Smart Structures and Systems, *Vol. 25, No. 2 (2020) 219-228*
DOI: https://doi.org/10.12989/sss.2020.25.2.219

Mechanical behaviors of piezoelectric nonlocal nanobeam with cutouts

Mohamed A. Eltaher*[1,2], Fatema-Alzahraa Omar [2a], Azza M. Abdraboh [3b],
Waleed S. Abdalla [2c] and Amal E. Alshorbagy [2d]

[1] *Mechanical Engineering Department, Faculty of Engineering, King Abdulaziz University, P.O. Box 80204, Jeddah 21589, Saudi Arabia*
[2] *Mechanical Design and Production Department, Faculty of Engineering, Zagazig University, Zagazig, 44519, Egypt*
[3] *Physics Department, Faculty of Science, Banha University, Banha, Egypt*

(*Received July 9, 2019, Revised September 30, 2019, Accepted October 14, 2019*)

Abstract. This work presents a modified continuum model to explore and investigate static and vibration behaviors of perforated piezoelectric NEMS structure. The perforated nanostructure is modeled as a thin perforated nanobeam element with Euler–Bernoulli kinematic assumptions. A size scale effect is considered by included a nonlocal constitutive equation of Eringen in differential form. Modifications of geometrical parameters of perforated nanobeams are presented in simplified forms. To satisfy the Maxwell's equation, the distribution of electric potential for the piezoelectric nanobeam model is assumed to be varied as a combination of a cosine and linear functions. Hamilton's principle is exploited to develop mathematical governing equations. Modified numerical finite model is adopted to solve the equation of motion and equilibrium equation. The proposed model is validated with previous respectable work. Numerical investigations are presented to illustrate effects of the number of perforated holes, perforation size, nonlocal parameter, boundary conditions, and external electric voltage on the electro-mechanical behaviors of piezoelectric nanobeams.

Keywords: perforated piezoelectric nanobeams; nonlocal elasticity; finite element method; static and dynamic behaviors; NEMS

1. Introduction

Piezoelectric material is a class of smart materials that has an interaction between mechanical and electrical properties, is widely used in many scientific and engineering fields. Piezoelectric materials have been employed in micro/nanostructures such as micro/ nanoelectromechanical systems (MEMS/NEMS) Lazarus *et al*. (2012), biosensors Murmu and Adhikari (2012), and nanoresonators Tanner *et al*. (2007). Due to wide applications of nanobeams in engineering, such as atomic force microscope (AFM), nanowires, and nanosensors they have attracted a lot of consideration Pei *et al*. (2004), Zand and Ahmadian (2009). Hence, many investigations have been carried out about bending, buckling, and vibration of nanobeams Li *et al*. (2010).

The size-effects are recognized to become more significant as the dimensions of structures reach to the nanoscale. The scale-independent concept of the classical continuum models causes some deficiencies and inaccurate results if they applied on those small-scale nanostructures. Microcontinuum field theories, including Micromorphic theory, Microstructure theory, nonlocal theory, and couple stress theory are the extensions of the classical field theories for the applications in micro/nanoscopic space and time scales, Chen and Liew (2004). Nonlocal continuum theory has obtained much popularity among the researchers, because of its efficiency as well as simplicity to analyze the behavior of various nanostructures. This work is concerned with models developed according to the widely used nonlocal elasticity theory of Eringen and Edelen (1972), Eringen (1983, 1984). Otherwise, perforation is a very common process in (MEMS/NEMS) fabrication.

Reddy (2007) advanced nonlocal theories for Euler–Bernoulli, Timoshenko, Reddy, and Levinson beams to study the bending, buckling and vibration behaviors of beams. Aydogdu (2009) presented numerical solution of nonlocal beam model to investigate the mechanical properties of nanobeams. Ansari *et al*. (2011) studied the free vibration behaviors of Euler–Bernoulli nonlocal nanobeams established in an elastic medium. A compact finite difference method is used for sixth-order discretization of the nonlocal beam model to achieve the fundamental frequencies of nanobeams. Juntarasaid *et al*. (2012) developed the nonlocal elasticity to study bending and buckling of nanowires including the effects of surface stress by using the analytical and numerical solutions. Eltaher *et al*. (2014) investigated numerically solution of nonlocal functionally graded (FG) Timoshenko beam model based on Eringen's nonlocal elasticity theory to study static and buckling behaviors of nanobeams. Fernández-Sáez *et al*. (2016) solved the paradox that appeared when solving the cantilever beam with the differential form of the

*Corresponding author, Ph.D., Professor,
E-mail: meltaher@kau.edu.sa; mmeltaher@zu.edu.eg
[a] Ph.D., E-mail: omarfatma5@gmail.com
[b] Ph.D., E-mail: aza.abdrabo@fsc.bu.edu.eg
[c] Ph.D., E-mail: wsamy@zu.edu.eg
[d] Ph.D., E-mail: alshorbagy62@gmail.com

http://www.techno-press.org/?journal=sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

Eringen model by using the integral form of the Eringen model. Zakeri *et al*. (2016) presented the accuracy and efficiency of the basic displacements functions elements in analysis of Eringen's nonlocal elasticity and Euler-Bernoulli beam theory. Eltaher *et al*. (2016) presented e effects of thermal load and shear force on the buckled of and post-buckled configurations of higher-order shear deformation beam theories using nonlocal theory.

Akbas (2017a, b) illustrated forced vibration behaviors of FG nanobeam by using modified couple stress theory with damping effect. Akbas (2017b, 2018a, b) studied free and forced vibration of edge cracked FG microscale beams based on the modified couple stress theory. Thai *et al*. (2018) exploited effect of nonlocal parameter on Postbuckling analysis of functionally graded nanoplates by using von Kámán's assumptions to account for the geometrical nonlinearity. Ansari *et al*. (2018) used finite element method to obtain thermal conductivity coefficients of SWCNTs reinforced polypropylene. Eltaher *et al*. (2018a, b) simulated the mechanical behaviors of the perforated nonlocal nanobeams. The formulation was based on the Euler– Bernoulli beam and Timoshenko beam with a nonlocal differential form of Eringen model. Akbas (2018c) investigated bending of a cracked FG nanobeam subjected to transversal point load at the free end of the beam. She *et al*. (2018a, b) investigated wave propagation of FG porous nanobeams and nanotube based on non-local strain gradient theory. Hamed *et al*. (2019) studied the effect of nonlocal parameter on the static bending of porous FG nanobeam by using finite element method. Faraji-Oskouie *et al*. (2019) derived the numerical solutions of original integral and differential formulations of Eringen's nonlocal model for static bending of Timoshenko beams. Abdalrahmaan *et al*. (2019) and Almitani *et al*. (2019) presented a unified analytical model to investigate free and forced vibration responses of perforated thin and thick beams. Akbas (2019) exploited the nonlocal elasticity theory to present axially forced vibration of a cracked nanorod under harmonic external dynamically load. She *et al*. (2019) studied the snap-buckling behaviors of FG porous curved nanobeams resting on three parameters elastic foundations.

Recently, the application of piezoelectric materials has been broadly spread in nano-structures including nonlocal effects. Yan and Jiang (2011) used the Euler–Bernoulli beam theory to study the influence of surface effects, and surface piezoelectricity on the vibrational and buckling behaviors of piezoelectric nanobeams. Ke *et al*. (2015) studied the vibration characteristics of piezoelectric nanoplate under various boundary conditions based on the Mindlin plate theory and the nonlocal theory. Jandaghian and Rahmani (2016) analyzed analytically the problems of free vibration behavior of piezoelectric nanobeams with Eringen's nonlocal theory. Kheibari and Beni (2017) studied the free vibration of piezoelectric nanotubes by using Love's cylindrical thin-shell model. Effect of size, electromechanical, and geometric were investigated for the natural frequency of piezoelectric nanotubes including the Euler–Bernoulli and consistent couple stress theories. Mahinzare *et al*. (2018) developed a formulation for the free vibration analysis of functionally graded circular nanoplate in two directions. It had shown that the angular velocity, external electric voltage, size dependency and power-law index had significant effects on the natural frequency. Eltaher *et al*. (2019a) demonstrated the coupling effects of nonlocal elasticity and surface properties on static and vibration characteristics of piezoelectric nanobeams with Eular-Bernolli beam theory. Mohamed *et al*. (2019) studied mechanical behaviors of SWCNTs beam by using energy equivalent model. Wang *et al*. (2019) obtained analytical solutions for vibrations of the piezoelectric inkjet print head. The governing equations for forced vibrations of the thin-plate were derived and solved analytically using the theory of piezoelectric plates and Rayleigh-Ritz method. Eltaher *et al*. (2019b, c) exploited energy equivalent method to investigate the effect of nanoscale on the vibrational behaviors of CNTs.

According to author's knowledge and literature review, there is no attempt to study mechanical behaviors of perforated piezoelectric nanobeam. So, the current work tries to fill this gab and to present a comprehensive model has capability of predicting the static bending and vibration behavior of perforated piezoelectric nanobeam structure. Kinematic relation of Euler–Bernoulli beam theory, which takes into consideration the influences of perforation size, number of holes for perforated beams, is proposed with a nonlocal differential equation of Eringen. The paper is structured as follows: in Sect. 2, the mathematical model is explained in detail. Sect. 3 contains numerical and solution techniques. Numerical results and discussion are presented and discussed in Sect. 4. Finally, at the end of the present work conclusions are summarized and listed in Sect. 5.

2. Mathematical formulation

2.1 Geometrical modification

A regular square perforation of piezoelectric nanobeam is presented in Fig. 1. As shown, length L, width b, and thickness h, with a pattern of square holes of spatial period l_s and side $l_s - t_s$, and a number of holes along the section is N. The filling ratio of the beam can be described by

$$\alpha = \frac{t_s}{l_s} \qquad 0 \le \alpha \le 1 \tag{1}$$

From previous equation, the beam is completely perforated at a filling ratio $\alpha = 0$ and completely filled at filling ratio α = 1. In perforated beam analysis, it is assumed that modified material properties of the beam. The normal stress will be abridged in the parts between holes, which will be under stressed with respect to the full beam case, and will be over-stressed in the remaining parts. By assuming that the total stress along the cross section is the same for both complete beam and perforated one and assuming a linear continuous stress distribution in the filled segments according to Luschi and Pieri (2014), the equivalent bending stiffness can be defined as

$$(EI)_{eq} = EI \times \frac{\alpha(N+1)(N^2+2N+\alpha^2)}{(1-\alpha^2+\alpha^3)N^3+3\alpha N^2 + (3+2\alpha-3\alpha^2+\alpha^3)\alpha^2 N+\alpha^3} \tag{2}$$

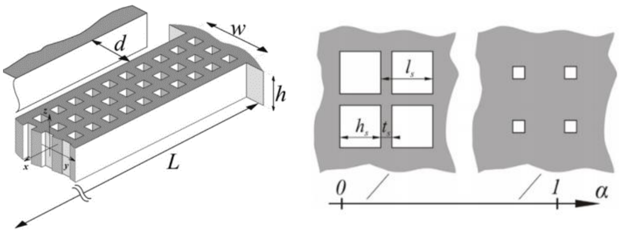


Fig. 1 Geometry of a perforated piezoelectric nanobeam with the coordinate system (Luschi and Pieri 2016)

By integrating over the beam segment, the average mass of the perforated beam per unit length can be written as

$$(\rho A)_{eq} = \rho A * \frac{[1 - N(\alpha - 2)]\alpha}{N + \alpha} \tag{3}$$

2.2 Nonlocal perforated piezoelectric nanobeam

Based on the Euler–Bernoulli beam theory (EBT), the displacement filed of any point of the beam are given by (Alshorbagy *et al*. 2011, Eltaher *et al*. 2019a, b)

$$u(x, z, t) = u_0(x, t) - z\frac{\partial w_0(x, t)}{\partial x} \tag{4}$$

$$w(x, z, t) = w_0(x, t) \tag{5}$$

where t is the time and $u_0(x, t)$ and $w_0(x, t)$ are displacement components in the mid-plane along the x and z. The nonzero strain ε_{xx} of the Euler-Bernoulli beam theory is

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0(x, t)}{\partial x^2} = \varepsilon_{xx}^0 - zk^0 \tag{6}$$

Like the displacement field, the distribution of electric potential for the piezoelectric nanobeam model is assumed to vary as a combination of a cosine and linear variation, which satisfies the Maxwell's equation as (Jandaghian and Rahmani 2015, Eltaher *et al*. 2019a,b)

$$\emptyset_{(x,z,t)} = -cos(\beta z)\varphi(x, t) + \frac{2V}{h}z \tag{7}$$

the non-zero components of electric field $\mathrm{E_x}, \mathrm{E_z}$ can be obtained as

$$\mathrm{E_x} = -\frac{\partial \varphi}{\partial \mathrm{X}} = \cos(\beta_z)\frac{\partial \varphi}{\partial \mathrm{X}} \tag{8}$$

$$\mathrm{E_z} = -\frac{\partial \varphi}{\partial \mathrm{Z}} = -\sin(\beta_z)\varphi - \frac{2\mathrm{V}}{\mathrm{h}} \tag{9}$$

in which $\beta = \pi/h$, $\varphi(x, t)$ is the electric potential function in the x−direction that must satisfy the electric boundary conditions, z is measured from the mid-plane of the nanobeam in the transverse direction, h is the thickness of the piezoelectric nanobeam, V is the external electric voltage. The constitutive relation of the one-dimensional piezoelectric beam can be written as

$$\sigma_x = C_{11}\varepsilon_x - e_{31}E_z \tag{10}$$

$$D_z = e_{33}\,\varepsilon_x + K_{33}\,E_z \tag{11}$$

where σ_x is axial stress, D_z is electric displacement, and C_{11}, e_{31} and K_{33} are elastic, piezoelectric and dielectric constants for the bulk medium. In the absence of free electric charges using Gauss's law

$$\frac{\partial D_z}{\partial X} = 0 \tag{12}$$

Substituting Eq. (11) into Eq. (12) and using Eqs. (6) and (9), and considering $\varphi(-h/2) = 0$ and $\varphi(h/2) = V$ as the electrical boundary conditions, the electric potential is expressed as, Gheshlaghi and Hasheminejad (2012)

$$\varphi_{(x,z)} = -\frac{e_{31}}{2k_{33}}\frac{\partial^2 w(x, t)}{\partial X^2}\left(Z^2 - \frac{h^2}{4}\right) + V\left(\frac{Z}{h} + \frac{1}{2}\right) \tag{13}$$

The axial stresses for the bulk in Eq. (10) can be written as

$$\sigma_x = c_{11}\varepsilon_0 - z\left(c_{11} + \frac{e_{31}^2}{k_{33}}\right)\frac{\partial^2 w(x, t)}{\partial X^2} + e_{31}\frac{V}{h} \tag{14}$$

Consequently, the Euler–Lagrange equation of motion of piezoelectric nanobeam can be described by, Eltaher *et al*. (2013a, b).

$$\frac{\partial N}{\partial x} = m_0\,\frac{\partial^2 u_0}{\partial t^2} \tag{15}$$

$$\frac{\partial^2 M}{\partial x^2} - \frac{\partial}{\partial x}\left(N_b\,\frac{\partial w_0}{\partial x}\right) = m_0\frac{\partial^2 w_0}{\partial t^2} \tag{16}$$

$$N = \int_A \sigma_{xx}\,dz\,, \quad M = \int_A z\,\sigma_{xx}\,dz \tag{17}$$

where N, M and N_b are axial force, bending moment resultants and the axial compressive force. The translated mass inertia of perforated beam is $m_0 = \int_A \rho\,dA = (\rho A)_{eq}$.

To impose the size effect of nanostructure, nonlocal piezoelectricity theory is proposed. This theory assumed that the stress tensor and the electric displacement at a reference point depend not only on the strain components and electric-field components at same position but also on all other points of the body Jandaghian and Rahmani (2016), the nonlocal constitutive relation can be written as

$$\sigma_{ij} - (e_0 a)^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_K \tag{18}$$

$$D_i - (e_0 a)^2\nabla^2 D_i = e_{ijkl}\varepsilon_{kl} + \epsilon_{ij}E_K \tag{19}$$

where σ_{ij}, ε_{ij}, D_i and E_i are the stress, strain, electric displacement and electric field, respectively; C_{ijkl}, e_{kij}, ϵ_{ij} are the fourth-order elasticity tensor, piezoelectric constants, dielectric constants. The constitutive equation for piezoelectric nanobeam based on the nonlocal Euler-Bernoulli theory reads.

$$M_{(x,t)} - \mu \frac{\partial^2 M_{(x,t)}}{\partial x^2} = -(EI)_{eff} \frac{\partial^2 w_{(x,t)}}{\partial x^2} \quad (20)$$

Direct substitution of Eq. (16) into Eq. (20), leads to the nonlocal equation of motion for perforated piezoelectric nanobeams

$$\begin{aligned}(EI)_{eff} \frac{\partial^4 w_{(x,t)}}{\partial x^4} + \left[1 - \mu \frac{\partial^2}{\partial x^2}\right]\Bigg[(\rho A)_{eq} \frac{\partial^2 w_{(x,t)}}{\partial t^2} \\ - \frac{d}{dx}\left((N)_{eff} \frac{\partial w_{(x,t)}}{\partial x}\right)\Bigg] = 0\end{aligned} \quad (21)$$

where $(EI)_{eff}$ and $(N)_{eff}$ are the effective bending rigidity and the effective axial load of the piezoelectric nanobeam expressed as

$$\begin{aligned}(EI)_{eff} = \Bigg\{\left[\frac{bh^3}{12}\left(C_{11} + \frac{e_{31}^2}{k_{33}}\right)\right] \\ \times \frac{\alpha(N+1)(N^2+2N+\alpha^2)}{(1-\alpha^2+\alpha^3)N^3+3\alpha N^2 + (3+2\alpha-3\alpha^2+\alpha^3)\alpha^2 N+\alpha^3}\Bigg\}\end{aligned} \quad (22)$$

$$(N)_{eff} = bh\varepsilon_0 c_{11} + bVe_{31} \quad (23)$$

3. Numerical formulation

This section is devoted to develop a numerical finite element model for nonlocal Euler–Bernoulli beam based on the Hamilton principle. The conventional Galerkin technique is employed to derive the weighted residual variation functional of the equilibrium. Denoting Galerkin's weight function by X, the variational formulation can be deduced by Eltaher *et al.* (2013) as

$$\begin{aligned}\int_0^T \sum_{e=1}^{ne} \Bigg(\int_0^L (EI)_{eff} \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 X}{\partial x^2} + \Big[(\rho A)_{eq} \frac{\partial w_0}{\partial t}\frac{\partial X}{\partial t} \\ - \left((N)_{eff} \frac{\partial w_0}{\partial x}\frac{\partial X}{\partial x}\right)\Big] - \mu(\rho A)_{eq} \frac{\partial^2 w_0}{\partial x \partial t}\frac{\partial^2 X}{\partial x \partial t} \\ + \mu (N)_{eff} \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 X}{\partial x^2}\Bigg) dxdt \\ + \int_0^t \left[\bar{N}\delta u_0 + \bar{V}\delta w_0 + \bar{M}\frac{\partial \delta w_0}{\partial x}\right]_0^L dt = 0.\end{aligned} \quad (24)$$

The deflection along the element in a local coordinate system, is given in terms of the Hermite interpolation functions as

$$\bar{w}(\bar{x}) = \sum_{i=1}^{4} N_i U_i \quad (25)$$

where U_i denotes the nodal degrees of freedom, representing the deflection and rotation at each terminal node of the element; and Ni, i = 1, 2, 3, 4, are the Hermite interpolation functions. By substituting Eq. (25) into the modified weak form, Eq. (24), and performing the integration, we get the following element equilibrium equation

$$\big[[M_l] + [M_{nl}]\big]\big\{\ddot{\bar{W}}\big\} + \big[[K_l] + [K_b]\big]\{\bar{W}\} = 0 \quad (26)$$

where M and K are the element mass and stiffness matrices, respectively. The subscripts l, nl, and b donate the local, nonlocal, and buckling.

4. Numerical results

This section shows the static and vibration characteristics of perforated piezoelectric nanobeams. The effects of a number of rows of holes, filling ratio, nonlocal parameter, and external electric voltage on static bending and natural frequency are presented and discussed. All material properties of the beam are proposed according to Yan and Jiang (2011). It is assumed that the nanobeam is made of one kind of lead zirconate titanate material, PZT-5H, the length to thickness ratio of the nanobeam is fixed at L/h = 20, and the beam width b is equal to the beam thickness h. The following parameters are used in computing the numerical values

$$C_{11} = 126\, GPa, \qquad \rho = 7.5\, x10^3\, kg\, m^{-3},$$
$$e_{31} = -6.5\, Cm^{-2}, \qquad k_{33} = 1.3\, x10^{-8}\, C\, V^{-1} m^{-1}$$

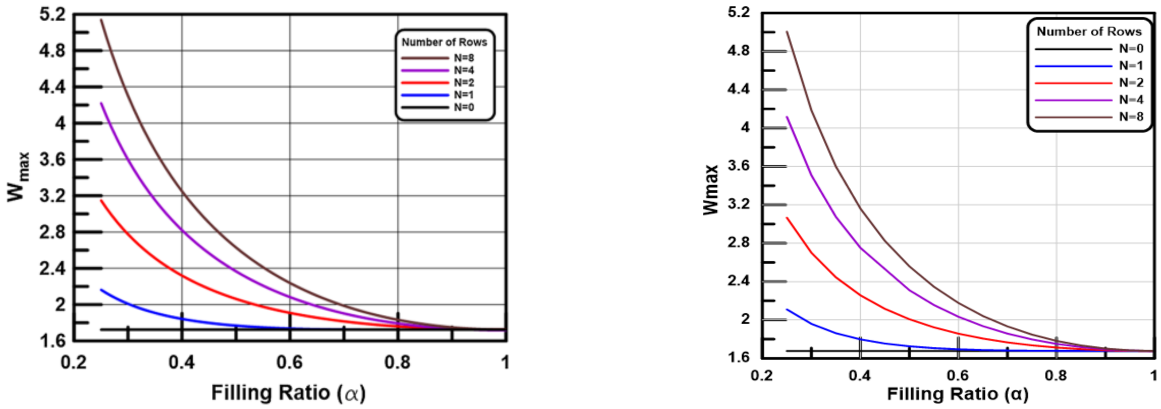
4.1 Static analysis

In this subsection, the maximum deflection of simply supported (S-S) beam under uniform load and nonlocal parameter effect is compared with previously published results. The model will be validated with results obtained by Reddy (2007) and shown in Table 1. The non-dimensional deflection is chosen in the following form $\bar{w}_{max} = 100 * \delta_{max} * \frac{EI}{q_0\, L^4}$. The coupling between the number of hole rows and the filling ratio the following plots are presented. The variation of the central deflection with respect to the filling ratio α for a different number of hole rows N is presented in Fig. 2. It is noted that, the numerical results are identical for analytical one of Eltaher *et al.* (2018a, b) for complete filled nanobeam (N = 0) and nonlocal parameter ($\mu = 3$).

Fig. 3 presents the variation of the maximum central

Table 1 Maximum non-dimensional deflection of S-S beams

L/h	μ	Analytical	Numerical
		Reddy (2007)	Present results
20	0	1.313	1.3020
	1	1.4487	1.4270
	2	1.5844	1.5520
	3	**1.7201**	**1.6770**
	4	1.8558	1.8020
	5	1.9914	1.9270



(a) Eltaher *et al.* (2018a) (b) Pressent result

Fig. 2 Max deflection Vs filling ratio at (μ = 3, V = 0)

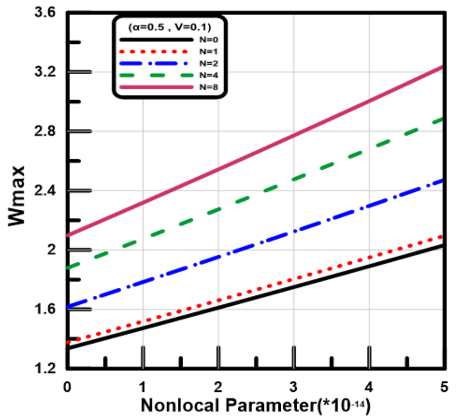


Fig. 3 Variation of normalized deflections Vs the nonlocal parameter at (α = 0.5, V = 0.1)

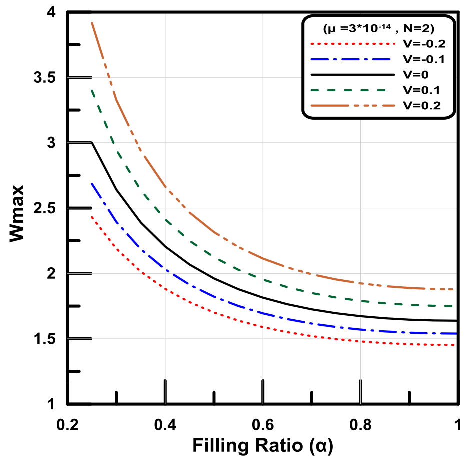


Fig. 4 Maximum deflection vs filling ratios at ($\mu = 3 \times 10^{-14}, N = 2$) and different voltage values

deflection with the nonlocal parameter for a different number of holes at (α = 0.5, V = 0.1). This figure shows that the deflection is affected by the number of holes in the perforation pattern of the piezoelectric beam. As the number of holes in the perforation pattern increases the maximum deflection also increase. The nonlocal parameter increases the maximum bending increases; this is because of the decrease of the bending stiffness of the beam with increasing the number of holes at the same filling ratio.

The variation of the normalized deflection with the filling ratios under different electrical loads, V, for the non-local parameter, μ, equals 1×10^{-14} and the number of holes, N, equals 4 is presented in Fig. 4. It is noted that the maximum deflection increases by increasing the applied voltage (–0.2, –0.1, 0, 0.1, 0.2) for the perforated piezoelectric nanobeam. Also, the filling ratios influence the normalized deflection of the perforated nanobeam within some range of applied voltage. When the filling ratio decreases, the maximum deflection increases due to the decrease of the bending stiffness.

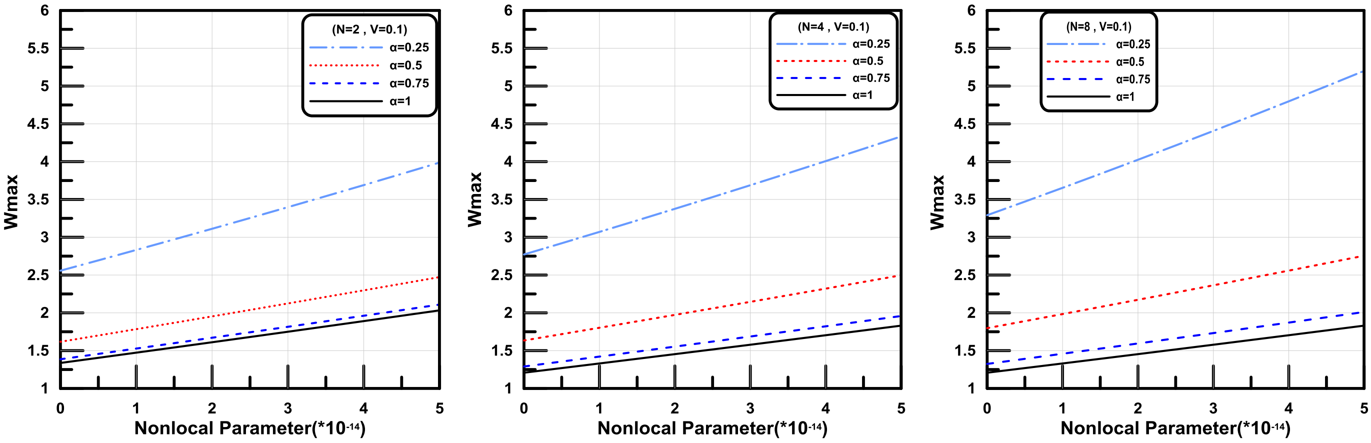
Fig. 5 shows the variation of the piezoelectric beam deflection against the nonlocal parameter for different values of the filling ratio. The results for three figures are obtained at a fixed value of the hole rows number (N = 2, 4, 8) to study the dependency of the beam response just on the filling ratio. By comparing Fig. 5(a) with Figs. 5(b) and (c), it can be concluded that the deflection increases with increasing the nonlocal parameter Because of the beam softening resulting from increasing the nonlocal parameter. The bending increases with decreasing the filling ratio. For any filling ratio, it is noted that the bending vary almost in a linear manner with the nonlocal parameter.

4.2 Vibration analysis

This section is devoted to validate the present model with the published work. The first flexural non-dimensional frequencies of the simply supported beam for different values of a nonlocal parameter are studied and compared with the results of Reddy (2007). The following equation should be solved to calculate fundamental frequencies

$$[\mathrm{K}][\overline{\mathrm{U}}] = \omega^2[\mathrm{M}][\overline{\mathrm{U}}] \tag{27}$$

where ω^2 represents the fundamental frequency of the beam, which has the nondimensional form $\lambda = \omega * \mathrm{L}^2 * \sqrt{\frac{\rho A}{EI}}$, and presented in Table 2. It is noted that the frequency of nanobeam is reduced by increasing the



(a) at N = 2 (b) at N = 4 (c) at N = 8

Fig. 5 Maximum deflection with the nonlocal parameter at various filling ratios

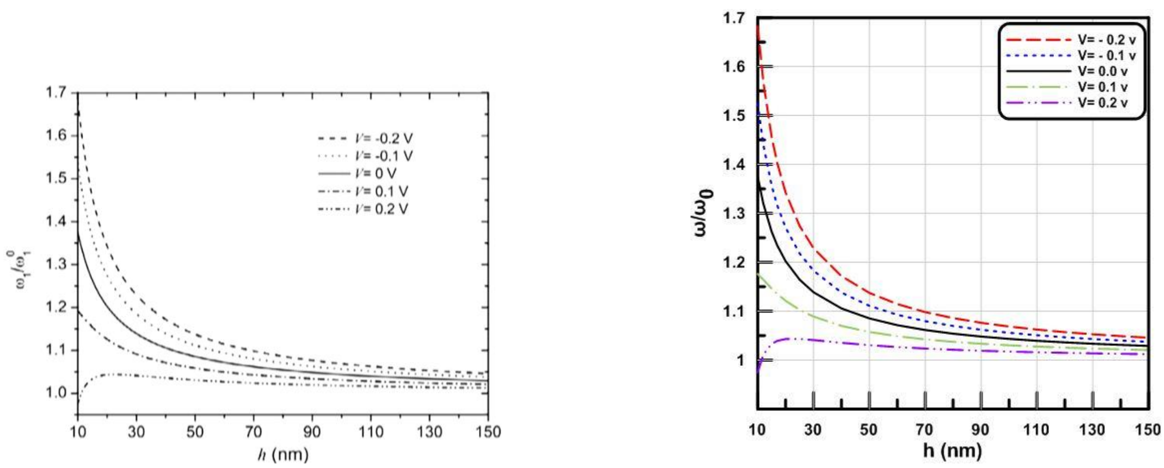
Table 2 Non-dimensional frequencies of the S-S beams for different nonlocal parameters

L/h	μ	Analytical	Numerical
		Reddy (2007)	Present results
20	0	9.8696	9.8797
	1	9.4159	9.4238
	2	9.0195	9.0257
	3	8.6693	8.6741
	4	8.3569	8.3606
	5	8.0761	8.0788

nonlocal parameter. This assures the significance of the nonlocal effect on the vibrational response of beams. Fig. **6** explains the variation of piezoelectric nanobeam with surface effect. This figure shows the normalized frequency against beam thickness for the simply supported case. The identical results of the proposed numerical model and analytical one of Yan and Jiang (2011) are observed.

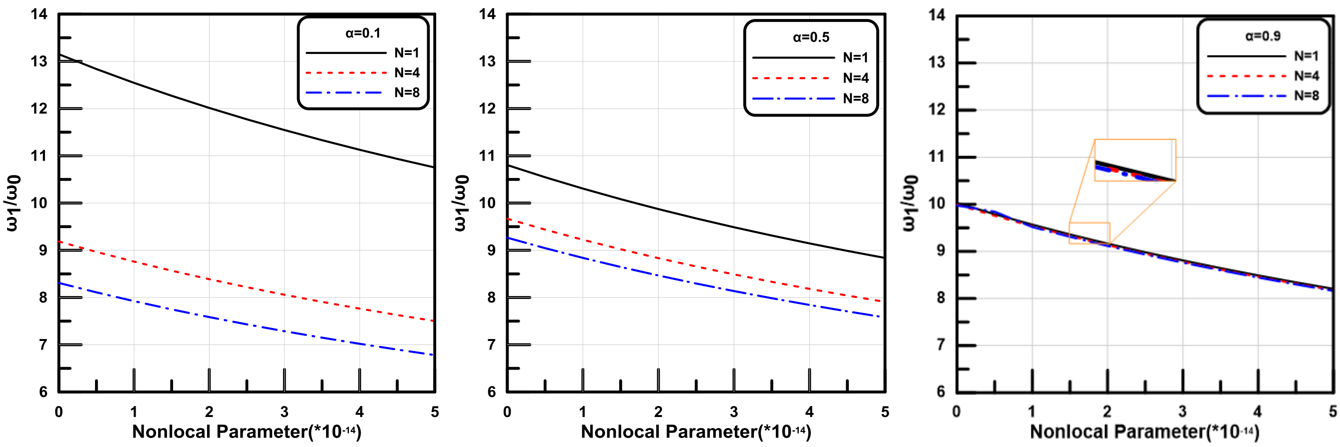
The coupling effects of nonlocality parameter and number of holes along cross section on the fundamental frequency of perforated piezoelectric nanobeams as depicted in Fig. 7. It is noted that the normalized frequency of the piezoelectric nanobeam model is decreased linearly as the nonlocal parameter increased for any value of filling ratio α or a number of holes N. So that, the effect of nonlocal parameter becomes more significant on the fundamental frequency. Also, the number of holes increased the normalized frequency decreased. But this reduction depends on the value of filling ratio.

Fig. 8 explains the variation of the normalized frequency against the filling ratio under different electrical loads (V) for a simply supported (S-S) nanobeam at ($\mu = 3 * 10^{-14}$, $N = 2$). It is observed that, with the increase of the applied positive voltage, the non-dimensional natural frequency increases. Also, the frequency drops down with the decrease of the filling ratio. It is also noted from this figure that the electromechanical coupling of piezoelectric materials can be explored for frequency tuning of nanobeams, as shown by the variation of the natural frequencies with the applied voltages.



(a) Yan and Jiang (2011) (b) Current results

Fig. 6 Variation of normalized resonant frequency against the beam thickness



(a) at α = 0.1 (b) at α = 0.5 (c) at α = 0.9

Fig. 7 Effect of nonlocal parameter and number of holes on the normalized fundamental frequencies of the piezoelectric nanobeams at (V = 0)

Fig. 9 shows the variation of the normalized frequencies with the nonlocal parameter for different values of the filling ratio at (V = 0.1, N = 4). The 1^{st} natural frequency decreases with increasing the nonlocal parameter. Also, in this figure, the normalized frequency increases with increasing the filling ratio.

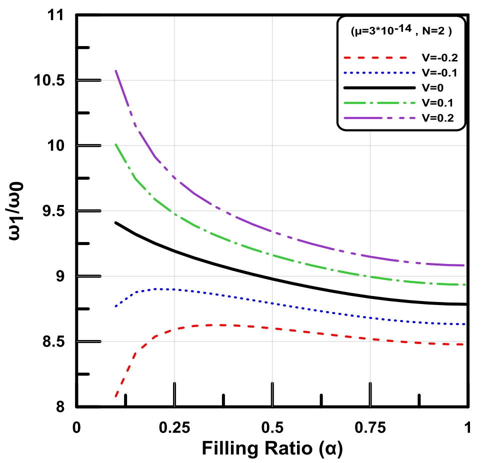


Fig. 8 Variation of the non-dimensional frequency with the filling ratio for piezoelectric nanobeam under different electrical loads, V

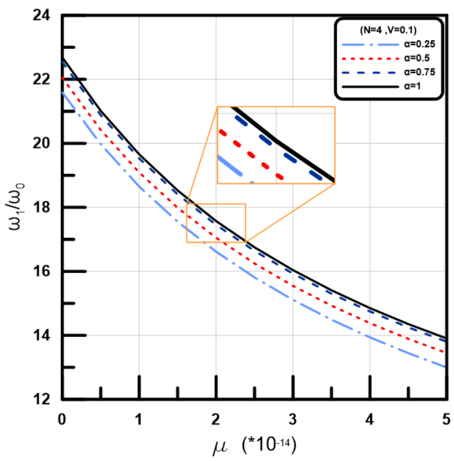
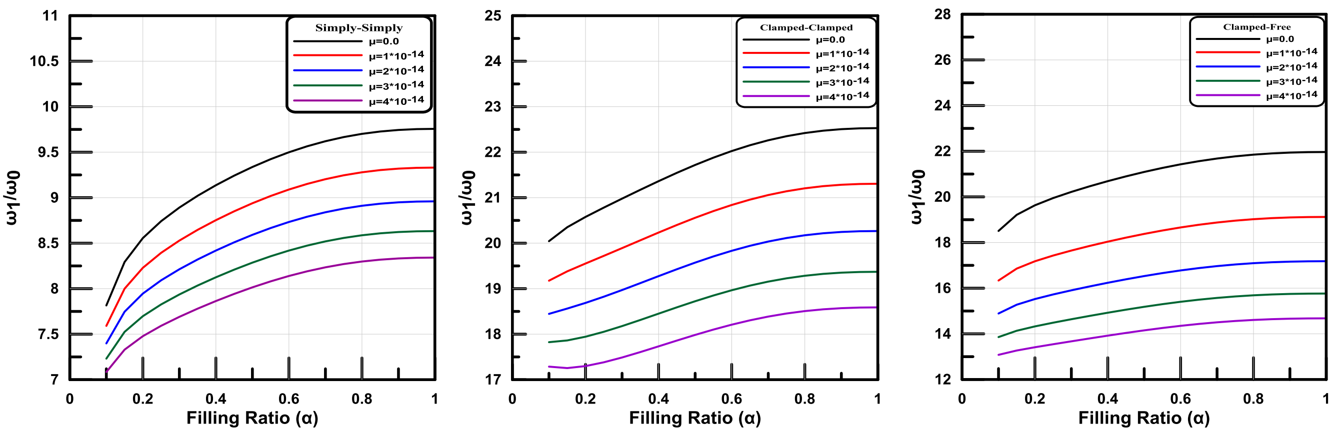


Fig. 9 Variation of the 1^{st} natural frequencies against the nonlocal parameter for various filling ratios at (V = 0.1, N = 4)



(a) Simply-simply Boundary conditions (b) Clamped-Clamped Boundary conditions (c) Clamped-Free Boundary conditions

Fig. 10 Effect of nonlocal parameter and filling ratio on the normalized fundamental frequency at (V = -0.1, N = 4)

Fig. 10 illustrates the coupling effect of the nonlocal parameter and filling ratio on the fundamental frequencies of perforated piezoelectric nanobeam at different boundary conditions. The boundary conditions are (a) simply supported (*S–S*); (b) the clamped- clamped (*C–C*); and (c) the cantilever (*C–F*) nanobeams. The normalized frequency is calculated for the perforated beam with the applied voltage V = − 0.1 V and the number of holes along cross-sectional N = 4. It is noted that the normalized frequency decreases for all the cases of boundary conditions with the increase of nonlocal parameter. By increasing the filling ratio from 0 to 1, the normalized frequency increases at a constant nonlocal parameter. The fundamental frequencies for clamped-clamped nanobeams are higher than those calculated for the simply supported and the cantilever nanobeams.

5. Conclusions

This work is exploited to present a novel modified continuum model to study mechanical behaviors (static and vibration) of perforated piezoelectric nanobeams. The proposed model is based on Euler-Bernoulli hypothesis with a nonlocal differential form of Eringen model. Numerical results illustrate the effects of perforation parameters (perforation size and a number of cutouts), nonlocal parameter, external electric voltage, and boundary conditions on the bending and dynamic characteristics of the perforated nanobeam. The main conclusions derived from the results are:

- For *static analysis*, concerning to the bending stiffness of the perforated piezoelectric nanobeam, the filling ratio is more significant than the number of hole rows.
- Because of softening companion to size-dependent the deflection increases linearly with increasing the nonlocal parameter for all values of the filling ratio and the hole number.
- For *vibrational analysis*, the length scale parameter tends to reduce frequencies of perforated piezoelectric nanobeam.
- The fundamental frequency is dependent on the coupling between the filling ratio and number of holes. The normalized frequencies decrease nonlinearly by decreasing the filling ratio or increasing the number of holes along the cross-sectional.
- The nonlocal parameter has significant effects with different BCs. With the increase of nonlocal parameter, the normalized frequency decreases for all boundary conditions, and the frequency calculated for (C–C) nanobeams are higher than those calculated for (S–S) and (C–F) nanobeam.

Acknowledgments

This work was supported by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under Grant no. (DF-063-135-1441). The authors, therefore, gratefully acknowledge the DSR technical and financial support.

References

Abdalrahmaan, A.A., Eltaher, M.A., Kabeel, A.M., Abdraboh, A.M. and Hendi, A.A. (2019), “Free and forced analysis of perforated beams”, *Steel Compos. Struct., Int. J.*, **31**(5), 489-502. https://doi.org/10.12989/scs.2019.31.5.489

Akbaş, Ş.D. (2017a), “Forced vibration analysis of functionally graded nanobeams”, *Int. J. Appl. Mech.*, **9**(7), 1750100. https://doi.org/10.1142/S1758825117501009

Akbaş, Ş.D. (2017b), “Free vibration of edge cracked functionally graded microscale beams based on the modified couple stress theory”, *Int. J. Struct. Stabil. Dyn.*, **17**(3), 1750033. https://doi.org/10.1142/S021945541750033X

Akbaş, Ş.D. (2018a), “Forced vibration analysis of cracked nanobeams”, *J. Brazil. Soc. Mech. Sci. Eng.*, **40**(8), 392. https://doi.org/10.1007/s40430-018-1315-1

Akbaş, Ş.D. (2018b), “Forced vibration analysis of cracked functionally graded microbeams”, *Adv. Nano Res., Int. J.*, **6**(1), 39-55. https://doi.org/10.12989/anr.2018.6.1.039

Akbaş, Ş.D. (2018c), “Bending of a cracked functionally graded nanobeam”, *Adv. Nano Res., Int. J.*, **6**(3), 219-242. https://doi.org/10.12989/anr.2018.6.3.219

Akbaş, Ş.D. (2019), “Axially Forced Vibration Analysis of Cracked a Nanorod”, *J. Computat. Appl. Mech.*, **50**(1), 63-68. https://doi.org/10.22059/jcamech.2019.281285.392

Almitani, K.H., Abdalrahmaan, A.A. and Eltaher, M.A. (2019), “On forced and free vibrations of cutout squared beams”, *Steel Compos. Struct., Int. J.*, **32**(5), 643-655. https://doi.org/10.12989/scs.2019.32.5.643

Alshorbagy, A.E., Eltaher, M.A. and Mahmoud, F.F. (2011), “Free vibration characteristics of a functionally graded beam by finite element method”, *Appl. Mathe. Model.*, **35**(1), 412-425. https://doi.org/10.1016/j.apm.2010.07.006

Ansari, R., Gholami, R., Hosseini, K. and Sahmani, S. (2011), “A sixth-order compact finite difference method for vibrational analysis of nanobeams embedded in an elastic medium based on nonlocal beam theory”, *Mathe. Comput. Model.*, **54**(11-12), 2577-2586. https://doi.org/10.1016/j.mcm.2011.06.030

Ansari, R., Rouhi, S. and Ahmadi, M. (2018), “On the thermal conductivity of carbon nanotube/polypropylene nanocomposites by finite element method”, *J. Computat. Appl. Mech.*, **49**(1), 70-85. 10.22059/JCAMECH.2017.243530.195

Aydogdu, M. (2009), “A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration”, *Physica E: Low-dimens. Syst. Nanostruct.*, **41**(9), 1651-1655. https://doi.org/10.1016/j.physe.2009.05.014

Chen, X. and Liew, K. (2004), “Buckling of rectangular functionally graded material plates subjected to nonlinearly distributed in-plane edge loads”, *Smart Mater. Struct.*, **13**(6), 1430. https://doi.org/10.1088/0964-1726/13/6/014

Eltaher, M., Alshorbagy, A.E. and Mahmoud, F. (2013a), “Vibration analysis of Euler–Bernoulli nanobeams by using finite element method”, *Appl. Mathe. Model.*, **37**(7), 4787-4797. https://doi.org/10.1016/j.apm.2012.10.016

Eltaher, M.A., Mahmoud, F.F., Assie, A.E. and Meletis, E.I. (2013b), “Coupling effects of nonlocal and surface energy on vibration analysis of nanobeams”, *Appl. Mathe. Computat.*, **224**, 760-774. https://doi.org/10.1016/j.amc.2013.09.002

Eltaher, M., Khairy, A., Sadoun, A. and Omar, F.-A. (2014), “Static and buckling analysis of functionally graded Timoshenko nanobeams”, *Appl. Mathe. Computat.*, **229**, 283-295. https://doi.org/10.1016/j.amc.2013.12.072

Eltaher, M.A., Khater, M.E., Park, S., Abdel-Rahman, E. and

Yavuz, M. (2016), "On the static stability of nonlocal nanobeams using higher-order beam theories", *Adv. Nano Res., Int. J.*, **4**(1), 51-64. https://doi.org/10.12989/anr.2016.4.1.051

Eltaher, M., Abdraboh, A. and Almitani, K. (2018a), "Resonance frequencies of size dependent perforated nonlocal nanobeam", *Microsyst. Technol.*, **24**, 3925-3937. https://doi.org/10.1007/s00542-018-3910-6

Eltaher, M., Kabeel, A., Almitani, K. and Abdraboh, A. (2018b), "Static bending and buckling of perforated nonlocal size-dependent nanobeams", *Microsyst. Technol.*, **24**(12), 4881-4893. https://doi.org/10.1007/s00542-018-3905-3

Eltaher, M.A., Omar, F.A., Abdalla, W.S. and Gad, E.H. (2019a), "Bending and vibrational behaviors of piezoelectric nonlocal nanobeam including surface elasticity", *Waves Random Complex Media*, **29**(2), 264-280. https://doi.org/10.1080/17455030.2018.1429693

Eltaher, M.A., Mohamed, N., Mohamed, S. and Seddek, L.F. (2019b), "Postbuckling of curved carbon nanotubes using energy equivalent model", *J. Nano Res.*, **57**, 136-157. https://doi.org/10.4028/www.scientific.net/JNanoR.57.136

Eltaher, M.A., Almalki, T.A., Ahmed, K.I.E. and Almitani, K.H. (2019c), "Characterization and behaviors of single walled carbon nanotube by equivalent-continuum mechanics approach", *Adv. Nano Res., Int. J.*, **7**(1), 39-49. https://doi.org/10.12989/anr.2019.7.1.039

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710. https://doi.org/10.1063/1.332803

Eringen, A.C. (1984), "Plane waves in nonlocal micropolar elasticity", *Int. J. Eng. Sci.*, **22**(8-10), 1113-1121. https://doi.org/10.1016/0020-7225(84)90112-5

Eringen, A.C. and Edelen, D. (1972), "On nonlocal elasticity", *Int. J. Eng. Sci.*, **10**(3), 233-248. https://doi.org/10.1016/0020-7225(72)90039-0

Faraji-Oskouie, M., Norouzzadeh, A., Ansari, R. and Rouhi, H. (2019), "Bending of small-scale Timoshenko beams based on the integral/differential nonlocal-micropolar elasticity theory: a finite element approach", *Appl. Mathe. Mech.*, **40**(6), 767-782. https://doi.org/10.1007/s10483-019-2491-9

Fernández-Sáez, J., Zaera, R., Loya, J. and Reddy, J. (2016), "Bending of Euler–Bernoulli beams using Eringen's integral formulation: a paradox resolved", *Int. J. Eng. Sci.*, **99**, 107-116. https://doi.org/10.1016/j.ijengsci.2015.10.013

Gheshlaghi, B. and Hasheminejad, S.M. (2012), "Vibration analysis of piezoelectric nanowires with surface and small scale effects", *Current Appl. Phys.*, **12**(4), 1096-1099. https://doi.org/10.1016/j.cap.2012.01.014

Hamed, M., Sadoun, A.M. and Eltaher, M.A. (2019), "Effects of porosity models on static behavior of size dependent functionally graded beam", *Struct. Eng. Mech., Int. J.*, **71**(1), 89-98. https://doi.org/10.12989/sem.2019.71.1.089

Jandaghian, A.A. and Rahmani, O. (2015), "On the buckling behavior of piezoelectric nanobeams: An exact solution", *J. Mech. Sci. Technol.*, **29**(8), 3175-3182. https://doi.org/10.1007/s12206-015-0716-7

Jandaghian, A. and Rahmani, O. (2016), "An analytical solution for free vibration of piezoelectric nanobeams based on a nonlocal elasticity theory", *J. Mech.*, **32**(2), 143-151. https://doi.org/10.1017/jmech.2015.53

Juntarasaid, C., Pulngern, T. and Chucheepsakul, S. (2012), "Bending and buckling of nanowires including the effects of surface stress and nonlocal elasticity", *Physica E: Low-dimens. Syst. Nanostruct.*, **46**, 68-76. https://doi.org/10.1016/j.physe.2012.08.005

Ke, L.-L., Liu, C. and Wang, Y.-S. (2015), "Free vibration of nonlocal piezoelectric nanoplates under various boundary conditions", *Physica E: Low-dimens. Syst. Nanostruct.*, **66**, 93-106. https://doi.org/10.1016/j.physe.2014.10.002

Kheibari, F. and Beni, Y.T. (2017), "Size dependent electro-mechanical vibration of single-walled piezoelectric nanotubes using thin shell model", *Mater. Des.*, **114**, 572-583. https://doi.org/10.1016/j.matdes.2016.10.041

Lazarus, A., Thomas, O. and Deü, J.-F. (2012), "Finite element reduced order models for nonlinear vibrations of piezoelectric layered beams with applications to NEMS", *Finite Elem. Anal. Des.*, **49**(1), 35-51. https://doi.org/10.1016/j.finel.2011.08.019

Li, C., Lim, C.W. and Yu, J. (2010), "Dynamics and stability of transverse vibrations of nonlocal nanobeams with a variable axial load", *Smart Mater. Struct.*, **20**(1), 015023. https://doi.org/10.1088/0964-1726/20/1/015023

Luschi, L. and Pieri, F. (2014), "An analytical model for the determination of resonance frequencies of perforated beams", *J. Micromech. Microeng.*, **24**(5), 055004. https://doi.org/10.1088/0960-1317/24/5/055004

Luschi, L. and Pieri, F. (2016), "An analytical model for the resonance frequency of square perforated Lamé-mode resonators", *Sensors Actuators B: Chem.*, **222**, 1233-1239. https://doi.org/10.1016/j.snb.2015.07.085

Mahinzare, M., Ranjbarpur, H. and Ghadiri, M. (2018), "Free vibration analysis of a rotary smart two directional functionally graded piezoelectric material in axial symmetry circular nanoplate", *Mech. Syst. Signal Process.*, **100**, 188-207. https://doi.org/10.1016/j.ymssp.2017.07.041

Mohamed, N., Eltaher, M.A., Mohamed, S. and Seddek, L.F. (2019), "Energy equivalent model in analysis of postbuckling of imperfect carbon nanotubes resting on nonlinear elastic foundation", *Struct. Eng. Mech., Int. J.*, **70**(6), 737-750. https://doi.org/10.12989/sem.2019.70.6.737

Murmu, T. and Adhikari, S. (2012), "Nonlocal frequency analysis of nanoscale biosensors", *Sensors Actuators A: Phys.*, **173**(1), 41-48. https://doi.org/10.1016/j.sna.2011.10.012

Pei, J., Tian, F. and Thundat, T. (2004), "Glucose biosensor based on the microcantilever", *Anal. Chem.*, **76**(2), 292-297. https://doi.org/10.1021/ac035048k

Reddy, J. (2007), "Nonlocal theories for bending, buckling and vibration of beams", *Int. J. Eng. Sci.*, **45**(2-8), 288-307. https://doi.org/10.1016/j.ijengsci.2007.04.004

She, G.L., Yan, K.M., Zhang, Y.L., Liu, H.B. and Ren, Y.R. (2018a), "Wave propagation of functionally graded porous nanobeams based on non-local strain gradient theory", *Eur. Phys. J. Plus*, **133**(9), 368. https://doi.org/10.1140/epjp/i2018-12196-5

She, G.L., Yuan, F.G. and Ren, Y.R. (2018b), "On wave propagation of porous nanotubes", *Int. J. Eng. Sci.*, **130**, 62-74. https://doi.org/10.1016/j.ijengsci.2018.05.002

She, G.L., Ren, Y.R. and Yan, K.M. (2019), "On snap-buckling of porous FG curved nanobeams", *Acta Astronautica*, **161**, 475-484. https://doi.org/10.1016/j.actaastro.2019.04.010

Tanner, S.M., Gray, J.M., Rogers, C., Bertness, K.A. and Sanford, N.A. (2007), "High-*Q* GaN nanowire resonators and oscillators", *Appl. Phys. Lett.*, **91**(20), 203117. https://doi.org/10.1063/1.2815747

Thai, S., Thai, H.-T., Vo, T.P. and Lee, S. (2018), "Postbuckling analysis of functionally graded nanoplates based on nonlocal theory and isogeometric analysis", *Compos. Struct.*, **201**, 13-20. https://doi.org/10.1016/j.compstruct.2018.05.116

Wang, Z.-l., Liu, J., Zheng, D.-c., Zhang, A.-b., Hu, K.-x. and Wang, J. (2019), "Vibration Analysis of Laminated Plates in a Piezoelectric Inkjet Printhead", *Proceedings of 2019 Symposium on Piezoelectrcity, Acoustic Waves and Device Applications* (*SPAWDA*), Harbin, China, January.

Yan, Z. and Jiang, L.Y. (2011), "The vibrational and buckling behaviors of piezoelectric nanobeams with surface effects", *Nanotechnol.*, **22**(24), 245703.

https://doi.org/10.1088/0957-4484/22/24/245703
Zakeri, M., Attarnejad, R. and Ershadbakhsh, A.M. (2016), "Analysis of Euler-Bernoulli nanobeams: A mechanical-based solution", *J. Computat. Appl. Mech.*, **47**(2), 159-180. https://doi.org/10.22059/JCAMECH.2017.140165.97
Zand, M.M. and Ahmadian, M. (2009), "Vibrational analysis of electrostatically actuated microstructures considering nonlinear effects", *Commun. Nonlinear Sci. Numer. Simul.*, **14**(4), 1664-1678. https://doi.org/10.1016/j.cnsns.2008.05.009

CC