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Nonlinear optimal control for reducing vibrations in civil structures using smart devices

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Abstract. The frequently excessive vibrations presented in civil structures during seismic events or service conditions may result in users' discomfort, or worst, in structures failure, producing economic and even human casualties. This work contributes in proposing the synthesis of a nonlinear optimal control strategy for semiactive structural control, with the main characteristic that the synthesis considers both the structure model and the semiactive actuator nonlinear dynamics, which produces a nonlinear system that requires a nonlinear controller design. The aim is to reduce the unwanted vibrations in the response of civil structures, by means of intelligent fluid semiactive actuator such as the Magnetorheological Damper (MRD), which is a device with a low level of power consumption. The civil structures for which the proposed control methodology can be applied are those admitting a state-dependent coefficient factorized representation model, such as buildings, bridges, among others. A scaled model of a three storey building is analyzed as a case study, whose dynamical response involves displacement, velocity and acceleration of each one of the storeys, subjected to the North-South component of the September 19th., 2017, Puebla-Morelos (7.1M), Mexico earthquake. The investigation rests on comparing the structural response over time for two different conditions: with no control device installed and with one MRD installed between the first floor and the ground, where a nonlinear optimal signal for the MRD input voltage is determined. Simulation results are presented to show the effectiveness of the proposed controller for reducing the building's dynamical response.

Keywords: structural control; nonlinear optimal control; MR damper; seismic protection; semiactive control

1. Introduction

In recent years, topics about vibration control in civil structures have been gaining more relevance due to the importance that represents having structures capable of resisting dynamic disturbances. Any type of civil structure localized in strong environmental vibrations, seismic or strong wind zones, including service loads, will be subject to vibrations along its service life. Vibrations may vary from harmless to severe, and may produce discomfort for users, structural harm or structural failure.

One way to protect civil structures from dynamic loads or to make them more comfortable for users, respect to vibrations, is by installing control devices, which are responsible for moving or controlling the structure (Xu and He 2017). Control devices play a critical role in smart civil structures and allow the alternation of structural characteristics as well as the reduction of structural responses in a passive, active or semiactive way (Xu and He 2017). The development of control systems in civil structures has produced an increasing interest in the smart structures field (Song *et al*. 2006); nevertheless, the first mention and use of the term structural control in civil engineering dates from few more than 40 years ago (Yao 1972). Protection to dynamic loads can be achieved through passive structural control (Maddaloni and Occhiuzzi 2014, Thenozhi and Yu 2013), active structural control (Soong and Spencer 2000, Luca *et al*. 2005) or semiactive structural control (Nguyen *et al*. 2007, Lin *et al*. 2010, Kori and Jangid 2009).

Structural passive control (Soong and Dargush 1997) uses passive devices that are non-controllable and require no additional energy to operate, due to the fact that they will not add energy to the system (Thenozhi and Yu 2013). The objective of these systems is to absorb a significant amount of seismic input energy, thus reducing the demand on the structural system. This kind of energy dissipation devices may take many forms and dissipate energy through a variety of mechanisms including the yielding of mild steel, viscoelastic action in rubber-like materials, shearing of viscous fluid, orificing of fluid and sliding friction (Symans and Constantinou 1999). Most of the passive devices can be tuned only to a particular structural frequency and damping characteristics, and sometimes these tuned values will not match the input excitation and the corresponding structure response, as these devices can not adapt to the structure response changes, thus they can not assure a successful vibration suppression (Fisco and Adeli 2011a). Such systems and their control schemes have been widely review by Constantinou *et al*. (1998) and recently applied to new materials by Flodén *et al*. (2015) and Saedi *et al*. (2017) among others.

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By the other hand, active and semiactive structural control, in a typical way, determinate the instantaneous control force necessary to stabilize a structure by reducing its dynamical response based on the feedback information from the measured response of the structure and/or feedforward information from the external excitation (Symans and Constantinou 1999), thus an effective control algorithm to compute the magnitude of forces to be applied to the structure is needed. Forces in active structural control (Spencer-Jr. and Soong 1999, Soong and Spencer-Jr. 2002) are generated, in some cases, through controllable active devices like electrohydraulic or electromechanical actuators demanding large power sources. An extensive review of devices for active structural control such as active tuned-mass dampers, active tendons and memory alloys is carried out in Korkmaz (2011) as well as applications to floor structures (Nyawako *et al*. 2015) and for wind and earthquake engineering (Hochrainer 2015). In the semiactive control case, supplemental damping devices (Spencer-Jr. and Sain 1997), also called smart control devices (Kerboua *et al*. 2014) are controllable, and combine the positive aspects of passive and active control ones, besides of requiring a small amount of energy to operate. Semiactive devices generally originate from a modified passive device, allowing to adjust its mechanical properties based on feedback from the excitation and/or from the measure response; however, the control forces are developed as a result of the motion of the structure itself, acting primarily to oppose the movement, and hence promote the global stability of the structure (Symans and Constantinou 1999); these devices include semiactive stiffness dampers, electrorheological dampers, magnetorheological dampers, piezoelectric dampers, tuned mass dampers and tuned liquid column dampers (Fisco and Adeli 2011a). There are promising devices in relation to its behavior, like devices with controllable fluids, named intelligent fluids, such as electrorheological or magnetorheological ones (Ha *et al*. 2013) that offer the adaptability and performance of the active devices, which represent an alternative as control mechanisms for energy dissipation, as well as being fail-safe control system, that is, in the case that the control system fails, they are no capable of applying external energy that may turn the structure unstable (Medina *et al*. 2008). Some control schemes for semiactive devices have been review in Symans and Constantinou (1999), Jung *et al*. (2004) and recently applied for magnetorheological dampers (Enriquez-Zarate *et al*. 2015) or in base isolation systems such as spherical friction pendulums (Weber *et al*. 2017).

Early attempts on structural control are based on the use of existing control algorithms developed in other fields, but research has shifted to modifying the existing control algorithms or developing new ones to suit the complex nature of civil structures (Fisco and Adeli 2011b). In the case of linear control of civil structures, proportional-integral-derivative (PID) control has been widely conducted for practical applications, for systems with one or two degrees of freedom. For multivariable systems, the control algorithm becomes complex, which make them unsuitable for the applications like vibration control of multi-degree of freedom flexible structures (Thenozhi and Yu 2013). Investigations showing ineffective or less efficient PID results respect to other controllers like fuzzy logic or sliding mode have been conducted by Nerves and Krishnan (1995), Guclu (2006), Guclu and Yazici (2008). On the other hand, the most basic and commonly used optimal controller is the linear quadratic regulator (LQR), where optimal control algorithms are based on the minimization of a quadratic performance index termed as cost functional, which evaluate the system state at a desired value with a minimum control effort (Nerves and Krishnan 1995). A modified LQR controller based on energy of the structural system is proposed in Alavinasab and Moharrami (2006). Sometimes states of the structures are measured indirectly using observers like Kalman filters; the addition of an observer to an LQR control strategy leads to what is termed as the linear quadratic Gaussian (LQG) controller (Zhang and Roschke 1999). Also, H_∞ control technique is one widely used linear robust control scheme in structural vibration control; this technique is insensitive with respect to the disturbances and parametric variations, which makes them suitable for the multiple-input multiple-output type structural control systems (Utkin 1990), but normally the design results in a higher order system which will make the implementation more difficult (Saragih 2010). A modified H_∞ controller with pole-placement is presented in Park *et al*. (2008). Note the previous controllers are based on linear control designs, which usually guarantee an adequate behavior close to a local region for which they are tuned. Nonetheless, nonlinear control methodologies, which take into account the nonlinearities of the system should improve the controller performance in a larger or global region. Nonlinear control techniques such as Takagi-Sugeno fuzzy (Chen *et al*. 2004), Takagi-Sugeno-Kang fuzzy inverse and Max-Min algorithms (Askari *et al*. 2016), sliding mode control (SMC), simple adaptive control (Javanbakht and Amini 2016) or neural network (NN)-based SMC have been reported by a number of authors (Nezhad and Rofooe 2007, Yakut and Alli 2011) where a switching control law is used to drive the system's state trajectory onto a predefined surface in the state-space, which in the case of structural vibration control, corresponds to a desired system dynamics. The robustness of the SMC against the uncertainties and parameter variations makes them a better choice for structural control applications, nevertheless the direct implementation of the control law will result in chattering effect due to the imperfection in the high-frequency discontinuous switching (Thenozhi and Yu 2013). Intelligent control techniques like NN, fuzzy logic, Wavelet-based algorithm and genetic control algorithm have also been proposed for vibration control of structures; a review of these control techniques or modifications to them can be found in Fisco and Adeli (2011b), Thenozhi and Yu (2013) and more recently in Reza and Fariba (2017). Among the revised control techniques, it is worth mentioning that the nonlinear optimal control has not been previously proposed, where in addition the MRD dynamics is considered in the controller design, which comprise a promising class of semiactive control devices for attenuation of vibrations and improving the seismic behavior of civil structures.

The main contribution of this work is the design of a nonlinear optimal controller for reducing vibrations in civil structures, where both the structure model as well as the nonlinear dynamics of the MRD are taken into account, which represents an important challenge in the control synthesis due to the intrinsic nonlinear nature of the system, and additionally, by being an optimal control scheme, the control effort to reduce vibrations, through the MRD, is minimized. The nonlinear systems (civil structures) for which the control methodology can be applied are those admitting a state-dependent coefficient factorized (SDCF) representation (Haessig and Friedland 2002), such as n-storeys buildings, vehicular bridges, pedestrian bridges, towers, among others. A detailed procedure for the control design is given. Simulation results are presented for a scaled model of a three storey building, including an MRD installed between the first floor and the ground, evidencing the effectiveness of using the proposed control strategy to achieve an adequate damping of the system, reducing the unwanted civil structure response.

The paper is organized as follows. Section II describes the civil structure mathematical model, by establishing the motion equation that describes its dynamical behavior. An optimal control scheme for nonlinear systems is presented in Section III. In Section IV the structural control using the proposed optimal controller, applied to a civil structure with MRD case study, is presented. Section V concludes the paper.

2. Description of the system

This section describes the model of a general civil structure, with the inclusion of the damper dynamics, named the structural system.

2.1 Civil structure dynamical model

For the control design of a civil structure, it is relevant to know its dynamic characteristics, through its mathematical model, which will allow to synthesize a control strategy to produce the expected dynamic behavior in the structure, within an operation region (Forrai *et al*. 2001, Zhang and Roschke 1999).

The equation of motion for a civil structure, such as a n-storey building, subject to a disturbance force applied in its base, represented by a one-dimensional ground acceleration $\ddot{x}_g$, can be modeled through the second Newton's movement law. The n-th storey of a building possess a m_n mass, a viscous damping coefficient c_n and a stiffness coefficient k_n These parameters are considered, respectively, in the mass matrix (M), the damping matrix (C_s) and the stiffness matrix (K) of the building, such that the structure dynamics from the Newton's movement law is described by (Chopra 2012)

$$M\ddot{x}_s + C_s\dot{x}_s + Kx_s = -M\lambda\ddot{x}_g \tag{1}$$

where n is the number of storeys; $x_s \in \mathbb{R}^n$ is the displacement vector; $\dot{x}_s$ is the velocity vector and $\ddot{x}_s$ is the structure's acceleration vector; λ is a vector of influence coefficients, which allows to specify which degrees of freedom are excited by the ground motion (Williams 2016), due to possible external disturbances produced by seismic events or strong environmental vibration. For illustrative purposes, this paper particularly deals with a scaled model of a three storey building, nonetheless the modeling methodology is applicable for an arbitrary number of storeys, or can be applied for modeling bridges, etc.

2.2 Structure model with damper input force

For the case that a n-degree of freedom structure is equipped with dampers, for instance an MRD, to produce an input force, system (1) results in

$$M\ddot{x}_s + C_s\dot{x}_s + Kx_s = -\Gamma f_d - M\lambda\ddot{x}_g \tag{2}$$

where f_d is the force generated by the damper installed in the structure. Term Γ is a vector that indicates the dampers' location in the structure. By defining a state-space vector $x = [x_s \quad \dot{x}_s]^T$, system (2) can be rewritten in a state-space representation as

$$\dot{x}_d = Ax + Bf_d + E\ddot{x}_g \tag{3}$$

where the corresponding matrices are given by

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_s \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1}\Gamma \end{bmatrix}, E = \begin{bmatrix} 0 \\ \lambda \end{bmatrix}.$$

Note that f_d is the damping input force that will be used to reduce the vibrations in the structure through the MRD.

2.3 Dynamical model of an MRD

The reactive force f_d in an MRD is produced through the change in the magnetorheological fluid viscosity, which varies accordingly with the magnetic field produced by an applied voltage to the damper, when the micron-sized magnetically polarizable particles dispersed in a carrier medium, form particle chains, and the fluid becomes semi-solid. According to currently accepted definitions, a semiactive control device, such as an MRD, is one that can not add mechanical energy to the structural system it is attached to, as no active forces are applied directly to the structure (Housner *et al*. 1997), but it produces a reactive or retarding force that exists only if there is a relative velocity between the two ends of the damper (Rao 2011), and has the property that it can be controlled to reduce the responses of the system. A passive component or semiactive actuator only takes energy out of the system (Wang *et al*. 2011).

A widely used model for describing the dynamical behavior of damping systems with hysteresis, is the Bouc-Wen model (Wen 1976), which can represent a wide variety of hysteretic models; nevertheless, a model that can represent in a better way the dynamic behavior of an MRD, is the model known as modified Bouc-Wen model (Spencer *et al*. 1997), schematically shown in Fig. 1. The expressions modeling the reactive force in an MRD via the modified Bouc-Wen model are (Spencer *et al*. 1997)

$$f_d = c_1 \dot{y}_d + k_1(x_p - x_0) \tag{4}$$

$$\dot{y}_d = \frac{1}{c_1 + c_0}\{\alpha\, z_d + c_0 \dot{x}_p + k_0(x_p - y_d)\} \tag{5}$$

$$\dot{z}_d = -\gamma|\dot{x}_p - \dot{y}_d|z_d|z_d|^{n-1} - \beta(\dot{x}_p - \dot{y}_d)|z_d|^n + A_d(\dot{x}_p - \dot{y}_d) \tag{6}$$

where y_d is an internal displacement; z_d is the variable that takes into account the dependency of the recorded responses; k_1 represents the accumulator stiffness, which is a deposit in the main body damper that contains pressurized nitrogen; c_0 is the viscous damping observed for high velocities; a damper represented by c_1, takes into account the damping at low velocities, k_0 controls the stiffness at high velocities; x_0 is the initial displacement for k_1 spring, associated with the nominal force due to the accumulator; x_p is the piston bar displacement which corresponds to the displacement of the structure at the point of attachment. The constants β, γ, n and A_d are damper's parameters.

The viscous damping parameters vary with the applied voltage to the damper's current driver, where, for the modified Bouc-Wen model, the following relations are used

$$\alpha = \alpha_a + \alpha_b u_d \tag{7}$$

$$c_0 = c_{0a} + c_{0b} u_d \tag{8}$$

$$c_1 = c_{1a} + c_{1b} u_d \tag{9}$$

being $\alpha_a, \alpha_b, c_{0a}, c_{0b}, c_{1a}$ and c_{1b} parameters related to a specific MRD. Term u_d represents a filtered voltage in the circuit, obtained from the dynamic involved in reaching the magnetorheological equilibrium in the fluid, established by the relationship

$$\dot{u}_d = -\eta(u_d - v) \tag{10}$$

where v is the applied voltage and η is a constant from the corresponding MRD electrical system (Kori and Jangid 2009).

Finally, the complete nonlinear model of a civil structure including an MRD, is described in Eqs. (3)-(10). This model will be used in the next section for control purposes.

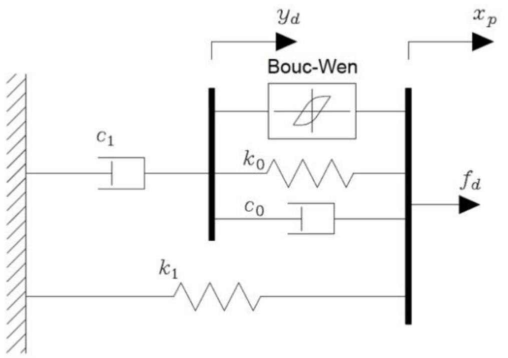


Fig. 1 Modified Bouc-Wen mechanical scheme for an MRD

3. Optimal control for reducing vibrations in civil structures

This paper considers the problem of designing an optimal control law u^* such that the structural system (3)-(10) output response minimizes the predetermined index, through which the system dynamical behavior is evaluated. It is worth mentioning the nonlinear optimal control is very complicated to be solved for general nonlinear systems, which is related to solve associated Hamilton-Jacobi-Bellman (HJB) equation (Sepulchre *et al.* 1997); however, for SDCF nonlinear systems determining the HJB equation solution is possible, and hence the optimal control solution.

To begin the controller's design, the structural system is firstly represented into the SDCF form, which is an important feature that will be used to obtain the solution of the optimal control via the state-dependent Riccati equation (SDRE). For the controller design, let us consider a nonlinear system given as

$$\dot{x} = f(x) + B(x)u + D \tag{11}$$

$$y = h(x) \tag{12}$$

where $x_s \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the system output; functions $f(x), B(x)$ and $h(x)$ are smooth maps of adequate dimensions. Term D is an external signal representing a known and bounded disturbance.

Assuming that $f(0) = 0$, $h(0) = 0$, $f(\cdot) \in C^1$ and $h(\cdot) \in C^1$, then functions $f(x)$ and $h(x)$ can be rewritten in a SDCF form as $f(x) = A(x)x$ and $h(x) = C(x)x$, as established in Cloutier (1997); system (11)-(12) results as

$$\dot{x} = A(x)x + B(x)u + D \tag{13}$$

$$y = C(x)x \tag{14}$$

As factorizations $A(x)x$ and $C(x)x$ are not unique, to accomplish well defined control schemes, these factorizations must be selected to guarantee controllability and observability properties (Banks *et al.* 2007).

The associated quadratic performance index to be minimized for system (13)-(14) is given by

$$J = \frac{1}{2}\int_{t_0}^{\infty} (e^T Q\, e + u^T R\, u)\, dt \tag{15}$$

where e is defined as $e = r - y$, and r is the system reference output, which in this case, $r = 0$ to maintain the building in the upright position. Q and R are symmetric and positive defined matrices. Matrix Q weights the time evolution of the tracking error e, while R is a matrix weighting the control effort expenditure; therefore these matrices are used to establish an equilibrium between the tracking performance and the control effort. The optimal control problem is related to obtain a control law u^*, in such a way that Eq. (15) is minimized.

Considering that system (13)-(14) is controllable and observable, then the control law (Ornelas *et al.* 2017)

$$u^*(x) = -R^{-1}\, B^T(x)(P\, x - z) \tag{16}$$

is a feedback optimal control law which guarantees a closed-loop asymptotic stability, performs stabilization and minimizes the associated cost functional J. Note that for the optimal control law, all of the states must be available for feedback to the controller (Kirk 2004, Anderson and Moore 1990), which can be obtained from direct measurements or state estimation as proposed in Anderson and Moore (1990) and Stengel (1994). Matrix P in Eq. (16) is the solution to the SDRE given as

$$\dot{P} = -C^T(x)\,Q\,C(x) + P\,B(x)R^{-1}\,B^T(x)P - A^T(x)\,P - P\,A(x) \tag{17}$$

and vector z is the solution of the differential equation

$$\dot{z} = -[A(x) - B(x)R^{-1}B^T(x)P]^T z - C^T(x)\,Q\,r + P\,D \tag{18}$$

The control problem for a civil structure turns to be a stabilization problem, where the reference, in the case of vertical structures, is the steady state upright position. The physical meaning of Eq. (15) is the minimization of the civil structure response, that is, minimizing the displacement (x_s), the velocity $(\dot{x}_s)$ and/or the acceleration $(\ddot{x}_s)$ in the structure's storeys with a minimum control effort. In this paper, an important feature of the civil structure modeling, is that the highly nonlinear MRD dynamics is taken into account for the nonlinear control design.

4. Civil structure response with an MRD using optimal control

In this section for analysis and simulation purposes an investigation is conducted for comparing the structural response over time of a scaled three storey building (Dyke 1996), where a first scenario that considers no additional damping to that provided by the scaled structure itself, is evaluated. A second scenario considers that a single MRD is installed between the first floor and the ground, as depicted in Fig. 2 and a nonlinear optimal controller is proposed for controlling the input voltage of the MRD.

4.1 Civil structure with the SDCF representation

System (3)-(10) represents the model of a general description of a civil structure with an MRD, whose state variables are defined as

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9]^T$$

where x_i, for i =1, 2, 3, are the corresponding i-th storey displacement; for $i =$ 4, 5, 6, are the corresponding velocities for the 1st, 2nd and 3rd storey, respectively, while the damper's variables are given by $x_7 = y_d, x_8 = z_d$ and $x_9 = u_d$, accordingly to Eqs. (5), (6) and (10). For the scaled civil structure Eqs. (3)-(10), the corresponding matrices for the SDCF representation Eqs. (13) and (14), are determined as

$$A(x) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} & a_{59} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} & a_{69} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} & a_{89} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\eta \end{bmatrix}$$

$$B(x) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \eta]^T$$

$$D = \left[0 \quad 0 \quad 0 \quad \frac{k_1 x_0}{m_{11}} - \ddot{x}_g \quad -\ddot{x}_g \quad -\ddot{x}_g \quad 0 \quad 0 \quad 0\right]^T$$

and $C(x)$ becomes the 9 x 9 identity matrix. Factorization for rows a_4, a_7 and a_8 is carried out factorizing state variables in the order $x_9, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$. The algebraic values of $A(x)$ are given in Tables 2, 3, 4 and 5 of Appendix, and its numerical values, as well as for $B(x)$ and D, are provided in Tables 6 and 7.

The mass, damping and stiffness for the scaled civil structure, represented in matrix form in M, C_s and K matrices, as well as the MRD parameters, obtained from Dyke *et al*. (1996) are shown in Table 6 and Table 7 of Appendix respectively.

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \text{kg} \tag{19}$$

$$C_s = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \frac{\text{N·sec}}{\text{m}} \tag{20}$$

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \text{x}\, 10^5 \frac{\text{N}}{\text{m}} \tag{21}$$

4.2 Controller synthesis

The modeling and control methodology is applied to a scaled three storey building depicted in Fig. 2. The optimal control problem for the scaled structure is related to find the control input u^* such that the structure rejects perturbation D and therefore is vertically stabilized, which is achieved by selecting $r = 0$ in Eq. (18). Since there is only one MRD, in the first storey, therefore vector Γ in Eq. (2) results as

$$\Gamma = [1 \quad 0 \quad 0]^T.$$

In the control law (16) and the SDRE (17), R is selected as $R = 1.5$ and the corresponding matrix Q as $Q = \text{diag}\,[100 \quad 100 \quad 100 \quad 2000 \quad 2000 \quad 2000 \quad 1 \quad 1 \quad 1]$, respectively.

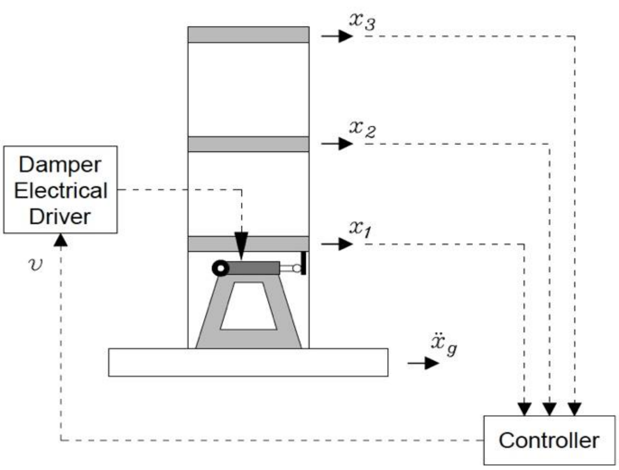


Fig. 2 Three storey building model with MRD

To excite the three degrees of freedom of the scaled model building, one for each storey, with the ground acceleration, in order to obtain a realistic mode shape and by considering displacements only in one direction (Williams 2016), the vector of influence coefficients λ in Eq. (2) is set as

$$\lambda = [1 \quad 1 \quad 1]^T$$

In simulation, one-dimensional ground acceleration $\ddot{x}_g$ from the North-South component of the September 19th. 2017, Puebla-Morelos (7.1M), Mexico earthquake [1] is applied to the scaled structure; considering the scaled size as one fifth of its normal height and the consequent reduction of the natural vibration period, then application take places at five times the original sampling rate (Dyke *et al*. 1996) to congruently shorten the original time-period of the ground acceleration. Acceleration timeseries for twelve seconds (one minute at no scaled time) is shown in Fig. 3 which contains the maximum peak ground acceleration of -2.20 m/s^2 registered in the station.

Table 1 presents the maximum peak responses obtained for the scaled structure subjected to the Puebla-Morelos earthquake. Information is divided into three columns. First data column shows peak responses when no additional damping exists to that provided by the scaled structure itself; second data column contains the peak responses for a scenario where an MRD is installed between the ground and the first storey and the proposed optimal control strategy for controlling the MRD input voltage is used. Responses shown are: relative to the ground displacement $x_i, i = 1, 2, 3$, interstorey displacement d_i, obtained through $d_i = x_i - x_{i-1}$, storey velocity $\dot{x}_i$ and storey acceleration $\ddot{x}_i$. Third data column shows percentages of response reduction when using control law (16), where it can be seen that the proposed controller reduces maximum peak responses of the structure in a significantly way. Additionally, Table 1 also presents the normalized-root-mean-square deviation (NRMSD) δx and $\delta \dot{x}$ for the corresponding response.

Fig. 4(a) shows the time response of the structure from scaled time second 1 to 6, according to earthquake registry of Fig. 3. Responses with No Control and Optimal control are shown. Displacement response for the first storey diminishes by the controller's action, with respect to the response obtained when no MRD is installed (No control). Similar behaviors are presented in Figs. 4(b) and 4(c) for the second and third storey responses, respectively. For the velocity response case, Figs. 4(d), 4(e) and 4(f) show, once again, that structural response is reduced in an important way throughout the time interval shown.

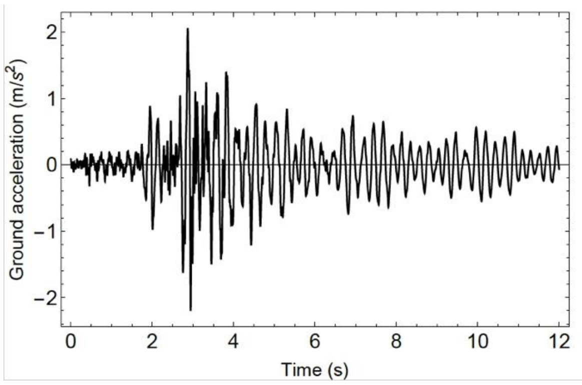
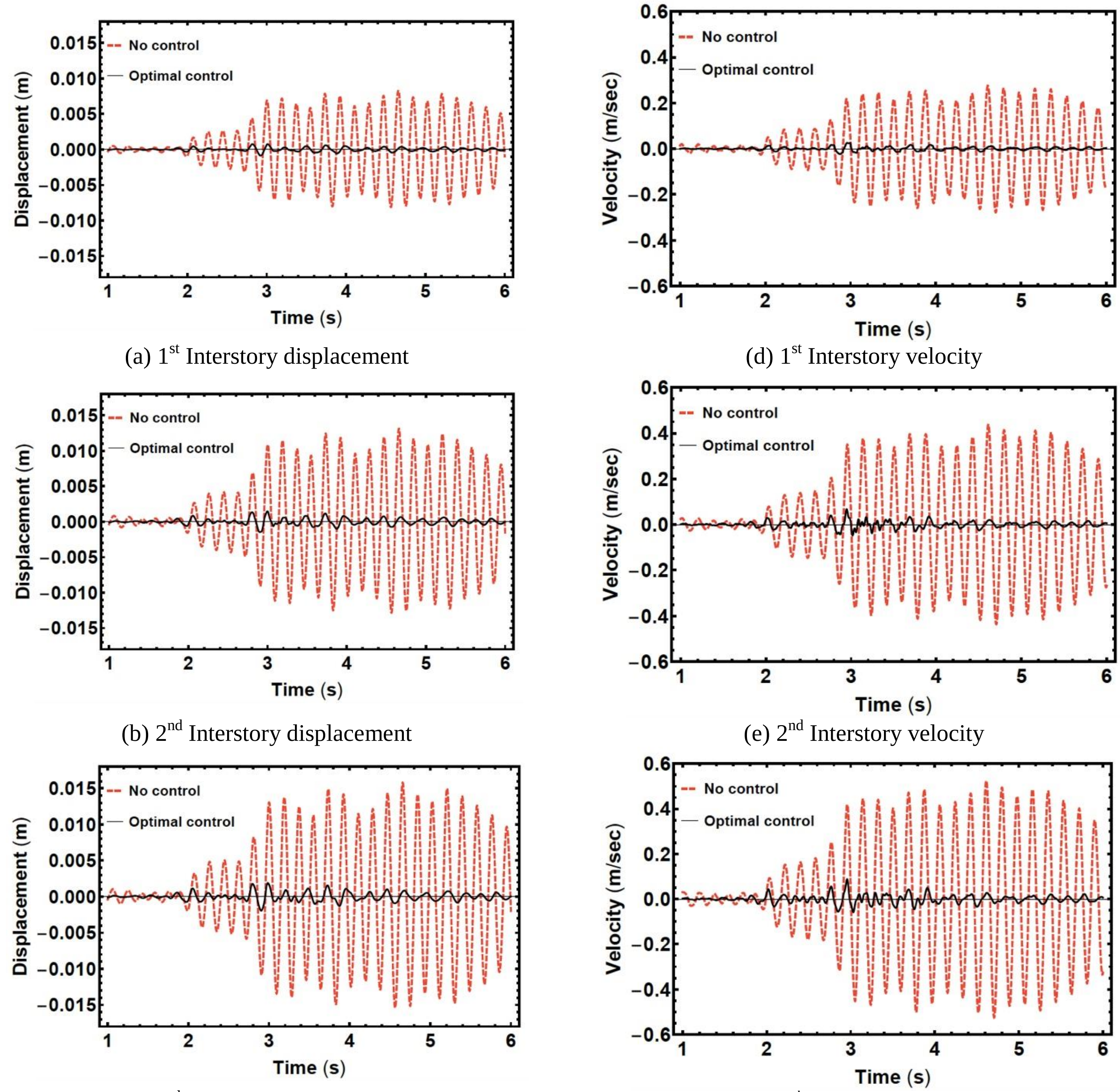


Fig. 3 Five times scaled ground acceleration timeseries for JC-54 Station from RACM, North-South component of the Puebla-Morelos (7.1M), Mexico earthquake, 2017

Table 1 Structure's maximum peak responses, response reduction percentage and NRMSD for each storey response

Response	***No control***	***Optimal control***	***Response reduction***
x_i (m)	0.0083	0.0009	89.6%
	0.0131	0.0015	88.8%
	0.0158	0.0019	87.7%
d_i (m)	0.0083	0.0009	89.6%
	0.0048	0.0006	87.3%
	0.0075	0.0011	85.6%
$\dot{x}_i$(m/s)	0.2780	0.0258	90.7%
	0.4378	0.0670	84.7%
	0.5242	0.0876	83.3%
$\ddot{x}_i$(m/s^2)	9.7937	3.9701	59.4%
	15.0024	3.3597	77.6%
	17.6050	4.1003	76.7%
δx_i	2.33	0.11	-
	3.70	0.20	-
	4.45	0.26	-
$\delta \dot{x}_i$	2.40	0.10	-
	3.80	0.19	-
	4.57	0.25	-
$\sum(\delta x_i + \delta \dot{x}_i)$	21.25	1.11	-

[1] Data obtained from the Instrumentation and Seismic Registry Centre A.C. (CIRES), with timeseries data from *Parque jardines de Coyoacan* station (JC54) from the Mexico's City Accelerographic Network (RACM).



(a) 1st Interstory displacement

(d) 1st Interstory velocity

(b) 2nd Interstory displacement

(e) 2nd Interstory velocity

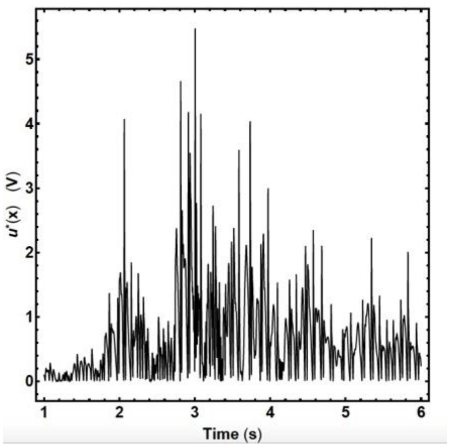
(c) 3rd Interstory displacement

(f) 3rd Interstory velocity

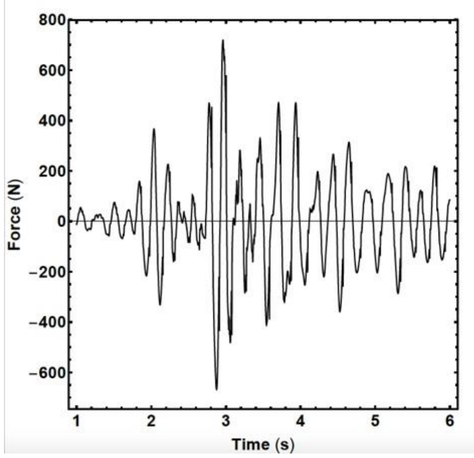
Fig. 4 Displacement and velocity response for the interstoreys

4.3 Results discussion

The peak values of the controlled response are significantly smaller than the uncontrolled one for all the cases, as shown from Figs. 4(a) to 4(f), as response profiles show both behaviors. Qualitatively established, optimal control meets its purpose of reducing the structure's response and has a good performance. For quantitative purposes, from Table 1, the peak values for the uncontrolled case are substantially larger than those obtained for the controlled case (e.g., 0.0158 m for the third floor, compared to 0.0019 m, which represents an 87.7% displacement reduction). Similar reductions are obtained for the rest of the profiles. For comparative purposes the maximum displacements obtained with the proposed optimal scheme are compared with the results obtained with a Proportional-Derivative controller, showing that for the first floor, the displacements are 0.0009 m vs 0.0020 m, respectively, and in a similar way for the second floor 0.0015 m vs 0.0034 m, and for the third floor 0.0019 m vs 0.0041 m, which in all cases the optimal controller produces a displacement reduction greater than 50%. A common disadvantage of using linear control methods, like PID controllers, is that their adequate performance cannot be ensured in a large operation region when the system is nonlinear or when the working conditions are different from the point for which the linear controller has been tuned, where both situations can be given in a civil structure due to unknown time-varying disturbances as earthquakes and different service conditions.



(a) Time history of the applied voltage



(b) Time history of the applied force

Fig. 5 Applied voltage and damping force

Beyond the peak response reduction and the graphical evidence, an additional study of the deviations is carried out for the cases of No control and Optimal control schemes. The deviations for the displacements and velocities from the position and velocity in steady state have been calculated for the scaled twelve seconds ground acceleration registry showed in Fig. 3 Through the corresponding root-mean-square deviation, deviations are calculated and normalized via the structural response range (Spiegel and Stephens 2008), considering the maximum response value minus the minimum response value, as

$$\delta x = \frac{\sqrt{(\sum_{t=0}^{N} x_t^2)/N}}{|x_{max} - x_{min}|} \tag{22}$$

$$\delta \dot{x} = \frac{\sqrt{(\sum_{t=0}^{N} \dot{x}_t^2)/N}}{|\dot{x}_{max} - \dot{x}_{min}|} \tag{23}$$

where δx and $\delta \dot{x}$ are the normalized root-mean-square deviation (NRMSD) for the displacement and velocity respectively. A value closer to zero represents a better performance due to minor building response. Individual NRMSD for each storey displacement and velocity, are given in the second part of Table 1 as well as the total. For example, the first floor displacement has a NRMSD of 2.33 for the uncontrolled case in comparison with 0.11 for the controlled case. Similar observations can be made for the rest of the floors and for its corresponding response. As can be seen, there is a substantial difference between the NRMSD values when the proposed control system is used, compared to the values when No control is applied.

Even though the study is carried out in the scaled building with only one MRD installed between the ground and the first floor, an experiment with three MRDs (one for each floor) was implemented. The obtained simulation results show a slight performance improvement by increasing the number of MRDs installed due to the coupling effect, nonetheless, response profiles presented in this paper have been selected for the one MRD case, by considering that the external force for the experiment is a ground motion given through its acceleration registry and applied directly to the first floor; in addition, the physical space required to install multiple MRDs and the relation cost-effective must be also taken into account in the implementation.

Additional to the structural response, Figs. 5(a) and 5(b), display the time history of the applied voltage and the corresponding damping force, respectively, generated by the controller for reducing the civil structure vibrations, where a low voltage is required for the MRD operation and thus a low energy consumption. It is important mentioning that for real applications, to prevent possible deformations, the operation range limits of the MRDs must be taken into account to size the damper device for a given structure such that an effective operation can be achieved.

5. Conclusions

This paper has demonstrated that it is possible to use a nonlinear optimal control scheme for reducing vibrations in civil structures through MRD. The proposed optimal nonlinear controller takes into account the structure model as well as the MRD dynamics for reducing the structural response in civil infrastructure, where an important characteristic is that MRD model directly depends on the structure variables (displacement and velocity), and hence, the inclusion of these dynamics results effective in the controller synthesis for the vibrations' reduction. Note that most of the actual proposals are based on determining the necessary force on the damper, however, the control of such force is through the applied voltage, which makes necessary to involve the nonlinear model of the MRD. From the obtained results, it is demonstrated the effectiveness of the proposed controller when a semiactive device is used. As a future work, this research is going to study important practical considerations to improve the controller robustness, such as the noise in the measurements and the design of state estimators to reduce the number of sensors.

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Appendix

Table 2 Fourth row algebraic values for *A(x)* matrix

Parameter	***Value***
a_{41}	$\frac{k_1 - k_{11}}{m_{11}} - \frac{c_{1a}k_0}{m_{11}(c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9)}$
a_{42}	$-\frac{k_{12}}{m_{11}}$
a_{43}	$-\frac{k_{13}}{m_{11}}$
a_{44}	$-\frac{c_{11}}{m_{11}} - \frac{c_{0a}c_{1a} + c_{0b}c_{1b}x_9^2}{m_{11}(c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9)}$
a_{45}	$-\frac{c_{12}}{m_{11}}$
a_{46}	$-\frac{c_{13}}{m_{11}}$
a_{47}	$\frac{c_{1a}k_0}{m_{11}(c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9)}$
a_{48}	$-\frac{c_{1a}\alpha_a + c_{1b}x_9^2\,\alpha_b}{m_{11}(c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9)}$
a_{49}	$-\frac{c_{1b}(c_{0a}x_4 + k_0(x_1 - x_7) + x_8\alpha_a) + c_{1a}(c_{0b}x_4 + x_8\,\alpha_b)}{m_{11}(c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9)}$

Table 3 Fifth and sixth row algebraic values for *A(x)* matrix

Parameter	*Value*	***Parameter***	*Value*
a_{51}	$-\frac{k_{21}}{m_{22}}$	a_{61}	$-\frac{k_{31}}{m_{33}}$
a_{52}	$-\frac{k_{22}}{m_{22}}$	a_{62}	$-\frac{k_{32}}{m_{33}}$
a_{53}	$-\frac{k_{23}}{m_{22}}$	a_{63}	$-\frac{k_{33}}{m_{33}}$
a_{54}	$-\frac{c_{21}}{m_{22}}$	a_{64}	$-\frac{c_{31}}{m_{33}}$
a_{55}	$-\frac{c_{22}}{m_{22}}$	a_{65}	$-\frac{c_{32}}{m_{33}}$
a_{56}	$-\frac{c_{23}}{m_{22}}$	a_{66}	$-\frac{c_{33}}{m_{33}}$
a_{57}	0	a_{67}	0
a_{58}	0	a_{68}	0
a_{59}	0	a_{69}	0

Table 4 Seventh row algebraic values for *A(x)* matrix

Parameter	*Value*
a_{71}	$\frac{k_0}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{72}	0
a_{73}	0
a_{74}	$\frac{c_{0a}}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{75}	0
a_{76}	0
a_{77}	$-\frac{k_0}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{78}	$\frac{\alpha_a}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{79}	$\frac{c_{0_b}x_4 + x_8\alpha_b}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$

Table 5 Eighth row algebraic values for *A(x)* matrix

Parameter	***Value***
a_{81}	$\frac{k_0(\beta\lvert x_8\rvert^n - A_d)}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{82}	0
a_{83}	0
a_{84}	$\frac{(c_{1a} + (c_{0b} + c_{1b})x_9)(A_d - \beta\lvert x_8\rvert^n)}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{85}	0
a_{86}	0
a_{87}	$\frac{k_0(A_d - \beta\lvert x_8\rvert^n)}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$
a_{88}	$-\frac{\alpha_a\beta\lvert x_8\rvert^n - A_d\alpha_a}{c_{0_a} + c_{1_a} + (c_{0_b} + c_{1_b})x_9} - \gamma\lvert x_8\rvert^{n-1}\left\lvert\frac{c_{1a}x_4 + k_0(x_7 - x_1) + c_{1b}x_4x_9 - x_8(\alpha_a + x_9\alpha_b)}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}\right\rvert$
a_{89}	$-\frac{(c_{0b}x_4 + x_8\alpha_b)(A_d - \beta\lvert x_8\rvert^n)}{c_{0a} + c_{1a} + (c_{0b} + c_{1b})x_9}$

Table 6 Structure's parameters

Mass	*Value*	***Damping***	*Value*	***Stiffness***	*Value*
m_{11}	98.3	c_{11}	175	k_{11}	12
m_{12}	0	c_{12}	-50	k_{12}	-6.84
m_{13}	0	c_{13}	0	k_{13}	0
m_{21}	0	c_{21}	-50	k_{21}	-6.84
m_{22}	98.3	c_{22}	100	k_{22}	13.70
m_{23}	0	c_{23}	-50	k_{23}	-6.84
m_{31}	0	c_{31}	0	k_{31}	0
m_{32}	0	c_{32}	-50	k_{32}	-6.84
m_{33}	98.3	c_{33}	50	k_{33}	-6.84

Table 7 Magnetorheological damper's model parameters

Parameter	*Value*	***Parameter***	*Value*
c_{0a}	21.0 N·sec/cm	α_a	140 N/cm
c_{0b}	3.5 N·sec/cm · u_d	α_b	695 N/cm · u_d
k_0	46.9 N/cm	γ	363 cm^{-2}
c_{1a}	283 N·sec/cm	β	363 N/cm^{-2}
c_{1b}	2.95 N·sec/cm · u_d	A_d	301
k_1	5.00 N/cm	n	2
x_0	14.30 cm	η	190 sec^{-1}