

# Numerical simulation of the effect of bedding layer geometrical properties on the shear failure mechanism using PFC3D

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**Abstract.** In this research the effect of bedding layer angle and bedding layer thickness on the shear failure mechanism of concrete has been investigated using PFC3D. For this purpose, firstly calibration of PFC3d was performed using Brazilian tensile strength. Secondly shear test was performed on the bedding layer. Thickness of layers were 5 mm, 10 mm and 20 mm. In each thickness layer, layer angles changes from 0° to 90° with increment of 25°. Totally 15 model were simulated and tested by loading rate of 0.016 mm/s. The results shows that when layer angle is less than 50°, tensile cracks initiates between the layers and propagate till coalesce with model boundary. Its trace is too high. With increasing the layer angle, less layer mobilize in failure process. Also the failure trace is very short. It's to be note that number of cracks decrease with increasing the layer thickness. The minimum shear test strength was occurred when layer angle is more than 50°. The maximum value occurred in 0°. Also, the shear test tensile strength was increased by increasing the layer thickness.

**Keywords:** bedding layer; shear test; anisotropy; tensile crack; PFC3D

## 1. Introduction

Anisotropy in concretes affects their mechanical properties and gained importance in different fields of civil and material engineering. It has been thoroughly demonstrated that many concretes have apparent anisotropic characteristics affecting their mechanical, thermal, seismic and hydraulic properties of these structural material. Various engineering application of concretes need to consider the anisotropic behavior of these brittle materials because these behaviors will produce some errors of different magnitudes depends on the extent of the rock on concrete anisotropy (Amadei 1982, 1983, 1996, Barla 1974, Pinto 1966, 1970, 1979, Rodrigues 1966, Salamon 1968).

The concretes or rock like materials may contains some planes of weakness in form of cracks, joints, schistosity, beddings, faults (Goodman 1993). It has been shown by many investigators that the compressive and tensile strengths of rocks, rock-like materials and concretes can be affected by their inherent anisotropic natures such as existence of weakness planes with in their structures which are sometimes known as transversely isotropic materials (Chen 1998, Chou 2008, Exadaktylos 2001, Nasser 1997, 2003, Ramamurthy 1993, Tien 2000). However, many experimental and numerical works have been carried out to study the cracks initiations and cracks propagations in many brittle materials due to the pre-existing cracks considering

different loading conditions (Zhou *et al.* 2014, Zhou *et al.* 2012, Lancaster *et al.* 2013, Ramadoss 2013, Pan *et al.* 2014, Mobasher *et al.* 2014, Sarfarazi *et al.* 2014, Noel and Soudki 2014, Haeri *et al.* 2014, Oliveira and Leonel 2014, Kim and Taha 2014, Tiang *et al.* 2015, Wan Ibrahim *et al.* 2015, Silva *et al.* 2015, Lee *et al.* 2015, Kequan and Zhoudao 2015, Haeri 2015a, b, Haeri *et al.* 2015a, b, c, Gerges *et al.* 2015, Liu *et al.* 2015, Wasantha *et al.* 2015, Li *et al.* 2015, 2016, Fan *et al.* 2016, Sardemir 2016, Sarfarazi *et al.* 2016, Sarfarazi and Haeri *et al.* 2016, Haeri *et al.* 2016a, b, c, Haeri and Sarfarazi 2016, Mohammad 2016, Shuraim 2016, Akbas 2016, Rajabi 2016, Yaylac 2016, Wang *et al.* 2016, 2017). The failure process pf these materials usually chanes with the orientation of these weakness planes with respect to the loading direction (Tien 2006, Tavallali 2010 a, b). Many numerical methods can be applied to investigate the effect of bedding Layer geometrical properties on the punch shear test, such as General Particle Dynamics (GPD) (Bi *et al.* 2017, Zhou and Yang 2012, Bi *et al.* 2015). Peridynamics (PD) (Silling 2000, Yunteng 2017, Wang 2018), The Extended Finite Element Method (Zhou *et al.* 2015a, b). It has been shown that two kinds of compressive failure modes known as internal compression shear failure and bedding plane sliding failure may exist for the layered concretes. On the other hand, three kinds of tensile failure modes may exist for these layered concretes i.e. the pure tensile failure, the pure shear failure or a mixed form of both tensile and shear failure. It has been shown experimentally that several rock types (especially those of sedimentary and metamorphic) contains inherent or structural anisotropy of different degrees (Saeidi *et al.* 2013, Hoek 1964, McLamore and

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Gray 1967, Horino and Ellickson 1970, Kwasniewski 1993, Nasser *et al.* 2003, Al-Harthi 1998). The spacing of bedding planes or rock layers is a measure of isotropic and anisotropic behaviors of sedimentary rocks on the other hand the metamorphic rocks are mostly anisotropic due to their natures affected by schistosity and cleavages (Singh *et al.* 1989, Ramamurthy 1993).

The Brazilian tensile strength (BTS) of different metamorphic rocks considering their schistosity orientations have been measured by several researchers (Berenbaum and Brodie 1959, Hobbs, 1963, Debecker and Vervoort 2009). The effects of relative orientation of layer planes with respect to the loading direction on the indirect tensile strength of some sedimentary rocks were investigated by Hobbs (1963) on siltstone, by McLamore and Gray (1967) on shale, by Tavallali and Vervoort (2010 a, b), as well as by Chen *et al.* (1998) on sandstone. Various failure modes of concrete samples were used in the literature to study, observe and classify the anisotropic rocks. Two modes of concrete failures proposed by Chen *et al.* (1998), i.e., the tensile splitting along the diameter of the loaded sandstone samples and shear failure along sandstone layers. Three types of failure process were identified by Tavallali and Vervoort (2010b) for the Brazilian disc-shaped specimens prepared from anisotropic rocks: i) The failure process activated nearly parallel to the direction of layers; ii) the failure process activated due to central fractures developed roughly parallel to the loading directions of the specimen; iii) the failure process developed due to non-central fractures.

However, in the concrete structures, investigating the effect of concrete anisotropy on their mechanical behaviors; thermal, seismic, and hydraulic properties under different loading conditions is of paramount importance.

The shear test proposed by Backers 2002, is method for determination of shear strength of concrete (Fig. 1).

In this test, a concrete cylinder is placed vertically between the loading platens of the machine and is compressed by two steel plates on the top and bottom surfaces of the cylinder. The specimen splits across many vertical diametric planes similar to the split-cylinder test, but the testing arrangement for the new test may be reduced. The relation is proposed by Backers 2002

$$\sigma = \frac{F}{A} \quad (3)$$

Where,  $\sigma$  is shear strength,  $F$  is failure load,  $A$  is area of bridge section.

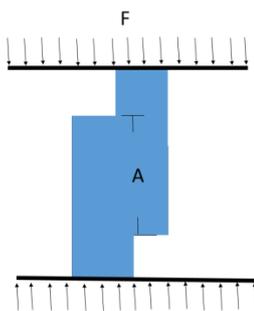


Fig. 1 Shear test, Backers 2002

## 2 Numerical modeling with PFC2D

This study presents the punch tests shearing stress results obtained from the laminated concrete specimens simulated numerically by a two-dimensional particle flow code (PFC2D). The effects of weak planes (laminations) on the failure strength and fracture patterns of these specimens are determined by a sophisticated discrete element code. In this procedure, a particular rock mass can be modeled as an assembly of rigid particles bonded to one another through some contact points (Cundall 1979, Potyondy and Cundall 2004). In the two-dimensional version of the code developed based on the discrete element method (DEM), the circular discs are considered to model the particles connected with cohesive and frictional bonds. The planar walls are modeled to confine each circular disc (particle) and also the particle assembly representing the whole rock mass specimen. The contacts in between the particles within the assembly are simulated by parallel bond model algorithm. The macro-strength of each sample and their fracturing patterns and failure process under various loading conditions are particularly influenced by the particular values assigned to the strength bonds. The frictional properties of the particles can be activated by specifying a reasonable friction coefficient which is mobilized as long as the particles stay in contacts at the contact points. The applied normal stress may exceed the specified normal bonding strength in between the particles where the tensile cracks are produced while the shear cracks are developed when the applied shear stress exceeds that of the specified shear bonding strength of particles due to rotation or in-plane shearing of the particles. During the failure process of a particular particles assembly, the shear strength of the assembly decreases to its residual value while the tensile strength at the contact points immediately drops to zero after the bonding breakage within the particles occurs (Itasca Consulting Group Inc 2004, Cho *et al.* 2007, 2008, Potyondy and Cundall 2004, Sarfarazi 2014). In PFC2D, the basic micro-parameters (such as bond strength, bond stiffness and contact friction coefficient) for a described contact problem are selected to represent all the microscopic behaviors of the particle assembly. These micro-parameters are selected such that they provide a reasonable macro-scale behavior for the material being modeled. An inverse modeling technique based on the trial and error approach is adopted in PFC3D (Itasca 1999) to determine the appropriate micro mechanical properties of the particle assembly from the macro mechanical properties given by the experimental tests for the numerical simulation of any geo-mechanical problem. The explicit finite difference scheme is adopted in PFC2D to solve the equations of force and motion based on the Newton's second law of motion, therefore, the initiation and propagation of induced cracks at the time of bond breaking within the assembly can be readily tracked (Potyondy and Cundall 2004). Table 1 represents the adopted micro-properties for a specific particle assembly by calibrating the PFC2D using the standard calibration process (Potyondy and Cundall 2004). The limitations of DEM are: (a) Fracture is closely related to the size of elements, and that is so called size effect.

Table 1 Micro properties used to represent the intact rock

Parameter	Value	Parameter	Value
Type of particle	disc	Parallel bond radius multiplier	1
Density (Kg/m <sup>3</sup> )	3500	Young modulus of parallel bond (GPa)	32
Minimum radius (mm)	0.27	Parallel bond stiffness ratio	2
Size ratio	1.56	Particle friction coefficient	0.5
Porosity ratio	0.08	Parallel bond normal strength, mean (MPa)	20
Damping coefficient	0.7	Parallel bond normal strength, SD (MPa)	2
Contact young modulus (GPa)	32	Parallel bond shear strength, mean (MPa)	20
Stiffness ratio	2	Parallel bond shear strength, SD (MPa)	2

Table 2 Micro properties used to represent the bedding interfaces

Parameter	Value	Parameter	Value
n_bond	1e3	s_bond	1e3
fric	0.25		

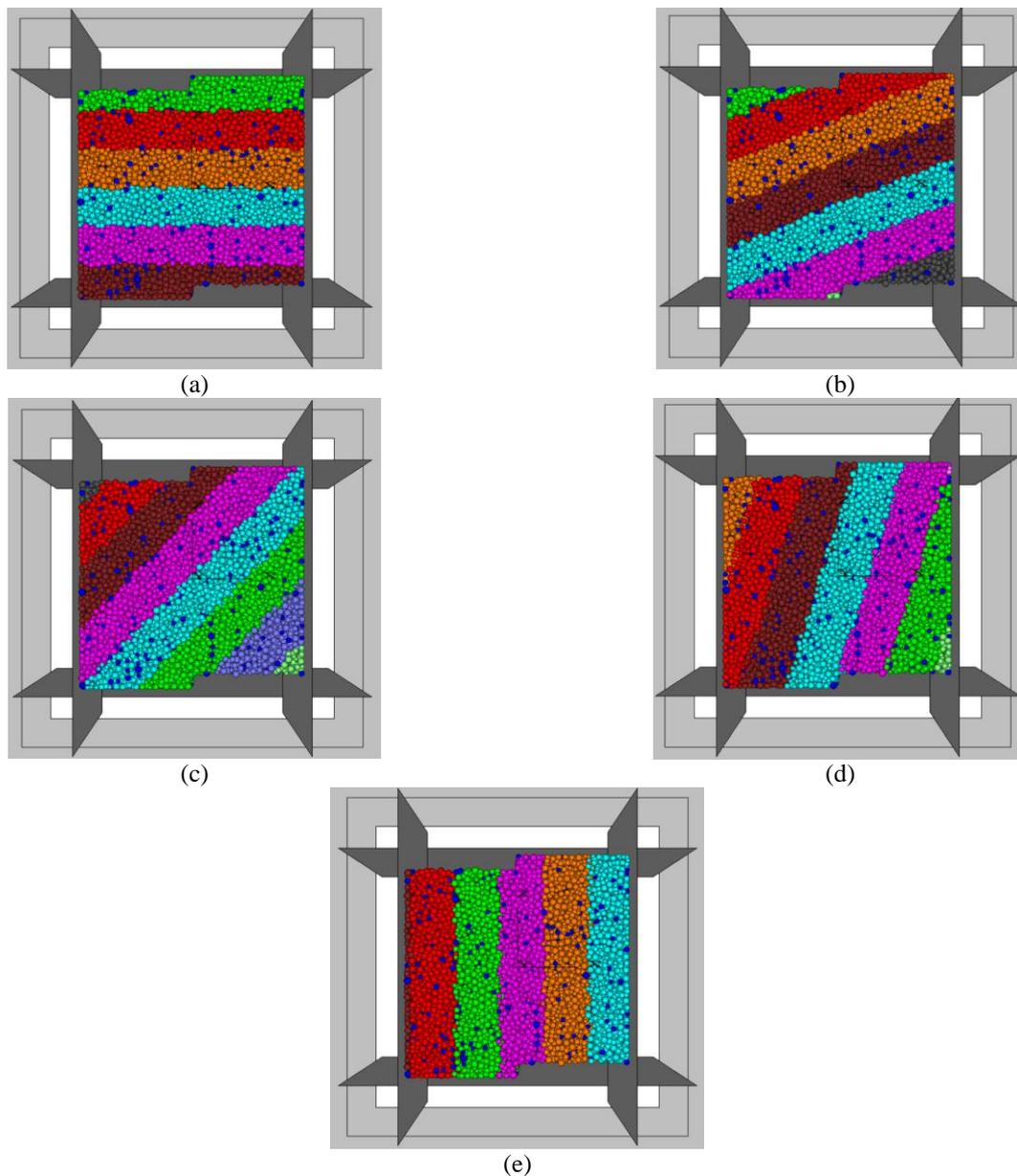


Fig. 3 Anisotropic concrete with Layers thicknesses of 10 mm and layer angle of (a) 0°, (b) 25°, (c)50°, (d) 75° and (e) 90°

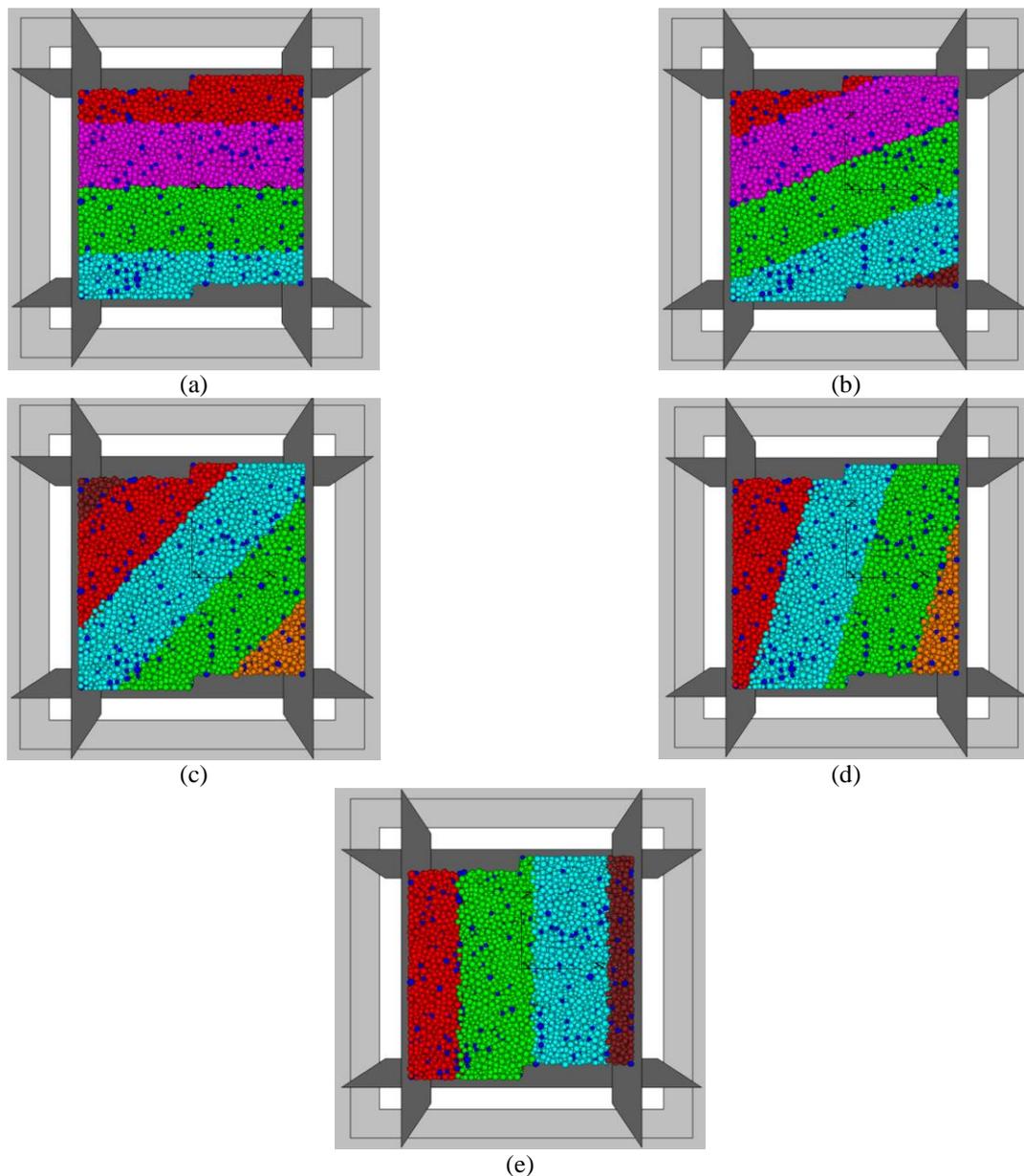


Fig. 4 Anisotropic concrete with Layers thicknesses of 20 mm and layer angle of (a)  $0^\circ$ , (b)  $25^\circ$ , (c)  $50^\circ$ , (d)  $75^\circ$  and (e)  $90^\circ$

(b) Cross effect exists because of the difference between the size and shape of elements with real grains. (c) In order to establish the relationship between the local and macroscopic constitutive laws, data obtained from classical geomechanical tests which may be impractical are used.

### 2.1 Numerical biaxial tests on non-persistent open joint

#### 2.1.1 Preparing the model

After calibrating PFC2D, shear tests in anisotropic concrete were numerically simulated by creating a rectangle model (Figs. 2-4). PFC specimen dimension were  $100\text{ mm} \times 100\text{ mm}$ . two rectangular band was removed from right and left of the model also one rectangular band was

deleted from middle of the model (Figs. 3-5). A total of 18,189 disks with a minimum radius of 0.27 mm were used to make up the specified specimen. Particles were surrounded by six walls. Bedding layers were formed in the model. Layers thicknesses were 5 mm, 10 mm and 20 mm. in constant layer thickness, the layer angularity changes from  $0^\circ$  to  $90^\circ$  with increment of  $25^\circ$ .

In total, 15 specimens containing different bedding layer were set up to investigate the influence of Layers thickness and layer angularity on failure behavior of models. Micro-properties for bedding layer interfaces was chosen too low (Table 2). An inverse modeling technique based on the trial and error approach is adopted in PFC3D (Itasca 1999) to determine the appropriate micro mechanical properties. Two upper and lower loading walls move toward each other by rate of 0.016 mm/s.

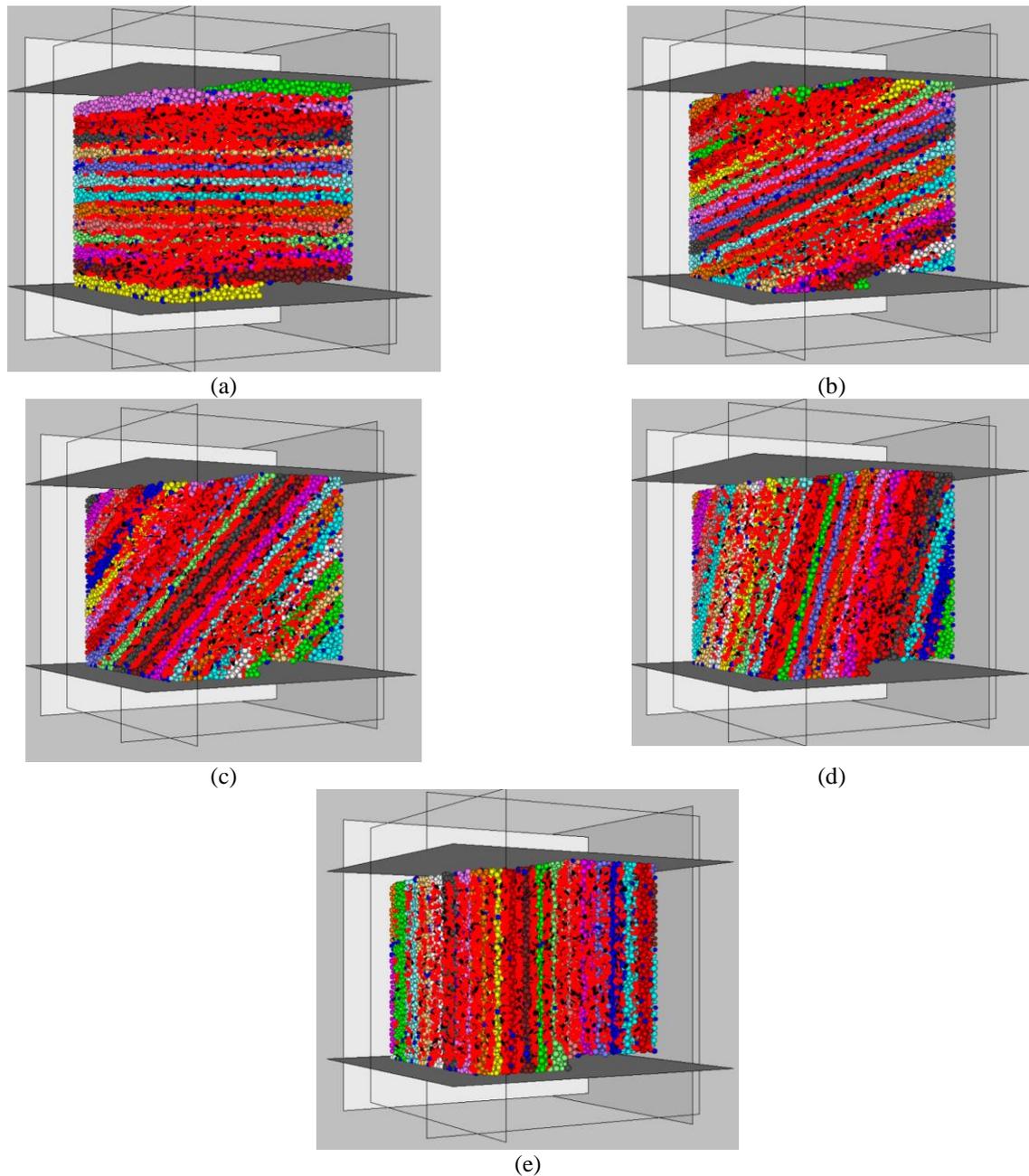


Fig. 5 Failure pattern in anisotropic concrete with layers thicknesses of 5 mm and layer angle of (a)  $0^\circ$ , (b)  $25^\circ$ , (c)  $50^\circ$ , (d)  $75^\circ$  and (e)  $90^\circ$

### 3. Results

#### 3.1 The effect of layer properties on the failure pattern of models

Figs. 5-7 shows the effect of layer thickness and layer angles on the failure pattern of models. Red line and yellow line represent the tensile crack and shear crack, respectively.

When layer angle is less than  $50^\circ$  (Figs. 5(e)-5(g); Figs. 6(e)-6(g), Figs. 7(e)-7(g)), tensile cracks initiates between the layers and propagate till coalesce with model boundary. Its trace is too high. With increasing the layer angle, less layer mobilize in failure process. Also the failure trace is

very short. Its to be note that number of cracks decrease with increasing the layer thickness (Figs. 5-7).

#### 3.2 The effect of bedding layer specification on the test tensile strength

Fig 8 shows the effect of bedding layer angle on the shear strength. Also the results of bedding layer thickness have been shown in this figure. The minimum test strength was occurred when layer angle is more than  $60^\circ$ . The maximum value occurred in  $0^\circ$ . Also, the shear strength was increased by increasing the layer thickness.

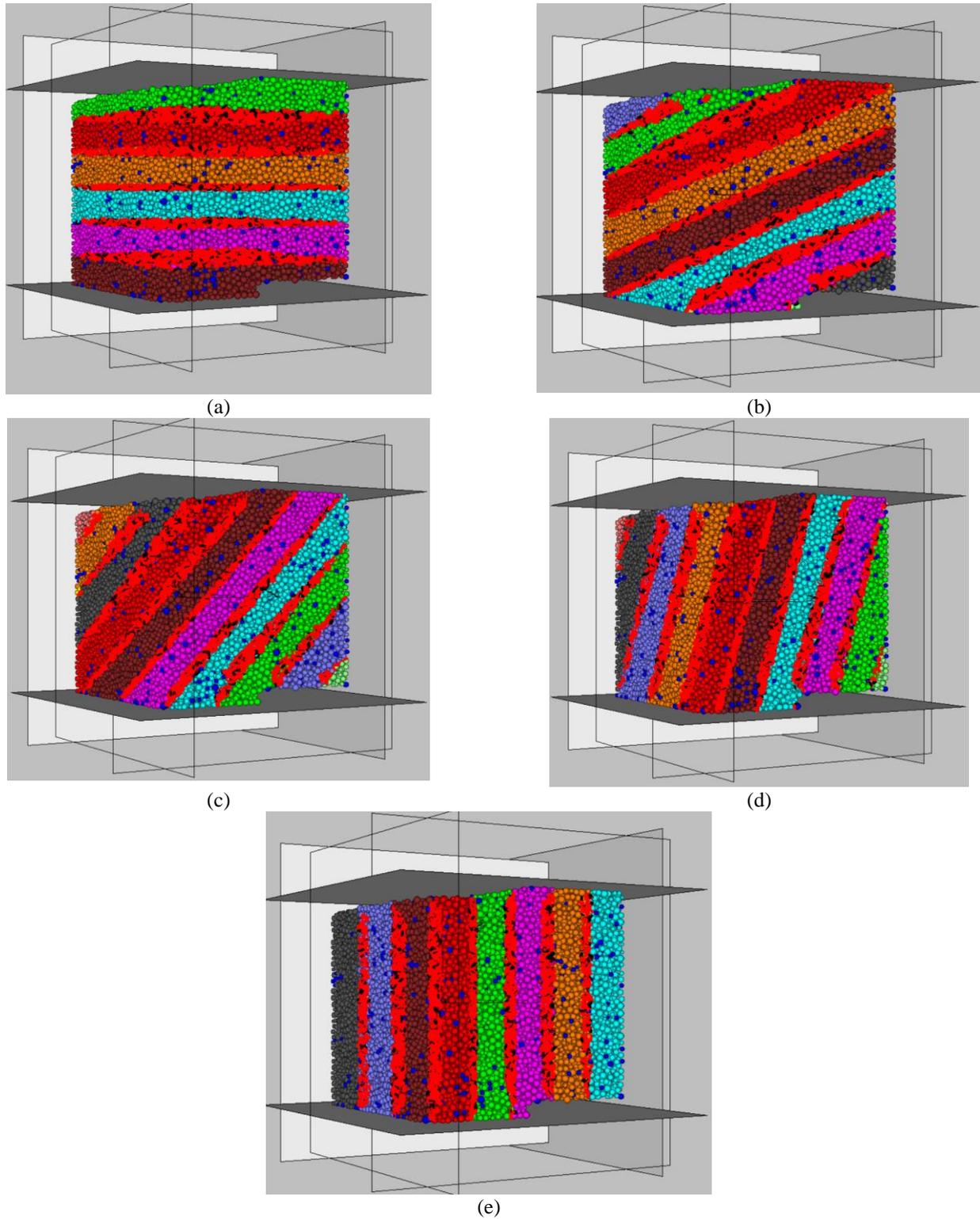


Fig. 6 Failure pattern in anisotropic concrete with layers thicknesses of 10 mm and layer angle of (a)  $0^\circ$ , (b)  $25^\circ$ , (c)  $50^\circ$ , (d)  $75^\circ$  and (e)  $90^\circ$

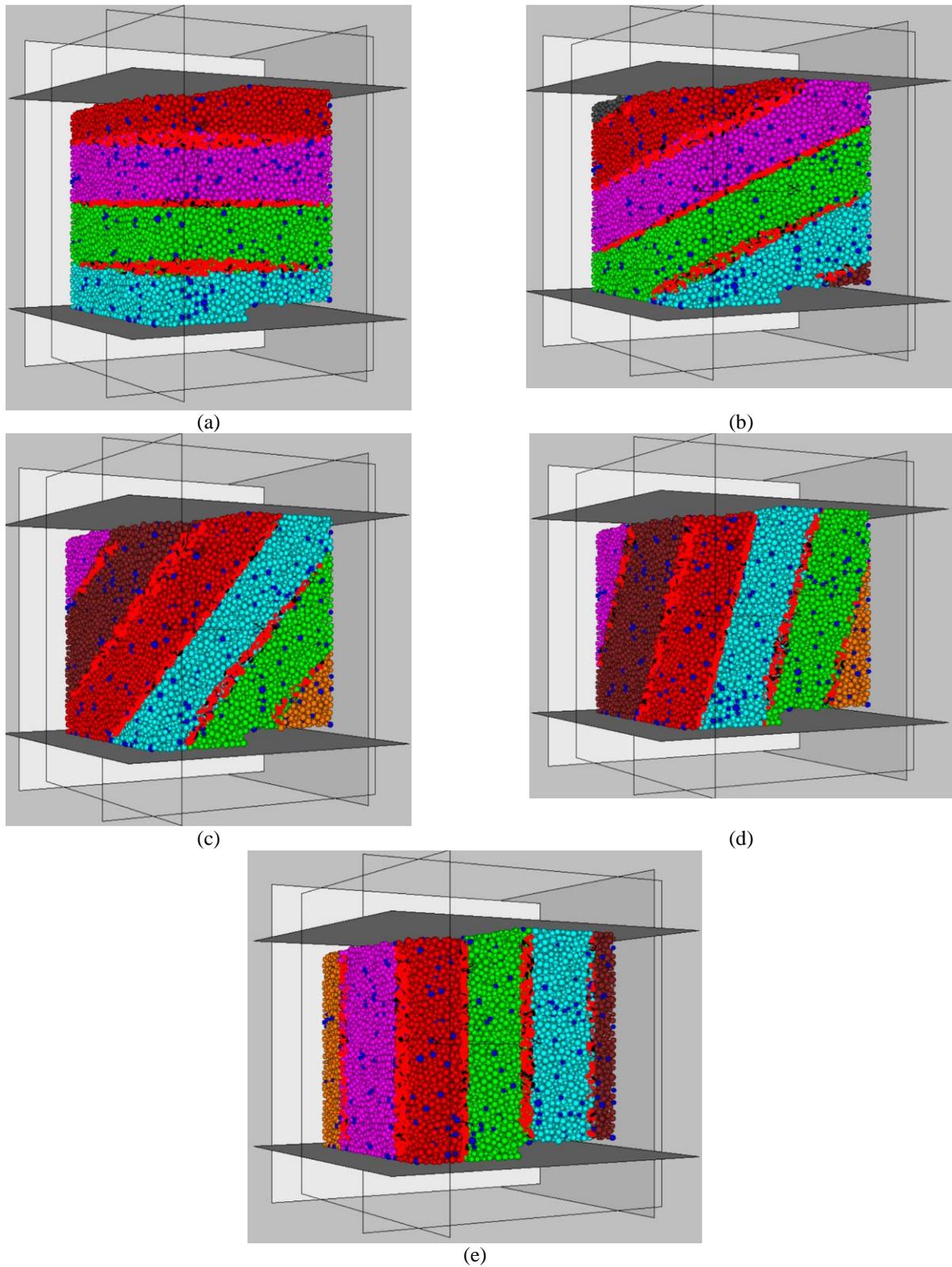


Fig. 7 Failure pattern in anisotropic concrete with layers thicknesses of 20 mm and layer angle of (a) 0°, (b) 25°, (c) 50°, (d) 75° and (e) 90°

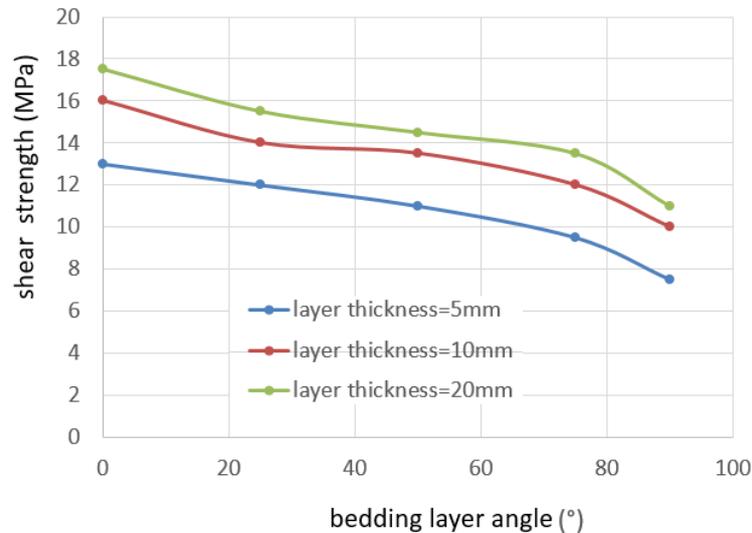


Fig. 8 The effect of bedding layer angle on the shear strength

#### 4. Conclusions

In this work the effect of bedding layers angle and layers thickness on the shear failure mechanism of concrete has been investigated using PFC3D. Firstly calibration of PFC3d was performed using laboratory Brazilian tensile strength. Secondly shear test was performed on the bedding layer. Thickness of layers were 5 mm, 10 mm and 20 mm. in each thickness layer, layer angles changes from 0° to 90° with increment of 25°. Totally 15 model were simulated and tested. The results shows that

- When layer angle is more than 60°, shear cracks initiates between the layers and propagate till coalesce with model boundary.
- When layer angle is less than 50°, tensile cracks initiates between the layers and propagate till coalesce with model boundary. Its trace is too high.
- With increasing the layer angle, less layer mobilize in failure process. Also the failure trace is very short.
- It's to be note that number of cracks decrease with increasing the layer thickness.
- The minimum shear test strength was occurred when layer angle is more than 50°. The maximum value occurred in 0°.
- Also, the shear strength was increased by increasing the layer thickness.

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