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Stochastic stability control analysis of an inclined stay cable under random and periodic support motion excitations

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Abstract. The stochastic stability control of the parameter-excited vibration of an inclined stay cable with multiple modes coupling under random and periodic combined support disturbances is studied by using the direct eigenvalue analysis approach based on the response moment stability, Floquet theorem, Fourier series and matrix eigenvalue analysis. The differential equation with time-varying parameters for the transverse vibration of the inclined cable with control under random and deterministic support disturbances is derived and converted into the randomly and deterministically parameter-excited multi-degree-of-freedom vibration equations. As the stochastic stability of the parameter-excited vibration is mainly determined by the characteristics of perturbation moment, the differential equation with only deterministic parameters for the perturbation second moment is derived based on the Itô stochastic differential rule. The stochastically and deterministically parameter-excited vibration stability is then determined by the deterministic parameter-varying response moment stability. Based on the Floquet theorem, expanding the periodic parameters of the perturbation moment equation and the periodic component of the characteristic perturbation moment expression into the Fourier series yields the eigenvalue equation which determines the perturbation moment behavior. Thus the stochastic stability of the parameter-excited cable vibration under the random and periodic combined support disturbances is determined directly by the matrix eigenvalues. The direct eigenvalue analysis approach is applicable to the stochastic stability of the control cable with multiple modes coupling under various periodic and/or random support disturbances. Numerical results illustrate that the multiple cable modes need to be considered for the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances, and the increase of the control damping rather than control stiffness can greatly enhance the stochastic stability of the parameter-excited cable vibration including the frequency width increase of the periodic disturbance and the critical value increase of the random disturbance amplitude.

Keywords: inclined cable; random disturbance; parametric excitation; stability control; matrix eigenvalue

1. Introduction

The dynamic stability analysis and control of randomly and periodically parameter-excited systems is a significant research subject in engineering. For example, the large-amplitude oscillation of cables in a cable-stayed bridge under support motion excitations such as deck disturbance is a parametrically excited vibration. The parametrically excited vibration of cables has the dynamic characteristics different from the conventional linear vibration, and the cable vibration stability is a main problem (Rega *et al.* 1984, Iyengar and Rao 1988, Takahashi 1991, Perkins 1992, Costa *et al.* 1996, Gonzalez *et al.* 2008, Luongo and Zulli 2012, Gattulli *et al.* 2002, Xia and Fujino 2006, Warminski *et al.* 2016). The unstable cable vibration results in a large-amplitude oscillation which can damage the cable-bridge structure. Thus the cable vibration stability needs to be analyzed and controlled. In fact, the cable support motion excitations such as deck disturbance include random component and deterministic component due to the dominant mode vibration and wide-band vibration of the deck. The random and deterministic combined support motions yield a stochastically and periodically parameter-excited cable stability problem. For the periodic parameter excitation, the parameter-excited systems can be expressed as the Hill equations or Mathieu equations with periodic time-varying parameters. The dynamic stability of the single Mathieu equation representing a single-degree-of-freedom parameter-excited system has been studied well by using the Floquet theorem (Nayfeh and Mook 1979). Several approximate numerical approaches and analysis methods for the dynamic stability of the coupled Mathieu equations representing a multi-degree-of-freedom parameter-excited system have been proposed (Hsu and Cheng 1974, Sinha and Wu 1991, Friedmann 1990, Ying *et al.* 2009, Stupnicka 1978, Lee 1976). In particular, the

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direct eigenvalue analysis approach has been developed based on the Floquet theorem and harmonic balance method, which converts the periodically parameter-excited stability problem into an eigenvalue problem and determines the stability directly by eigenvalues (Takahashi 1981, Ying *et al.* 2006). This approach has been applied to inclined stay cables with and without control as periodically parameter-excited multi-degree-of-freedom systems to obtain unstable regions (Ying *et al.* 2006, 2007).

However, small random parameter excitations can result in a remarkable influence on the dynamic stability of parameter-excited systems. Thus the random parameter excitation is necessary to be considered for the stochastic and periodic combined parameter-excited vibration of cables (Bolotin 1984, Ibrahim 1985, Dimentberg 1988, Manohar and Ibrahim 1999, Zhou *et al.* 2010, Giaccu *et al.* 2015). The stability of inclined stay cables under random and deterministic combined support motion excitations is a stochastically parameter-excited stability problem. The infinite hierarchy equations for the response moment of single-degree-of-freedom linear systems with Gaussian stationary parameter excitations have been presented, and the mean-square stability of the randomly parameter-excited systems was studied based on the hierarchy closure procedure (Bobryk *et al.* 2005, Bolotin 1984). The stochastic averaging method and the Lyapunov exponent combined with the perturbation method have been applied to the stability analysis of randomly parameter-excited systems which have harmonic and white noise excitations (Roberts and Spanos 1986, Lin and Cai 1994, Ariaratnam *et al.* 1991, Kozin and Zhang 1991, Namachchivaya 1991, Xie 2006). These methods are presently difficult to be applied to the stochastic stability analysis of high-degree-of-freedom parameter-excited systems. Recently, a direct eigenvalue analysis approach to the stability of randomly and periodically parameter-excited systems with multi-degree-of-freedom has been developed by the combination of the stochastic response moment stability and deterministic eigenvalue analysis approach (Ying and Ni 2017).

Most researches on the cable vibration control are concentrated on the response reduction. The passive control of cable vibration responses has been studied largely, for example, by using supplemental dampers such as viscous oil damper, viscoelastic rubber damper and nonlinear magneto-rheological damper (Pacheco *et al.* 1993, Xu *et al.* 1998, Krenk 2000, Main and Jones 2001, Wang *et al.* 2005). To reduce further responses, the active and semi-active controls (Soong and Spencer 2002, Spencer and Nagarajaiah 2003, Casciati *et al.* 2012, Nagarajaiah and Jung 2014, Or *et al.* 2008) of cable vibration responses have been studied based on the optimal control strategy (Fujino *et al.* 1993, Gehle and Masri 1998, Bossens and Preumont 2001, Dyke *et al.* 1996, Symans and Constantinou 1999, Johson *et al.* 2003, Duan *et al.* 2005, 2006, Duan *et al.* 2018, Duan *et al.* 2019). For example, the active boundary control modeled a controlled cable as bilinear system and then determined the control law according to the Lyapunov stability (Susumpow and Fujino 1995, Baicu *et al.* 1996). The semi-active control using magneto-rheological dampers determined the control law of cable vibration responses based on the clipped linear quadratic control strategy (Johson *et al.* 2003, Diouron *et al.* 2003, Wang *et al.* 2015). However, these researches on the response reduction considered the controlled cables as linear systems or self-exciting systems. Several researches have considered the controlled cable as deterministically parameter-excited system and have discussed on the instability control of the inclined stay cable with magneto-rheological damper under periodic support motion excitations (Ying *et al.* 2007, Ouni *et al.* 2012). As the stability of inclined stay cables under random and deterministic combined support motion excitations is a stochastically parameter-excited stability problem, the stochastic stability analysis and control of the randomly and periodically parameter-excited cable vibration need to be studied further.

In this paper, the stochastic stability control of the parameter-excited vibration of the inclined stay cable with multiple modes coupling under the random and periodic combined support motion excitations is studied by using the direct eigenvalue analysis approach based on the response moment stability, Floquet theorem, Fourier series and matrix eigenvalue analysis. First, the differential equation with time-varying parameters for the transverse vibration of the inclined cable with control under random and deterministic support disturbances is derived. By using the Galerkin method, the partial differential equation is converted into the ordinary differential equations which describe the stochastically and deterministically parameter-excited multi-degree-of-freedom cable vibration. Second, for the stochastic stability, the differential equation for the perturbation second moment is derived based on the Itô stochastic differential rule, which is a deterministic matrix equation with only periodic time-varying parameters. The stochastically and deterministically parameter-excited vibration stability is converted into the response moment stability of the deterministic parameter-varying system. Third, based on the Floquet theorem, the periodic parameters of the perturbation moment equation and the periodic component of the characteristic perturbation moment expression are expanded into the Fourier series, and then the eigenvalue equation is obtained which determines the perturbation moment behavior. The stochastic stability of the parameter-excited controlled cable vibration under the random and periodic combined support disturbances is converted into the eigenvalue problem which is determined directly by the matrix eigenvalues. Finally, numerical results on the unstable regions of the controlled cable vibration under the random and periodic support disturbances with various excitation and control parameters are given to illustrate the stochastic stability of the multiple modes coupling vibration, the effects of the random and periodic support disturbances on the stochastic stability, and the control effectiveness of the stochastic stability by increasing the cable damping and stiffness.

2. Randomly and periodically parameter-excited vibration equations of a controlled cable

Engineering structures such as taut cables are induced frequently the parameter-excited vibration by support disturbances, and their dynamic stability control is a significant problem. Consider an inclined stay cable with control under support disturbances as shown in Fig. 1. The differential equations of motion in plane of the cable can be expressed as (Irvine 1981, Ying *et al.* 2007)

$$m\ddot{u}+c_{11}\dot{u}+c_{12}\dot{v}=\frac{\partial}{\partial s}[T_s\frac{\partial u}{\partial s}+EA\varepsilon(\frac{\partial u}{\partial s}+\frac{\mathrm{d}x}{\mathrm{d}s})]+f_{cx} \quad (1)$$

$$m\ddot{v}+c_{21}\dot{u}+c_{22}\dot{v}=\frac{\partial}{\partial s}[T_s\frac{\partial v}{\partial s}+EA\varepsilon(\frac{\partial v}{\partial s}+\frac{\mathrm{d}y}{\mathrm{d}s})]+f_{cy} \quad (2)$$

where u and v are respectively the horizontal and vertical cable displacements, m is the mass per unit length, c_{11}, c_{12}, c_{21} and c_{22} are damping coefficients, T_s is the static tension, E is the elastic modulus, A is the cross-sectional area, ε is the nonlinear longitudinal strain, s, x and y are respectively the curvilinear, horizontal and vertical coordinates, f_{cx} and f_{cy} are respectively the horizontal and vertical control force components. The cable boundary conditions are

$$u=u_A(t)=u_{Ad}(t)+u_{Ar}(t),\quad v=0,\quad \text{for}\quad s=0 \quad (3)$$

$$u=0,\quad v=v_B(t)=v_{Bd}(t)+v_{Br}(t),\quad \text{for}\quad s=L \quad (4)$$

where u_A is the horizontal displacement of end A, v_B is the vertical displacement of end B, subscripts d and r denote respectively deterministic and random components, L is the cable length, and t is the time variable. The cable vibration is caused by the support disturbances u_A and v_B. By using displacement transformations, eliminating the longitudinal cable displacement and neglecting small high-order displacement terms, the vibration equation for the transverse cable displacement v_s can be derived from Eqs. (1) and (2) (Ying *et al.* 2006, 2007). The equation for the dimensionless transverse cable displacement w and the corresponding cable boundary conditions are given by

$$\ddot{w}+c\dot{w}-[k_1+k_{2d}(t)+k_{2r}(t)]\frac{\partial^2 w}{\partial z^2}+k_3\int_0^1 w\mathrm{d}z = f_v(t)+f_c \quad (5)$$

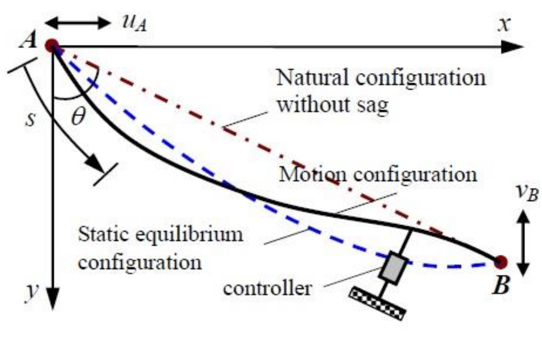

Fig. 1 An inclined sagged stay cable with control under support disturbances in plane

$$w=0,\quad \text{for}\quad z=0;\qquad w=0,\quad \text{for}\quad z=1 \quad (6)$$

where $w=v_s/L$, $z=s/L$, c is the damping coefficient, f_v is the excitation dependent on support disturbances u_A and v_B, f_c is the cable-transverse control force, and parameters

$$\begin{aligned} k_1 &= \frac{T_x}{mL^2\sin\theta} \\ k_{2d}(t) &= \frac{EA}{L^3}[v_{Bd}(t)\cos\theta-u_{Ad}(t)\sin\theta] \\ k_{2r}(t) &= \frac{EA}{L^3}[v_{Br}(t)\cos\theta-u_{Ar}(t)\sin\theta] \\ k_3 &= \frac{EA}{mL^2}(\frac{mgL}{T_x})^2 \end{aligned} \quad (7)$$

in which T_x is the horizontal static tension, θ is the angle between axis y and AB, k_{2d} and k_{2r} are respectively the deterministic and random time-varying parameters due to the support disturbances. Eq. (5) is a partial differential equation with time-varying parameters, which describes the parameter-excited transverse vibration of the inclined cable under random and deterministic support disturbances.

In general, the control force produced by a controller such as active mass damper or semi-active magneto-rheological damper based on the optimal feedback strategy includes damping force and elastic force (Fujino *et al.* 1993, Krenk 2000, Ying *et al.* 2007). Under the energy equality, the control force can be expressed equivalently as

$$f_c=-(c_{eq}\dot{w}+k_{eq}w)\delta(z-z_c) \quad (8)$$

where c_{eq} and k_{eq} are respectively the equivalent damping and stiffness coefficients, δ is the Dirac delta function, and z_c is the position coordinate of the control force. Substituting Eq. (8) into Eq. (5) yields the parameter-varying differential equation for the controlled cable

$$\ddot{w}+[c+c_{eq}\delta(z-z_c)]\dot{w}-[k_1+k_{2d}(t)+k_{2r}(t)]\frac{\partial^2 w}{\partial z^2} + k_{eq}w\delta(z-z_c)+k_3\int_0^1 w\mathrm{d}z = f_v(t) \quad (9)$$

By expanding displacement w based on the boundary conditions and using the Galerkin method, Eq. (9) can be converted into ordinary differential equations with time-varying parameters. The matrix equation and displacement expansion are

$$\ddot{\mathbf{Q}}+\mathbf{C}\dot{\mathbf{Q}}+[\mathbf{K}_0+\mathbf{K}_d(t)+\mathbf{K}_r(t)]\mathbf{Q}=\mathbf{F}(t) \quad (10)$$

$$w(z,t)=\sum_{j=1}^{n}q_j(t)\sin j\pi z \quad (11)$$

where n is the modal number, q_j is the j-th modal displacement, $\mathbf{Q}=[q_1, q_2, \ldots, q_n]^{\mathrm{T}}$, $\mathbf{F}=[f_1, f_2, \ldots, f_n]^{\mathrm{T}}$, $\mathbf{C}$ is the damping coefficient matrix including cable and controller parts, and parameter matrices

$$\mathbf{K}_0 = \begin{bmatrix} k_1(i\pi)^2\delta_{ij} + 2k_{eq}\sin i\pi z_c \sin j\pi z_c \\ + \dfrac{2k_3}{ij\pi^2}[1-(-1)^i][1-(-1)^j] \end{bmatrix}_{n\times n}$$
$$\mathbf{K}_d(t) = \left[k_{2d}(t)(i\pi)^2\delta_{ij}\right]_{n\times n} \tag{12}$$
$$\mathbf{K}_r(t) = \left[k_{2r}(t)(i\pi)^2\delta_{ij}\right]_{n\times n}$$
$$f_i(t) = \int_0^1 2f(t)\sin i\pi z \mathrm{d}z$$

Eq. (10) describes the stochastically parameter-excited multi-degree-of-freedom vibration of the controlled inclined cable under random and deterministic support disturbances. The support disturbances are considered generally as the combination of periodic part and random part (Ying and Ni 2017). Then the stiffness parameter k_{2d} or $\mathbf{K}_d$ is the time function of period T, and the stiffness parameter k_{2r} or $\mathbf{K}_r$ is the random process which is assumed as the linear combination of independent Gaussian white noises of two supports. The dynamic stability and control of the cable under random and periodic parameter excitations are an important problem to be studied.

3. Response moment stability of parameter-excited controlled cable vibration

The stability of parameter-excited multi-degree-of-freedom system (10) is determined by the dynamic behavior of perturbation $\mathbf{Q}_p$ of modal displacement $\mathbf{Q}$. The differential equation for the perturbation can be obtained from Eq. (10) as

$$\ddot{\mathbf{Q}}_p + \mathbf{C}\dot{\mathbf{Q}}_p + [\mathbf{K}_0 + \mathbf{K}_d(t) + \mathbf{K}_r(t)]\mathbf{Q}_p = 0 \tag{13}$$

Eq. (13) describes the multi-degree-of-freedom perturbation behavior with random and periodic parameter excitations. For the stability analysis, the equation is rewritten in the following state equation

$$\dot{\mathbf{X}} = [\mathbf{A}_d(t) + \mathbf{A}_r(t)]\mathbf{X} \tag{14}$$

where $\mathbf{X}=[\mathbf{Q}_p^{\mathrm{T}},\ \dot{\mathbf{Q}}_p^{\mathrm{T}}]^{\mathrm{T}}$ is the perturbation state vector, $\mathbf{A}_d$ is the deterministic parameter matrix with period T, $\mathbf{A}_r$ is the random parameter matrix, and

$$\mathbf{A}_d = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{K}_0 - \mathbf{K}_d & -\mathbf{C} \end{bmatrix},\quad \mathbf{A}_r = \begin{bmatrix} 0 & 0 \\ -\mathbf{K}_r & 0 \end{bmatrix} \tag{15}$$

in which $\mathbf{I}$ is the identity matrix. Based on Eqs. (7), (12) and (15), the random parameter matrix can be expressed as $\mathbf{A}_r=\mathbf{A}_{r1}\xi_A(t)+\mathbf{A}_{r2}\xi_B(t)$, where $\mathbf{A}_{r1}$ and $\mathbf{A}_{r2}$ are constant matrices, ξ_A and ξ_B are respectively the independent Gaussian white noises with intensities D_A and D_B.

Eq. (14) is a randomly and periodically parameter-varying matrix differential equation for the perturbation. Based on the Itô stochastic differential rule, the equation can be converted into a stochastic differential equation and the Wong-Zakai correction terms need to be included due to the parametric white noise excitations (Bolotin 1984, Ibrahim 1985, Dimentberg 1988). By the expectation operation, the differential equation for the mean perturbation is obtained as

$$\frac{\mathrm{d}E[\mathbf{X}]}{\mathrm{d}t} = \mathbf{A}_s(t)E[\mathbf{X}] \tag{16}$$

where $E[\cdot]$ is the expectation operator, and $\mathbf{A}_s=\mathbf{A}_d+(D_A\mathbf{A}_{r1}^2+D_B\mathbf{A}_{r2}^2)/2$ is the parameter matrix with period T. Eq. (16) is a deterministic differential equation with periodic parameters, and then the stability of the mean system is the same as the corresponding deterministic system under only periodically parametric excitations. The stability analysis of deterministic systems with periodic parameters has been given in references (Takahashi 1981, Ying *et al.* 2006). However, the stability of the randomly parameter-excited system is mainly determined by the characteristics of the perturbation moment. Based on the Itô stochastic differential rule, the differential equation for the product of perturbations is derived from Eq. (14) with the Wong-Zakai correction terms as

$$\begin{aligned} \mathrm{d}(\mathbf{X}\mathbf{X}^{\mathrm{T}}) &= [\mathbf{A}_s(\mathbf{X}\mathbf{X}^{\mathrm{T}}) + (\mathbf{X}\mathbf{X}^{\mathrm{T}})\mathbf{A}_s^{\mathrm{T}} \\ &\quad + D_A\mathbf{A}_{r1}(\mathbf{X}\mathbf{X}^{\mathrm{T}})\mathbf{A}_{r1}^{\mathrm{T}} + D_B\mathbf{A}_{r2}(\mathbf{X}\mathbf{X}^{\mathrm{T}})\mathbf{A}_{r2}^{\mathrm{T}}]\mathrm{d}t \\ &\quad + [\mathbf{A}_{r1}(\mathbf{X}\mathbf{X}^{\mathrm{T}}) + (\mathbf{X}\mathbf{X}^{\mathrm{T}})\mathbf{A}_{r1}^{\mathrm{T}}]\mathrm{d}W_A(t) \\ &\quad + [\mathbf{A}_{r2}(\mathbf{X}\mathbf{X}^{\mathrm{T}}) + (\mathbf{X}\mathbf{X}^{\mathrm{T}})\mathbf{A}_{r2}^{\mathrm{T}}]\mathrm{d}W_B(t) \end{aligned} \tag{17}$$

where W_A and W_B are the independent unit Wiener processes. The expectation operation of Eq. (17) yields

$$\dot{\mathbf{Y}} = \mathbf{A}_s\mathbf{Y} + \mathbf{Y}\mathbf{A}_s^{\mathrm{T}} + D_A\mathbf{A}_{r1}\mathbf{Y}\mathbf{A}_{r1}^{\mathrm{T}} + D_B\mathbf{A}_{r2}\mathbf{Y}\mathbf{A}_{r2}^{\mathrm{T}} \tag{18}$$

where the symmetric matrix $\mathbf{Y}=E[\mathbf{X}\mathbf{X}^{\mathrm{T}}]$ is the second moment of perturbation $\mathbf{X}$. Eq. (18) is a deterministic differential matrix equation with periodic parameters, which determines the perturbation second moment.

For the stochastic stability analysis or the characteristics of the perturbation second moment, the direct eigenvalue analysis approach can be applied (Ying *et al.* 2006, Ying and Ni 2017). Rearrange matrix $\mathbf{Y}$ into vector $\mathbf{V}$ and Eq. (18) is rewritten as

$$\dot{\mathbf{V}} = \mathbf{B}_d(t)\mathbf{V} \tag{19}$$

where $\mathbf{V}$ consists of independent elements in $\mathbf{Y}$, and $\mathbf{B}_d$ is the parameter matrix with period T which is composed of elements in $\mathbf{A}_s$, $\mathbf{A}_{r1}$ and $\mathbf{A}_{r2}$. The stochastic stability of system (14) is determined by the characteristics of the second moment $\mathbf{Y}$ or $\mathbf{V}$, and $\mathbf{V}$ is determined by deterministic Eq. (19) with periodic parameters. Based on the Floquet theorem, the fundamental solution of Eq. (19) can be expressed as the product of periodic and exponential components (Ying *et al.* 2006, Nayfeh and Mook 1979). Expanding the periodic component and periodic parameters into the Fourier series yields

$$\mathbf{B}_d(t) = \frac{1}{2}\mathbf{D}_{c0} + \sum_{j=1}^{\infty}(\mathbf{D}_{sj}\sin j\omega t + \mathbf{D}_{cj}\cos j\omega t) \tag{20}$$

$$\mathbf{V}(t) = e^{\lambda t}[\frac{1}{2}\mathbf{C}_{c0} + \sum_{k=1}^{\infty}(\mathbf{C}_{sk}\sin k\omega t + \mathbf{C}_{ck}\cos k\omega t)] \quad (21)$$

where λ is a characteristic constant, $\omega=2\pi/T$ is the parametric varying frequency, $\mathbf{D}_{sj}$ and $\mathbf{D}_{cj}$ are constant matrices, $\mathbf{C}_{sk}$ and $\mathbf{C}_{ck}$ are coefficient vectors. Substituting Eqs. (20) and (21) into Eq. (19) and balancing each harmonic term yield a series of algebraic equations. The equations are assembled into the following matrix equation

$$\mathbf{D}_{cd}(\lambda)\mathbf{Z} = 0 \quad (22)$$

where $\mathbf{Z} = [\mathbf{C}_{c0}^{\mathrm{T}}, \mathbf{C}_{c1}^{\mathrm{T}}, \mathbf{C}_{s1}^{\mathrm{T}}, \cdots]^{\mathrm{T}}$ is a coefficient vector to be solved, matrix $\mathbf{D}_{cd} = \lambda\bar{\mathbf{I}} - \bar{\mathbf{D}}$ depends on parameter λ, $\bar{\mathbf{D}}$ is composed of $\mathbf{D}_{sj}$ and $\mathbf{D}_{cj}$, and $\bar{\mathbf{I}}$ is the identity matrix. For a non-trivial solution to Eq. (22), the determinant of matrix $\mathbf{D}_{cd}$ is equal to zero, that is, $\left|\lambda\bar{\mathbf{I}} - \bar{\mathbf{D}}\right| = 0$. Thus the λ is an eigenvalue of matrix $\bar{\mathbf{D}}$.

According to the above analysis, if the real part of one eigenvalue of $\bar{\mathbf{D}}$ has a positive value, then the moment vector $\mathbf{V}$ or matrix $\mathbf{Y}$ tends to infinity for $t\to\infty$, perturbation $\mathbf{X}$ or $\mathbf{Q}_p$ becomes boundless, and thus the modal displacement $\mathbf{Q}$ or stochastic system response is unstable; otherwise the stochastic system response is stable. Therefore, the stochastic stability of the parameter-excited vibration of the cable under random and periodic parameter excitations is determined directly by the eigenvalues of constant matrix $\bar{\mathbf{D}}$ in Eq. (22). The matrix eigenvalues can be obtained by using conventional numerical algorithms. The stochastic stability control of the cable under random and periodic parameter excitations can be analyzed by comparing the stabilities (or unstable regions) of the controlled and uncontrolled cables.

As an example, consider an inclined stay cable in a cable-support bridge, which random and deterministic support disturbances are produced by the deck of end B and the tower of end A. In general, the support disturbance of end B is larger than the support disturbance of end A. Based on the test data, the dominant periodic vibration frequency of end A is twice that of end B (Ying *et al.* 2006), and the random disturbances of two ends are modeled as independent Gaussian white noises ξ_A and ξ_B. Then the dimensionless support disturbances can be expressed as

$$w_B(t) = \frac{v_B}{L} = \mu_1\sin(\omega t + \gamma) + \mu_2\xi_B(t)$$
$$w_A(t) = \frac{u_A}{L} = \varepsilon_1\sin 2\omega t + \varepsilon_2\xi_A(t) \quad (23)$$

where μ_1, μ_2, ε_1 and ε_2 are the amplitudes of disturbance components, γ is the phase difference, and ω is the periodic disturbance frequency. For other cases, the above analysis method is still applicable and similar results can be obtained. Substituting Eq. (23) into Eq. (7) yields the deterministic and random time-varying parameters

$$k_{2d}(t) = k_{21}\sin\omega t + k_{22}\cos\omega t + k_{23}\sin 2\omega t$$
$$k_{2r}(t) = k_{24}\xi_A(t) + k_{25}\xi_B(t)$$
$$k_{21} = \frac{EA}{L^2}\mu_1\cos\gamma\cos\theta, \quad k_{22} = \frac{EA}{L^2}\mu_1\sin\gamma\cos\theta$$
$$k_{23} = -\frac{EA}{L^2}\varepsilon_1\sin\theta \quad k_{24} = -\frac{EA}{L^2}\varepsilon_2\sin\theta \quad (24)$$
$$k_{25} = \frac{EA}{L^2}\mu_2\cos\theta$$

By using parameters in Eq. (24), the parameter matrices in Eqs. (12) and (15) can be obtained finally. The stochastic stability of the inclined stay cable is determined by the characteristics of its perturbation second moment in Eq. (19). The characteristics of the second moment are determined by the eigenvalues of the constant matrix in Eq. (22). Thus the stochastic stability of parameter-excited vibration of the cable under random and periodic support disturbances is determined directly by the matrix eigenvalues, and then the stochastic stability control can be evaluated by comparing the controlled and uncontrolled cables.

4. Numerical results on stability of parameter-excited controlled cable vibration

Consider the inclined stay cable with control as given above which has parameter values L=129.2 m, $A=71.97\times10^{-4}$ m^2, θ=0.984 rad, m=58.9 kg/m, $E=2.0\times10^5$ MPa, $T_x=3300\sin\theta$ kN and ζ=0.002 (modal damping ratio) (Ying *et al.* 2006). The random and periodic support disturbances have basic dimensionless parameter values $\mu_1=1.5\times10^{-3}$, $\mu_2=0.05\times10^{-3}$, $\varepsilon_1=\varepsilon_2=0$, $\gamma=0$ and $D_A=D_B=1$. The control force has the equivalent damping c_{eq} and stiffness k_{eq}, and the dimensionless position coordinate z_c=0.92. The first five natural frequencies of the uncontrolled cable are 0.93, 1.83, 2.75, 3.66 and 4.58 Hz. Numerical results on the stochastic stability of the parameter-excited controlled cable vibration under the random and periodic support disturbances are given in Figs. 2-17.

For the uncontrolled cable with only the first mode considered, Fig. 2 shows the unstable region on the plane (ω, μ_1) of periodic support-disturbance frequency and amplitude ($\mu_2=0.05\times10^{-3}$), which consists of unstable points and is verified through numerical simulation. It is seen that the maximal unstable region is around the twice first frequency and the other unstable region around the fractional twice first natural frequency is very small. There is not the unstable region for the disturbance frequency larger than the third natural frequency under certain random support disturbance amplitude such as $\mu_2<0.05\times10^{-3}$. However, the first several modes of the stay cable cannot be neglected based on the factual observation. For the uncontrolled cable with (e.g., first six modes) multiple modes considered, Fig. 3 shows the unstable region on the same plane (ω, μ_1), which is verified through numerical simulation (μ_2=0.05×10^{-3}). It is obtained by comparing Figs. 2 and 3 that the multiple mode coupling enlarges remarkably the unstable region and makes the unstable region around the combination of natural frequencies produced. There is the unstable region for the disturbance frequency larger than the third natural frequency under certain random support disturbance amplitude. Figs. 4 and 5 show the unstable regions on the plane (ω, μ_2) of periodic support-disturbance frequency and random support-disturbance amplitude for the uncontrolled cable with the first mode and multiple modes considered, respectively (μ_1=1.5×10^{-3}). It is obtained again by comparing Figs. 4 and 5 that the multiple mode coupling enlarges remarkably the unstable region. In particular, Fig. 5 illustrates that there is a lower bound (e.g., μ_2=0.06×10^{-3}) of the unstable region of the uncontrolled cable with multiple modes for wide periodic support-disturbance frequency, that is the critical value of random disturbance amplitude (μ_2) of the completely stochastic instability. Thus the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances depends mainly on the random disturbance amplitude larger than the critical value, and also depends mainly on the periodic disturbance frequency and amplitude when the random disturbance amplitude is smaller than the critical value. The stay cable with the first mode considered overestimates the stochastic stability, and then the multiple modes need to be considered for the cable stability analysis.

The active and semi-active feedback controls can produce artificial damping and then provide largely supplemental damping for the controlled cable. For the equivalent damping coefficient rising to ζ_{eq}=0.01, Fig. 6 shows the unstable region on the plane (ω, μ_1) of periodic support-disturbance frequency and amplitude for the controlled cable with multiple modes under the random and periodic support disturbances (μ_2=0.05×10^{-3}). For the equivalent damping coefficient ζ_{eq}=0.05, Fig. 7 shows the unstable region on the same plane (ω, μ_1) for the controlled cable with multiple modes under the random and periodic support disturbances (μ_2=0.05×10^{-3}). It is obtained by comparing Figs. 3, 6 and 7 that the unstable region including lower bound and frequency width is reduced largely by the increase of the equivalent control damping as shown in Fig. 8. Figs. 9 and 10 show the unstable regions on the plane (ω, μ_2) of periodic support-disturbance frequency and random support-disturbance amplitude for the controlled cable under the random and periodic support disturbances with the equivalent damping coefficient ζ_{eq}=0.01 and ζ_{eq}=0.05, respectively (μ_1=1.5×10^{-3}). It is obtained again by comparing Figs. 5, 9 and 10 that the unstable region is reduced largely by the increase of the equivalent control damping. The lower bound of the unstable region for wide periodic disturbance frequency in Fig. 5 or the critical value of random disturbance amplitude (μ_2) of the completely stochastic instability is heightened by the increase of the equivalent control damping. Thus the feedback control as damping force can greatly enhance the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances.

The active and semi-active feedback controls can also provide supplemental stiffness for the controlled cable. For the equivalent stiffness coefficient k_{eq}=0.1k_1 and k_{eq}=0.2k_1 [k_1 is a stiffness coefficient given in Eq. (7)], Figs. 11 and 12 show the unstable regions on the plane (ω, μ_1) of periodic support-disturbance frequency and amplitude for the controlled cable with multiple modes under the random and periodic support disturbances, respectively (μ_2=0.05×10^{-3}). Fig. 13 shows the comparison of the unstable regions for the controlled cable with the equivalent stiffness coefficient k_{eq}=0.1k_1 and k_{eq}=0.2k_1 and the uncontrolled cable with k_{eq}=0. It is seen that the increase of the equivalent control stiffness can heighten slightly the lower bound of several unstable sub-regions at lower frequency band, but diminish remarkably the critical value of random disturbance amplitude (μ_2) of the completely stochastic instability. Figs. 14 and 15 show the unstable regions on the plane (ω, μ_2) of periodic support-disturbance frequency and random support-disturbance amplitude for the controlled cable under the random and periodic support disturbances with the equivalent stiffness coefficient k_{eq}=0.1k_1 and k_{eq}=0.2k_1, respectively (μ_1=1.5×10^{-3}). It is obtained by comparing Figs. 5, 14 and 15 that the critical value of random disturbance amplitude (μ_2) of the completely stochastic instability is reduced nonlinearly (e.g. from μ_2=0.06×10^{-3} to μ_2=0.05×10^{-3}) by the increase of the equivalent control stiffness. For certain equivalent stiffness, the critical value of the completely stochastic instability is insensitive to the stiffness change and several unstable sub-regions at lower frequency band can be reduced. Thus the stiffness of the feedback control needs to be designed suitably for the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances.

For the feedback control including both the equivalent damping ζ_{eq}=0.01 and equivalent stiffness k_{eq}=0.1k_1, Fig. 16 shows the unstable region on the plane (ω, μ_1) of periodic support-disturbance frequency and amplitude for the controlled cable with multiple modes under the random and periodic support disturbances (μ_2=0.05×10^{-3}). Fig. 17 shows the comparison of the unstable regions for the controlled cable with the equivalent damping ζ_{eq}=0.01 and equivalent stiffness k_{eq}=0.1k_1, only the equivalent stiffness k_{eq}=0.1k_1 and only the equivalent damping ζ_{eq}=0.01. It is seen that the unstable region is reduced and in particular, the critical value of the completely stochastic instability is heightened by the increase of the equivalent control damping rather than the equivalent control stiffness. The combination of the equivalent control damping and stiffness can reduce further unstable sub-regions at certain frequency band. Thus the feedback control by supplemental damping can greatly increase the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances. The supplemental stiffness of the feedback control needs to be designed suitably for increasing further the stochastic stability of the parameter-excited cable vibration.

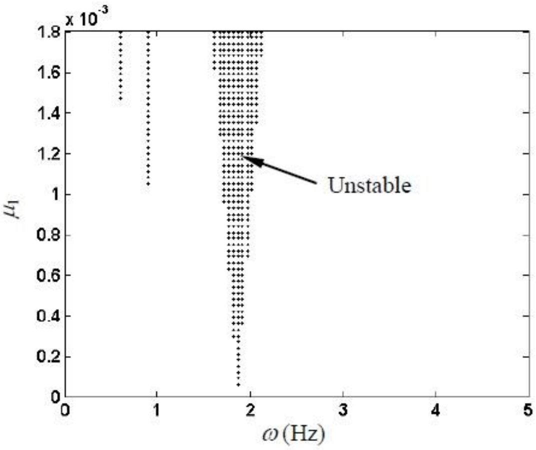

Fig. 2 Unstable region on the plane (ω, μ_1) of the uncontrolled cable with single mode under random and periodic disturbances (μ_2=0.05×10^{-3})

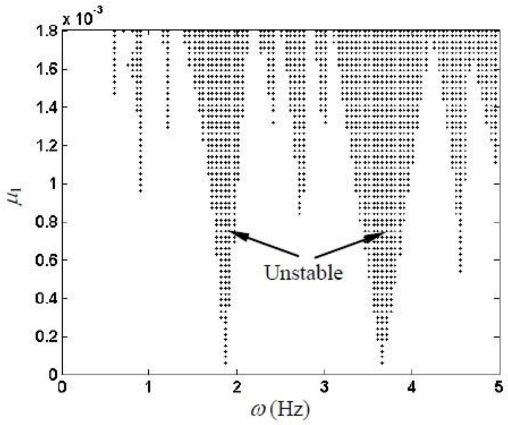

Fig. 3 Unstable region on the plane (ω, μ_1) of the uncontrolled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3})

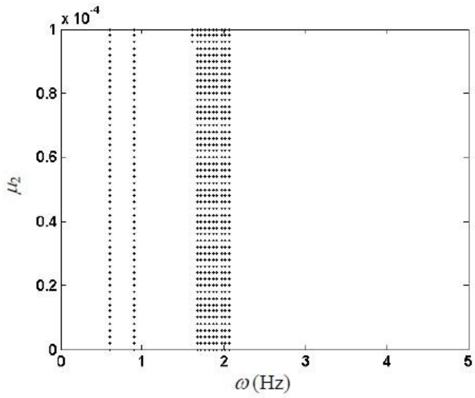

Fig. 4 Unstable region on the plane (ω, μ_2) of the uncontrolled cable with single mode under random and periodic disturbances (μ_1=1.5×10^{-3})

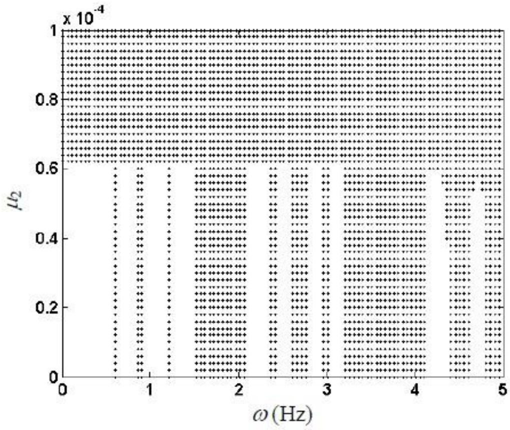

Fig. 5 Unstable region on the plane (ω, μ_2) of the uncontrolled cable with multiple modes under random and periodic disturbances (μ_1=1.5×10^{-3})

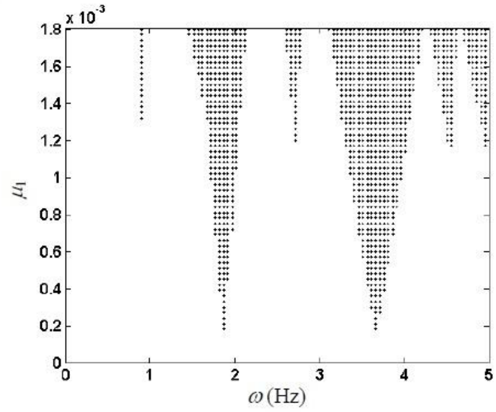

Fig. 6 Unstable region on the plane (ω, μ_1) of the controlled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3}, ζ_{eq}=0.01)

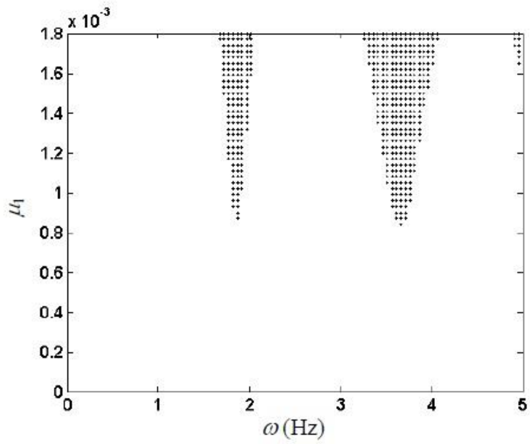

Fig. 7 Unstable region on the plane (ω, μ_1) of the controlled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3}, ζ_{eq}=0.05)

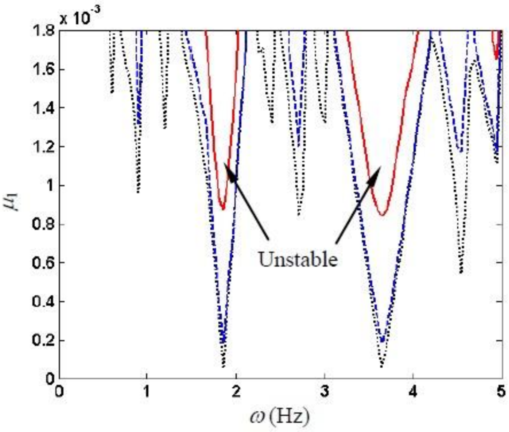

Fig. 8 Unstable region on the plane (ω, μ_1) of the controlled cable compared with the uncontrolled cable (μ_2=0.05×10^{-3}) (solid line: ζ_{eq}=0.05; dashed line: ζ_{eq}=0.01; dotted line: ζ=0.002)

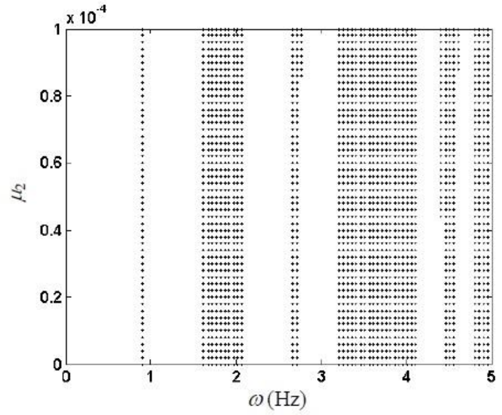

Fig. 9 Unstable region on the plane (ω, μ_2) of the controlled cable with multiple modes under random and periodic disturbances (μ_1=1.5×10^{-3}, ζ_{eq}=0.01)

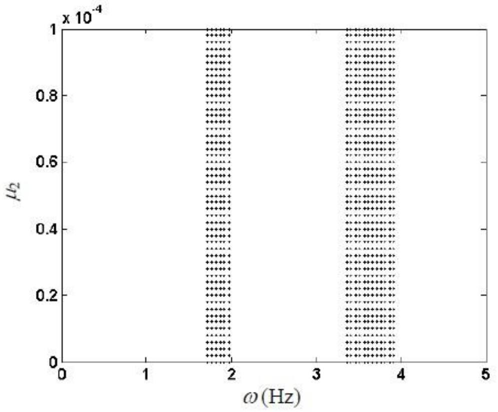

Fig. 10 Unstable region on the plane (ω, μ_2) of the controlled cable with multiple modes under random and periodic disturbances (μ_1=1.5×10^{-3}, ζ_{eq}=0.05)

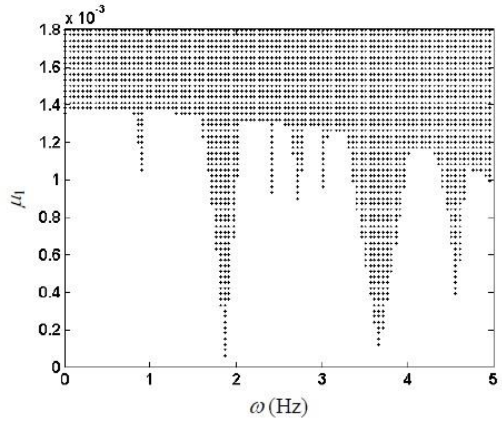

Fig. 11 Unstable region on the plane (ω, μ_1) of the controlled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3}, k_{eq}=0.1k_1)

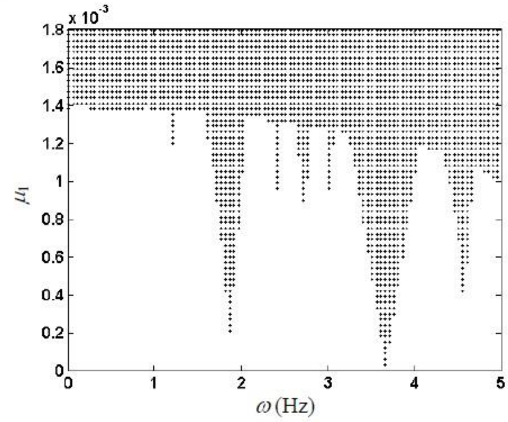

Fig. 12 Unstable region on the plane (ω, μ_1) of the controlled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3}, k_{eq}=0.2k_1)

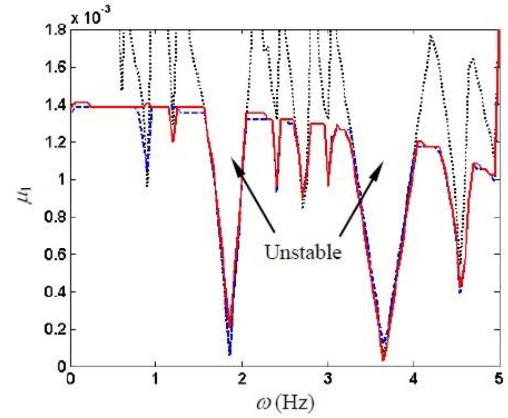

Fig. 13 Unstable region on the plane (ω, μ_1) of the controlled cable compared with the uncontrolled cable (μ_2=0.05×10^{-3}) (solid line: k_{eq}=0.2k_1; dashed line: k_{eq}=0.1k_1; dotted line: k_{eq}=0)

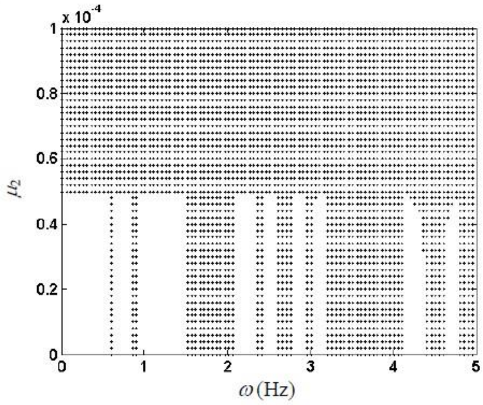

Fig. 14 Unstable region on the plane (ω, μ_2) of the controlled cable with multiple modes under random and periodic disturbances (μ_1=1.5×10^{-3}, k_{eq}=0.1k_1)

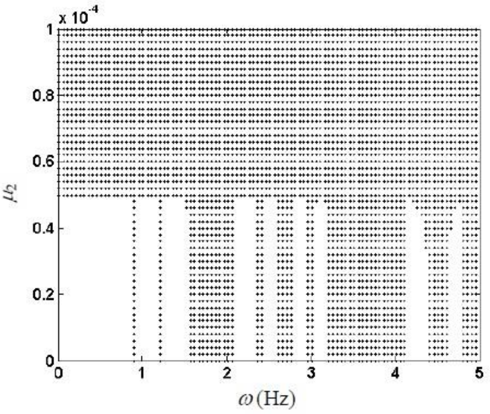

Fig. 15 Unstable region on the plane (ω, μ_2) of the controlled cable with multiple modes under random and periodic disturbances (μ_1=1.5×10^{-3}, k_{eq}=0.2k_1)

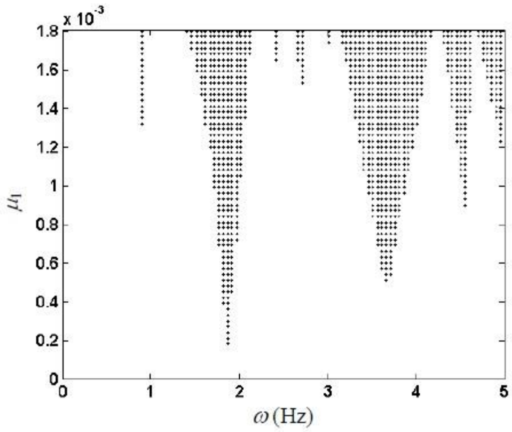

Fig. 16 Unstable region on the plane (ω, μ_1) of the controlled cable with multiple modes under random and periodic disturbances (μ_2=0.05×10^{-3}, ζ_{eq}=0.01, k_{eq}=0.1k_1)

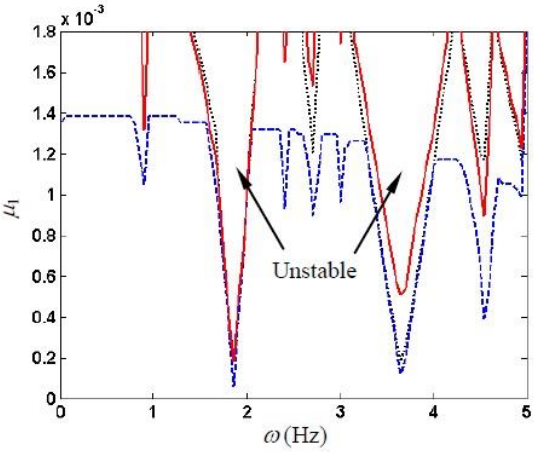

Fig. 17 Unstable region on the plane (ω, μ_1) of the controlled cable with various control damping and stiffness (μ_2=0.05×10^{-3}) (solid line: ζ_{eq}=0.01, k_{eq}=0.1k_1; dashed line: ζ=0.002, k_{eq}=0.1k_1; dotted line: ζ_{eq}=0.01, k_{eq}=0)

5. Conclusions

The stochastic stability control of the parameter-excited vibration of the inclined stay cable with multiple modes coupling under the random and periodic combined support disturbances has been analyzed by using the direct eigenvalue analysis approach based on the response moment stability, Floquet theorem, Fourier series and matrix eigenvalue analysis. The differential equation with time-varying parameters for the transverse vibration of the inclined cable with control under random and deterministic support disturbances has been derived and converted into the ordinary differential equations for the stochastically and deterministically parameter-excited multi-degree-of-freedom vibration. The dynamic stability of the randomly parameter-excited system is mainly determined by the characteristics of the perturbation moment. The differential equation with only deterministic parameters for the perturbation second moment has been derived based on the Itô stochastic differential rule, and then the stochastically and deterministically parameter-excited vibration stability is transformed into the deterministic parameter-varying response moment stability. Based on the Floquet theorem and Fourier series, the eigenvalue equation has been derived from the periodically parameter-varying equation for the perturbation second moment. Thus the stochastic stability of the parameter-excited cable vibration under the random and periodic combined support disturbances is determined directly by the matrix eigenvalues. The direct eigenvalue analysis approach to the stochastic stability of the controlled cable with multiple modes coupling under the random and periodic combined support disturbances has the following main advantages: (a) the stochastically and periodically parameter-excited stability of cables with multiple modes coupling can be determined directly by the resulting matrix eigenvalues; (b) the parameter-excited stability of cables under various periodic and/or random support disturbances can be analyzed uniformly.

Numerical results on unstable regions of the parameter-excited cable vibration under the random and periodic support disturbances have illustrated that: (a) the multiple cable modes need to be considered for the stochastic stability of the parameter-excited vibration under the random and periodic support disturbances; (b) the stochastic stability of the parameter-excited cable vibration under the random and periodic support disturbances depends mainly on the random disturbance amplitude for larger random disturbance and depends mainly on the periodic disturbance frequency and amplitude for smaller random disturbance; (c) the increase of the control damping can largely reduce the unstable region of the parameter-excited cable vibration under the random and periodic support disturbances and greatly enhance the stochastic stability; (d) the increase of the control stiffness can nonlinearly diminish the critical value of the random disturbance amplitude and reduce the completely stochastic instability, but the suitable combination of the control damping and stiffness can increase further the stochastic stability of the parameter-excited cable vibration for the periodic support disturbance in certain frequency band.

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