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Modal flexibility based damage detection for suspension bridge hangers: A numerical and experimental investigation

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Abstract. This paper addresses the problem of damage detection in suspension bridge hangers, with an emphasis on the modal flexibility method. It aims at evaluating the capability and the accuracy of the modal flexibility method to detect and locate single and multiple damages in suspension bridge hangers, with different level of severity and various locations. The study is conducted numerically and experimentally on a laboratory suspension bridge mock-up. First, the covariance-driven stochastic subspace identification is used to extract the modal parameters of the bridge from experimental data, using only output measurements data from ambient vibration. Then, the method is demonstrated for several damage scenarios and compared against other classical methods, such as: Coordinate Modal Assurance Criterion (COMAC), Enhanced Coordinate Modal Assurance Criterion (ECOMAC), Mode Shape Curvature (MSC) and Modal Strain Energy (MSE). The paper demonstrates the relative merits and shortcomings of these methods which play a significant role in the damage detection of suspension bridges.

Keywords: suspension bridge hangers; stochastic subspace identification; modal flexibility; damage detection

1. Introduction

In recent years, the improvement in construction materials and construction technology, computing capability, and above all a better understanding of the physics of the complex phenomena which control the external loads acting on structures, have revolutionized the civil engineering community, enabling the construction of large and elegant civil infrastructures. With the rise of large-scale civil engineering structures, particularly bridges, as recently sadly illustrated by the sudden collapse of the Morandi bridge in Genoa, the structural health monitoring (SHM) problems have become a crucial scientific issue (Mei *et al*. 2017, Wang *et al*. 2016, Wu and Casciati 2014, Casciati *et al*. 2017). The goal is to be able to detect, locate and assess the extent of damage in a structure so that its remaining life can be known and possibly extended. As a critical component of suspension bridges, hanger structures are vulnerable to adverse environmental effects, such as corrosion and fatigue, resulting in hidden dangers for suspension bridges (Ko *et al*. 2009). Therefore, damage detection in hangers is an indispensable part in SHM of suspension bridges.

Current cable monitoring techniques are integrated with a variety of damage detection methods, such as acoustic emission inspection (Zejli *et al*. 2012), magnetic flux leakage detection (Xu *et al*. 2012), and evaluating cable tensile forces (Kim 2013) can be effective to detect anomalies. With the development of the nondestructive test (NDT), some advanced techniques such as digital image processing technique (Kim *et al*. 2013, Vanniamparambil *et al*. 2013) and inspection robots (Cho *et al*. 2013) have been used for cable damage detection. In addition, Lin et al. (2017) applied the virtual distortion method (VDM) to damage detection in cable structures in a 3D FEM of a bridge. Although those NDT methods are effective, the techniques are costly or complicated to operate, and only can monitor the local damage on the cables where the sensors are located.

As an alternative to the current local detection methods of bridge cables, the vibration-based damage detection (VBDD) methods (which are based on the changes in modal characteristics) have been widely applied (Bouaanani 2006, Turmo and Luco 2010, Cho and Jang *et al*. 2010). The VBDD method is divided into two types. One type is to directly measure the dynamic characteristics of cables and through the changes in the modal parameters of cables as a possible indicator of damage, such as frequency measurement (Lepidi *et al*. 2009), evaluate the amplification effects of the bridge cable produced by the moving load application (Lonetti and Pascuzzo 2014) and a

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model-free test method which is based on the variation of the acceleration response of damaged cables (An *et al*. 2015).

The other type is the global monitoring technique. The basic idea behind this technique is that cables/hangers as the main load-bearing elements of deck, once damaged, produce changes in modal parameters (frequencies, mode shapes and modal damping) which provide sufficient information to detect the damaged position. However, the effectiveness of detection often depends on the extent of damage, and, for large civil engineering structures, the structure modal data are little sensitive to tiny damages. In such a case, finding an effective indicator for damage detection or damage location becomes particularly important. For this purpose, many traditional modal parameters-based methods have been proposed, such as Coordinate Modal Assurance Criterion COMAC (Wahab and De Roeck 1999), Enhanced Coordinate Modal Assurance Criterion ECOMAC (Hunt 1992), Mode Shape Curvature MSC (Pandey *et al*. 1991), Modal Strain Energy MSE (Stubbs *et al*. 1995, Li *et al*. 2006) and Modal Flexibility method MF (Pandey and Biswas 1994, Jaishi and Ren 2006, Perera and Manzano 2007, Shih *et al*. 2009, Meruane and Heylen 2011).

Among different traditional modal parameters-based methods, the modal flexibility technique is a more effective approach which incorporates the natural frequencies and mode shapes. It has been widely used by a number of researchers for detecting damage in bridge structures, such as girder bridges (Toksoy and Aktan 1994, Huth *et al*. 2005, Catbas and Brown *et al*. 2006), arch bridges (Jaishi and Kim *et al*. 2007), truss bridges (Wang and Chan *et al*. 2013), cable-stayed bridges (Ko and Sun *et al*. 2002, Ding *et al*. 2010) and suspension bridges (Chen *et al*. 2014). Koo *et al.* (2008) used the MF method to detect cracks in a 10 m bridge model with a steel box-girder. Catbas *et al.* (2008) investigated the effectiveness of deflection and curvature obtained from modal flexibility as damage-sensitive features for bridge SHM. Reynders and De Roeck (2010) also used the changes in flexibility to develop a new algorithm called local flexibility (LF) method to locate and quantify damage in a concrete bridge. Shih *et al*. (2011) used the modal flexibility and modal strain energy changes before and after damage as the indices for the assessment of truss bridge structural health state. Chen *et al.* (2014) explored a novel damage detection technique based on stress influence lines (SILs) of bridge decks and validated the efficacy of the technique in deck damage detection through a case study of the Tsing Ma suspension bridge. Ni *et al*. (2008) developed a method based on MF method to detect damage in long stay cables. It was successfully applied to identify damage in the Ting Kau Bridge numerically for single damage scenarios under ambient effects. Talebinejad and Fischer *et al.* (2011) also conducted a numerical study to detect damage in stay cables. They were successful in locating 95% damage in single and multiple damage cases under 2% RMS noise. The literature confirms that the MF method has a wide variety of applications in damage detection of bridge structures but, so far, not in detecting damage in the hangers of suspension bridges.

At present, the modal flexibility-based study for health monitoring of the hangers is rare. Talebinejad and Sedarat *et al.* (2014) conducted a numerical evaluation of four damage detection techniques in assessment of the Alfered Zampa Memorial suspension bridge, including the flexibility matrix, the stiffness matrix, the uniform load surface and uniform load surface curvature methods; damages are simulated by reducing the modulus of elasticity of different bridge components, where the stiffness reduction levels of a hanger are 100%, 90%, 75%, and 50%. The results confirmed that the modal flexibility method can detect the above damage without considering the influence of noise. In addition, An *et al.* (2015) simulated 50% damage in hangers and the numerical results indicated that the damage cannot be detected using the MF method. Currently, Wickramasinghe *et al*. (2016) carried out a similar study for detecting the damage in main cables of suspension bridges. Based on the FE model updating technique, they simulated the damage by reducing the Young's modulus of the main cables with two levels, respectively, of 10% and 20%, and 5% random noise was introduced to the mode shapes generated from the FE model. The results demonstrated that the MF method can detect and locate the damage successfully in both single and multiple damage cases.

While these studies have made great progress, challenges still exist, such as the effect of noise should be considered in the process of hanger's damage detection, and the effectiveness of the MF method needs further verification by experiment. To our knowledge, at present, no experimental study based on the MF method has been used for damage detection in suspension bridge hangers. In this paper, we aim to use a modal flexibility technique to detect and locate single and multiple damages in hangers with varied locations and severity on a laboratory mockup of a suspension bridge. The modal parameters as the important components of MF method are extracted from output-only measurements. As a robust and accurate modal identification algorithm, the stochastic subspace identification (SSI) method (Van Overschee and De Moor 2012) is very efficient to apply for high noise and harsh operating environments (Wu *et al*. 2016). Two types of implementation are available: the covariance-driven implementation, SSI-cov (Peeters and Ventura 2003) and the data-driven implementation, SSI-data (Peeters and De Roeck 1999).

The paper is organized as follows: Section 2 presents the SSI-cov and MF methods for damage index estimation. Section 3 describes the suspension bridge mock-up and its finite element modelling; in addition, the vibration characteristics of the bridge mock-up are shown in this section. Section 4 mainly focuses on the numerical validation of the SSI-cov and the damage index in the presence of noise. Section 5 mainly focuses on the experimental verification of the proposed damage index in single and multiple damages with varied locations and severity in the hangers of the suspension bridge. Section 6 gives a comparison between the modal flexibility method and the traditional modal parameters-based methods. Finally, findings and conclusions of the study are summarized at the end.

2. Methodology and algorithm

The Modal Flexibility (MF) method is a widely used technique in damage detection; it associates the damage to the change in the modal parameters of the structure (assuming an experimental modal analysis can be conducted). For large civil structures, such as suspension bridges, only ambient vibration, induced by the environment, can be measured. Hence, in order to use the MF method, one should first identify the modal parameters using existing blind methods, based on output-only measurements.

2.1 Covariance-driven stochastic subspace identification (SSI-cov)

The use of output-only measurements (ambient vibration) to extract the modal parameters of civil structures is a well-established practice, which has been widely used in the past few decades (Rainieri and Fabbrocino 2014). For the time-domain signal, the covariance-driven stochastic subspace identification method (SSI-cov) is one of the most robust and accurate system identification methods for output-only modal analysis of mechanical structures. In this section, the SSI-cov is briefly described.

Consider a linear time-invariant structure governed with its equation of motion

$$M\ddot{q}(t)+C_{\mathrm{d}}\dot{q}(t)+Kq(t)=f(t) \tag{1}$$

where M, C_{d}, and K are constant mass, damping, and stiffness matrices, respectively; $f(t)$ is the external force vector, and $q(t) = [q_1(t), \ldots, q_m(t)]^T$ is the displacement vector.

Through using model reduction, sampling and considering the noise, Eq. (1) can be converted to the discrete-time stochastic state-space model without the input term

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{2}$$

where x_k is the discrete-time state vector, A is the discrete state matrix, C is the output matrix. w_k is the process noise from the disturbances and modelling inaccuracies and v_k is the measurement noise from the sensor inaccuracy. They are both unmeasurable vector signals but can be assumed stationary with zero mean $E[w_k]=0,\ E[v_k]=0$. Further the stochastic process x_k is stationary with zero mean $E[x_k x_k^T]=\Sigma$, $E[x_k]=0$ where the state covariance matrix Σ is independent of the time k. w_k and v_k are independent of the actual state $E[x_k w_k^T]=0,\ E[x_k v_k^T]=0$. The output covariance matrices R_i are defined as

$$R_i = E\left[y_{k+i} \quad y_k^T\right] \tag{3}$$

The next state-output covariance matrix G is defined as

$$G=E\left[y_{k+1} \quad y_k^T\right] \tag{4}$$

From these definitions the relationship between A, C and R_i (i=1,2⋯) is easily deduced

$$R_i = CA^{i-1}G \tag{5}$$

Eq. (5) is the core expression of the SSI-cov method. It can be seen from this formula that the output covariance matrices R_i is directly related to the structure system matrices A and C, which is the key step to identify the modal parameters of the structures.

By applying the output covariance matrices, constructing the Hankel matrix and Toeplitz matrix (see Van Overschee and De Moor 2012 for a detailed derivation), the modal parameters are obtained from the discrete state matrix A and the output matrix C. Through conversion, an eigenvalue decomposition of the discrete-time state matrix can be obtained by

$$A=\Psi\Lambda_d\Psi^{-1} \tag{6}$$

where Λ_d is a diagonal matrix containing the discrete-time complex eigenvalues and Ψ contains the eigenvectors as columns. The eigenvalues of A occur in complex conjugated pairs and can be written as:

$$\lambda_i,\lambda_i^* = -\omega_i\xi_i \pm j\omega_i\sqrt{1-\xi_i^2} \tag{7}$$

where ξ_i are the modal damping ratios and ω_i are the eigenfrequencies (rad/s). The mode shapes ϕ_i at the sensor locations, defined as columns of Φ, are found as

$$\Phi=C\Psi \tag{8}$$

Finally, for further improving the accuracy of the SSI-cov method, stabilization diagrams are often applied to distinguish spurious modes. A whole detailed derivation of the identification procedure can be found in (Van Overschee and De Moor 2012).

2.2 Modal flexibility-based damage index

Based on structural flexibility matrix, Pandey and Biswas (1994) proposed the MF method to detect the damage of a wide-flange steel beam. Modal flexibility of a structure converges rapidly as the frequency increases, it can be calculated using only a few lower-order modes. Unlike the traditional methods, which require an analytical model of the structures to evaluate the flexibility, the MF method can use only the experimental data collected from the structure. However, for the large civil structures, the mass-normalized mode shapes data are not available. In order to apply the MF method in large civil structures, one of the simplest methods is used to calculate the MF with ambient vibration measurements. The modal flexibility at a

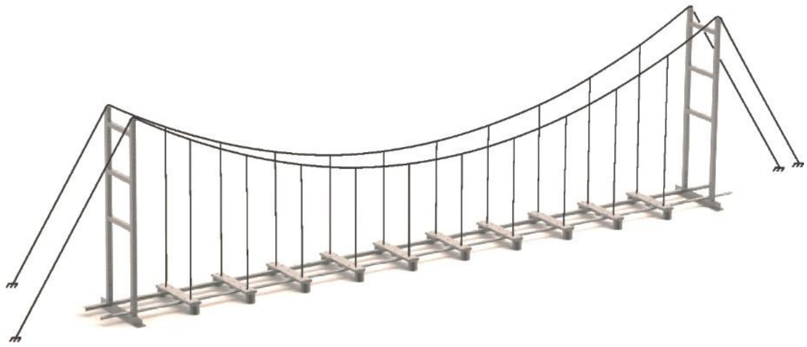


Fig. 1 CAD view of the studied suspension bridge

position x of the damaged and undamaged structure can be defined as

$$F_x = \sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T \tag{9}$$

$$F_x^* = \sum_{i=1}^{m} \frac{1}{\omega_i^{*2}} \phi_{xi}^* \phi_{xi}^{*T} \tag{10}$$

where i (i=1, 2, 3 …m) is the considered mode number; ω_i and ω_i* are the i^{th} frequencies, while ϕ_{x_i} and $\phi_{x_i}^*$are the i^{th} mode shapes of the damaged and undamaged structure at position x, respectively. All of the above modal parameters are assumed to be obtained from the SSI-cov method.

The stiffness of a damaged structures is, in general, smaller compared to the healthy structure, and inversely for the flexibility. The variation of the flexibility matrix, with respect to the nominal structure, can be obtained as

$$\Delta F = F_x \text{-} F_x^* \tag{11}$$

In this study, ΔF is normalized by the F_x and hence the damage index for locating damage in a structure is defined as in Eq. (12). The normalized damage index DI is determined by the relative change of the modal flexibility

$$DI = \frac{|\Delta F|}{F_x} \tag{12}$$

Generally, the dynamic characteristics of a suspension bridge often include with lateral, vertical, torsional and coupled modes (Huang *et al*. 2005). The lateral and vertical mode shapes as the significant components can be directly used to detect the damage in suspension bridges. In Wickramasinghe's method, they define and test two corresponding damage indices. One index is based on vertical components of mode shapes and the other is based on its lateral components. Their results confirm that using the vertical components of the mode shapes are much better than the lateral ones. Eq. (12) is hence rewritten as Eq. (13) to accommodate the damage index (DI), where the subscripts v denotes the vertical components of the mode shapes.

$$DI_v = \left| \frac{\Delta F}{F_x} \right|_v \tag{13}$$

3. The suspension bridge mock-up and its numerical model

The suspension bridge laboratory model is shown in Fig. 1. It has a span of 2.2 m and two articulated towers (pylons) of 0.62 m, the main steel cables (catenary) have a diameter of 1 mm and the 2×10 hangers have a diameter of 0.5 mm; the deck is free to rotate at both ends and is attached to the catenary by the two rows of hangers. It is described in details in (Preumont *et al*. 2016). A finite element model has been developed; it was used to obtain the vibration properties in both damaged and undamaged states to verify numerically the proposed damage detection methodology.

3.1 Numerical model

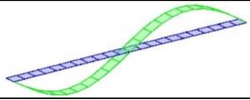
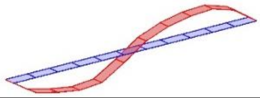
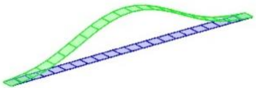
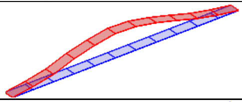
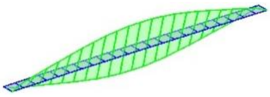
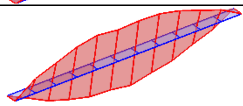
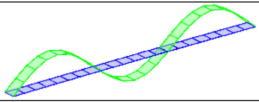
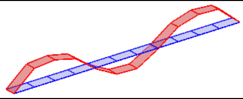
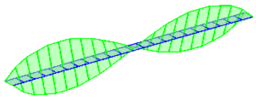
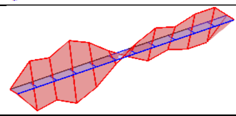
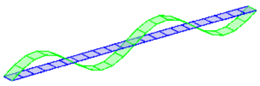
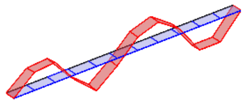
The mock-up consists of a flexible deck supported with a set of prestressed cables. We assume that the dynamics of the cables can be neglected and their interaction with the deck is restricted to the longitudinal tension (this assumption has been verified in previous studies). Note that this assumption doesn't affect the result, while it simplifies the modelling. Assuming a classical finite element formulation, the equation governing the dynamic response of the system is

$$M\ddot{X} + KX = Bf_d \tag{14}$$

where X is the vector of global coordinates of the finite element model, M and K are respectively the mass and stiffness matrices of the structure, including a linear model of the cables; the geometric stiffness due to the prestress in the hanger is included in the model and the structural damping is neglected to simplify the presentation. f_d is the vector of external disturbances (expressed in global coordinates).

The model of the bridge of Fig. 1 is developed using SAMCEF Finite Element software, and exported to MATLAB, where a state space model is built. The deck is

Table 1 Comparison between the numerical and the experimental natural frequencies and mode shapes. The natural damping has been estimated between 0.1% and 0.8%

Mode #	Numerical [Hz]	Experimental [Hz]	Numerical Mode shape	Experimental Mode shape
1st B	4.7	5.1		
2nd B	6.5	6		
1st T	9.8	10.1		
3rd B	12.3	11.8		
2nd T	12.1	13.2		
4th B	17.6	19.1		

modelled with finite elements of beams, while the hangers and the main cable (catenary) are modelled with bars.

3.2 Validation of the numerical model

To validate the numerical model, vibration tests were conducted to obtain the numerical and experimental modal parameters (natural frequencies and mode shapes) of the bridge mock-up. The setup of the experiment will be presented in the section 5.1. A comparison of results between numerical model and laboratory mock-up is given in Table 1.

Table 1 shows the first few vibration modes of the FE model and compares them to those measured experimentally (only the deck is shown). The agreement between the model and the experiment is fairly good. From the above comparisons, it can be concluded that the FE model is a good representation of the laboratory mock-up.

4. Numerical simulation

In this section the main objective is to verify numerically the feasibility of DI_v to detect damage in the suspension bridge hangers. Therefore, the effective model is not only considered as the undamaged baseline but also used to simulate various damage scenarios. To extract the modal parameters of those undamaged and damaged models, the SSI-cov method is used, then the damage indices are formed by Eqs. (9)-(13). The specific process is as follows.

4.1 Modal identification

The bridge is excited on its deck using a point force, as indicated in Fig. 2; a stationary Gaussian white noise (GWN) is used. The vertical acceleration of the deck is measured at 20 points, close the hangers. The numbering of the hangers from 1 to 10 on one side, starting from the right, and from 11 to 20 on the opposite side.

In order to use the SSI-cov method for modal identification, the ambient vibration of the structure is measured during at least 1000 to 2000 periods of the lowest natural frequency

$$T_{\min} = \frac{1000 \sim 2000}{f_{\min}} \text{ sec},$$

where $f_{\min}$ is the lowest natural frequency in Hz (Cantieni 2005, Wickramasinghe *et al*. 2016). Based on the preliminary FRF analysis, $f_{\min}$ is close to 5 Hz. Therefore, the recording time was selected as 200sec. To simulate the actual test environment, a zero-mean Gaussian white noise is added to the accelerometer outputs, where a standard deviation of 1% to 5% of the signal level is considered.

In order to minimize the effect of the uncertainties on the identified modes, the concept of stabilization diagram is introduced to help in the manual selection of the modes that are more likely to represent the physical modes (Goi and Kim 2016) and eliminate the spurious modes by using the stabilization criteria (Reynders *et al*. 2008). The frequencies f_i and damping ξ_i ratios are expected to be obtained in certain ranges $f_{min} \le f_i \le f_{max}$ and $\xi_{min} \le \xi_i \le \xi_{max}$; all the modes out of these ranges will be discriminated. Only modes verifying these criteria are plotted in the stabilization diagram. In the classical implementation of the stabilization

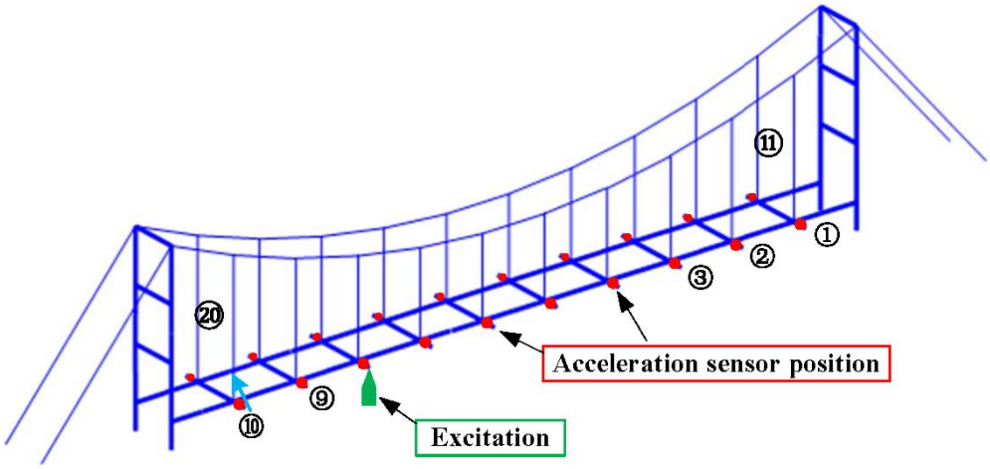


Fig. 2 Sensor layout and excitation locations in the bridge deck

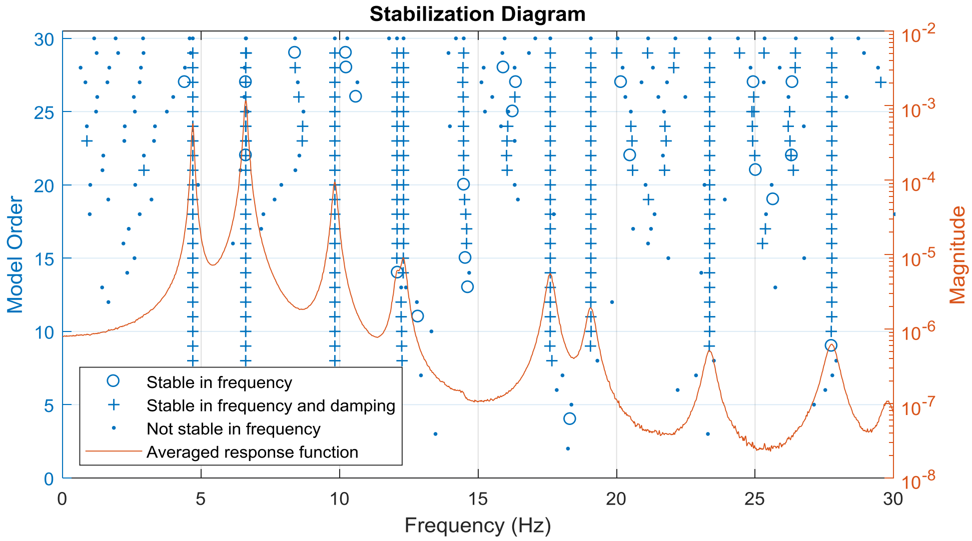


Fig. 3 Stabilization diagram. The criteria are: 0.5% for frequencies, 10% for the damping ratios

Table 2 Identification results of numerical model by SSI-cov in noisy random vibration (1%-5% RMS noise added in the vibration data)

Mode #	**Frequency (Hz)**		**Damping ratio**		**MAC**
	Theoretical	Identified	Theoretical	Identified	
1	4.69	4.65	0.01	0.013	0.9967
2	6.51	6.55	0.01	0.013	0.9997
3	9.83	9.80	0.01	0.014	1
4	12.07	11.97	0.01	0.012	0.8399
5	12.31	12.30	0.01	0.011	0.9977
6	17.60	17.59	0.01	0.009	0.9994
7	19.07	19.12	0.01	0.012	0.9971
8	23.35	23.36	0.01	0.012	0.9993

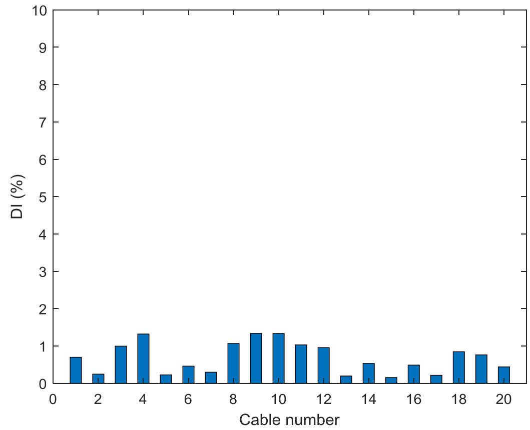
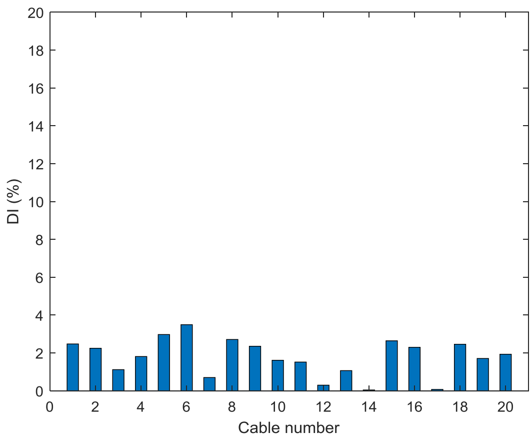
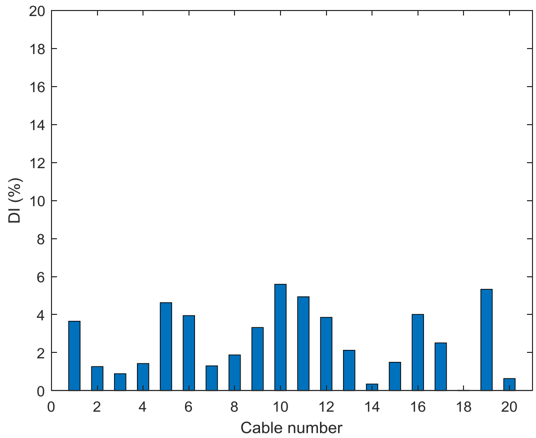


Fig. 4 Effect of modal identification error (with SSI-cov) on the Damage Index



(a) DI_v with 1% RMS noise



(b) DI_v with 5% RMS noise

Fig. 5 Effect of measurement noise on the damage index (for a healthy structure)

diagram, the typical stability criteria values are as follows: ε_f= 0.5 % for frequency, ε_ξ= 10% for damping. Two modes identified in certain two orders, "i" and "i + 1," will be plotted in the stabilization diagram if (Mrabet *et al*. 2014)

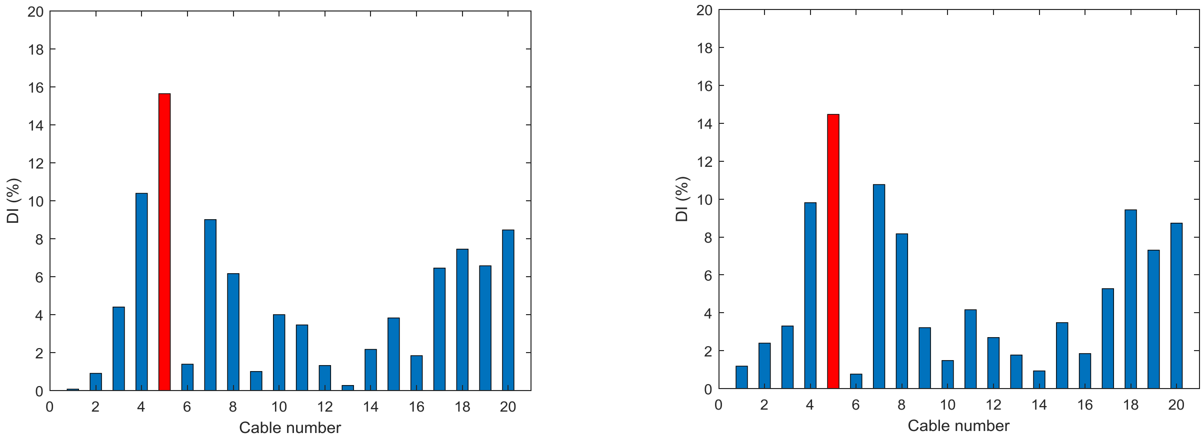
$$\left|\frac{f_i - f_{i+1}}{f_i}\right| \le \varepsilon_f\,, \qquad \left|\frac{\xi_i - \xi_{i+1}}{\xi_i}\right| \le \varepsilon_\xi \tag{15}$$

A typical stabilization diagrams is shown in Fig. 3. In the frequency range from 0 to 30 Hz, at least 9 modes are present. The theoretical values are obtained by computing the FE model and the identified modal parameters of the numerical model are given in Table 2. There are two pairs of rather close modes: 11–13 Hz. Although these modes are too closely spaced, SSI-cov method is successful to identify the frequencies in such a pair. Another advantage is that the SSI-cov method can also identify the damping ratios directly. However, comparing the identified mode shapes with MAC (Modal Assurance Criterion) values, the identified fourth mode shape cannot be consistent with the theoretical value. Considering that the identified fourth mode shape may affect the accuracy of damage indices, only the other seven quite high MAC modes will be used to calculate the vertical damage index (DI_v) in the following section.

4.2 Damage detection

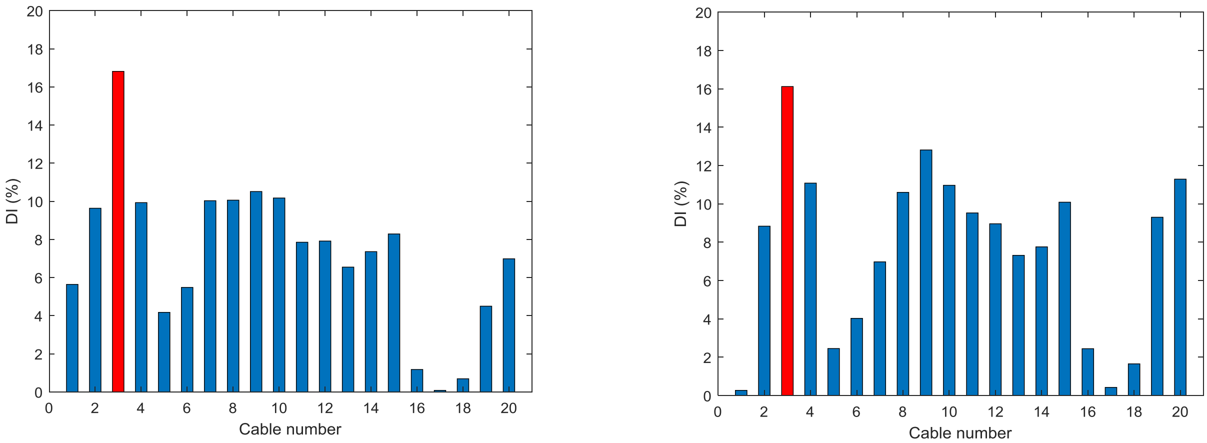
Damage can be simulated by changing the Young's modulus, changing the cross-section area or simply removing the elements at the damage location. In this study, we simulate damage in the hanger cables by reducing their Young's modulus. It is worth noting that a bad signal and a mistaken identification affects the damage index DI and may lead to wrong damage detection results. In order to figure out the extent of the signal noise and the identification error, we applied the damage detection method on a healthy bridge.

Fig. 4 illustrates the effect of modal identification error on the damage index DI level, where the SSI-cov method has been used to extract the modal parameters (natural frequencies and mode shapes); it shows that a damage index of 1.5% could be measured due to the identification error. Fig. 5 shows the effect of measurement noise, where two levels of the noise are considered. It is evident that these



(a) DI_v with 1% RMS noise (b) DI_v with 5% RMS noise

Fig. 6 Damage indices DI for Simulation-case 1 (damage in cable 5)



(a) DI_v with 1% RMS noise (b) DI_v with 5% RMS noise

Fig. 7 Damage indices for DI Simulation-case 2 (damage in cable 3)

two figures have clear deviations and demonstrate the influence of the noise and identification error on the damage detection accuracy.

To verify the feasibility of the MF method, a 95% damage is simulated numerically in hanger 3 and 5, where two levels of measurement noise are considered. Then the damage indices DIs are constructed by using the SSI-cov method to extract the modal parameters of the damaged and undamaged model. The results are shown as follow:

- Simulation-case 1: damage cable at the mid-span

In this damage case, the stiffness of the hanger cable 5 (mid-span) is reduced by 95%, then the modal parameters are identified from ambient vibration. The damage indices (a) DI_v with 1% standard deviation measurement noise and (b) DI_v with 5% standard deviation noise are shown in Fig.6. The DI_v for the damaged cable reaches its maximum value at the nodes of the damaged location. In Fig. 6(b), although the noise level is relatively high, the DI_v still shows its maximum value at the damaged location.

- Simulation-case 2: damage cable at one third of the span

Fig. 7 illustrates the DI_v for the second damage case, after reducing by 95% the stiffness of cable 3 (at one third of the span). Observe that the plot of DI_v peaks at the exact damage location for both levels of noise.

Based on the examination of the two examples, it can be concluded that incorporating the vertical components of mode shapes for detecting damage in a hanger for this bridge structure is a successful approach. The performance of DI_v in single and multiple damage scenarios with different damage levels is evaluated through experimental study in next section.

5. Experimental implementation

5.1 Experimental setup

The laboratory mock-up of the suspension bridge (Fig. 8) is used to investigate the capability of the modal flexibility method. It consists of two articulated towers of 0.62 m

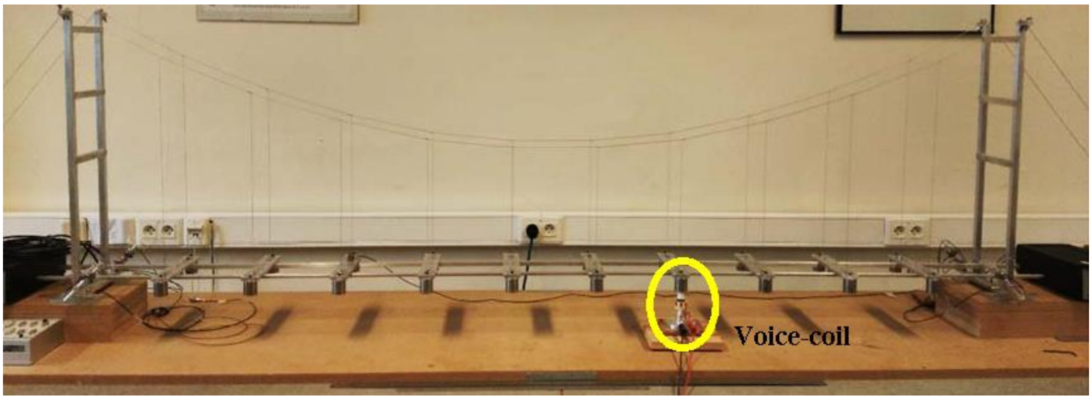
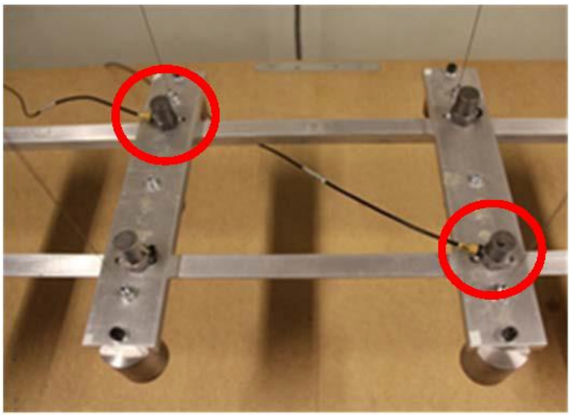
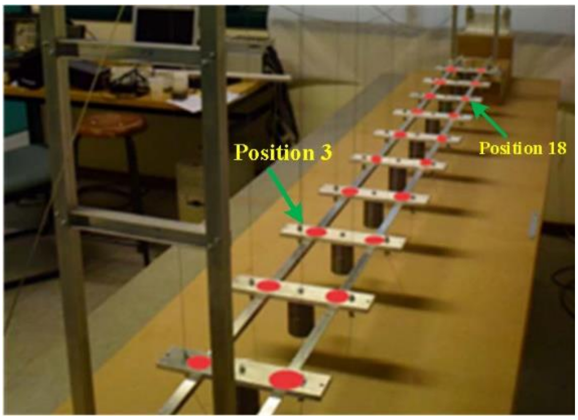


Fig. 8 Laboratory mock-up of the suspension bridge



(a) Accelerometer



(b) Position on the deck

Fig. 9 Detail information of accelerometer

distant of 2.2 m; the deck is free to rotate at both ends and is attached to the catenary by two rows of 10 hangers. This mock-up was used previously for demonstrating active damping with stay cables (Preumont *et al*. 2016).

The catenary consists of a steel cable with a diameter of 1mm and the hangers are made of steel cables of 0.5 mm; the tension T_0 in the catenary and the hangers can be adjusted with screws. It is measured indirectly from the lateral bending natural frequency f_s according to the string formula

$$f_s = \frac{1}{2L}\sqrt{\frac{T_0}{\rho A}} \tag{16}$$

f_s being measured by a non-contact custom made laser sensor (Achkire and Preumont 1998), L is the length of the cable, ρ is its mass density and A is its cross section. In this way, it was possible to distribute the tension in the hangers uniformly.

As shown in Fig. 8, a small magnet is attached to the deck and a voice coil is used to apply a disturbance to the structure (band-limited white noise). Prior to vibration measurement, the data acquisition system was established, which involves 4 single-axial B&K 34371V accelerometers, positioned to measure vertical accelerations. Layout of accelerometers is illustrated in Fig. 9. The sample setting is the same as the section 4.1. The output data are obtained by repeated measurements at all positions on the deck (Fig. 9(b)). The modal parameters of the damaged and undamaged bridge are estimated by the same method (SSI-cov) as in section 4. The experimental frequencies and mode shapes of the healthy bridge are shown in Table 1.

5.2 Damage scenarios

The damage in the hanger cables is simulated by reducing the pre-stress in the cables in the mock-up. Cable numbers are shown in Fig. 2. The numbering of the hangers from 1 to 10 on one side, starting from the right, and from 11 to 20 on the opposite side. Table 3 below presents the details of the damage scenarios considered in this section. For each damage case, the SSI-cov method is used for modal identification, and the damage index is computed.

5.3 Results and discussion

5.3.1 Single damage scenario

- Damage Case 1

Experiment results of the damage index for Case 1 are shown in Fig. 10 (damage at cable 5). In Fig. 10, the

Table 3 Different damage scenarios

Damage scenarios	Location	Damage Intensity
Single damage scenario		
Case 1	Cable 5	Reduce 95% tension
Case 2	Cable 3	Reduce 95% tension
Case 3	Cable 3	Reduce 80% tension
Case 4	Cable 7	Reduce 90% tension
Case 5	Cable 19	Reduce 90% tension
Multiple damage scenario		
Case 6	Cable 15 and Cable 16	Reduce 90% tension
Case 7	Cable 4 Cable 18	Reduce 80% tension Reduce 90% tension
Case 8	Cable 6 Cable 9	Reduce 90% tension Reduce 95% tension

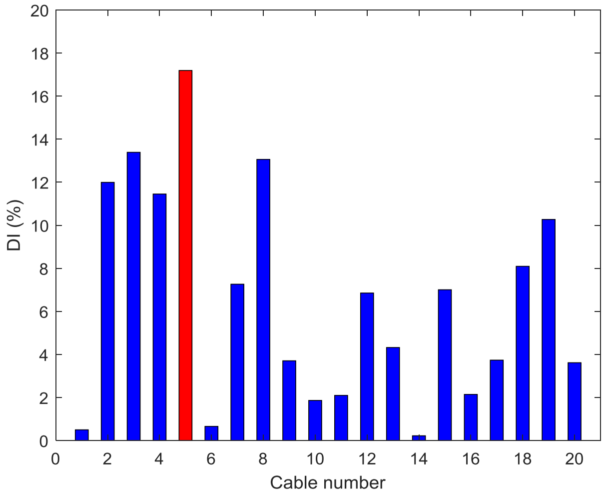
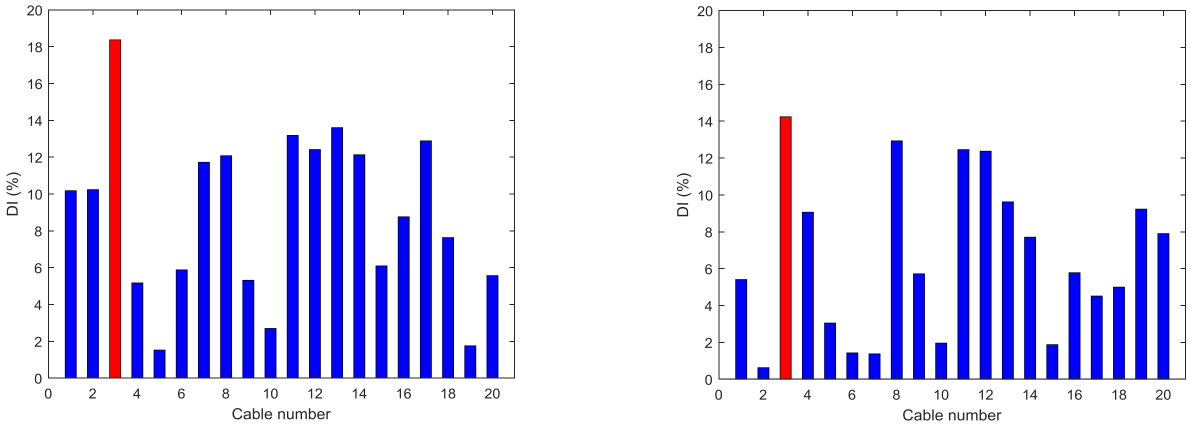


Fig. 10 Damage indices for damaged cable 5 (95%)

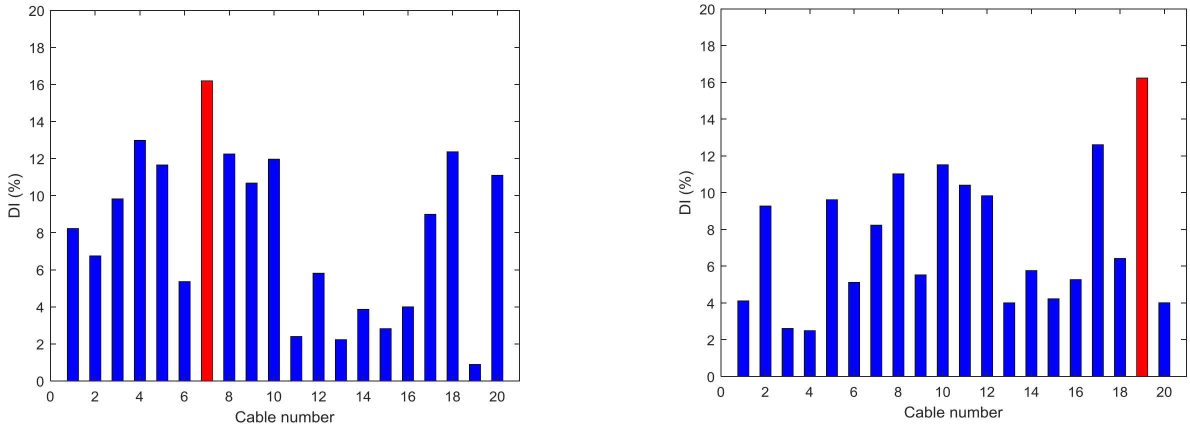


(a) Case 2 (95%)

(b) Case 3 (80%)

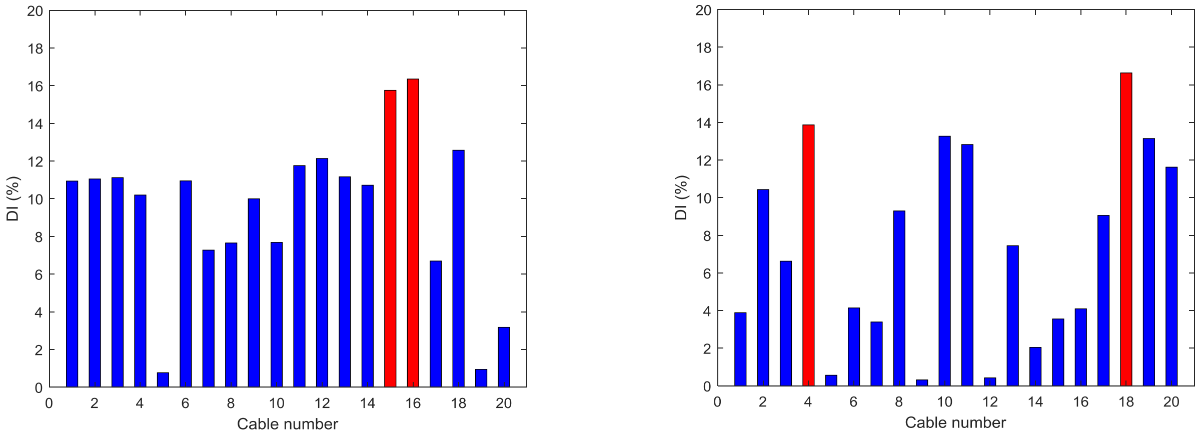
Fig. 11 Damage indices for damaged cable 3

vertical damage index DI_v for the damaged cable reaches its maximum value at the damaged location. In this damage case, the vertical damage index is able to detect the 95% damage of the cable successfully at the middle of span and confirm the actual location of the damage.



(a) Damage indices for damaged cable 7 (90%) (b) Damage indices for damaged cable 19 (90%)

Fig. 12 Results of Case 4 and 5



(a) Damage indices for damaged cable 15 and 16 (b) Damage indices for damaged cable 4 and 18

Fig. 13 Results of Case 6 and 7

- Damage Case 2 and 3

Fig. 11 illustrates the DI_v for Case 2 and Case 3, where the tension of cable 3 is reduced (95%). The damage is easily distinguished from DI_v peak for Case 2. However, when the damage intensity reduces to 80% in Case 3, the DI_v shows its maximum values at various locations, and hence the results are unreliable.

Based on the examination of these three cases, it can be concluded that using the vertical damage index is a successful approach to detect severe damages in the hanger cables.

- Damage Case 4 and 5

In this scenario, there are two 90% tension reductions near the middle and four-fifths of the span (cable 7 and cable 19), respectively. From Fig. 12(a), it is evident that the damage index (DI_v) gives a correct prediction of the damage location. Similar trends are observed in Fig. 12(b). Therefore, the results of these two cases demonstrate the effectiveness of this approach for damage detection, with severe intensities.

5.3.2 Multiple damage scenario

- Multiple damage Case 6

In this damage case, the hanger tension is reduced by 90% at the middle span (cable 15 and cable 16); the results are shown in Fig. 13(a) and it can be seen that DI_v detects accurately the damage.

- Multiple damage Case 7

In damage Case 7, there are two damage positions, one near the mid-span (cable 4) with 80% tension reduction, another one near two-thirds of the span (cable 18) with 90% tension reduction. The results are shown in Fig. 13(b) and the method is able to locate accurately the damage in two-thirds of the span but fail to detect the damage in the middle span.

- Multiple damage Case 8

In this damage case, 90% tension reduction is made near the mid-span (cable 6) and 95% tension reduction is also made near four-fifths of the span (cable 9). The results are shown in Fig.14 and it can be seen that DI_v locates the damage correctly.

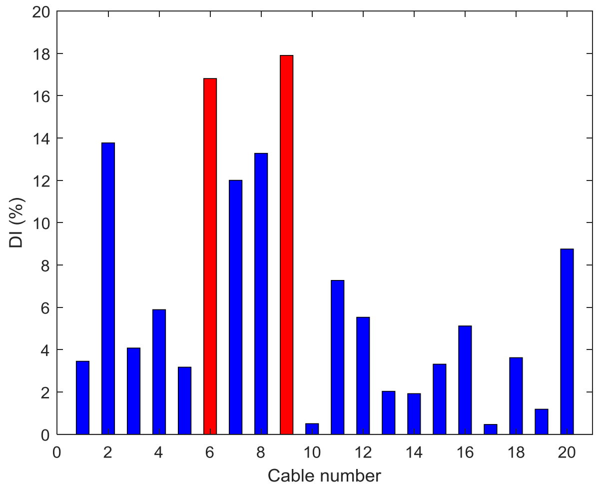
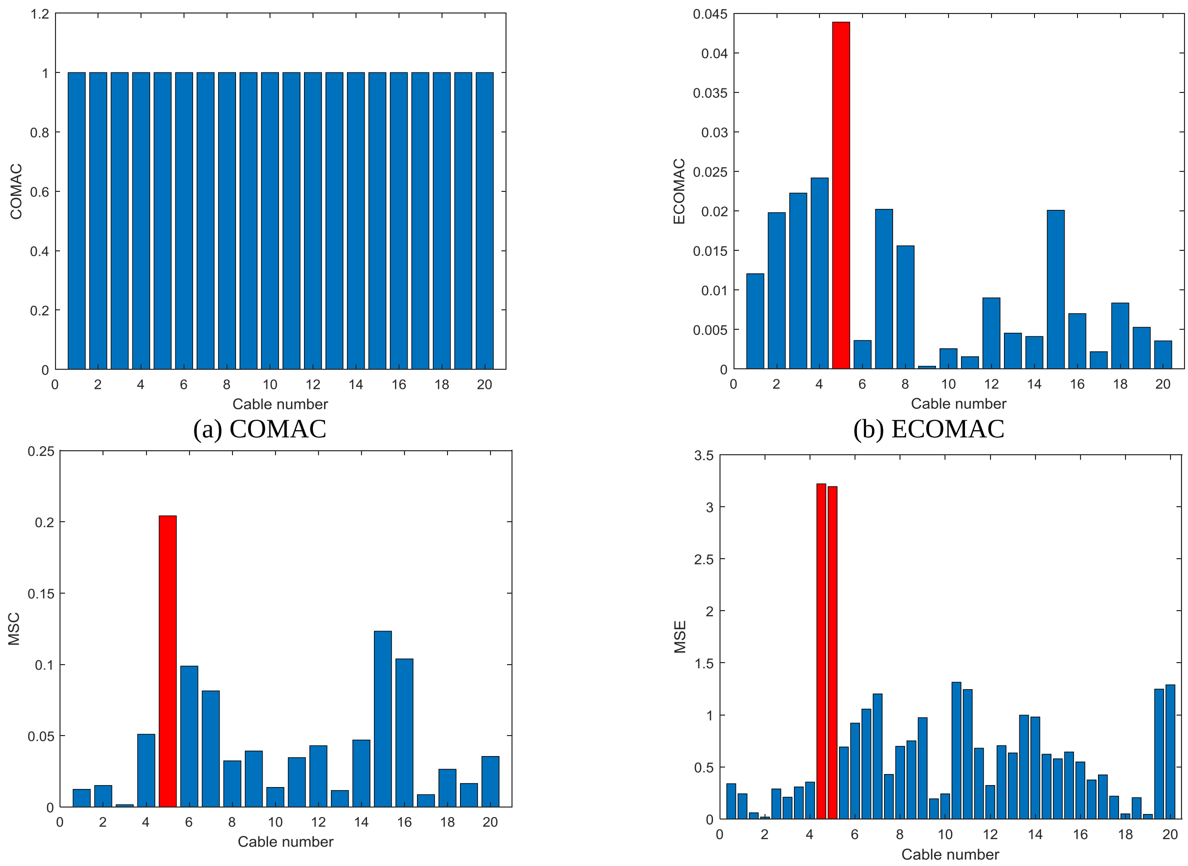


Fig. 14 Damage indices for damage Case 8 (damage at cable 6 and 9)



(a) COMAC (b) ECOMAC (c) MSC (d) MSE

Fig. 15 Detailed results for Case 1. MSE method is realized by computing the numerical difference and a series of local integral of the mode shapes. To avoid the loss of cable information (cable 1, 10 and 11, 20), double bars are added in (d)

5. Comparison with traditional methods

In this section, the traditional modal parameters-based damage detection methods: Coordinate Modal Assurance Criterion COMAC (Wahab and De Roeck 1999), Enhanced Coordinate Modal Assurance Criterion ECOMAC (Hunt 1992), Mode Shape Curvature MSC (Pandey *et al*. 1991) and Modal Strain Energy MSE (Stubbs *et al*. 1995, Li *et al*. 2006) are introduced to detect the damage for the scenarios of Table 3. The experimental results are summarized in Table 4.

Table 4 Comparison of damage detection methods

Case #	COMAC	ECOMAC	MSC	MSE	MF
1	X	**O**	**O**	**O**	**O**
2	X	X	OO	**O**	**O**
3	X	X	OO	OO	OO
4	X	OO	OO	**O**	**O**
5	X	X	X	X	**O**
6	X	OO	**O**	**O**	**O**
7	X	OO	X	OO	OO
8	X	OO	OO	OO	**O**

***O**: damage detected; X: damage not detected; OO: damage detected not accurately

As one observes in Table 4, most of the methods cannot detect the low-level damages, corresponding to a loss of 80% of the hangers (Case 3 and Case 7). When the damage intensity is high, up to 95% tension loss, and close to the mid-span (Case 1), three traditional methods: ECOMAC, MSC and MSE are able to detect the damage successfully and confirm the actual location of the damage. The detailed results are shown in Fig. 15 for Case 1. However, through comparing the experimental results (Table 4) of these four traditional methods we can find that once the damage intensity down to 90% and the damage position deviates from the mid-span, the performance of the MSC and MSE methods are better than the COMAC and ECOMAC methods (Case 2, Case 4 and Case 6). A reasonable explanation for this outcome is the fact that both the COMAC and ECOMAC methods depend on the mode shape amplitudes of the damaged structure, while the MSC and MSE methods are calculated based on the second derivative of the mode shapes, which is more sensitive to local the damage (Talebinejad *et al*. 2011). As a result, these two methods are relatively successful to detect the damage. For the 90% damage level near the mid-span (Case 4 and Case 6), the MSE method has the same performance as MF method. However, when the damage occurs near the edge of the span (Case 5 and Case 8), the performance of the MSC and MSE methods are inferior to the MF method. Therefore, the MF method is more applicable than others.

6. Conclusions

This study compared numerically and experimentally several damage detection methods on a laboratory suspension bridge mock-up, with an emphasis on the Modal Flexibility method. It presented a successful, numerical and experimental implementation of the method for damage detection in the hangers of suspension bridges. The effectiveness is demonstrated by a number of numerical simulations and experiments. At present, the modal flexibility-based study for detecting the damage in suspension bridge hangers is rare. This study is the first experimental verification in the MF method-based damage detection in suspension bridge hangers. In addition, the comparison between the MF method and the traditional methods (COMAC, ECOMAC, MSC and MSE) is carried out for eight experimental cases. Modal parameters including natural frequencies and mode shapes of the healthy structure and the damaged structure are extracted by the SSI-cov method. Based on the obtained results and their interpretations, the main findings are presented below:

- The DI_v was capable for successfully detecting single and multiple damage (more than 90% intensity) in the hangers of suspension bridges. In practice of SHM, the DI as an indicator can be evaluated regularly and significant changes would be used as a warning flag.
- The results obtained with the DI_v suggest some superior capability compared with the traditional modal parameters-based methods.
- The performance of the MSC and MSE methods were better than the COMAC and ECOMAC methods. Once the damage occurs at the edge of the span, the performance of the MSC and MSE methods were inferior to that of the MF method. However, only high-intensity damages were detectable by the modal flexibility method. This is due to the very low contribution of the single hanger stiffness to the global stiffness matrix.

Finally, the findings of this study must be extended and validated to field tests on a real and full-scale suspension bridge.

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