of copula parameter, an obvious decline is also observed in Fig. 27. The change in copula parameter is in association with the change in modal parameters. This further proves the applicability of copula parameter for indicating structural damages from the recorded time series data.

One should note the use of the concept of copula in the evaluation of monitored data may have some limitations. First of all, the copula itself has a complicated formulation. If the data does not show very clear nonlinear dependences, the copula function may lose its feature in identifying the anomalous data. Second, the selection of candidate copula models is a challenging job. There is a huge number of copula families available in literature that could be adopted. The question of which type should fit the problem the best is not easy to be answered. Moreover, copula-ARMA is applicable only for modeling two time series data. For more time series data, the use of copula-ARMA may need extension or further development. The conclusions drawn from this study should be seen in the light of these limitations.

6. Conclusions

This study proposes a long term measurement based bridge structural health monitoring method by utilizing the fundamental Gaussian copula. In the case study, bivariate copula model is employed to check the changes in modal parameter time series data. Although the modal parameter time series data does not turn out to be an adequate indicator, the results showed that the change in frequencies from two sensors is able to be used as the structural health condition indicator through copula parameter. When proper window size is utilized for calculating copula parameters, the changes in copula model parameters can agree well with the change in modal parameters. Through use of long term measurement, it is possible to derive valuable information in order to increase the reliability of bridge structures. With appropriate linkage between data analysis and risk management, long term bridge vibration monitoring will facilitate engineers to extract more useful information for bridge inspection and optimal maintenance planning.

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