Performance of passive and active MTMDs in seismic response of Ahvaz cable-stayed bridge

Seyed Mehdi Zahrai^{*} and Mohammad Froozanfar^a

Center of excellence for Engineering and Management of civil Infrastructures, School of Civil Engineering, College of Engineering, University of Tehran, P.O. Box 11155-4563, Tehran, Iran

(Received April 6, 2018, Revised April 7, 2019, Accepted April 9, 2019)

Abstract. Cable-stayed bridges are attractive due to their beauty, reducing material consumption, less harm to the environment and so on, in comparison with other kinds of bridges. As a massive structure with long period and low damping (0.3 to 2%) under many dynamic loads, these bridges are susceptible to fatigue, serviceability disorder, damage or even collapse. Tuned Mass Damper (TMD) is a suitable controlling system to reduce the vibrations and prevent the threats in such bridges. In this paper, Multi Tuned Mass Damper (MTMD) system is added to the Ahvaz cable stayed Bridge in Iran, to reduce its seismic vibrations. First, the bridge is modeled in SAP2000 followed with result verification. Dead and live loads and the moving loads have been assigned to the bridge. Then the finite element model is developed in OpenSees, with the goal of running a nonlinear time-history analysis. Three far-field and three near-field earthquake records are imposed to the model after scaling to the PGA of 0.25 g, 0.4 g, 0.55 g and 0.7 g. Two MTMD systems, passive and active, with the number of TMDs from 1 to 8, are placed in specific points of the main span of bridge, adding a total mass ratio of 1 to 10% to the bridge. The parameters of the TMDs are optimized using Genetic Algorithm (GA). Also, the optimum force for active control is achieved by Fuzzy Logic Control (FLC). The results showed that the maximum displacement of the center of the bridge main span reduced 33% and 48% respectively by adding passive and active MTMD systems. The RMS of displacement reduced 37% and 47%, the velocity 36% and 42% and also the base shear in pylons, 27% and 47%, respectively by adding passive and active systems, in the best cases.

Keywords: cable-stayed bridge; seismic; MTMD; near-field and far-field earthquakes; vibration control; passive and active; genetic algorithm; fuzzy logic

1. Introduction

The idea of cable-stayed bridges was proposed by Fausto Veranzio (1595) for the first time in a book named "Machinae Novae". This idea wasn't considered noteworthy until the 2nd World war, by which, the steel became rare and this method was used to fix the bridges that their piers were intact. The Sweden "Stromsund" bridge (1968 and 1991) built in 1955 is known as the first modern cablestayed bridge in the world. Its designers realized that cablestayed bridges need less material for the cables and the deck, in comparison with suspension bridges.

Nowadays, the specific properties of cable-stayed bridges like beauty, less need to material usage, less building expenses, the possibility of crossing the impassable barriers, less harm to the environment and tourist attraction potential have made this kind of bridges as one of the most important and widely used bridges in many countries around the world. Since different kinds of loads like: dead and live loads, vehicle, human (walking, running and jumping) or train moving loads, wind and its different kinds

^aMSc, Research Assistant E-mail: mohammad.froozan@ut.ac.ir of effects and related phenomena and also earthquake may be applied to the cable-stayed bridges, they are susceptible to many effects like fatigue in their different parts, serviceability disorder, uncomforting sense in users, reduction of their beneficial life, light damage or even total collapse. Such events have the sequels of wealth loss, loss of life, disorder in transportation, harm to the environment and many other consequences; so, the matter of using control devices and systems to mitigate the vibrations in cable-stayed bridges, has been a vital issue for research purposes.

Tuned mass damper (TMD) idea was introduced by Frahm (1911) for the first time in a paper named "Device for damping vibrations bodies", to reduce the rocking vibrations of ships. Ormondroyd and Den Hartog (1928) presented the first mathematical theory on dynamic vibration absorber by connecting viscous damping to the TMD. Den Hartog (1940) on a book named "Mechanical vibrations", proposed close form introduction of optimal parameters of TMD, frequency and damping ratio, for an undamped SDOF system. The performance of TMD system in controlling the vibrations depends deeply on its optimal parameters. This will lead to the case that TMD be in negative phase with the main structure inducing a force in opposite side of structures displacement. Since then, many researches have conducted studies on TMD systems among them: Den Hartog (1956), Snowdon (1959), Falcon et al. (1967) and Ioi and Ikeda (1978).

^{*}Corresponding author, Professor E-mail: mzahrai@ut.ac.ir

In TMD systems, there is always the risk of detuning the values of parameters of TMD, due to dynamic loads which may harm the structure and change its properties like stiffness and also may harm the TMD itself. If the TMD becomes detuned, the controlling system will not work correctly and even it may operate against the structure. Another matter is that the TMD can only control the vibrations with frequency near the natural frequency of one mode (mostly first mode) and it is incapable in higher modes. For this reason, Multi Tuned Mass Damped (MTMD) systems have been proposed to be more efficient than the single TMD in vibration control covering wider frequency band of the dynamic loads. Xu and Igusa (1992) proposed multiple substructure system with closely spaced natural frequencies, combined with the main structure. Their results showed that this system is equivalent to a single viscous damping, added to the damping of structure. Igusa and Xu (1994) examined MTMD capabilities in controlling the vibrations. Their results showed that the MTMD is more effective and more robust than a single TMD with the same total mass. Since then, many researchers have conducted studies on MTMD systems, like Wu and Chen (2000) and Dehghan-Niri et al. (2010). Elias and Matsagar (2017) conducted a state-of-the-art review on the TMD and MTMD systems in response control of structures. Their research was about the theoretical background and developments of MTMD systems.

Although the performance of MTMD system is more trustful than a single TMD, but there is still the concern of detuning the TMDs and also the system might not have good performance in many values of the loading frequencies. It considers neither the characteristics of load exerting to the structure nor the response conditions. The idea of using Active Tuned Mass damper (ATMD) came up to solve this problem. Li and Liu (2002) worked on Active Multiple Tuned Mass Damper (AMTMD) under the ground acceleration. The result of that research was that AMTMD can remarkably improve the performance of the MTMD and has higher effectiveness than ATMD. Casado et al. (2010) conducted a research on the control of excessive footbridge vibrations, via passive and active TMD systems, which showed positive performance. Wen and Sun (2011) worked on the optimal design of hybrid system with multi distributed TMDs and ATMDs in 3rd Nanjing Bridge which showed good efficiency for the wind-induced vibration control. Wen and Sun (2014) conducted the same research but this time with ATMDs which performed well. Nagarajaiah and Jung (2014) researched on the smart tuned mass dampers (STMDs), among them active case, and discussed about the implementation of such systems in tall buildings and bridges.

The parameters of TMDs must be optimized to reach the best performance and also an algorithm is needed to calculate the optimum forces for active TMDs. Frans and Arfiadi (2015) studied the optimum locations and properties of MTMD systems, using Genetic algorithm. Alonso *et al.* (2015) assessed the performance of TMD, using Genetic algorithm and then compared it to the classical Den Hartog's proposal. Their results showed that this method is more effective than the classical one. Salvi and Rizzi (2016)

studied on the derivation of optimum tuning formulas for passive TMDs which their results are discussed in detail and easy to be used. Lavan (2017) performed multiobjective design of TMDs for base excitation and external load via Genetic algorithm by two approaches to solve the problem. Li et al. (2014) presented a method for vibration control of a building by ATMD and some controlling algorithms, among them Fuzzy Logic Algorithm (FLA). Ramezani et al. (2017) used Fuzzy system to determine optimal parameters of TMDs for seismic responses control of tall buildings. Nazarimofrad and Zahrai (2018) presented a mathematical model to obtain the seismic performance of an irregular multi-story building having two ATMDs at center of mass on the top floor. They employed the model to investigate the seismic response of 10 and 15-story asymmetric plan buildings in different cases using fuzzy logic and LQR forces for those two ATMDs.

In this paper, the performance of multiple passive and active TMDs is evaluated in controlling the seismic vibrations of Ahvaz cable-stayed bridge. The parameters of TMDs have been optimized using Genetic algorithm, while the Fuzzy logic algorithm is utilized for ATMD.

2. Equation of motion of passive and active MTMD systems

The MTMD system consists of a few TMDs that are built of mass blocks (made of concrete, steel, ice storage and so like), springs and dampers connecting the masses to the main structure. The main structure can be a building, bridge, chimney or any other kind of structures. The block's mass is related to the total mass of the main structure, with a parameter named "mass ratio" (µ) (TMD mass to the structure mass) with proposed values between 1 and 10%. The system can be in various forms; i.e., it may be translational or pendulum acting in horizontal or vertical directions, be a single or a multi TMDs system, be passive, semi-active, active or hybrid, be designed to control moving load produced vibrations, wind induced vibrations or the seismic (or the base acceleration motion) vibrations. In this paper, the translational MTMD system is used to control the seismic vibrations, through two cases of passive and active control systems.

The equation of motions for the uncontrolled structure is

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t)$$
(1)

where: **M**, **C** and **K** are the mass, damping and stiffness matrix of the structure, respectively, and the parameters, $\ddot{u}(t)$, $\dot{u}(t)$ and u(t) are the acceleration, velocity and displacement vectors of the structure, respectively. **F**(*t*) is the external load, inserted to the structure.

The structure can be equipped with the passive MTMD system and its equation of motions becomes

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\{\mathbf{r}\}\ddot{u}_{q}(t)$$
(2)

where the parameters are like the previous equation, this time for the sum of main structure and the TMDs, and also



Fig. 1 Side view of Ahvaz cable stayed bridge and the dimensions



Fig. 2 Some sections of the Ahvaz cable stayed bridge

 $\{r\}$ is the ground acceleration impact vector and the $\ddot{u}_g(t)$ is the ground acceleration vector.

If the structure is equipped with active MTMD system, the equation of motions becomes

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{E}\mathbf{F}(t) + \mathbf{D}\mathbf{V}(t)$$
(3)

where the parameters are defined like the passive equation. E and D, are the matrices and F(t) and V(t) are the vectors for the position and values of the external load and the control force, respectively.

The equation of motion in active system can be solved either in frequency domain or the space state. In this paper, the equation has been solved in space state.

3. Analytical model and verification

3.1 SAP2000 model of Ahvaz 8th bridge and verification

Ahvaz 8th bridge is the 8th bridge of Ahvaz City built over the most important river in Iran, Karun River. The bridge is a cable-stayed bridge with the total length of 641.9 m and the main span of 212 m long and 22 m wide. The deck is made of both precast and in situ concrete construction. The pylon type is diamond and made of concrete. The bridge has 2 pylons, each with two bundles of 8 cables in each side and the height of pylons is 81 m from the foundation. The side spans are maintained by the round concrete columns. The cost of building of Ahvaz 8th bridge was 40 million dollar, in its opening time in 2011. Fig. 1 shows the side view of the bridge, and some sections of the bridge are shown in Fig. 2.

In order to evaluate the performance of passive and active MTMD systems in controlling the seismic vibrations of the Ahvaz 8th bridge, the model of the bridge was built in SAP2000 (2008) verified with the main model of Hexa consulting company, the designer of the bridge, and the Y-LCP model proposed by Casado (2011), which was very



Fig. 3 Ahvaz cable stayed bridge model built in SAP2000



Fig. 4 Y-LCP model by Casado

Table 1 SAP2000 model verification of Ahvaz cable stayed bridge

Mode number	Paper model period (s)	Main model period (s)	Period error (%)	Y-LCP model period (s)	Paper model to Y-LCP model period ratio
1	2.09	2.14	2.33	2.05	1.019
2	1.46	1.50	2.66	1.56	0.935
3	1.44	1.48	2.70	1.30	1.107
4	1.20	1.25	4.00	1.16	1.034
5	1.07	1.11	3.60	0.99	1.080
6	0.91	0.92	1.08	0.91	1.00
7	0.88	0.9	2.22	0.90	0.977

similar to the Ahvaz 8th bridge. The model in SAP2000 is shown in Fig. 3 and the Y-LCP model in Fig. 4. The model verification is done by comparing the period values of different modes, in Table 1. The results showed that the model of this paper was assembled correctly in SAP2000 and the error in modeling was low and acceptable. Comparing the model of this study to the Y-LCP model showed that the period ratios in first 7 modes are almost equal, with little error. The differences between the periods of the current model and main bridge model have occurred due to different modeling methods, and the differences between periods of the current model and Y-LCP model are due to different characteristics of the models.

3.2 OpenSees model and verification

Although SAP2000 is a reliable and widely used program in civil engineering, it is not suitable for running a nonlinear time-history analysis, especially when the model is massive, like the current model; so, it is necessary to improve the finite element model. For this reason, the model is built in powerful finite element software, OpenSees (2008). First, the model is defined in a 3-D, 6 degrees of freedom space. Then the nodes, the nodal masses (no rotational masses), and the states of the supports are considered. The supports are fixed in all directions and accomplished by fix command. The 12 states of the geometric transformation functions, 6 for the linear and 6 for the p-delta, are used. Then the material types are defined. The decks, the pylons and the piers are made of C35 concrete and the girders and secondary beams of the deck in mid spans are made of S355J0 steel and all are arranged by uniaxial material command. The cables are also made of special steel with $F_u=1860$ N/mm². Concrete material is defined as Concrete01 only enduring the compression and the steel material is defined by Steel01, which has a bilinear diagram with a strain hardening. The cables material is perfectly plastic gap material which can consider the



Fig. 5 Ahvaz cable stayed bridge model in OpenSees, shown by OSP

Mode number	OpenSees model period (s)	SAP2000 model period (s)	Period error (%)
1	2.03	2.09	2.87
2	1.36	1.46	6.84
3	1.32	1.44	8.33
4	1.07	1.20	10.83
5	0.87	1.07	18.69
6	0.83	0.91	8.79
7	0.74	0.88	15.90

pretensioning, via its strain gap. Then the sections of the bridge parts have been defined. The bridge consists of 22 types of sections for the decks, pylons, piers, cables, piles and the caps piles. These sections are defined by fiber section and the characteristics are defined by patch quad and the reinforcing bars by layer command. The elements are written by force BeamColumn command which links the start node to the end node, with a number of integration points, section tag and a geometric transformation tag.

Some of the codes written in OpenSees are as

- 1. model BasicBuilder -ndm 3
- 2. node 1 228.0 10.1 -11.8
- 3. mass 93 209473.2 209473.2 209473.2 0.0 0.0 0.0
- 4. fix 525 1 1 1 1 1 1
- 5. uniaxialMaterial Steel01 1 353e6 200000e6 0.01
- 6. section Fiber 1 -GJ 689101 {
- patch quad 1 6 2 0.58 -0.3 0.58 0.3 0.56 0.3 0.56 -0.3

patch quad 1 3 6 0.56 -0.005 0.56 0.005 -0.56 0.005 -0.56 - 0.005

patch quad 1 6 2 -0.56 -0.3 -0.56 0.3 -0.58 0.3 -0.58 -0.3 } 7. geomTransf PDelta 4 -1 0 0

8. element forceBeamColumn 27 29 30 6 3 12

The model of the OpenSees is shown in the Fig. 5, which has been shown by OSP (OpenSees Post Processing).

After modeling the bridge in OpenSees, the model was verified. The verification has been done by several methods.

The first method is to use the TCL Editor to write the model codes in which if the orders are wrong, they are not shown in colors. The second method is to use the "puts" command. In this method, after the desired parts, the "puts" order is used to display the arbitrary phrase, if there is no error in modeling. The third way is to compare the modal periods of the model of OpenSees to the model of SAP2000 of Ahvaz Bridge. The comparison is presented in Table 2. The results showed that the model is constructed with acceptable error which is common, due to differences in modeling methods of SAP2000 and OpenSees. The fourth method is to run a time-history analysis in SAP2000 and the OpenSees and compare the responses of the models in two programs. The result of the time-history analysis which is here the displacement at the midpoint of the main span in longitudinal and transverse directions, under the record of the Manjil earthquake (Iran) in Abbar station is shown in Fig. 6. The results showed that the models are completely collaborative.

The reasons why the model periods in SAP2000 and OpenSees and also the time-history responses have little difference are:

1. The different types of mass assignment in two programs and the totally different way of building the



Fig. 6 Displacement time-history of OpenSees bridge model verified via SAP2000 model

Table 3 Earthquake records chosen for time-history analysis

Record	Record type	Earthquake	Country	Year	Magnitude (Richter)	Station
1		Manjil	Iran	1990	7.37	Rudsar
2	Far-field	Kobe	Japan	1995	6.9	HIK
3		Northridge	U.S.	1994	6.69	Featherly Park
4		Manjil	Iran	1990	7.37	Abbar
5	Near-field	Kobe	Japan	1995	6.9	Kobe University
6		Northridge	U.S.	1994	6.69	Arleta-Nordhoff fire sta.

models. 2. The differences between material introductions; as in SAP2000 material types are linear while in OpenSees, materials include nonlinear zone which is more logical. 3. The differences in introduction of elements. 4. The different analysis characteristics such as: number of integration points, the solution methods and so on. The verification showed that the models had acceptable condition and the differences were natural.

3.3 Earthquake records

In order to evaluate the performance of passive and active MTMD systems, a nonlinear time-history analysis is performed. For this purpose, 3 far-field and 3 near-field records are chosen (Table 3). The records are from the Manjil earthquake in Iran, the Kobe earthquake in Japan and the Northridge earthquake in U.S., all of them among the most famous and destroying earthquakes in those countries. The reason of choosing the records in two bundles of far-field and near-field is to evaluate the performance of MTMD systems against the earthquake record type.

3.4 TMD placements

Multi Tuned Mass Damper (MTMD) is used in the Ahvaz cable stayed bridge to control the seismic vibrations. The system is passive in case 1 and active in case 2. The number of TMDs changes between 1 and 8 here, to investigate the influence of this parameter on the performance of the system. The TMD is used to declare the horizontal vibrations of the bridge, especially in transverse direction. The displacement of the midpoint of the bridge main span is selected as the goal response to be decreased. In each case, the TMDs are placed in specific points of the bridge to be tuned with a special mode. As an example, Fig. 7 and Table 4 show the TMD locations in case of 8 TMDs. To prove the feasibility of the installation locations, a step by step sample calculation is followed here. If $\mu=0.1$ (the highest µ value), the mass of each TMD will be 727 ton. If the material is steel, the volume of each TMD is 93 m³ and a 5.5 m×5.5 m×3 m steel block can lead to this goal. It must be mentioned that in the practical applications, μ value will be less than 0.05 and the TMD sizes will be far less. So, the locations are reasonable in all cases.

	++
TMD8 TMD6 TMD	TMD1 TMD5 TMD7
TMD4	тмдз

Fig. 7 TMD locations in case of 8 TMDs

Table 4 The placement of MTMDs (X direction locations)

TMD No.	1	2	3	4	5	6	7	8
X location (m)	+6	-6	+12	-12	+20	-20	+45	-45

Table 5 Optimized values of TMDs parameters in state of 8 TMDs and total mass ratio of 0.1

Row	Total mass ratio (µ)	TMD No.	Mass (ton)	Frequency (rad/s)	Damping ratio (%)
1		TMD1	727.42	4.759	15.54
2		TMD2	727.42	4.782	16.89
3		TMD3	727.42	4.630	16.31
4	0.1	TMD4	727.42	4.596	17.08
5	0.1	TMD5	727.42	4.178	16.67
6		TMD6	727.42	4.248	17.73
7		TMD7	727.42	5.798	23.57
8		TMD8	727.42	5.841	24.19

3.5 Genetic algorithm

A good performance in TMD systems is expected only when its parameters are optimized. The mass ratios of TMDs are chosen equal and total mass ratios are 0.01, 0.02, 0.04, 0.06, 0.08 and 0.1; so the mass ratio is predefined. The frequency and damping ratio of the dampers are the parameters that must be optimized. From several methods of optimization, Genetic Algorithm (GA) is used for this purpose. GA is based on the revolution theory, proposed by Charles Darwin and nowadays it is widely used in many sciences, among them structural control. This algorithm deals with the chromosomes, gens, crossover, mutation and also the natural selection. The "desirable factor" is achieved by the objective function. The steps of optimization by this method are as below:

1. Some random answers (chromosomes) of the problem are added to the algorithm as the first generation. The number of these non-optimized answers depends on the problem.

2. On some of the first generation members, the crossover occurs and the objective functions of the answers are defined. The parents are chosen based on the random selection methods, such as roulette wheel selection.

3. The mutation occurs on some of the answers, produced by crossover. The rate of the mutation is often low, for example 0.02.

4. The mixed population is made by putting last generation, child produced by crossover and child produced by mutation, together.

5. From the mixed population, based on one of the methods, the m (number of the chromosomes in each

generation which is stable) most desired chromosomes are chosen to make the new generation.

6. The chromosomes are evaluated to clarify answer to this question: Are the chromosomes (answers) of the current generation close enough to the optimized one? If "yes", the algorithm ends and the best chromosome is selected as the answer. If "no", the algorithm will be iterated by the next generations until the answer becomes, "yes". The closure of the answers is assessed by predefined methods like specified number of iterations or generation, specified penalty value.

In order to follow such algorithm, a link between the programs OpenSees and MATLAB is needed. OpenSees includes the model of the bridge and the nonlinear timehistory analysis is run and the optimization is achieved by codes in MATLAB. The method is that the bridge is analyzed in OpenSees. Then the values of a defined response of the bridge (here, maximum displacement of midpoint of the main span in transverse direction) is transferred to the MATLAB and chosen as the goal. The parameters of the TMDs, i.e., damping and stiffness are changed through GA and then the new values are transferred to the OpenSees. Again the analysis is done with new parameters and the values of response are transferred to the MATLAB, again. This cycle is repeated in an iterative manner until it reaches acceptable answer.

After conducting the optimization steps, the values of the optimized parameters are achieved in each case of number of TMDs and each case of total mass ratio of TMDs. As an example, Table 5 shows the optimized values of the case of 8 TMDs with total mass ratio of 0.1. The total effective mass of the structure is 58194 tons and the TMDs



Fig. 8 Membership functions and the fuzzifier rules

masses are equal. The optimization parameters and information are as this

Find: md1, cd1, kd1,, md8, cd8, kd1				
MaxIt=300;	%number of iterations			
nPop=20;	%number of population			
pc=0.8;	%percentage of crossover			
<pre>nc=2*round(pc*nPop/2);</pre>	%number of crossovers			
gamma=0.05;	%selection parameter			
pm=0.3;	%percentage of mutation			
nm=round(pm*nPop);	%number of mutations			
mu=0.02;	% mutation rate			

The optimization of the parameters of TMDs and the active control has been executed by linking OpenSees to MATLAB. There are several methods in this way, among them calling OpenSees from MATLAB, calling MATLAB from OpenSees and the third way, using TCP/IP, from TCL language which is used in network connections.

3.6 Fuzzy Logic Control (System) (FLC)

In this paper, the 2nd case of control system deals with active TMDs. The active system includes:

1. Some TMDs (8 TMDs here) with optimized parameters, 2. Some sensors to achieve the entering load or acceleration if the system is open and some sensors to record the responses of different points of the structure if the system is close and both sensors, if the system is open-closed loop control. 3. A processor (control computer) to run the active control rules and to achieve the optimum control forces and 4. Some actuators to apply the control forces to the TMDs.

The steps of the active control system are as following: 1. A time-history analysis is run and the predefined responses are recorded only after one time step (for example 0.02s). 2. The analysis is stopped and the responses and exerted excitations, recorded by the sensors, are transmitted to the active control managing computer. 3. This active computer consists of an active control algorithm. The algorithm receives and processes the information and then the optimized forces are calculated. 4. The optimized forces are exerted to the structure via the actuators. Then the analysis goes to the next time step i.e., the time-history analysis is done for another time step. This process is repeated until the time reaches the end of analysis.

In this paper, the Fuzzy Logic Control (FLC) is chosen as the active control algorithm. The fuzzy logic algorithm was invented by Lotfizadeh (1965) and deals with the real world propositions, against the computer logic system which is based on true or false or 1 and 0. The FLC consists of:

1. Fuzzifier. This is the first step of this system converting the mathematical information to the language of Fuzzy logic. In this paper, the displacement of the midpoint in the main span of the bridge and the velocity are recorded by the sensors. Then these records are fuzzified as shown in Fig. 8. The exaggeration factor for the displacement is 0.4 m and for the velocity is 2 m/s. Membership value is a number between 0 and 1, instead of being 0 or 1.

The terms PL means positive and large, PS means positive and small, Zero means the zero values domain, NS means negative and small and the NL means negative and large. From this function, the values of displacement and velocity are converted to the discretional terms.

2. Fuzzy Inference Engine. After the entrance values fuzzified, there must be an engine to make decision about the output forces. The decision is made by the engine of fuzzy logic control, either Mamdani or Takagi-Sugeno engine.

3. Fuzzy Rule Base. The decision matrix is made of the fuzzy rules. Fuzzy rules are made based on "if-then" paraphrases. In this paper, Mamdani engine with a 5×5 Fuzzy search table is used. Table 6 shows the "if-then" rules, based on fuzzified values of displacement and velocity. As an example, if the velocity is PS and the displacement is NS, then the force must be PM. The membership function of the output force is shown in Fig. 9. The terms in Table 6 and Fig. 9 are as given in Fig. 8 and here the NM and PM terms mean negative and mild and positive and mild, respectively.

4. Defuzzifier. The terms achieved in step 3 are in fuzzy logic language and must be transformed again to the mathematical numbers in order to be exerted to the finite element model. Defuzzifier carries this duty. There are several methods for this purpose, such as Centroid of area



Fig. 9 Membership function of output force

Table 6	5×5	Fuzzy	search	table
---------	-----	-------	--------	-------

		Velocity			_	
NL	NS	Zero	PS	PL		
NL	NL	NM	NS	NS	PL	
NL	NM	NM	NM	NS	PS	
NS	Zero	Zero	Zero	PS	Zero	Displacement
PS	PM	PM	PM	PL	NS	
PS	PS	PM	PL	PL	NL	

Table 7 Variable parameters used in this paper

Row	Variable	Range
1	State of control system	Passive and active
2	Number of TMDs	1 to 8
3	Total mass ratio of TMDs	From 0.01 to 0.1
4	Type of seismic record	Far-field and near-field
5	Record peak acceleration	0.25 g, 0.4 g, 0.55 g and 0.7 g

(COA), Bisector of area (BOA), Smallest maximum (SM), Largest maximum (LM), Mean of maximum (MOM), Weighted average (WA) and so on. In this paper, the Centroid of area method is used which calculates the centroid of area of membership function as the output value. Eq. (4) is used in the COA method.

$$z_0 = \frac{\int \mu_C(z) z dz}{\int \mu_C(z) dz} \tag{4}$$

For example, if the displacement and the velocity from OpenSees, at a moment are 0.15 m and -1.0 m/s respectively, then the Fuzzifier terms are PS and NL from membership function. From the fuzzy search Table the output force is NL. Defuzzifier will change this term to an unscaled force between -0.8 and -1.0. Then this force is exerted to the model after being scaled.

4. Numerical results and discussion

4.1 Outline

The objective of this paper is to evaluate the performances of passive and active MTMD systems in controlling the vibrations of Ahvaz cable-stayed bridge. For this purpose, some parameters are changed to evaluate their effect on the performance of controlling systems. Also, some of the bridge responses are detected to be controlled. Table 7 shows the varying parameters used in this paper. The responses monitored in this paper are: Displacement and velocity of the midpoint of the main span, Root Mean Square (RMS) of the displacement, base shear of the pylons and their normalized values. The default PGA was 0.4 g.

4.2 Effect of number and mass ratio of TMDs

In this paper, the total mass ratio varies from 0.01 to 0.1 and also the number of TMDs changes from 1 to 8, to



Fig. 10 Effect of total mass ratio and number of TMDs on bridge displacement control



Fig. 11 Bridge deck displacement time-history for the Northridge earthquake (Featherly Park) in the transverse direction

investigate the effect of these parameters. The aim of using such system is to control horizontal vibrations of the main span. The TMDs are tuned to the different modes, according to their number. Fig. 10 shows the mass ratio-number of TMDs effect in passive case on decreasing the displacement of midpoint of main span in the transverse direction. The results for the longitudinal direction are alike.

The results showed that by increasing the values of total mass ratio of TMDs and also the number of TMDs, the system controls the displacement of the bridge deck in a better way. For example, the decrease of displacement improves from 15% to 28%, when the μ changes from 0.01 to 0.1, in 8 number of TMDs; but, the performance improvement of the MTMD system gets less until it reaches 0, around μ =0.1. From this value on, increasing mass ratio of TMDs has no effect on the control. As another example, the decrease amount changes from 19% to 26%, when the number of TMDs changes from 1 to 8, in μ =0.06.

4.3 The effect of earthquake records

Three far-field and three near-field records are exerted to the bridge to see the earthquake type effect on the control. Fig. 11 shows the effect of control systems on the displacement time-history of Featherly Park station of the Northridge earthquake record and Fig. 12 shows the decrease of maximum displacement values using passive and active cases, for the earthquake records in horizontal directions in two bundles of far-field and near-field records.

Fig. 11 shows that both passive and active MTMDs have controlled the displacement time-history during the earthquake. An important result from this figure was that both systems couldn't decrease the vibrations in first seconds of earthquake, which is a negative point of these systems. A shining point is that the uncontrolled structure, due to its low damping, couldn't dissipate the free vibrations; but the control devices helped it to dissipate the vibrations immediately, preventing the structure from more damages, especially when many elements have entered the plastic domain.

The results in Fig. 12 show that the active system controls the vibrations more effectively, in all records. The average decrease for the passive case was 28% and 25%, under the far-field and near-field records, respectively, and for the active system, 44% and 40%, under the far-field and near-field records, respectively. The results show that both cases of TMDs are more effective against far-field records in comparison with the cases under near-field records. Also, the diversity of control effectiveness is more against near-field records, 17% for Abbar station to 32% for Arleta-Nordhoff fire station, in passive case. This means that the active case is less related to the incoming records, in comparison with passive system showing its safety and robustness. Also, no especial relation was observed between the direction of the vibrations and the controlling power.



Fig. 12 Decrease of maximum displacement at the midpoint of the bridge main span under various earthquake records



Fig. 13 Root Mean Square (RMS) of displacement time-history at the midpoint of the bridge main span for the Featherly Park record



Fig. 14 Decrease of displacement RMS at the midpoint of the bridge main span under different earthquakes

4.4 RMS of displacement

Root Mean Square (RMS) of the displacement is monitored as another parameter to investigate the effect of control systems. The results of this evaluation are shown in Figs. 13 and 14.

Fig. 13 shows that using MTMDs decreases the values of RMS of displacement time-history throughout the earthquake. The active system was more effective than the passive one. Especially, the systems were successful in controlling the peak values of RMS, here in second 28. From second 0, the RMS diagrams for uncontrolled, with passive system and with active system, got far away from each other, showing that the control systems are working better after some few seconds from the beginning of excitation.

The results in Fig. 14 were almost the same as those for displacement. The active system worked more effectively and the results for the far-field records were better than those under the near-field records. In average, the decrease for the passive system was 26%, and 39% for the active system. The decrease rates under the near-field records was



Fig. 15 Bridge deck velocity time-history for the Northridge earthquake (Featherly Park) in transverse direction



Fig. 16 Normalized peak values of bridge deck velocity for the earthquake records in different cases

24% and 36%, by using passive and active systems, respectively, and 30% and 42% under the far-field records. The diversity of the results in near-field records was high, more in passive system, showing the uncertainty against these kind of records.

4.5 Velocity

Although the first aim was to control the deck displacement, but the decrease in velocity time-history is also detected. Fig. 15 shows the velocity time-history of Featherly Park record of the Northridge earthquake and Fig. 16 shows the normalized peak values of the velocity of the records, in both horizontal directions.

The results of Fig. 15 show that the systems controlled the velocity, however not as effective as the case for displacement. The control by the active system is considerable, contrary to the passive system. The results also show that the control systems are effective to dissipate the free vibrations, where the bare structure (structure without any control systems) is unable to terminate the remained vibrations, immediately.

Fig. 16 shows the normalized values of the maximum velocity along its time-history. As shown, the active system alleviated the values from 1.0 to 0.725 in average, where the passive system could reduce the values from 1.0 to only 0.873. This occurred because of the fact that the parameters

of the TMDs were optimized based on the displacement, not velocity. As in the past responses, the systems were more prosperous against the far-field excitations.

4.6 Base shear

Another important factor, base shear, is also monitored as a controlled response, in this paper. Decease in values of base shear leads to decreasing the forces entering the structure and so, decreases the demands exerted to the elements in the structure, preventing them to get nonlinear.

Fig. 17 shows the normalized peak values of base shear of the pylons, in horizontal directions. The results showed that both active and passive systems were promising, mostly the active system. Active system decreased the base shear from 1.0 to 0.69 averagely and the passive system decreased from 1.0 to 0.83. Also, the results showed that the systems could control the base shear more effectively, for the Featherly Park record, which was used to optimize the MTMDs parameters.

4.7 Peak record acceleration effect

By scaling the PGA of the records, to 0.25 g, 0.4 g, 0.55 g and 0.7 g, the effect of this parameter is considered in this study. For this purpose, the Abbar record of the Manjil earthquake is chosen. The effect in both directions is



Fig. 17 Normalized peak values of bridge pylon base shears for different earthquake records



Fig. 18 The effect of PGA on the performance of MTMD system

considered by the decrease of maximum values of displacement via the passive MTMD system. The results (see Fig. 18) showed that in longitudinal direction, the PGA has less effect where from 0.5 g on, the control efficiency has decreased from 25% to 23%. In transverse direction, the control values had no change from 0.25 g to 0.4 g, then it decreases from 32% to 27% and then the steep gets lower and diagram reaches 25% (Fig. 18). The cause of this change is that in transverse direction, the structure sustains more demands and some of the elements enter plastic range. This causes internal damping in those elements, so that the share of the MTMD system becomes lower, in total damping of the structure. Another reason is that the TMDs may get damaged and detuned, in such high acceleration.

4.8 Acceleration

The other response of the bridge that was influenced by adding the MTMDs is the acceleration time-history. The acceleration inserted to the structure will be impressed by the structural elements characteristics and an acceleration response history is achieved in every single point of the structure. The acceleration at midpoint of the main span is chosen to detect the impact of using MTMDs.

As shown in Fig. 19, the peak values of the reference point in three cases: without MTMDs, with passive MTMDs and the structure with passive and active MTMDs against six earthquakes and in two directions are under comparison. Fig. 19 shows that the controlling system succeeds in reducing the values in all cases, but not as well as the peak values of the displacement and velocity. The results also showed that the performance of the system in reducing the acceleration values somehow resembles to the pylon base shear impressed by the system. The best performance of the system was against the Featherly park earthquake record, especially at the longitudinal direction. The difference of the performance under different records shows that the time-history analysis is sensitive to the chosen records and thus precise selection must be made for design purposes. In the best case the values reduced from normalized 1.0 to 0.59 and 0.74, respectively by PTMD and ATMD systems.

5. Robustness of the system

For the structures enhanced with the control systems, there is always concern about the uncertainties, among them, the changes in the structure's parameters, the changes in the controlling system's parameters and uncertainties about the



Fig. 19 Normalized peak values of acceleration for the earthquake records in different cases

Table 8 Reduction percent of the bridge responses by the passive MTMDs system, when the lateral stiffness and damping ratio change by $\pm 5\%$, $\pm 10\%$, $\pm 20\%$, $\pm 50\%$ and $\pm 100\%$, in case of 8 TMDs and μ =0.1

	Lateral stiffness or	Lateral	stiffness	Damping ratio		
Row	damping ratio change (%)	Displacement reduction (%)	Base shear reduction (%)	Displacement reduction (%)	Base shear reduction (%)	
1	-100	0.2	0	3.3	4.1	
2	-50	2.1	1.6	4.5	6.3	
3	-20	11.6	9.3	16.6	11.9	
4	-10	19.5	12.5	21.1	14.8	
5	-5	23.7	16.1	24.3	16.2	
6	0	26.5	17.0	26.5	17.0	
7	5	22.8	15.9	26.0	16.6	
8	10	18.2	12.8	24.9	14.7	
9	20	13.3	8.8	20.7	12.4	
10	50	1.7	1.0	6.8	6.0	
11	100	0.3	0.1	4.5	5.5	

input loads. The uncertainties of the structure are mostly about the lateral stiffness of the structure, related to the pylons, piers and the cables and also about the amount of inherent damping of the structure. In the design of the structures, there are always some parameters that for the real structure, they may have different amounts, in comparison with the design values. These differences come from: simplified design assumptions, differences or changes in the mechanical parameters, any errors during building the structure and so on. Thus, in one hand, for the real structure, the amount of the lateral stiffness and the damping ratio may be different from those considered in the design, by any changes when building the structure or by the effects of the loads especially the seismic loads that can enforce the structure enter the inelastic zone. On the other hand, the controlling system (MTMD) parameters are set and tuned with the design values and any changes of such values in the real structure, can make the system detuned. So, the system might not have the expectable performance and there is a need to testify robustness of the system.

In this paper, the lateral stiffness and damping ratio of the Ahvaz cable-stayed bridge are changed by $\pm 5\%$, $\pm 10\%$, $\pm 20\%$, $\pm 50\%$ and $\pm 100\%$, to test the robustness. About the lateral stiffness, although it is logical that for the real structure, its values be less than the design values due to the cracking, it is considered to be even two times bigger, due to the human errors in estimating and considering the mechanical properties of the sections, especially the compressive strength of the concrete impressing the elasticity modulus. Tables 8 and 9 present the impact of such changes on the robustness of the passive and active control systems. In these tables, the robustness is tested in two cases, i.e., the changes in lateral stiffness and damping. In each case, among different responses of the bridge, two of them are considered, i.e., the displacement of the midpoint of the main span and the base shear of the bridge. For these responses, the average impact of the system in reducing the excitations under the six earthquake timehistories, in the case of 8 TMDs and μ =0.1 is compared

Row	Lateral stiffness or damping ratio change (%)	Lateral stiffness		Damping ratio	
		Displacement reduction (%)	Base shear reduction (%)	Displacement reduction (%)	Base shear reduction (%)
1	-100	4.1	3.8	6.0	5.2
2	-50	17.9	15.6	19.9	17.0
3	-20	33.2	24.9	34.0	25.8
4	-10	38.9	28.7	38.4	29.6
5	-5	41.6	30.4	41.7	30.6
6	0	42.0	31.0	42.0	31.0
7	5	41.2	30.1	41.4	30.3
8	10	37.5	28.3	38.9	29.1
9	20	32.4	24.5	34.4	25.3
10	50	18.3	15.8	20.2	16.4
11	100	5.5	3.3	7.2	4.9

Table 9 Reduction percent of the bridge responses by the active MTMDs system, when the lateral stiffness and damping ratio change by $\pm 5\%$, $\pm 10\%$, $\pm 20\%$, $\pm 50\%$ and $\pm 100\%$, in case of 8 TMDs and μ =0.1



Fig. 20 Robustness of normalized reduction of maximum bridge displacement to lateral stiffness reductions under different earthquake records and control cases (A: active – P: passive)

with the bridge with no controlling system and the response reduction percent is monitored. Then this reduction percent is compared to the case with no changes of the lateral stiffness and damping ratio to check the robustness.

These results showed that the MTMD had its best performance right where the system was tuned and no changes in the values of lateral stiffness and damping of the structure occurred. As much as the changes in structural parameters increase, the efficiency of the system reduces. For the passive case, the effectiveness of the system decreased with a higher steep in comparison with that of the active system. This showed that the passive system was more sensitive to the changes in structural parameters, and it needs to be tuned exactly. For the passive system, the reduction percent of the displacement and base shear, decreased from 26.5% and 17% to 19.5% and 12.5%, respectively, when the lateral stiffness changed -10%. These amounts were 11.6% and 9.3% respectively, by -20% changes in lateral stiffness. This happened smoother for the active system where displacement and base shear reductions get lower from 42% and 31% to 38.9% and 28.7%, respectively, by -10% changes in lateral stiffness and to 33.2% and 24.9%, respectively by -20% change of the lateral stiffness of the bridge. By higher change of parameters, i.e., \pm 50% and \pm 100%, the efficiency of the system reduces extremely. Similar results were observed for the changes in damping.



Fig. 21 Robustness of normalized reduction of maximum bridge base shear to lateral stiffness reductions under different earthquake records and control cases (A: active – P: passive)



Fig. 22 Robustness of normalized reduction of maximum bridge velocity to lateral stiffness reductions under different records and control cases (A: active – P: passive)

For further clarification, the system robustness is tested for different values of the MTMD parameters as shown in Figs. 20-23. Figs. 20-22 highlight the robustness in different cases for the state of the system, the records, responses and mass ratio of the system, i.e., all the cases studied in the previous parts. The normalized reductions (to properly controlled system) of maximum values of displacement, base shear and velocity of the bridge, are detected by 5%, 10%, 20% and 50% lateral stiffness reductions. To investigate the effect of mass ratio of the MTMDs on the robustness of the system, two amounts of μ values, 0.02 and 0.1, are considered and the normalized peak displacement reduction at deck midpoint is monitored. The structural stiffness is reduced by 5%, 10%, 20%, 50% and 100% for this aim compared to properly controlled case.

Based on obtained results, the less the structural parameters have errors for any reason, the more the system is robust. Active system is generally more robust, especially when the change percent is less.



Fig. 23 Robustness of the system for MTMD mass ratio (0.02 and 0.1) to different lateral stiffness reductions

Another result is that as the algorithm was set to the displacement to achieve the system's parameters, the robustness against the displacement is more, in comparison with the velocity. Fig. 23 proves that for the higher values of the mass ratio, the system acts more efficiently and is more reliable. For the mass ratio of 0.02, the system could not decrease the responses for 50% and 100% lateral stiffness change, even for the active control and the efficiency of the system declines with a higher steep.

Comparing the robustness of the system against the changes in lateral stiffness and damping of the structure with each other showed that if both these structural parameters change, the efficiency of the system gets even lower, whether this change be positive or negative.

The results proved that the robustness of the active MTMDs system (Table 9) is higher than that of the passive system (Table 8) and also proved that the system still could control the excitations, even if it gets a bit detuned.

6. Conclusions

This paper evaluated the performance of passive and active MTMD in controlling the seismic vibrations of Ahvaz 8th bridge. For this purpose, the model of the bridge was built in SAP2000 and then verified with the main model of the consulting company of the bridge, Hexa co. Then the model was made in finite element software, OpenSees in order to perform a nonlinear time-history analysis verifying the model made in SAP2000. Three farfield and three near-field records, from the Manjil, Kobe and Northridge earthquakes were chosen to study the effect of the record type. Then the bridge equipped by: case1, passive MTMD and case 2, active MTMD. The parameters of the TMDs were optimized by the Genetic Algorithm (GA) and the active system was performed via Fuzzy Logic Control (FLC). Then the response of the bridge was detected to evaluate the performance of the systems: 1. Midpoint of the main span displacement and the effect of number and total mass ratio of TMDs. 2. RMS of

displacement and also velocity at the midpoint of the main span and 3. The pylon base shear. The effect of the earthquake records and PGA of the records also was considered. The results showed that:

1. Both passive and active MTMD systems decreased the displacement. The effectiveness of the active system was more, as expected and the diversity of the results in active case and against the far-field records was lower, showing the more safety and robustness

2. By increasing the total mass ratio from 0.01 to 0.1, the systems worked more effectively. However, the steep of the diagram reaches almost 0, in μ =0.1 which means that since then, increasing mass ratio is not logical. By adding the number of TMDs from 1 to 8, the systems performance improved. As an example, the decrease of displacement in mass ratio of 0.04 reached from 17% for 1 TMD to 24% for 8 TMDs.

3. The passive system decreased the displacement 28% and 25% averagely, against far-field and near-field excitations, respectively and 44% and 40%, by the active system, showing that the active system was more successful and the systems performed a bit better against the far-field earthquakes. The control systems could decrease the RMS of the displacement 30% and 42% by the passive and active systems against far-field records and 24% and 36% by passive and active systems, respectively, against the near-field records.

4. The normalized velocity at the midpoint of the main span decreased from 1.0 to 0.873 and 0.725, using the passive and active systems, in average. The results showed that although the parameters were optimized by the displacement, the velocity also was controlled, especially with active systems. The pylon normalized base shear reached from 1.0 to 0.83 and 0.69, by the passive and active systems, respectively, which showed that the systems could control the entering excitation and reduce the demands on the bridge members.

5. The results achieved by changing the Abbar record's PGA from 0.25 g to 0.4 g, 0.55 g and 0.7 g, showed that this parameter has low effect in the longitudinal direction. In

transverse direction, the performance of the MTMD systems got lower, with a variable steep, maybe by detuning the TMDs.

6. The peak values of the acceleration were also studied for which, the system succeeded to reduce the values, not as well as the displacement and velocity. At the best case, the PTMD and ATMD decreased the normalized values from 1.0 to 0.59 and 0.74, respectively.

7. The robustness of the MTMDs through changing the lateral stiffness and damping of the bridge showed that the system still could control the excitations if it gets detuned due to the uncertainties of these structural parameters and the efficiency of the control system reduces with increasing such uncertainties. The results also showed that the active system is more robust than the passive case.

References

- Casado, A.K. (2011), Seismic behavior of cable-stayed bridges: Design, analysis and seismic devices, *Doctoral thesis*, Department of continuum mechanics and theory of structures, Universidad Politechnica de Madrid.
- Casado, C.M., Diaz, I.M., Sebastian, J., Poncela, A.V. and Lorenzana, A. (2010), "Implementation of passive and active vibration control on an in-service footbridge", *Struct. Control Health Monit.*, 20(1), 70-87. https://doi.org/10.1002/stc.471.
- Dehghan-Niri, E., Zahrai, S.M. and Mohtat, A. (2010), "Effectiveness-robustness objectives in MTMD system design: An evolutionary optimal design methodology", *Struct. Control Health Monit.*, **17**(2), 218-236. https://doi.org/10.1002/stc.297.
- Den Hartog, J.P. (1947), *Mechanical vibrations*, 3rd Ed., New York: McGraw-Hill.
- Elias, S. and Matsagar, V. (2017), "Research developments in vibration control of structures using passive tuned mass dampers", *Annu. Rev. Control*, **44**, 129-156. https://doi.org/10.1016/j.arcontrol.2017.09.015.
- Falcon, K.C., Stone B.J., Simcock, W.D. and Andrew, C. (1967), "Optimization of vibration absorbers: A graphical method for use on idealized systems with restricted damping", *J. Mech. Eng.* Sci., 9(5), 374-381. https://doi.org/10.1243/JMES_JOUR_1967_009_058_02.
- Frahm, H. (1909), "Devices for Damping Vibration of Bodies", US Patent.
- Frans, R. and Arfiadi, Y. (2015), "Designing optimum locations and properties of MTMD systems", *Civil Eng. Innov. Sustain*,, **125**, 892-898. https://doi.org/10.1016/j.proeng.2015.11.079
- Igusa, T. and Xu, K. (1994). "Vibration control using multiple tuned dampers", J. Sound Vib., **175**(4), 491-503. https://doi.org/10.1006/jsvi.1994.1341.
- Ioi, T. and Ikeda, K. (1978), "On the dynamic vibration damped absorber of the vibration system", *Bull. Jap. Soc. Mech. Eng.*, 21(151), 64-71. https://doi.org/10.1299/jsme1958.21.64.
- Jimenez-Alonso, J.F. and Saez, A. (2015), "Estimating robust optimum parameters of tuned mass dampers using multiobjective genetic algorithms", *Proceedings of the 3rd International Conference on Mechanical Models in Structural Engineering (CMMoST)*, Sch. Architecture, Seville, Spain.
- Lavan, O. (2017), "Multi-objective optimal design of tuned mass dampers", *Struct. Control Health Monit.*, 24 (11), https://doi.org/10.1002/stc.2008
- Li, C. and Liu, Y. (2002). "Active multiple tuned mass dampers for structures under the ground acceleration", *Earthq. Eng. Struct. D.*, **31**(5), 1041-1052. https://doi.org/10.1002/eqe.136.
- Li, Z.J., Zuo, S.Y. and Liu, Y.Y. (2014), "Fuzzy Sliding Mode

Control for Smart Structure with ATMD", *Proceedings of the* 3rd Chinese Control Conference (CCC), Nanjing, 21-25.

- Nagarajaiah, S. and Jung, H.J. (2014), "Smart tuned mass dampers: recent developments", *Smart Struct. Syst.*, **13**(2), 173-176. https://doi.org/10.12989/sss.2014.13.2.173.
- Nazarimofrad, E. and Zahrai, S.M. (2018). "Fuzzy control of asymmetric plan buildings with ATMD considering soil structure interaction", *Soil Dynam. Earthq. Eng.*, **115**, 838-852. https://doi.org/10.1016/j.soildyn.2017.09.020.
- OpenSees (2008), "Open system for earthquake engineering simulation", Pacific earthquake engineering research center, University of California, Berkeley, USA.
- Ormondroyd, J. and Den Hartog J.P. (1928), "The theory of dynamic vibration absorber", *Transaction of the ASME*, **50**, 9-22.
- Ramezani, M., Bathaei, A. and Zahrai, S.M. (2017), "Designing fuzzy systems for optimal parameters of TMDs to reduce seismic response of tall buildings", *Smart Struct. Syst.*, **19**(3), 61-74. https://doi.org/10.12989/sss.2017.19.3.269.
- Salvi, J. and Rizzi, E. (2016), "Closed-form optimum tuning formulas for passive Tuned Mass Dampers under benchmark excitations", *Smart Struct. Syst.*, **17**(2), 231-256. https://doi.org/10.12989/sss.2016.17.2.231.
- SAP2000 (2008), "Integrated structural analysis and design software", *Computers and Structures Inc.* Berkeley, California, USA.
- Snowdon, J.C. (1959), "Steady-state behavior of the dynamic absorber", J. Acoust. Soc. Am., 31(8), 1096-1103. https://doi.org/10.1121/1.1907832.
- Tori, K., Ikeda, K. and Nagasaki, T. (1968). "A non-iterative optimum design method for cable-stayed bridges", In Processing of Japan Society of Civil Engineers, 115-123.
- Veranzio, F. (1595), Machinae Novae, Heinz Moos Verlag.
- Wen, Y.K. and Sun, L.M. (2011), "Research on wind response control of large cable-stayed bridge under construction by using hybrid system of TMDs and ATMDs", *Eng. Mech.*, 28(7), 171-179.
- Wen, Y.K. and Sun, L.M. (2015), "Distributed ATMD for buffeting control of cable-stayed bridges under construction", *Struct. Stab. Dynam.*, 15(3). https://doi.org/10.1142/S0219455414500540.
- Wilson, J. and Gravelle, W. (1991). "Modeling of a cable-stayed bridge for dynamic analysis", *Earthq. Eng. Struct. D.*, 20,707-771. https://doi.org/10.1002/eqe.4290200802.
- Wu, J.N. and Chen, G.D. (2000), "Optimization of multiple tuned mass dampers for seismic response reduction", *Proceedings of* the American Control Conference, Chicago, IL, U.S.
- Xu, K. and Igusa, T. (1992), "Dynamic characteristics of multiple substructures with closely spaced frequencies", *Earthq. Eng. Struct.* D., **21**, 1059-1070. https://doi.org/10.1002/eqe.4290211203.

HJ