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Investigations on state estimation of smart structure systems

J. Arunshankar*

Department of Instrumentation and Control Systems Engineering, *PSG College of Technology*, *Coimbatore - 641004*, *India*

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Abstract. This paper aims at enlightening the properties, computational and implementation issues related to Kalman filter based state estimation algorithms and sliding mode observers, by applying them for estimating the states of a smart structure system. The Kalman based estimators considered in this work are Kalman filter and information filter and, the sliding mode observers considered are Utkin observer and higher order sliding mode observer. A fourth order linear time invariant model of a piezo actuated beam is used in this work. This structure is embedded with four number of piezo patches, of which two act as sensors, one as disturbance actuator and the other as control actuator. The performance of the state estimation algorithms is evaluated through simulation, for the first two vibrating modes of the piezo actuated structure, when the structure is maintained at first mode and second mode resonance.

Keywords: smart structure; piezo actuated structure; state estimation; Kalman filter; information filter; Utkin observer; higher order sliding mode observer

1. Introduction

The unavailability of a subset of state variables of systems, for measurement imposes challenges for the design of controllers. The reason for the unavailability may be due to the lack of measurement system or sometimes these variables may not be accessible for measurement. This drives the need to develop a dynamical system called as observer or estimator, which uses the mathematical model of the system and, the system output to generate the state variables. Sometimes the states of the system will be generated to monitor the performance of the system.

Kalman filter is a recursive linear estimator which successively calculates a minimum variance estimate of a state that evolves over time, on the basis of periodic observations that are linearly related to this state. The Kalman filter estimator minimizes the mean squared estimation error and is optimal with respect to a variety of important criteria, under specific assumptions about process and observation noise. The earlier implementation of the Kalman filter algorithm presented few drawbacks for practical implementation. Information filter is claimed to be the algebraic equivalent of Kalman filter. It is essentially a Kalman filter expressed in measures of information about state estimates and their associated covariance. The use of information filter for state estimation can be justified, for the case where the initial conditions of the states to be estimated are known poorly. There are no gain or innovation covariance matrices involved in information filter, and the maximum dimension of a matrix to be inverted is the state dimension. Information filter is computationally simpler and it is a more direct and, a natural method of dealing with multisensory data fusion problems. It has a special advantage in decentralised sensor networks, because it provides a direct interpretation of node observation and contribution in terms of information, as reported in Mutambara (1999).

The sliding mode observers are widely used due to the finite-time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation, as presented in Edwards and Spurgeon (1998) and Utkin (1992), and it is based on Variable Structure Control (VSC). VSC with the sliding mode is an established method for controlling uncertain dynamic systems. In spite of high accuracy and robustness with respect to various internal and external disturbances, VSC has a main drawback called chattering effect, which is a dangerous high-frequency vibration of the controlled system. The concept of VSC later was employed for applications including the problem of state estimation. The earliest work by Utkin (1992) utilizes a discontinuous switched component within an observer.

A new distributed fusion filtering algorithm for linear multiple time-delayed systems, with multisensory distributed fusion filter formed by the summation of, Local Kalman Filters having Time Delays (LKFTDs) in both the system and measurement models, is proposed by Lee *et al.* (2012a, b). Nonlinear state estimation performed using unscented transformation with certain parts of classic Kalman filter, resulting in a comparatively higher accurate method is presented by Rao *et al*. (2009). An internal model based method to estimate the structural displacements and velocities under ambient excitation using only acceleration measurements, using the standard Kalman filtering technique, is proposed by Ma *et al*. (2014), it has been demonstrated and evaluated via numerical simulations on an eight-story lumped mass model and, experimental data of a three-story frame excited, by the ground accelerations

*Corresponding author, Ph.D., Professor,
E-mail: j_arunshankar@yahoo.com

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of actual earthquake records. Estimation of dynamic displacement with high accuracy by blending, high-sampling rate acceleration data with low-sampling rate displacement measurement, using a two-stage Kalman estimator is presented in Kim *et al*. (2016). A new structural damage detection algorithm based on substructure approach using sequential extended Kalman estimator, from the degradation of the identified substructural element stiffness values, for large size structural systems with limited input and output measurements is proposed by Lei *et al*. (2013). An extension of classical Kalman filter for real time estimation of structural state and unknown inputs, without using collocated acceleration measurements is proposed by Lei *et al*. (2016). Further data fusion of acceleration and displacement or strain measurements is used to prevent the drifts in the identified structural state and unknown inputs in real time.

An extension of the classical Kalman filter for real time joint estimation of structural states and the unknown inputs, by fusing the data of partially measured displacement and acceleration responses to prevent the drifts is proposed by Liu *et al*. (2016). A response estimation technique based on the Kalman state estimator applied for the structural health monitoring of a simply-supported beam, which successfully estimated the strain responses at unmeasured locations with the highest performance by fusing acceleration, strain and tilt, by minimizing the intrinsic measurement noise, under non-zero mean input excitations is presented by Palanisamy *et al*. (2015). A two-stage and two-step algorithm is proposed in Lei *et al*. (2015) for the identification of structural damage as well as unknown external excitations, where in stage-one, structural state vector and unknown structural parameters are recursively estimated by a two-step Kalman estimator approach, with the unknown external excitations are estimated sequentially by least-squares estimation in stage-two. Here the number of unknown variables to be estimated in each step is reduced, which simplify the identification problem and reduces computational efforts significantly. Application of Kalman based estimators for the experimental evaluation of the closed loop performance of the reaching law based discrete sliding mode controller with Multisensor Data Fusion (MSDF) in real time, by controlling the first two vibrating modes of a piezo actuated structure is presented by Arunshankar *et al*. (2013).

A robust H_∞ sliding mode descriptor observer for simultaneous state and disturbance estimation of uncertain system is developed by Lee *et al*. (2012a). A sliding-mode observer where the switching terms are designed such that the faults are tracked and reconstructed from their respective sliding surfaces, with a feature to perform the fault reconstruction online along with the state estimation is proposed by Veluvolu and Soh (2011). A new approach which is based on the utilisation of a network of two interconnected sliding mode observers, with the first used for fault diagnosis and the second used for estimation of unknown inputs for fault diagnosis and estimation of unknown inputs, in a class of non-linear systems is presented by Sharma and Aldeen (2011). Sliding mode observer for generating the slosh states, which are otherwise difficult to measure is developed by Kurode *et al*. (2013).

A sliding-mode controller equipped with a sliding-mode observer is synthesized and applied to a novel three-axis, four-wire optical pickup for the purpose of sensorless tilt compensation by Chao and Shen (2009). A higher order sliding mode observer for asymptotic identification of the full state vector and the vector of unknown inputs for MIMO nonlinear causal systems with unstable internal dynamics is proposed by Shtessel *et al*. (2010). To detect actuator faults, on a robot manipulator a higher order sliding-mode unknown input observer is proposed by Capisani *et al*. (2012).

The main contribution of this paper is, it brings the inherent features, which talks about the computational and implementation issues of the Kalman based estimators and the sliding mode observers. The paper highlights the importance of process noise and measurement noise, which will affect the performance of the estimators, selection of initial values of the estimates and state covariance matrix, which may lead to computational complexity and estimation error. The information presented will be useful to select an estimator for generating the states of smart structure using appropriate estimator, for its health monitoring and control, for the given conditions.

The structure of the paper is as follows. In Section 2 review of observers used in this work is presented. Section 3 presents the mathematical model of the smart structure system, whose states are to be estimated. Results and discussion are presented in Section 4. Conclusions are drawn in Section 5.

2. Review of observers

2.1 Kalman filter

Consider a system described in linear form

$$x(k+1) = Ax(k) + w(k) \tag{1}$$

where $x(k)$ are states of interest at time k, A the state transition matrix from time (k) to $(k+1)$, and $w(k)$ the associated process noise modelled as an uncorrelated white sequence with

$$E[w(i)w^T(j)] = \delta_{ij}Q(i) \tag{2}$$

where $Q(i)$ is process noise covariance matrix.

The system is observed according to the linear equation

$$z(k) = Hx(k) + v(k) \tag{3}$$

where $z(k)$ is the vector of observations made at time k, H the observation matrix and $v(k)$ the associated observation noise modelled as an uncorrelated white sequence with

$$E[v(i)v^T(j)] = \delta_{ij}R(i) \tag{4}$$

where $R(i)$ is measurement noise covariance matrix.Also

$$E[v(i)w^T(j)] = 0 \tag{5}$$

The state estimate and covariance estimate at time t_k $\hat{x}(k|k)$ and $P(k|k)$ as presented in Kalman (1960) are,

State and covariance prediction

$$\hat{x}(k+1|k) = A\hat{x}(k|k) + w(k) \tag{6}$$

$$P(k+1|k) = AP(k|k)A' + Q \tag{7}$$

Measurement Prediction

$$\hat{z}(k+1|k) = H\hat{x}(k+1|k) \tag{8}$$

Innovation Covariance

$$M(k+1) = HP(k+1|k)H' + R \tag{9}$$

Measurement Residual

$$e(k+1) = z(k+1) - \hat{z}(k+1|k) \tag{10}$$

Filter Gain

$$K(k+1) = P(k+1|k)H'M(k+1)^{-1} \tag{11}$$

State and covariance updation

$$\begin{aligned} &x(k+1|k+1) \\ &= x(k+1|k) + \mathrm{K}(k+1)e(k+1|k) \end{aligned} \tag{12}$$

$$\begin{aligned} &P(k+1|k+1) \\ &= P(k+1|k) - K(k+1)H'P(k+1|k) \end{aligned} \tag{13}$$

2.2 Information filter

Information filter is essentially a Kalman filter expressed in terms of measures of information about the states of interest, rather than direct state estimates and their associated covariances as presented in Mutambara (1999). The two key information-analytic variables are information matrix $\hat{Y}(i|j)$ and information state vector $\hat{y}(i|j)$, with information matrix being the inverse of state covariance matrix (P)

$$Y(i|j) = P^{-1}(i|j) \tag{14}$$

The information state vector is the product of the inverse of state covariance matrix and state estimate $\hat{x}(i|j)$

$$\hat{y}(i|j) = P^{-1}(i|j)\hat{x}(i|j) \tag{15}$$

The update equation for the information state vector

$$\hat{y}(k|k) = \hat{y}(k|k-1) + H^T R^{-1} z(k) \tag{16}$$

The expression for information matrix associated with the above estimate is

$$Y(k|k) = Y(k|k-1) + H^T R^{-1} H \tag{17}$$

The information state contribution *i(k)* from an observation *z(k)*, and its associated information matrix *I(k)* are

$$i(k) = H^T R^{-1} z(k) \tag{18}$$

$$I(k) = H^T R^{-1} H \tag{19}$$

The information propagation coefficient $L(k|k-1)$, which is independent of the observation made is given by

$$L(k|k-1) = Y(k|k-1)AY^{-1}(k-1|k-1) \tag{20}$$

Prediction

$$\hat{y}(k|k-1) = L(k|k-1)\hat{y}(k-1|k-1) \tag{21}$$

$$Y(k|k-1) = [AY^{-1}(k-1|k-1)A^T + Q(k)]^{-1} \tag{22}$$

Estimation

$$\hat{y}(k|k) = \hat{y}(k|k-1) + i(k) \tag{23}$$

$$Y(k|k) = Y(k|k-1) + I(k) \tag{24}$$

2.3 Utkin observer

Consider a linear system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{25}$$

$$y(t) = Cx(t) \tag{26}$$

Where $A \in \Re^{n\times n}, B \in \Re^{n\times m}, C \in \Re^{pxn}$ an $p \geq m$. Assume that the matrices *B* and *C* are of full rank and pair (*A*,*C*) is observable, as given in Edwards and Spurgeon (1998) and, Utkin (1992). Since the outputs are to be considered, it is logical to effect a change of coordinates, so that the output appear as components of the states. One possibility is to consider the transformation $\boldsymbol{x} \mapsto \boldsymbol{T_c x}$ where

$$T_c = \begin{bmatrix} {N_c}^T \\ 0 \end{bmatrix} \tag{27}$$

With the columns of $N_c \in R^{n\times(n-p)}$span the null space of *C*. This transformation is nonsingular, and with respect to this new coordinate system, the new output distribution matrix is

$$CT_c^{-1} = [0 \quad I_P] \tag{28}$$

Where *p* is the number of output from the system and $n$is the order of the system, with

$$T_c \ \mathrm{A}T_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{29}$$

$$T_c B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{30}$$

The nominal system is

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1 u(t) \tag{31}$$

$$y(t) = A_{21}x_1(t) + A_{22}y(t) + B_2 u(t) \tag{32}$$

$$T_c x = \begin{bmatrix} x_1 \\ y \end{bmatrix} \quad (33)$$

The observer proposed by Utkin has the form

$$\dot{\hat{x}}_1(t) = A_{11}\hat{x}_1(t) + A_{12}\hat{y}(t) + B_1 u(t) + Lv \quad (34)$$

$$\dot{\hat{y}}(t) = A_{21}\hat{x}_1(t) + A_{22}\hat{y}(t) + B_2 u(t) - v \quad (35)$$

Where $(\hat{x}_1, \hat{y})$ represent the state estimates for $(x_1, y), L \in R^{(n-p)\times p}$is a constant feedback gain matrix and, the discontinuous vector v is defined component wise by

$$v_i = M sign(\hat{y}_i - y_i) \quad (36)$$

Where $M \in R_+$. If the errors between the estimates and the true states are $e_1 = \hat{x}_1 - x_1$ and $e_y = \hat{y} - y$, then from Eqs. (31)-(35) the following error system is obtained

$$\dot{e}_1(t) = A_{11}e_1(t) + A_{12}e_y(t) + Lv \quad (37)$$

$$\dot{e}_y(t) = A_{21}e_1(t) + A_{22}e_y(t) - v \quad (38)$$

Since the pair (A, C) is observable, the pair (A_{11}, A_{21}) is also observable. As a consequence, L can be chosen to make the spectrum of $A_{11} + LA_{21}$ lie in C_-. Define a further change of coordinates, dependent on L by

$$\tilde{T} = \begin{bmatrix} I_{n-p} & L \\ 0 & I_p \end{bmatrix} \quad (39)$$

and let $\tilde{e}_1 = e_1 + Ly$. The error system with respect to the new coordinate is

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) + \tilde{A}_{12}e_y(t) \quad (40)$$

$$\dot{e}_y(t) = A_{21}\tilde{e}_1(t) + \tilde{A}_{22}e_y(t) - v \quad (41)$$

Where $\tilde{A}_{11} = A_{11} + LA_{21}, \tilde{A}_{12} = A_{12} + LA_{22} - \tilde{A}_{11}L$ and $\tilde{A}_{12} = A_{22} - A_{21}L$.

It follows from Eq. (41) that in the domain

$$\begin{aligned} \Omega = \{(e_1, e_y)&: \|A_{21}e_1\| \\ &+0.5\lambda_{max}(\tilde{A}_{22} + \tilde{A}_{22}^T)\|e_y\| < M - \eta\} \end{aligned} \quad (42)$$

Where $\eta < M$ is some small positive scalar, the reachability condition is satisfied.

$$e_y^T \dot{e}_y < -\eta\|e_y\| \quad (43)$$

Consequently, an ideal sliding motion will take place on the surface

$$S_o = \{(e_1, e_y): e_y = 0\} \quad (44)$$

It follows that after a finite time t_s and, for all subsequent time, $e_y = 0$and $\dot{e}_y = 0$, Eq. (40) then reduces to

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) \quad (45)$$

Which by choice of L, represents a stable system and so $\tilde{e}_1 \to 0$, consequently $\hat{x}_1 \to x_1$ as $t \to \infty$. Eq. (45) represents reduced order sliding mode dynamics.

2.4 Higher order sliding mode observer

An observer itself is a dynamical system, which is driven by a control input and, by the difference between the output of the observer and the output of the plant, called output error, which should ideally become zero. In sliding mode observer, the idea to generate a sliding mode on the subspace for which the output error is zero is applied, Edwards and Spurgeon (1998). Consider a nominal linear system given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Er(t) \\ y(t) &= Cx(t) \end{aligned} \quad (46)$$

Where $A \in \Re^{nxn}, B \in \Re^{nxm}, C \in \Re^{pxn}, E \in \Re^{nxq}$ with $q \leq p \leq n$ and, matrices C and E are of full rank, and it is assumed that only the signals $u(t)$ and $y(t)$ are available. The objective is to synthesize an observer to generate a state estimate $\hat{x}(t)$ and output estimate $\hat{y}(t) = C\hat{x}(t)$, such that a sliding mode is attained in which the output error

$$e_y(t) = \hat{y}(t) - y(t) \quad (47)$$

is forced to zero in finite time. The particular observer structure that will be considered can be written in the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - G_l e_y(t) + G_n v \quad (48)$$

where $G_l, G_n \in \Re^{nxp}$ are appropriate gain matrices and, v represents a discontinuous switched component to induce a sliding motion.

Considering the dynamical system given by Eq. (46), a sliding mode observer of the form given by Eq. (48), which rejects the uncertainty, will exist if and only if the nominal linear system, defined by the triple (A,E,C) satisfies: Rank*(CE)* = *q*, invariant zeros of the triple *(A,E,C)* must lie in C_-. For a square system, where *p* = *q*, it should be noted that the above two conditions fundamentally require the triple *(A,E,C)* to be relative degree one and minimum phase. Under these assumptions, there exists a linear change of coordinates$x \mapsto Tx$ such that in the new coordinate system

$$\begin{aligned} \dot{x}_1(t) &= A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t) \\ \dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t) + Er(t) \\ y(t) &= x_2(t) \end{aligned} \quad (49)$$

Where $x_1 \in \Re^{(n-p)}, x_2 \in \Re^p$ and the matrix A_{11} has stable eigenvalues. The coordinate system above will be used as a platform for the design of a sliding mode observer.

Consider a dynamical system of the form

$$\begin{aligned} \dot{\hat{x}}_1(t) &= A_{11}\hat{x}_1(t) + A_{12}\hat{x}_2(t) + B_1u(t) \\ \dot{\hat{x}}_2(t) &= A_{21}\hat{x}_1(t) + A_{22}\hat{x}_2(t) + B_2u(t) \\ &\quad -(A_{22} - A_{22}^S)e_y(t) + v \\ \hat{y}(t) &= \hat{x}_2(t) \end{aligned} \quad (50)$$

where A_{22}^{s} is a stable design matrix and the discontinuous vector v is defined by a higher order sliding mode control law, named as super-twisting controller, Rolink *et al*. (2006), which has the advantage of chattering attenuation. If the state estimation errors are defined as $e_1 = \hat{x}_1 - x_1$ and $e_2 = \hat{x}_2 - x_2$ then it is shown that

$$\begin{aligned}\dot{e}_1(t) &= A_{11}e_1(t)\\ \dot{e}_y(t) &= A_{21}e_1(t) + A_{22}^{S}e_y(t) + v - Er\end{aligned} \tag{51}$$

Since in this situation $e_y = e_2$. The nonlinear error system given by Eq. (51) is quadratically stable and a sliding motion takes place forcing $e_y = 0$ in finite time. The dynamical system given by Eq. (50) is thus regarded as an observer for the system given in Eq. (46). It follows that, after transformation of the system

$$G_l = T^{-1}\begin{bmatrix} A_{12} \\ A_{22} - A_{22}^{s}\end{bmatrix} \quad \text{and} \quad G_n = T^{-1}\begin{bmatrix} 0 \\ I_p\end{bmatrix} \tag{52}$$

Hence the observer given in Eq. (50) can be written in terms of the original coordinates in the form of Eq. (48). The review of super twisting controller is given in the following sub section.

2.5 Super-twisting controller

The Super-Twisting controller depends only on the actual value of the sliding variable, and it is effective only for chattering attenuation, as reported Pisano and Usai (2011). It however, does not require this output derivative to be measured, but it has been originally developed and analysed for systems with relative degree one with respect to the input, as presented in Khan *et al*. (2003), which is given by

$$\dot{s} = \varphi(s,t) + \gamma(s,t)u \tag{53}$$

Where $0 < |\varphi(.)| \le \Phi$ and $0 < \Gamma_m \le \gamma(.) \le \Gamma_M$. The control law *u(t)* is a combination of two terms

$$\begin{aligned} u(t) &= u_1(t) + u_2(t)\\ \dot{u}_1(t) &= \begin{cases} -u & if \quad |u| > 1\\ -W sign(s) & if \quad |u| \le 1\end{cases}\\ u_2(t) &= \begin{cases} -\lambda |s_0|^{\rho} sign(s) & if \quad |s| > s_0\\ -\lambda |s|^{\rho} sign(s) & if \quad |s| \le s_0\end{cases}\end{aligned} \tag{54}$$

Where $|s| < s_0$.The trajectories of the controller 'twist' around the origin in the phase portrait of the sliding variable, the corresponding sufficient conditions for the finite time convergence to the sliding manifold are

$$\begin{aligned} W &> \frac{\Phi}{\Gamma_m}\\ \lambda^2 &\ge \frac{4\Phi}{\Gamma_m^2}\frac{\Gamma_M(W+\Phi)}{\Gamma_m(W-\Phi)}\\ 0 &< \rho \le 0.5\end{aligned} \tag{55}$$

This controller may be simplified when the controlled system is linearly dependent on control, u does not need to be bounded and $s_0 = \infty$

$$\begin{aligned} u &= -\lambda|\sigma|^{\rho} sign(s) + u_1\\ \dot{u}_1 &= -W sign(s)\end{aligned} \tag{56}$$

The super-twisting controller does not need any information on the time derivative of the sliding variable, which makes it less complex and can be easily realized in real time. The choice $\rho = 0.5$ ensures that the maximal possible 2-sliding realization for real-sliding order 2 is achieved.

3. Mathematical model of smart structure system

The piezo actuated structure considered in this work is shown in Fig. 1, with its mathematical model taken from Arunshankar and Umapathy (2012). The smart structure is a cantilever beam made of aircraft grade aluminium, whose dimensions and properties are given in Table 1.

Two piezo ceramic patches, which act as sensors are surface bonded on the bottom surface of the beam, one at a distance of 10 mm and the other at a distance of 105 mm from the fixed end. Another pair of piezo patches is surface bonded on the top surface of the beam, one at a distance of 10 mm and another at a distance of 375 mm from the fixed end, to act as control and disturbance actuators respectively. Excitation input is applied to the structure through the

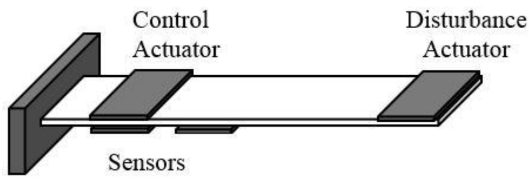

Fig. 1 Schematic of the piezo actuated structure

Table 1 Properties and dimensions of aluminum beam

Length (m)	0.45
Width (m)	0.0135
Thickness (m)	0.001
Young's modulus (Gpa)	71
Density (kg/m^3)	2700
First mode frequency (Hz)	5.5
Second mode frequency (Hz)	30.4

Table 2 Properties and dimensions of piezoceramic sensor/actuator

Length (m)	0.0765
Width (m)	0.0135
Thickness (m)	0.0005
Young's modulus (Gpa)	47.62
Density (kg/m^3)	7500
Piezoelectric strain constant (mV^{-1})	-247×10^{-12}
Piezoelectric stress constant (VmN^{-1})	-9×10^{-3}

disturbance actuator. The dimensions and properties of piezo ceramic patches are given in Table 2.

The unknown parameters of the smart structure dynamics are estimated using Recursive Least Squares (RLS) estimation algorithm, an online model identification technique. The smart structure is designed to demonstrate the performance of sliding mode controller, for controlling the vibration, as presented in Arunshankar *et al*. (2013). Since the structure is designed to perform vibration control, realizing the fact that the first few vibration modes play a vital role in structural dynamics, the model of the structure is selected to represent the dynamics of first two modes of vibration, which resulted in the smart structure to be represented as a fourth order linear time invariant model. The smart structure consists of two piezo sensors, the purpose of including the second sensor is to demonstrate the benefits of data fusion, which contributed for the improvement in the vibration control.

To identify the unknown parameters of the structure, it is excited by a sinusoidal signal by sweeping the frequency in the range of (0-50 Hz), which is inclusive of the first two natural frequencies of the beam, through disturbance actuator and, a square wave signal as an input to the control actuator. With a sampling frequency of 200 Hz, the input – output data of the cantilever beam is collected, for identifying the model. The model thus obtained is validated by observing the convergence of identified parameters, matching between actual plant and model response, and closeness of the natural frequencies of the identified model with that of the experimentally measured one, using a data set that is different from the data used to calculate the model parameters. The continuous time state space model of the smart structure system thus obtained is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Er(t) \\ y(t) &= Cx(t) \end{aligned} \tag{57}$$

where A is the system matrix, B is control input vector, E the disturbance vector, C the output matrix, x is the state vector and y the system output.

$$A = \begin{bmatrix} 92.1084 & 64.5070 & -39.8911 & 65.1749 \\ -159.5286 & 14.3813 & 112.5734 & -118.4229 \\ 116.4182 & -111.6173 & -15.247 & 160.9807 \\ -63.1027 & 39.0227 & -63.7560 & -93.4438 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.5220 \\ 0.2457 \\ -0.3766 \\ 0.7240 \end{bmatrix} \quad E = \begin{bmatrix} -0.0141 \\ -0.0387 \\ 0.0421 \\ 0.0058 \end{bmatrix} \quad \rightleftarrows C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

The model is discretized for a sampling interval of 0.01 sec.

4. Results and discussion

The performance of observers is evaluated through simulation. Excitation signal is a sinusoidal signal with amplitude of 10 Vpp, applied to the disturbance actuator. The frequency of the disturbance is maintained at 5.5 Hz for the first 3 secs, and maintained at 30.4 Hz, for the next 3 secs, which is done to maintain the structure at first and second mode resonance. The response of four state variables X1, X2, X3 and X4 is shown in Fig. 2.

Kalman filter is implemented, by selecting zero initial state vector, initial state error covariance matrix as $0.1\times I_{4x4}$, process noise covariance Q as 1, and measurement noise covariance R as 0.001. States estimated by Kalman filter is shown in Fig. 3 and Kalman filter error is shown in Fig. 4.

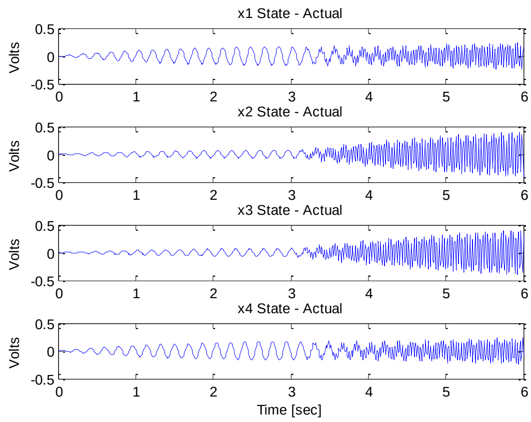

Fig. 2 Response of the system when excited by first and second mode frequency

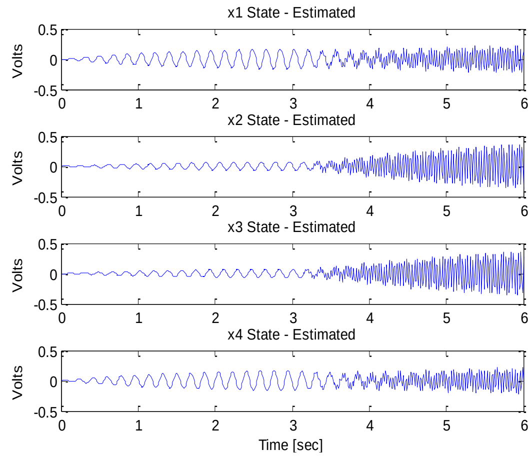

Fig. 3 States estimated using Kalman filter

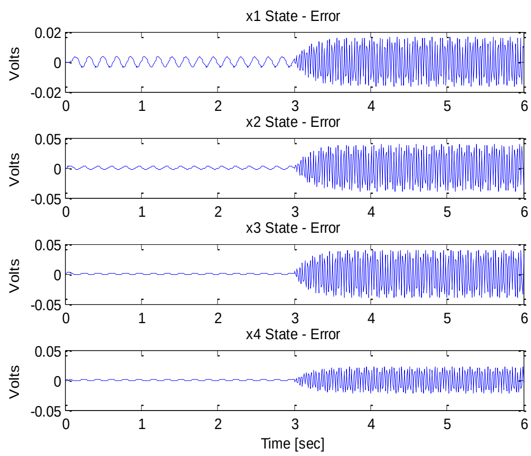

Fig. 4 Kalman filter error

Information filter is implemented, by selecting zero initial state vector, initial state error covariance matrix as I_{4x4}, process noise covariance matrix Q as $0.1\times I_{4x4}$, and measurement noise covariance R as 0.001. States estimated by information filter is shown in Fig. 5 and the information filter error is shown in Fig. 6.

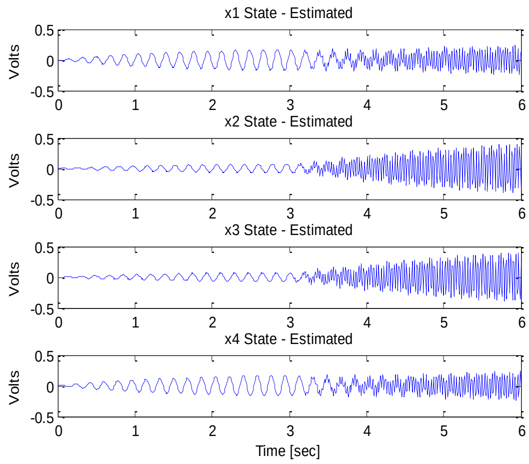

Fig. 5 States estimated using information filter

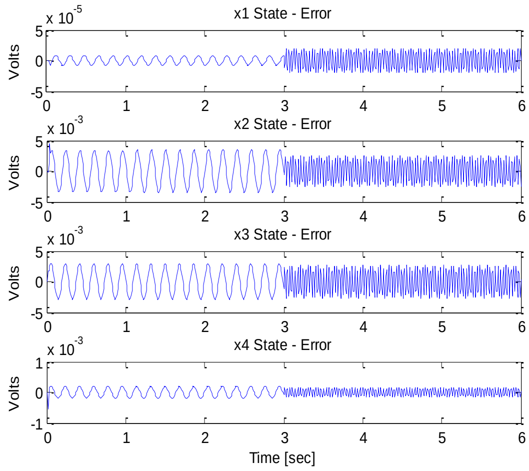

Fig. 6 Information filter error

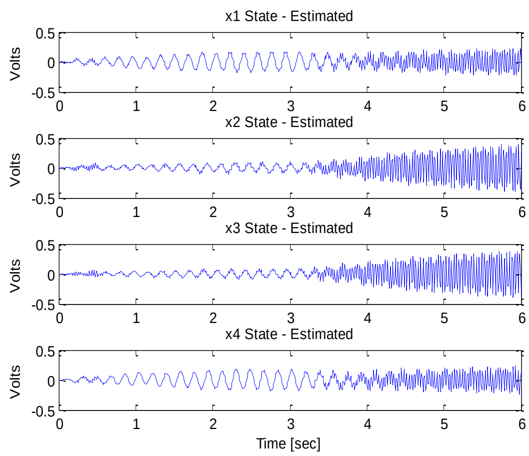

Fig. 7 States estimated using Utkin observer

Utkin observer is implemented, by selecting zero initial state vector, with M = 0.01 and L = 0.2. The states estimated by Utkin observer is shown in Fig. 7 and the Utkin observer error is shown in Fig. 8.

Higher Order Sliding Mode observer is implemented, by selecting zero initial state vector, with m = 1, λ = 14, W = 4, ρ = 2000. The weight matrices G_l and G_n associated with the observer are given below. States estimated by Higher Order Sliding Mode observer is shown in Fig. 9 and the Higher order sliding mode observer error is shown in Fig. 10.

$$G_l = \begin{bmatrix} 1.0004 \\ -1.4260 \\ 1.1850 \\ -0.9758 \end{bmatrix} \qquad G_n = \begin{bmatrix} -0.0003906 \\ 0.00006040 \\ -0.0002699 \\ 0.0001930 \end{bmatrix}$$

In Kalman filter implementation, the process noise covariance matrix Q and the measurement noise covariance R are taken as tuning parameters for the performance of the filter, which are obtained by trial and error. The values of Q and R are to be otherwise determined by methods proposed by Mehra (1970), Yao *et al.* (2002), Wolin and Ho (1993), Zhou and Luecke (1995). Moreover performance of the Kalman filter depends upon the selection of initial state

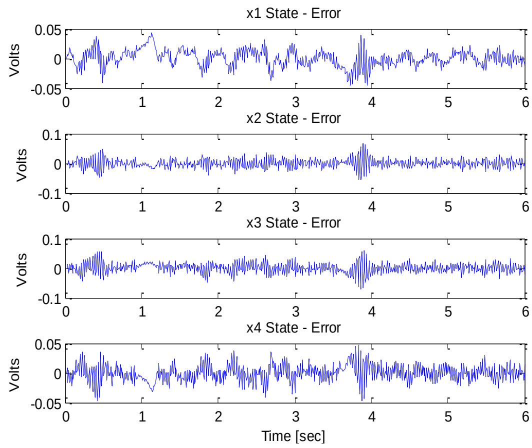

Fig. 8 Utkin observer error

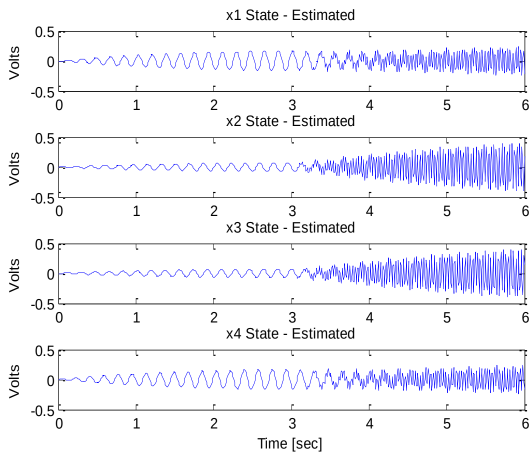

Fig. 9 States estimated using higher order sliding mode observer

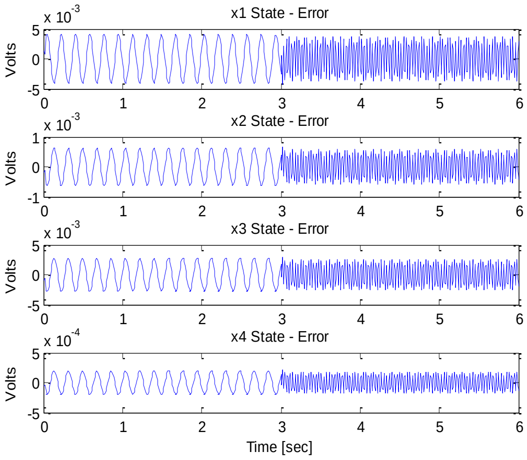

Fig. 10 Higher order sliding mode observer error

vector and the initial state covariance matrix. Selecting a very high initial state covariance matrix indicates that the filter is purposefully challenged with large initial errors, but in any case, the filter will perform well, which can be noticed by the estimation error reaching closer to zero, even though initial errors are larger. In this work, since the aim is to compare the performance of estimators, the elements of initial state covariance matrices are selected as smaller values.

The advantage of information filter is that it is very easy to initialize. The value of the initial state vector can be taken as zero and the initial covariance matrix can be taken as unity matrix. Similar to Kalman filter the values of Q and R are taken as tuning parameters. The computational complexity involved in the information filer is less, since comparatively less number of matrix inversions is computed. It is easy to decentralize information filter, hence it can be applied for control of larger structural systems.

Implementation of sliding mode observer does not require the values of process noise and measurement noise covariance.

5. Conclusions

This paper presents the application of Kalman filter based state estimation algorithms and sliding mode observers, for estimating the states of a smart structure system. The performance of these algorithms, when applied for estimating the states of fourth order linear time invariant model of a smart structure system are compared.

- It is seen that the estimation error obtained with Kalman and information filter is lesser when compared with the estimation error obtained with Utkin observer. This is because of the switching action taking place in the sliding mode observer leads to chattering effect.
- With the higher order sliding mode observer, since the chattering effect is reduced, the observer error is minimised.

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