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An inverse approach based on uniform load surface for damage detection in structures

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Abstract. In this paper, an inverse approach based on uniform load surface (ULS) is presented for structural damage localization and quantification. The ULS is excellent approximation for deformed configuration of a structure under distributed unit force applied on all degrees of freedom. The ULS make use of natural frequencies and mode shapes of structure and in mathematical point of view is a weighted average of mode shapes. An objective function presented to damage detection is discrepancy between the ULS of monitored structure and numerical model of structure. Solving this objective function to find minimum value yields damage's parameters detection. The teaching-learning based optimization algorithm has been employed to solve inverse problem. The efficiency of present damage detection method is demonstrated through three numerical examples. By comparison between proposed objective function and another objective function which make use of natural frequencies and mode shapes, it is revealed present objective function have faster convergence and is more sensitive to damage. The method has good robustness against measurement noise and could detect damage by using the first few mode shapes. The results indicate that the proposed method is reliable technique to damage detection in structures.

Keywords: damage detection; uniform load surface; inverse approach; modal parameters; teaching-learning based optimization

1. Introduction

Unforeseen loading, Interaction with corrosive agents, temperature variation, material ageing and other events are affecting a structure during operation and consequently introduce undesired damage. Structural health monitoring and on time damage detection is one of the common interests in different fields of engineering in order to preventative measures to avoid human casualties and financial detriment. In this way, damage detection using vibratory data become interesting and popular due to low cost and the ability to monitor the whole of the structure by measuring a few limited points. Vibration based damage detection techniques are established based on the fact that, the modal parameters such as natural frequencies, mode shapes and damping ratio are proportional to physical and geometrical parameters of the structure and any change in modal parameters is originated from change in physical or geometrical properties. Also, dynamic response or forced vibration response of the structure affected by damage and becomes different. Detecting of damage is classified into four levels as follows (Carden and Fanning 2004, Meruane and Heylen 2011):

Level 1. Determination that damage is exist or not;

Level 2. Determination of the geometric location of the damage;

Level 3. Quantification of the severity of the damage;

Level 4. Estimation of the residual lifetime.

First and second levels does not need to any knowledge about model of structure. These two levels known as response based method. Although, second level need to know spatial data of system to detecting damage location. An analytical or numerical model of structure is necessary to performing third level. Fourth level needs complete information of third level and also needs information about damage propagation and fracture mechanics. Third and fourth levels are known as model based method in damage detection. Researchers' attention mainly focused on second and third levels, because early damage detection with suitable accuracy could help operator to make good decision about rehabilitation or stop utilization of structure.

Model-based methods attempt to modify or update a primary numerical model of the damage structure until the produce model characteristics become similar to monitored structure. A comprehensive review about different model-based techniques for structural damage detection using vibration analysis has been presented by Fan and Qiao (2011). One of the interesting approaches to damage detection by using the model of the structure is inverse method. An objective functions is required for damage detection by inverse method. The objective function is defined as discrepancy between modal parameters or dynamic response of the model of the structure and the monitored structure. Whenever objective function value becomes zero or very small, the damage parameters are detected. Important issue in inverse method is the selection of appropriate damage index for developing objective

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function. Several objective functions proposed in literature for damage detection. Khiem and Lien (2004) were used natural frequencies to detect multi-crack in beam by problem formulation in the form of nonlinear optimization. Ruotolo and Surace (1997) were defined an objective function by using natural frequencies, modal curvature and normalized mode shapes for crack detection in the beam-like structure and the genetic algorithm was used to solve the optimization problem. Raich and Liszkai (2007) were proposed an objective function by using frequency response function to damage detection in structures. Other researches on vibration based damage detection by using inverse method could be found in (Panigrahi *et al*. 2009, Perera *et al*. 2013, Kourehli *et al.* 2013, Hosseinzadeh *et al.* 2016, Fatahi and Moradi 2018, Seyedpoor *et al*. 2018, Nobahari *et al.* 2017, Vosoughi 2015).

As explained by Fan and Qiao (2011), usually, damage detection methods are verified by beam-type or plate-type structures; because most structures or their major components in engineering can be simplified as a beam or plate. In this way, researchers to investigate on damage detection methods make use of beam (Khiem and Lien 2004, Ruotolo and Surace 1997) or plate (Corrado *et al.* 2015) for numerical simulation and experimental test. Some researchers applied truss as test structure to validate their methods (Lim and Kashangaki 1994). In some researches frame structure was object of damage detection (Ovanesova and Suarez 2004). Moradi and Tavaf (2013) applied an inverse approach to damage detection in circular cylindrical shells. A double-beam system made of two parallel beams connected together through an elastic layer considered as an approximate model for the sandwich beam in which the shearing behavior of soft core is neglected (Mirzabeigy *et al.* 20017). Nguyen (2016) tries to detect crack like damage in a double-beam system by using auxiliary mass. When researchers wants to show effectiveness of their proposed method for damage detection, validate it against at least one of the mentioned structures.

This paper mainly focuses on validate performance and effectiveness of a new objective function for damage detection in structures. The objective is defined by using uniform load surface (ULS). The ULS is make use of natural frequencies and mode shapes and fuse data to estimate structural deflection under uniform load distributed over all degree of freedoms of structure. After illustration of ULS and presenting mathematical formulation, an objective function developed by using it. Then teaching-learning based optimization (TLBO) algorithm is introduced in order to solve objective function. Finally, the effectiveness and performance of developed objective function examined against three numerical examples and effect of different parameters studied.

2. Proposed method

After discritizing a structure - without any energy dissipation - by means of finite elements, the modal analysis could be performed to determine the natural frequencies and mode shape of the structure. The mathematical representation of modal analysis is as follows

$$(K-\omega_i^2 M)\varphi_i = 0,\ i = 1,2,...,N_f \tag{1}$$

where K and M are global stiffness and mass matrices of the structure, respectively; ω_i is the i th natural frequency and φ_i is corresponding mode shape. N_f is total number of measured degrees of freedom or total number of sensors. In structural health monitoring, change in natural frequencies and mode shapes is sign of damage. Commonly, these modal parameters applied for constructing objective function and it is expect by maximizing or minimizing objective function the damage parameters will be identified. There is some concept in literature which found based on natural frequencies and mode shapes. Modal flexibility is one of them. The transformation of the modal parameters to modal flexibility matrix using r modes is as following

$$F_r = \Phi_r \Lambda_r^{-1} \Phi_r^T \tag{2}$$

where Φ is a matrix consist of mass-orthogonal and mass-normalized mode shape vectors as following

$$\Phi_r = [\varphi_1\ \varphi_2\ ...\ \varphi_r] \tag{3}$$

Λ is a square matrix with zero off diagonal elements as follows

$$\Lambda_r = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_r^2 \end{bmatrix},\ \omega_1 < \omega_2 < ... < \omega_r \tag{4}$$

Each components of flexibility matrix in Eq. (2) is given by

$$F_{i,j} = \sum_{k=1}^{r} \frac{\varphi_k(i)\varphi_k(j)}{\omega_k^2} \tag{5}$$

physical interpretation of $F_{i,j}$ refers to deformation at point i of the structure if unit load applied at point j. If the structure loaded by a uniform unit load distribute all over the structure, the deformation at point i is summation of modal flexibility between point i and all degrees of freedom as following

$$u(i) = \sum_{j=1}^{N_f} F_{i,j} = \sum_{j=1}^{N_f}\left(\sum_{k=1}^{r} \frac{\varphi_k(i)\varphi_k(j)}{\omega_k^2}\right) = \sum_{k=1}^{r} \frac{\varphi_k(i)\sum_{j=1}^{N_f}\varphi_k(j)}{\omega_k^2} \tag{6}$$

Eq. (6) is called uniform load surface (ULS) of structure. The ULS could be considered as a weighted average of mode shapes where weight factor is the inverse of the square of the natural frequencies. This weight factor leads to the less contribution of higher order modes and the ULS converges to exact deformation only by the first few mode shapes.

The ULS was first investigated by Zhang and Aktan (1998) and then applied in damage detection problems. Sung *et al.* (2013) were studied the effect of damage in slender beam on curvature of ULS. The results show that change in ULS curvature only occur at damage elements. Masoumi and Ashory (2014) were refine the ULS obtained from damage structure by means of stationary wavelet transform and then applied continuous wavelet transform to localize damage.

As illustrated in introduction, the inverse methods are interesting to damage detection in structures. Also, the ULS of a structure has good information due to data fusion of frequencies and mode shapes. As a result, an objective function is developed for structural damage detection as follows

$$Obj = \sum_{i=1}^{N_f} \left| 1 - \frac{ULS^m(i)}{ULS^*(i)} \right| \tag{7}$$

where ULS^m and ULS^* are uniform load surface from FE model and uniform load surface of system with unknown damage, respectively.

One way for damage modeling in structure is modulus of elasticity degradation. The modulus of elasticity of damaged element related to intact one through the following formula

$$E_i^d = (1-\beta_i)\, E_i \tag{8}$$

where E_i^d, E_i and β_i are elasticity modulus of element after damage, before damage and damage severity, respectively. The damage severity value change between [0,1], where 0 is correspond to healthy element and 1 is correspond to complete destruction of element. By this definition, the design variables of objective function become

$$Obj(\beta_1, \beta_2, ..., \beta_{N_f}) = \sum_{i=1}^{N_f} \left| 1 - \frac{ULS^m(i)}{ULS^*(i)} \right| \tag{9}$$

An optimization method is required for solving Eq. (9) in order to find possible minimum value. In next section, Teaching-Learning based optimization method is introduced and will apply in numerical studies.

3. Teaching-Learning based optimization

Teaching-Learning based optimization (TLBO) is population based, metaheuristic algorithm inspired by the education procedure in a classroom. TLBO was proposed by Rao *et al.* (2012) and applied in different area of sciences and engineering (Singh *et al*. 2013, Basu 2014, Jordehi 2015). Logic behind TLBO is that learners (students) want to get better scores through education, so, teacher has great role. A good teacher could better educate learners and help to raise their scores or marks. The teacher quality could evaluate by mean value of class scores and the best teacher is who that his or her knowledge equals to mean value of class or teacher and learners have same knowledge. Except teacher education, learners could educate themselves by collaboration and reciprocate knowledge. TLBO simulate education from teacher and collaborative learning among students for finding global optimum. TLBO is parameter free and does not need to any parameters tuning, it requires only common controlling parameters like population size and number of generations for its working. In this algorithm different decision variables are analogous to different subjects offered to students and student's overall result is analogous to the values of objective function. The procedure of TLBO is divided into two phases, the Teacher phase and the Learner phase.

3.1 Teacher phase

The first step of knowledge sharing in the TBLO is teacher phase which attempts to simulate teacher's influence on the student. During this phase the learners or students are motivated by the teacher and try to promote their knowledge which consequently yields to increase the mean result of the class in the subject taught by teacher. Consider N number of learner in the class and J number of subject teaches to them. These are representing the population size of N with J design variables. At any iteration, the mean result of the class in a specific subject $j\,(1,2,3,\ldots,J\,)$ given by

$$M_j = \frac{1}{N} \sum_{k=1}^{N} X_k(j) \tag{10}$$

where $X_k(j)$ is the grade of learner k in subject j.

The learner with the best overall grade taking all the subjects (or best fitness value) is designated as teacher and other learners move toward teacher to enhance their own overall grade via following equation

$$X_k^{new}(j) = X_k^{old}(j) + r\,(X_{Teacher}(j) - T_F\, M_j) \tag{11}$$

where $X_k^{new}(j)$ and $X_k^{old}(j)$ are new and old grade of learner k in subject j, r is the random number in the range $[0,1]$, $X_{Teacher}(j)$ is the teacher grade in subject j and T_F is teaching factor could be either 1 or 2 with equal probability. X_k^{new} accept instead of X_k^{old} if it gives better fitness function. At the end of each teaching cycle, the current best student will become the teacher for the next iteration.

3.2 Learner phase

After knowledge achievement by learners under teacher conduction, they could become more fit by mutual discussions and interactive learning. In this phase, for each student p from the class, another student q is randomly select, in such a way $p \neq q$. After fitness evaluation of both student if student p be better than student q, then

$$X_p^{new}(j) = X_p^{old}(j) + r(X_p^{old}(j) - X_q(j)),\quad j = 1,2,...,J \tag{12}$$

otherwise

$$X_p^{new}(j) = X_p^{old}(j) + r(X_q(j) - X_p^{old}(j)),\quad j = 1,2,...,J \tag{13}$$

where r is the random number in the range $[0,1]$. X_p^{new} accept instead of X_p^{old} if it gives better fitness function.

4. Numerical examples

This section is devoted to show applicability of proposed method in damage detection in structures. Three numerical cases are considered. As stated before, the TLBO is an algorithm for maximizing an objective function, while for solving objective function in Eq. (9), minimizing is desired. Therefore, the objective function applied in TLBO must be in the form of maximizing problem as follows

$$Obj_{TLBO} = \frac{1}{1+Obj} \tag{14}$$

where Obj was present in Eq. (9). It is obvious when Obj_{TLBO} become maximum, the value of Obj is minimum and damage's parameters are determined.

The population of the students in the TLBO is considered 30 through all examples. Whenever the value of the objective function becomes less than 10^{-8} or the number of iterations becomes more than 1500, the optimization algorithm terminated and results of algorithm considered as damage parameters.

4.1 A beam

A beam with clamp-simply support boundary conditions is considered as first numerical example and depicted in Fig. 1. The physical and geometrical properties of this beam are as following:

$$E = 70\,GPa, \rho = 2700\,kg/m^3,$$
$$h = 60\,mm, w = 100\,mm, L = 1.5\,m$$

where E, ρ, h, w and L are modulus of elasticity, density, beam's height, beam's width and length, respectively. Based on different theories for beam modeling, the thin or Euler-Bernoulli beam theory could apply when the length of beam at least 10 times larger than the height. By considering the value of geometrical properties, the thin beam theory is adapted to modeling the system in Fig. 1.

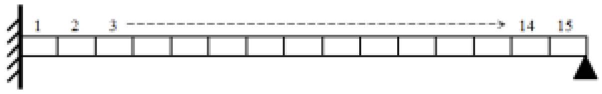


Fig. 1 Finite element model of the clamp-simply support beam

The beam divided into 15 equal length elements. The stiffness and mass matrix of thin beam element is presented in Finite element books like Liu and Quek (2013). After assembling elements and deriving global stiffness and mass matrix, modal parameters like natural frequencies and mode shapes could be calculated and consequently the uniform load surface of system is achieved.

In this numerical example, the main aim is to comparison between inverse method for damage detection using ULS and another inverse method which make use of natural frequencies and mode shapes in objective function as follows

$$Obj : \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\omega_i^m - \omega_i^e}{\omega_i^e}\right)^2} + \sum_{i=1}^{n}(1 - MAC_i) \tag{15}$$

where ω_i^m and ω_i^e are ith natural frequency of system from FE model and from system with unknown damage, respectively. n is total number of measured natural frequencies and mode shapes. MAC is the modal assurance criterion that gives the correlation between two vectors. For damage detection, MAC is calculated by

$$MAC_i = \frac{(\varphi_i^{m^T}\varphi_i^e)^2}{(\varphi_i^{m^T}\varphi_i^m)(\varphi_i^{e^T}\varphi_i^e)} \tag{16}$$

where φ_i^m and φ_i^e are ith mode shape vector of system from FE model and from system with unknown damage, respectively. T is for vector transpose. The value of MAC is a scalar quantity between [0,1]. Where zero means no correlation or orthogonality condition between vectors while one indicates two vectors are identical. Objective function in Eq. (15) applied in different damage detection problems although sometimes a little change has been made by authors (Nanda *et al*. 2014, Oh *et al*. 2015, Wei *et al*. 2018).

Two different damage scenarios are presumed by reducing in modulus of elasticity of damaged elements, which are presented by detail in Table 1. It is assumed, the numbers of damages are given in this example. Single damage detection does not considered, because both objective functions have same efficiency in single damage detection. When the input data are noise free, both objective functions are able to correctly localize and quantify damaged elements based on scenario 1, therefore we study the effect of number of modes on convergence rate of objective function. The algorithm is performed five times for each objective function and best convergence rate considered. The convergence of the proposed objective function value is plotted in Fig. 2. As observed, using three modes in objective function, accelerate algorithm in finding damage, it is reasonable because more applied mode shapes provide more information. Also, the convergence of the objective function value in Eq. (15) is plotted in Fig. 3. As seen, by using three modes, significant change in convergence rather than using two modes is observed. By comparison between Figs. 2 and 3, it is revealed the objective function based on ULS has rapid convergence rate rather than objective function in Eq. (15). After damages detection in scenario number 1, the second scenario has been investigated. In this scenario damages are not severe and located near supports. Objective function in Eq. (15) is not able to detect the damages in the scenario number 2 by use of two mode shapes. Optimization algorithm was conducted 10 times, but damages were not detected correctly. Also, the optimization problem in Eq. (15) was solved by genetic algorithm (GA), but no success was achieved in detecting the damages correctly. When the number of mode shapes increases from two to three, damages were successfully detected for 6 times by conducting the algorithm for 10 times. When four modes are used, algorithm could detect the damages in all performances. Unlike the objective function of Eq. (15), proposed objective function by using two mode shapes, is able to detect the damages in the scenario number 2 during all running processes.

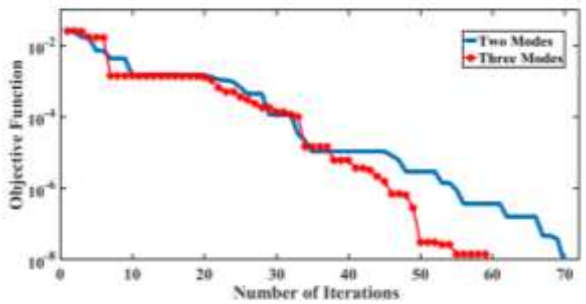


Fig. 2 Effect of modes numbers on convergence rate of proposed objective function in damage scenario 1 detection in the beam

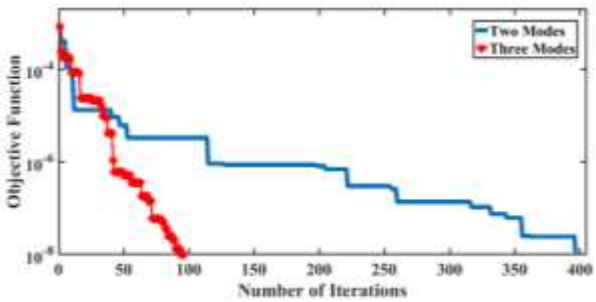


Fig. 3 Effect of modes numbers on convergence rate of objective function in Eq. (15) in damage scenario 1 detection in the beam

Table 1 Assumed damage scenarios in the beam

Damage Scenario 1		Damage Scenario 2	
Element no.	β	Element no.	β
3	0.1	2	0.05
8	0.1	13	0.05

As already said, the objective function of Eq. (15) is not able to detect the damage in the scenario number 2 by using two mode shapes. In Table 2, a number of solutions for optimization problems of Eq. (15) are given after meeting the condition of algorithm termination (1500 iterations). By using the values obtained for location and intensity of damages, value of objective functions in Eq. (15) and proposed objective function are calculated. As it is seen, proposed objective function has higher values, indicating this objective function is able to highlight the impact of damage by combining the information and passes the local extremum points more easily.

4.2 A six-story shear building

This example is devoted for damage detection in a six-story shear building. The schematic of the building is shown in Fig. 4. The Physical characteristics of this shear building like mass and stiffness are presented in Table 3. The masses and stiffnesses of stories are chosen in non-uniform pattern. This pattern yields structural irregularity and hampers simplicity in solving inverse problem. Two damage scenarios are presumed by reducing in story-stiffness, which are presented by detail in Table 4. The first scenario is devoted for single damage with small deterioration. The second scenario is for dual damage in different stories of shear building. The first and second mode shapes of shear building before and after damage of first scenario are graphed in Fig. 5. Each mode shape normalized with respect to undamaged mode shape. As seen the effect of damage is insignificant and direct using of mode shapes for damage detection may lead to incorrect results. Normalized uniform load surface of the building by using first two mode shapes, before and after damage occurrence of first scenario is depicted in Fig. 6. As expected, the difference between healthy and damaged state become more visible by using the ULS due to data fusion. The ULS is calculated by using first two mode shapes and applied for damage detection.

When information lacks any noise, the proposed objective function is able to detect the damages with high speed. In order to account measurement noise, the natural frequencies are contaminated with random noise by using the following formula

$$\omega_i^{Noise} = \omega_i (1 + NL \times Rndn) \qquad (17)$$

where ω_i^{Noise} and ω_i are ith natural frequencies with and without noise, respectively. NL is noise level and $Rndn$ is a random number between [-1,1]. Also, mode shapes contamination is done by using the following

Table 2 Comparison between different objective function in damage scenario 2 detection in beam

	Identified damage from objective function in Eq. (15) after 1500 iterations				Objective function value in Eq. (15)	Proposed objective function value
	First damage		Second damage			
	Element no.	β	Element no.	β		
First run	8	0.0397	12	0.0484	0.000010523	0.0265862702
Second run	10	0.0439	13	0.0486	0.000018436	0.0357274493

relationship

$$\phi_i^{Noise} = \phi_i\,(1 + NV) \tag{18}$$

$$NV = \sqrt{\frac{\sigma^2}{e^{SNR\times(\frac{\ln(10)}{10})}}} \times RV \tag{19}$$

where ϕ_i^{Noise} and ϕ_i are ith mode shapes with and without noise, respectively. σ^2 is the variance of noiseless mode shape, SNR is signal to noise ratio and RV is a zero-mean vector with random elements between [-1 1]. The relationship for noise vector in Eq. (19) is adopted from Nguyen (2016). For examining the efficiency of this method versus the noise, two different noise patterns are taken into account as per the Table 5. Results of algorithm implementation for the first scenario of damage are given in Fig. 7. As it is observed, in spite of low intensity of damage, algorithm can detect the damage with a good accuracy in presence of noise with various patterns. Algorithm convergence is drawn in Fig. 8, indicating that the objective function takes less value in presence of noise with first pattern. Results of algorithm implementation for the second scenario of damage are given in Fig. 9. As it can be seen, in both patterns of noise application, damage is detected with higher accuracy in the third-story than fifth-story. Algorithm convergence for the second scenario of damage is plotted in Fig. 10. Similar to the first scenario of damage, objective function takes less value in presence of noise with first pattern.

As it is seen in this example, the proposed objective function is able to detect the damages with low intensity and multiple damages in shear building using two mode shapes and in presence of noise with various intensities.

Table 3 Physical characteristics of the six-story shear building

Story number	Mass (kg)	Stiffness (N/m)
1	200	10000
2	100	10000
3	180	8000
4	150	5000
5	100	7000
6	150	6000

Table 4 Assumed damage scenarios in the six-story shear building

Damage Scenario 1		Damage Scenario 2	
Story no.	Stiffness reduction	Story no.	Stiffness reduction
2	0.05	3	0.1
		5	0.1

Table 5 Different noise patterns considered for data contamination in the six-story shear building

Noise Pattern 1		Noise Pattern 2	
SNR	NL	SNR	NL
30	3%	50	7%

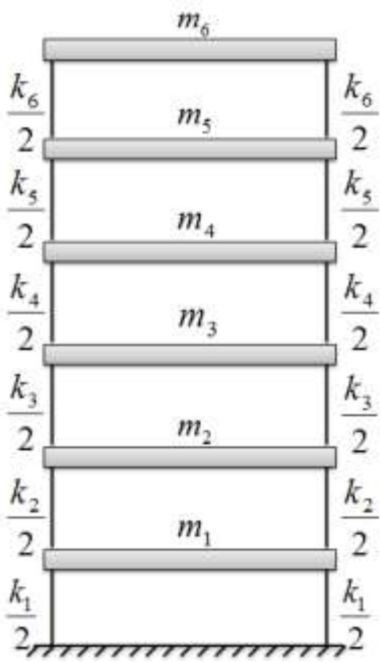
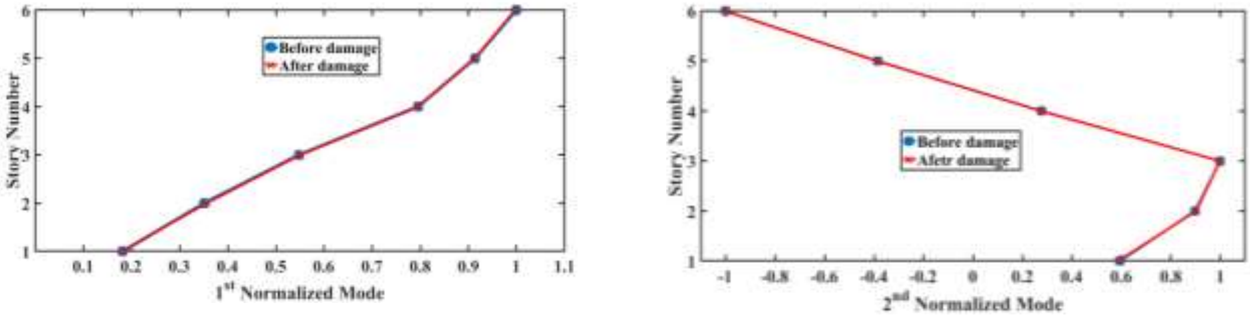


Fig. 4 Six-story shear building



(a) First mode shape (b) Second mode shape

Fig. 5 Mode shapes of the shear building before and after damage occurrence of the first scenario

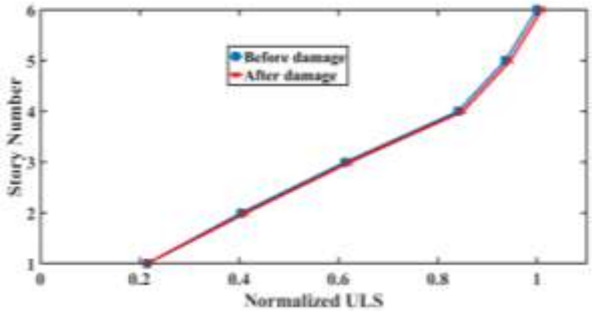


Fig. 6 The ULS of the shear building by using first two modes, before and after damage occurrence of the first scenario

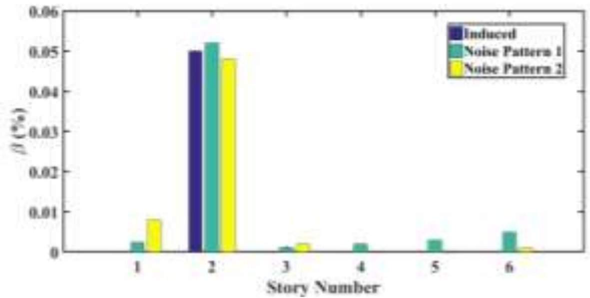


Fig. 7 Damage identification result of the six-story building for damage scenario 1 in presence of noise

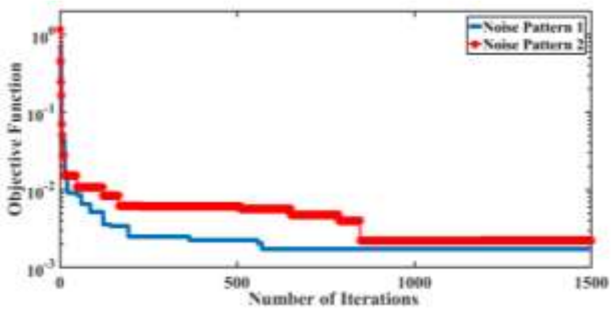


Fig. 8 Convergence rate of proposed method with different noise patterns in first scenario damage detection in shear building

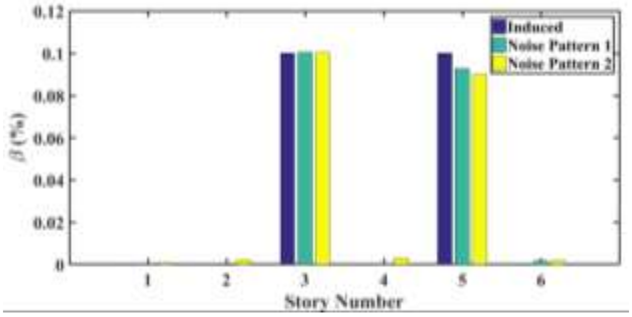


Fig. 9 Damage identification result of the six-story building for damage scenario 2 in presence of noise

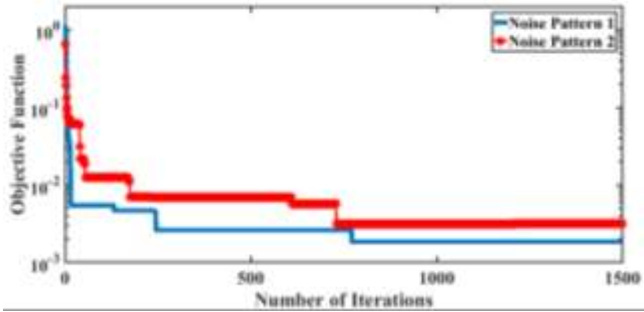


Fig. 10 Convergence rate of proposed method with different noise patterns in second scenario damage detection in shear building

4.3 A Double-beam system

As stated in introduction, a double-beam system with elastic inner layer is an approximate model for sandwich beam. There are limited researches on damage detection in this system. So, we test proposed method for damage detection in a double-beam system. Schematic of system is presented in Fig. 11, each beam divided into 10 elements; therefore the whole system is composed of 20 elements. More detail about this system could be found in Oniszczuk (2000). Both beams are similar and physical and geometrical properties of system are as following:

$$E = 70\,GPa, \rho = 2700\,kg/m^3, h = 50\,mm,$$
$$w = 100\,mm, k = 10^5\ N/m^2, Length = 1m$$

k is stiffness of inner elastic layer which is modeled by Winkler type elastic layer. Three different scenarios are considered by elasticity modulus degradation of beam's element, which are presented by detail in Table 6. The first scenario is devoted for dual damage in one of the beams. In the second scenario, each beam has single damage. The third scenario is devoted for triple damage in both beams. The ULS is calculated by using first two mode shapes and applied for damage detection. Noise contamination is considered by adding 2% noise level to natural frequencies and noise vector by SNR 70 to mode shapes. Damages detection results for different scenarios are given in Figs. 12-14. As seen, in the case of noiseless data, the method could detect damages with high accuracy. In the presence of noise the accuracy is good and acceptable and the results approve applicability of proposed method for damage detection in structure composed of different parts with different materials.

Table 6 Assumed damage scenarios in the double-beam system

Damage Scenario 1		Damage Scenario 2		Damage Scenario 3	
Element no.	Stiffness reduction	Element no.	Stiffness reduction	Element no.	Stiffness reduction
7	0.1	12	0.1	2	0.1
14	0.1	15	0.1	7	0.15
				13	0.1

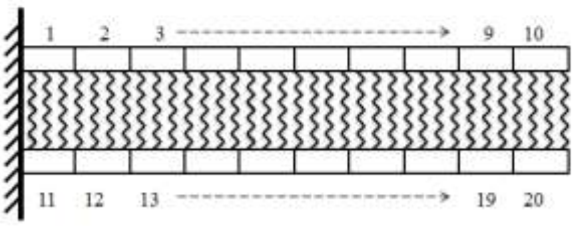


Fig. 11 Finite element model of the double-beam system

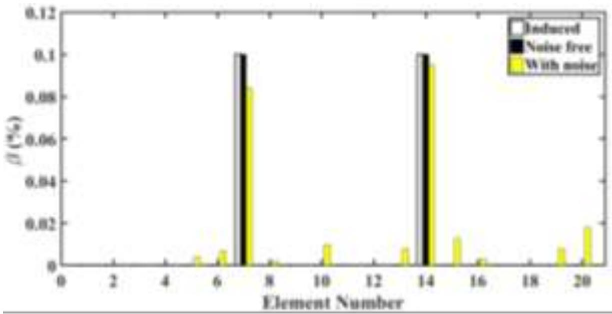


Fig. 12 Damage identification result of the double-beam system for damage scenario 1 in presence of noise

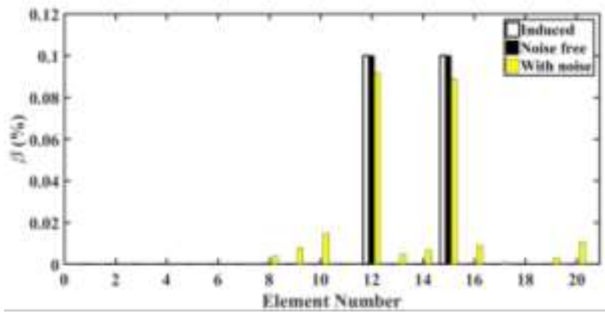


Fig. 13 Damage identification result of the double-beam system for damage scenario 2 in presence of noise

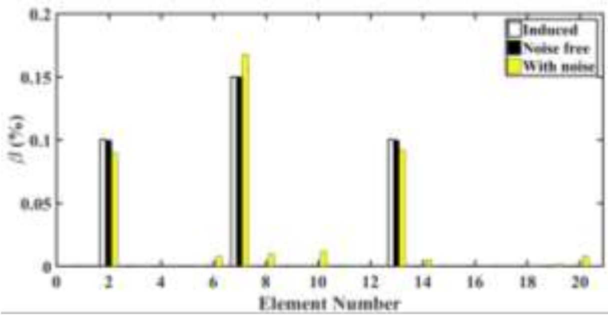


Fig. 14 Damage identification result of the double-beam system for damage scenario 3 in presence of noise

5. Conclusions

In the present study, detecting the damage in structures by using inverse method was taken into account and a new objective function was introduced for such purpose. An objective function proposed to damage detection as discrepancy between the ULS of monitored structure and numerical model of structure. The ULS could be considered as a weighted average of mode shapes where weight factor is the inverse of the square of the natural frequencies. The values of parameters in which this objective function become minimum, are correspond to damage's parameters. The Teaching-Learning based optimization was used to solve objective function. A numerical model of structure was constructed using finite element method and damage was considered as stiffness degradation in elements. The efficiency of proposed method was investigated through three numerical examples. In the first example, a beam with hinged fixed support conditions was investigated. In this example, efficiency of proposed objective function is compared to another objective function which uses the natural frequencies and mode shape functions. Results showed that the proposed objective function is more sensitive to damage. When another objective function by using two mode shapes is not able to detect the damages in the beam, the proposed objective function detects easily the damages by use of two mode shapes. It was also indicated that the proposed objective function has more convergence speed and easily escapes the local extremum points. In the second example, a six-story shear building was investigated. In such example, impact of damage on the first and second mode shape was taken into account. Meanwhile, the ULS was calculated by using the first two-mode-shapes before and after the damage. Results show that ULS can magnify the impact of damage and provides more information than the mode shapes. In this example, damage detection was tracked for two scenario of damage as well as for various intensities of noise, which led to a good accuracy in detecting the damage. In the third example, a double-beam system as an approximate model for soft core sandwich structure was considered. For several scenarios of damages and in presence of noise the algorithm could detect the damages with acceptable accuracy. It could be conclude that, the ULS is good fusion between natural frequencies and mode shapes data and have good robustness against measurement noise and could confidently applied for other types of structures.

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