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Structural modal identification and MCMC-based model updating by a Bayesian approach

F.L. Zhang[1,2,3a], Y.P. Yang[3b], X.W. Ye[*4], J.H. Yang[3c] and B.K. Han[3d]

[1]Key Laboratory of Earthquake Engineering and Engineering Vibration, Institute of Engineering Mechanics, China Earthquake Administration, Harbin 150086, China
[2]School of Civil and Environmental Engineering, Harbin Institute of Technology, Shenzhen 518055, China
[3]College of Civil Engineering, Tongji University, Shanghai 200092, China
[4]Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China

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Abstract. Finite element analysis is one of the important methods to study the structural performance. Due to the simplification, discretization and error of structural parameters, numerical model errors always exist. Besides, structural characteristics may also change because of material aging, structural damage, etc., making the initial finite element model cannot simulate the operational response of the structure accurately. Based on Bayesian methods, the initial model can be updated to obtain a more accurate numerical model. This paper presents the work on the field test, modal identification and model updating of a Chinese reinforced concrete pagoda. Based on the ambient vibration test, the acceleration response of the structure under operational environment was collected. The first six translational modes of the structure were identified by the enhanced frequency domain decomposition method. The initial finite element model of the pagoda was established, and the elastic modulus of columns, beams and slabs were selected as model parameters to be updated. Assuming the error between the measured mode and the calculated one follows a Gaussian distribution, the posterior probability density function (PDF) of the parameter to be updated is obtained and the uncertainty is quantitatively evaluated based on the Bayesian statistical theory and the Metropolis-Hastings algorithm, and then the optimal values of model parameters can be obtained. The results show that the difference between the calculated frequency of the finite element model and the measured one is reduced, and the modal correlation of the mode shape is improved. The updated numerical model can be used to evaluate the safety of the structure as a benchmark model for structural health monitoring (SHM).

Keywords: structural modal identification; model updating; Bayesian method; Markov Chain Monte Carlo algorithm; structural health monitoring

1. Introduction

Modal parameters of a structure generally include natural frequencies, mode shapes and damping ratios. When the excitation frequencies are close to the natural frequencies of a structure, the structure is prone to resonance, which may cause potential damage to the structure; the mode shape reflects the stiffness and mass distribution of the structure while the damping ratio characterizes the ability to dissipate energy (Au and Zhang 2016, Zhang and Au 2016). Modal parameters play an important role in model updating, damage detection, structural health monitoring, seismic design, etc.

There are several ways to analyze the structural performance such as establishing numerical models and performing field tests. For example, the finite element model (FEM) can be used to acquire dynamic characteristics of a structure, so that engineers can analyze possible failure modes as well as the dynamic response under different excitations, such as earthquake excitations. Although the precision of the FEM can be improved by refining the meshing size or selecting a higher order shape function, modeling errors still exist due to the simplification of the model, the discretization of the structure, and the error of material properties. In addition, for the existing structures, material aging, possible structural damage, etc. may also change the value of modal parameters, so that the initial FEM cannot accurately simulate the in-situ structure.

Another way is to carry out field tests to collect dynamic response of structures such as acceleration data or displacement data (Ye *et al.* 2013, Ye *et al.* 2016a, Ni and Zhang 2019). Efficient applications have been applied to long-span bridges (Ye *et al.* 2016b, Ni *et al.* 2019) and high-rise buildings (Zhang *et al.* 2016). Such data can be used to assess the structural condition (Ye *et al.* 2017) as well as the structural reliability (Ye *et al.* 2015).

With the combination of field tests and model updating, a more accurate numerical model can be obtained. On one hand, the operational response of the structure can be obtained by ambient vibration tests, which reflects actual

*Corresponding author, Professor
E-mail: cexwye@zju.edu.cn
[a] Professor
[b] MEng
[c] Assistant Professor
[d] Professor

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boundary conditions of the structure (Zhang *et al.* 2019). On the other hand, the initial FEM can be updated based on the measured modal parameters to obtain a more accurate numerical model whose calculated results can better match test results. By making full use of the numerical model and in-situ test results, the modeling error of the initial FEM can be significantly reduced, and the dynamic characteristics of the structure under operational environments can be obtained. The updated FEM can provide a benchmark model for the structural reliability analysis, structural damage identification, as well as structural health monitoring (SHM).

Model updating methods can be divided into deterministic methods and probabilistic methods depending on whether the parameter to be updated is regarded as a random variable. The deterministic model updating method refers to improving the mass matrix, stiffness matrix and design parameters of FEM through the test results, so that the calculated modal parameters can match the measured ones (Natke and Schulze 1981, Kabe 1985, Zhang and Zhang 1992). It was first studied in the mid-1960s when the matrix updating method was proposed, which was the earliest comprehensive and complete method for model updating (Berman and Flannelly 1971). However, for an in-situ structure, the number of degrees of freedom (DOF) is large, so the complete stiffness matrix and mass matrix are usually not available. In addition, due to limitations of test conditions, the measured modes are usually also incomplete. Therefore, the solution derived from deterministic methods by increasing the number of constraint equations or reducing the number of unknowns is usually the local optimal solution.

In addition, for in-situ structures uncertainties often exist. On one hand, there are uncertainties in the field test, such as the systematic error of the instrument and equipment, the error caused by the test conditions, and so on. On the other hand, uncertainties also exist in FEM caused by mathematical assumptions, discretization of structures, and errors of values of material parameters. The existence of these uncertainties makes the updated model derived from the deterministic method can only reproduce the results of a particular case and cannot quantify the uncertainties of updated parameters. By considering the parameter to be updated as a random variable, the Bayesian model updating theory combines the prior information and the test results to derive the posterior probability density functions (PDFs) of the parameters to obtain a more accurate FEM and can also quantify the uncertainties, which has been an important method of model updating in recent years.

Beck and Katafygiotis (1998) introduced the Bayesian statistical theory in dynamic system identification, and established the benchmark model updating method based on Bayesian theory. The posterior PDFs of parameters can be obtained by weighted Gaussian distribution based on a series of finite points (Beck and Katafygiotis 1998, Katafygiotis and Beck 1998). Due to ill-conditional computation problems, early Bayesian model updating method was limited in its application in practical engineering, so it was often applied to some numerical models. Besides, for in-situ structures, the posterior PDFs are usually in the form of high-dimensional and not standard, so the results estimated by the Gaussian distribution may not be accurate especially when the modeling errors are large.

Beck and Au (2002) proposed an enhanced model updating method based on Markov Chain Monte Carlo (MCMC) algorithm (Metropolis *et al.* 1953, Hastings 1970), and divided the entire process into multiple levels by kernel density estimation (Au and Beck 1999), which improved the computation efficiency. Based on this, Lam *et al.* (2015) proposed a novel stopping criterion, which introduces the uncertainty of the relative error in the objective function as a parameter in the updating process and introduces the convergence procedure for the structure with different complexity. The Bayesian method based on MCMC makes it possible to implement the application of complex structures. Based on the data obtained from the field test, Lam *et al.* (2015) utilized the enhanced MCMC sampling method to update the FEM of a coupled floor system. Lam *et al.* (2017) carried out the ambient vibration test of a 14-story industrial building in Hong Kong and updated the interlayer stiffness of the structure based on the enhanced Bayesian model updating method.

Compared with the deterministic methods, the Bayesian probabilistic method provides a tool to quantify the uncertainty, which can better reflect the uncertainty nature of the problem. Therefore, it has become a research hotspot in recent years. However, the Bayesian updating method is mainly used in numerical models and some other simple structures, and has less application to traditional Chinese style pagodas.

This paper presents the work of the field test, modal identification and model updating of a rebuilt Chinese pagoda. The pagoda is a reinforced concrete frame structure, which further will be retrofitted. The initial finite element model was established based on the design information. Field tests were carried out to collect dynamic responses of the structure under ambient conditions, and modal parameters of the structure were identified by the enhanced frequency domain decomposition method. Model updating was carried out based on the modal parameters identified. The posterior PDFs of the parameters were determined from the MCMC sampling algorithm and the uncertainties are also quantified based on the Bayesian updating method, so that the updated model can better match the dynamic performance of the actual structure. The updated model can evaluate the seismic performance of the structure and provide an analytical model for subsequent reinforcement.

2. Description of the investigated structure

The objective pagoda is 56.64 m high with eight floors. The typical plane shape is a regular octagon. The pagoda is a reinforced concrete frame structure with bored pile foundation. The main structure is shown in Fig. 1. As the owner intends to remove the column at the center of the pagoda from -4.90 to -0.05 m, it is necessary to perform

Fig. 1 The objective pagoda

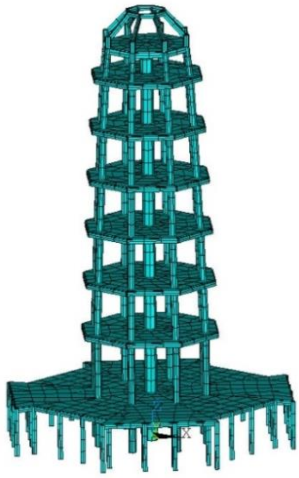


Fig. 2 The finite element model of the pagoda

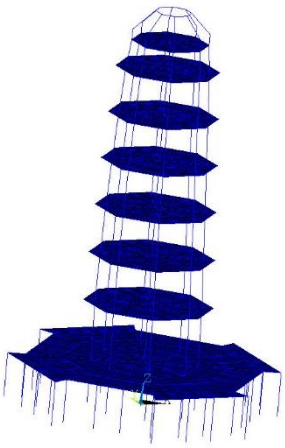
structural monitoring to obtain the dynamic characteristics of the structure and establish a benchmark model for the structural performance evaluation.

3. Finite element analysis

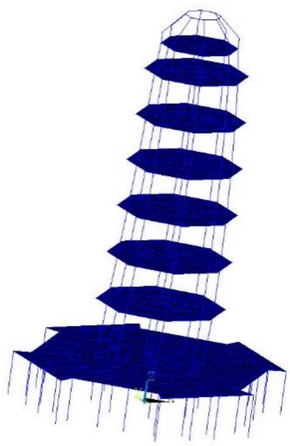
The FEM was established using the software ANSYS 15.0. According to the design data, the concrete compression strength is C40 for all columns and C30 for beams and slabs. The elasticity modulus values are based on Chinese National Standards GB 50010-2010, i.e., 3.25×10^4 MPa and 3.0×10^4 MPa, respectively. The Beam188 unit was used to simulate beams and columns, and the Shell181 unit was used to simulate slabs. Considering that the pile foundation is placed at the bottom of the structure, bottom constraint is then set to be fixed. There are 1359 beam elements and 816 shell elements in total. The total number of finite element model elements is 2175 and the number of nodes is 2236. The overall model is shown in Fig. 2.

Table 1 Calculated modes of the finite element model

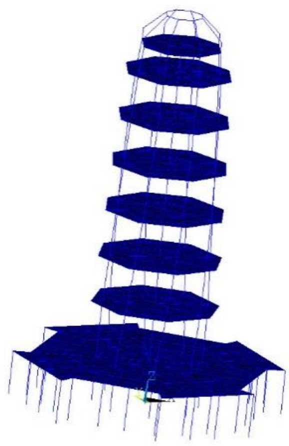
Modes	Frequency/Hz	Period/s	Mode shape
1	1.031	0.970	1st translational mode in Y direction
2	1.032	0.969	1st translational mode in X direction
3	1.468	0.681	1st torsional mode
4	3.297	0.303	Translational mode in 1st and 3rd quadrant
5	3.301	0.303	Translational mode in 2nd and 4th quadrant
6	4.104	0.244	2nd torsional mode
7	5.766	0.173	Translational mode in 1st and 3rd quadrant
8	5.771	0.173	Translational mode in 2nd and 4th quadrant
9	6.370	0.157	3rd torsional mode



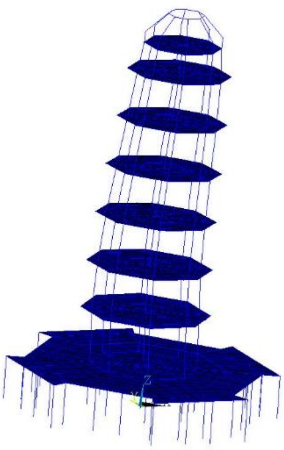
(a) 1st mode shape



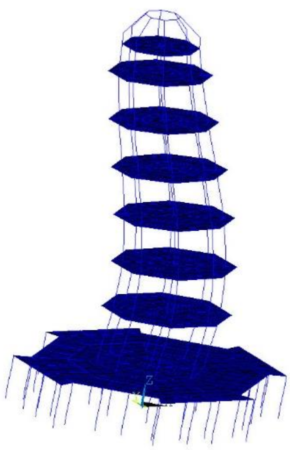
(b) 2nd mode shape



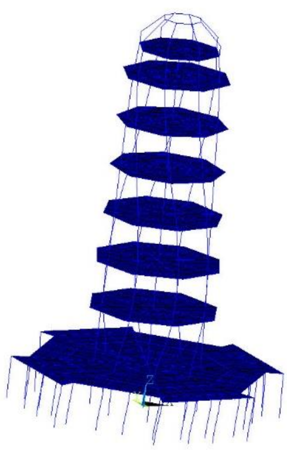
(c) 3rd mode shape



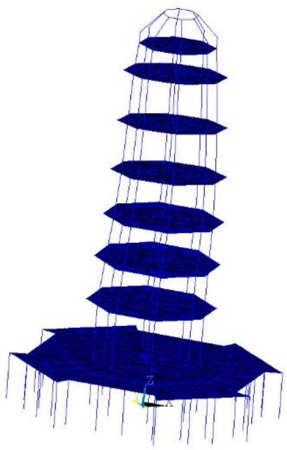
(d) 4th mode shape



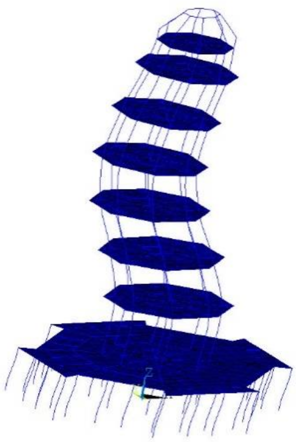
(e) 5th mode shape



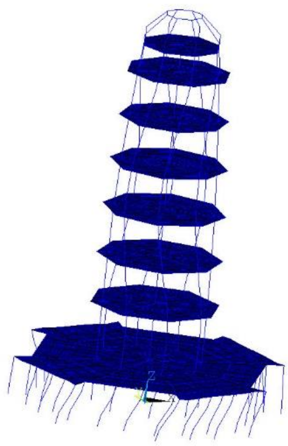
(f) 6th mode shape



(g) 7th mode shape



(h) 8th mode shape



(i) 9th mode shape

Fig. 3 The calculated mode shape of FEM

The subspace iteration method is used to calculate the first nine modal frequencies of the structure. The result is shown in Table 1, and the mode shapes of the structure are shown in Fig. 3.

4. Field test

Measurement points were located near staircases from the second floor to eighth floor along the height of the structure. Two single-axis piezoelectric accelerometers

(Lance 0132T) were arranged for each measurement point to acquire the acceleration response in the X and Y directions respectively. Due to the limited number of sensors, the whole test was divided into six setups. The measurement point in the eighth floor was selected as the reference point, i.e., the two accelerometers in the eighth floor remained the same in each setup. The tests were completed by moving other sensors in other floors. Piezoelectric accelerometers used in the test are shown in Fig. 4. The frequency range of the sensor is 0.05~500 Hz. The acceleration data collected by the sensor is outputted to the data acquisition instrument in the form of electrical signal, and the measured acceleration amplitude can be converted by the calibrated sensitivity value of corresponding sensor. The data acquisition system used for data collection is DEWESoft®, which has eight channels with an accuracy of 0.03%. The data acquisition system is shown in Fig. 5.

According to the sampling theorem, in order to prevent spectral aliasing, the sampling rate needs to be at least twice the interested frequency, so the acquisition frequency is set to 500 Hz, and each setup lasts 15 min. Table 2 shows the setup configuration. The numbers in Table 2 denote the location number. The first digit of each number indicates the floor number where the measuring point is located, and the last two digits indicate the measuring point number. For example, 801 indicates the measuring point No. 1 in the eighth floor. The layout of the measurement points is shown in Fig. 6 and the yellow circle in the figure represents the location of measurement points.

Fig. 4 Accelerometers (Lance 0132T)

Fig. 5 Data acquisition system

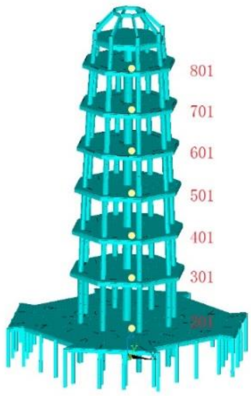


Fig. 6 Measurement locations

Table 2 Setup configuration

Setup	Measurement locations	
1	801	701
2	801	601
3	801	501
4	801	401
5	801	301
6	801	201

Table 3 Identified natural frequencies

Mode	Frequency/Hz
1	1.363
2	1.390
3	2.686
4	4.468
5	4.528
6	6.219
7	7.592
8	7.764

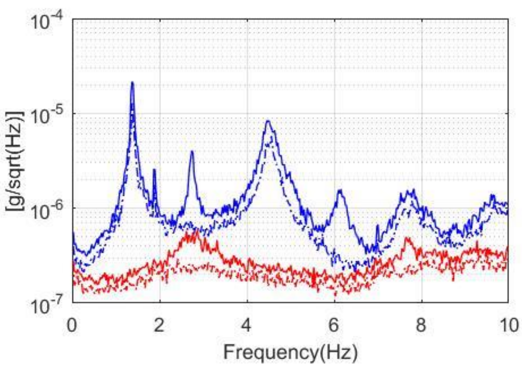


Fig. 7 Singular value spectrum

Table 4 Calculated frequencies f_a and measured frequencies f_m

Mode	1	2	4	5	7	8
$f_a(Hz)$	1.031	1.032	3.297	3.301	5.766	5.771
$f_m(Hz)$	1.363	1.390	4.468	4.528	7.592	7.764
$\frac{f_a - f_m}{f_m}$	24.36%	25.76%	26.21%	27.10%	24.05%	25.67%

Table 5 MAC between calculated mode shapes and measured mode shapes

Mode	1	2	4	5	7	8
1	0.9686	--	--	--	--	--
2	--	0.9565	--	--	--	--
3	--	--	0.8375	--	--	--
4	--	--	--	0.5952	--	--
5	--	--	--	--	0.6787	--
6	--	--	--	--	--	0.5573

In ambient vibration tests, the input information is often unknown. However, since ambient vibration source is mainly composed of wind loads and micro tremors, the input is usually regarded as wide stationary random process. Therefore, as long as the test is long enough, the statistical characteristics can be estimated by analysing the samples obtained from the field test. By assuming that the input follows the Gaussian noise with the constant power spectral density, modal parameters of the structure can be identified by analysing the response collected in the field test. In this study, the enhanced frequency domain decomposition method was used to identify the modal parameters, and the singular value spectrum calculated is shown in Fig. 7, where clear peaks can be found in the spectra.

The first eight frequencies identified are shown in Table 3. The basic frequency of the structure is 1.363 Hz, i.e., the fundamental natural period is about 0.734 s. The adjacent translational frequencies are relatively close, indicating that the mass and stiffness distribution of the structure in the two test directions are relatively uniform.

Correlation analysis is an effective method used to compare the calculated results from FEM with the measured results. For modal frequencies, the differences can be compared directly. For mode shapes, modal assurance criterion (MAC) is often used for correlation analysis. The calculation of the MAC value between the two modes is as follows

$$\mathrm{MAC}_{ij} = \frac{\left|\widehat{\boldsymbol{\psi}}_i^T \boldsymbol{\psi}_j(\boldsymbol{\theta})\right|^2}{\left(\widehat{\boldsymbol{\psi}}_i^T \widehat{\boldsymbol{\psi}}_i\right)\left(\boldsymbol{\psi}_j^T(\boldsymbol{\theta})\boldsymbol{\psi}_j(\boldsymbol{\theta})\right)} \tag{1}$$

where $\widehat{\boldsymbol{\psi}}_i$ is the i-th order test mode shape; and $\boldsymbol{\psi}_j(\boldsymbol{\theta})$ is the j-th order FEM calculated mode shape based on the design parameters $\boldsymbol{\theta}$. It can be known from Eq. (1) that the MAC value is a number between 0 and 1. The closer the value is to 1, the better the correlation between the two modes is.

Due to the limited numbers of measurement points, the DOFs of the measured modes are not the same as the DOFs of the FEM. An effective way is to extract the node displacement in the calculated mode shapes corresponding to the measurement points, and by normalizing the extracted modes, the DOFs of the two could be the same, which facilitates subsequent correlation analysis.

Table 4 shows the difference between the initial calculated frequencies and the measured frequencies. Table 5 shows the MAC matrix between the mode shapes. Because in each floor, only one point was measured so that the torsional mode shape cannot be obtained, thus in subsequent analysis only translational modes are used.

As can be seen from Table 4, the relative frequency error is around 25%, and the maximum difference is 27.10% (the fifth-order mode). It can be seen from Table 5 that the MAC values between the calculated and identified mode shapes of the first three translation modes are all above 0.8, indicating that the correlation between the calculated mode shape and the corresponding measured mode shape is good. However, MAC values of the latter three modes are relatively low. Model updating is necessary to obtain an accurate FEM.

5. Model updating

In order to carry out the model updating, a MCMC sampling method is used in this paper. Beck and Au (2002) proposed a multiple-level adaptive sampling algorithm to improve the efficiency of the model updating by improving the proposed distribution in the MH algorithm. Lam *et al.* (2015) further enhanced the method. This method is introduced briefly as follows.

Since the natural frequencies and mode shapes are relatively easy to identify, the posterior PDFs of the parameters $\boldsymbol{\theta}$ can be derived based on the measured modal parameters $\mathbf{D}$. The Bayesian formula shows

$$p(\boldsymbol{\theta}|\mathbf{D}) = cp(\mathbf{D}|\boldsymbol{\theta}) \tag{2}$$

where $p(\boldsymbol{\theta}|\mathbf{D})$ is the posterior probability density function (PDF) of $\boldsymbol{\theta}$; $p(\mathbf{D}|\boldsymbol{\theta})$ is the likelihood function; and c is a constant.

Since the modal parameters between different modes are independent of each other, the likelihood function can be regarded as the product of each mode

$$p(\mathbf{D}|\boldsymbol{\theta}) = \prod_{i=1}^{m} p(\hat{f}_i, \widehat{\boldsymbol{\Psi}}_i|\boldsymbol{\theta}) = \prod_{i=1}^{m} p(\hat{f}_i|\boldsymbol{\theta})\, p(\widehat{\boldsymbol{\Psi}}_i|\boldsymbol{\theta}) \tag{3}$$

where $\hat{f}_i$ is the measured natural frequency of the i-th mode; and $\widehat{\boldsymbol{\Psi}}_i$ is the corresponding measured mode shape.

Define the difference between measured natural frequency and calculated frequency as

$$\varepsilon_{f,i} = \frac{\hat{f}_i - f_i(\boldsymbol{\theta})}{\hat{f}_i} \tag{4}$$

where $f_i(\boldsymbol{\theta})$ is the calculated natural frequency based on the parameters $\boldsymbol{\theta}$. Assuming that $\varepsilon_{f,i}$ follows a Gaussian distribution whose mean is zero and standard deviation is κ, then

$$p(\hat{f}_i|\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\kappa^2}} exp\left\{-\frac{1}{2\kappa^2}\left[\frac{\hat{f}_i - f_i(\boldsymbol{\theta})}{\hat{f}_i}\right]^2\right\} \tag{5}$$

According to the reference (Lam *et al.* 2015), the fraction error of mode shape can be defined as

$$\varepsilon_{\psi,i} = \left\{1 - \left|\widehat{\boldsymbol{\Psi}}_i^{\,T} \boldsymbol{\psi}_i(\boldsymbol{\theta})\right|^2\right\}^{1/2} \tag{6}$$

where the modal shape is normalized. Assuming that the difference of the mode shape follows a zero-mean Gaussian distribution, then

$$p(\widehat{\boldsymbol{\Psi}}_i|\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\kappa^2}} exp\left\{-\frac{1}{2\kappa^2}\left[1 - \left|\widehat{\boldsymbol{\Psi}}_i^{\,T} \boldsymbol{\psi}_i(\boldsymbol{\theta})\right|^2\right]\right\} \tag{7}$$

Then the objective function can be defined as

$$J(\boldsymbol{\theta}) = \sum_{i=1}^{m}\left(\left(\frac{\hat{f}_i - f_i(\boldsymbol{\theta})}{\hat{f}_i}\right)^2 + \left(1 - \left|\widehat{\boldsymbol{\Psi}}_i^{\,T} \boldsymbol{\psi}_i(\boldsymbol{\theta})\right|^2\right)\right) \tag{8}$$

By using the software ANSYS and MATLAB, the direct updating of the parameters can be performed conveniently.

The elastic modulus of columns, beam and slabs are selected as the parameters to be updated, i.e., $x(1)$~$x(7)$ represents the ratio of elastic modulus of columns in the 1st to 7th floor to their initial values respectively, and $x(8)$ is the change ratio of elastic modulus of beams and slabs.

For the selected parameters, the initial distribution space is defined as (0.5, 3.0), and the initial sample points are shown in Fig. 8, and the sample point of the second level and the last level (the eighth) are shown in Figs. 9 and 10, respectively.

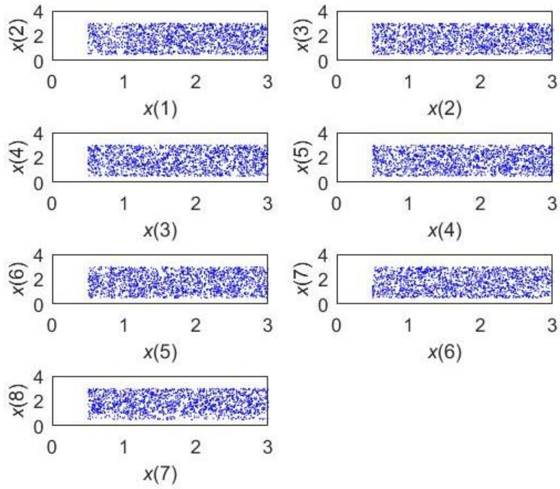


Fig. 8 Sample points in level 1

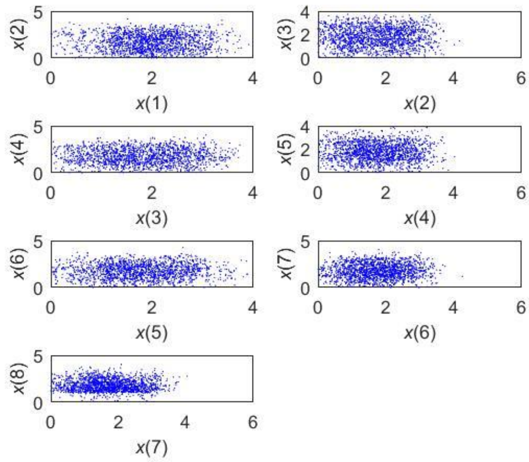


Fig. 9 Sample points in level 2

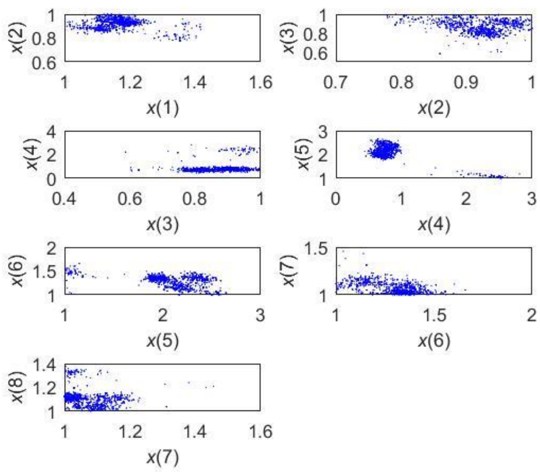


Fig. 10 Sample points in Level 8

Table 6 Optimal value and uncertainties of model parameters

	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$	$x(8)$
Optimal value	1.160	0.940	0.875	0.720	2.110	1.320	1.030	1.100
COV(%)	5.65	4.39	7.75	54.82	15.27	9.12	5.49	6.30

Table 7 Difference between calculated modes and measured modes

Modes	1	2	4	5	7	8
Calculated frequency/Hz	1.434	1.434	4.370	4.370	7.495	7.501
Measured frequency/Hz	1.363	1.390	4.468	4.528	7.592	7.764
Difference	5.17%	3.20%	2.20%	3.49%	1.28%	3.39%
MAC	0.9737	0.9592	0.8814	0.8699	0.8355	0.8265

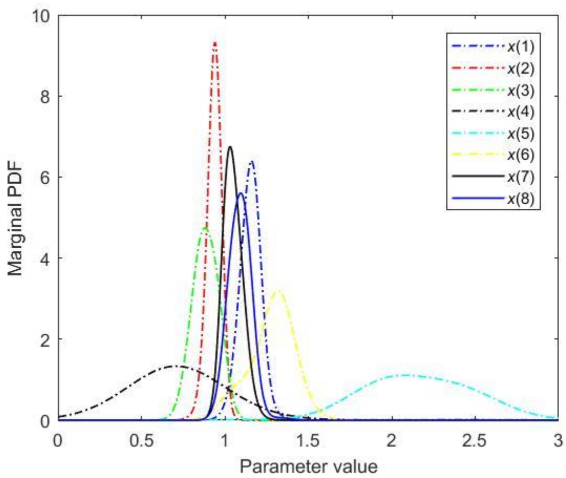


Fig. 11 Posterior PDF of the model parameters

The posterior marginal PDFs of the eight model parameters are shown in Fig. 11. As can be seen in this figure, the posterior marginal PDFs of model parameters are similar to the Gaussian distribution, i.e., the farther away from the most probable value, the smaller the probability is. The optimal value of the updated parameters and the corresponding uncertainty are shown in Table 6, where the coefficient of variation (COV) represents for the ratio of the standard deviation to the optimal value. The optimal value is then substituted into the finite element model, and the difference of the natural frequency between the modified FEM and the measured one are obtained. The updated natural frequencies and MAC values are shown in Table 7.
It is worth mentioning that the optimal value in Table 6 is the ratio of the updated parameters to the initial ones. As can be seen from Table 6, the elastic modulus of the columns in the 2nd floor, 3rd floor and 4th floor is slightly lower than that before the updating, when others are a bit larger than initial values. Compared with the initial FEM calculated modes, the difference of the natural frequency is reduced to 2%~5%. Compared with Table 5, the MAC value of the last three modes is improved, indicating that the modeling error of the initial FEM and the measured results is reduced by the selected parameters.

6. Conclusions

In this paper, the modal parameters of a pagoda are identified using data collected in the ambient vibration tests with multiple setups. The finite element model of the pagoda is established, and it is found that there is a large gap between the modal parameters identified from collected data and calculated from FEM. Model updating is performed based on a multi-level MCMC method using the measured results. The posterior PDF and uncertainty of the parameters are obtained. The optimal value of the updated parameter is substituted into the finite element model, and the comparison shows that the difference between the updated calculated modes and the measured one is effectively reduced. The MCMC Bayesian model updating method was well applied on the model updating of a pagoda. Note that after the field test, the central column removal work started. The updated model can be used as the benchmark model for the subsequent analysis.

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