**Smart Structures and Systems**, *Vol. 22, No. 5 (2018) 631-641*
DOI: https://doi.org/10.12989/sss.2018.22.5.631


# Effective Heterogeneous Data Fusion procedure via Kalman filtering

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*(Received July 22, 2018, Revised October 11, 2018, Accepted November 10, 2018)*

**Abstract.** This paper outlines a computational procedure for the effective merging of diverse sensor measurements, displacement and acceleration signals in particular, in order to successfully monitor and simulate the current health condition of civil structures under dynamic loadings. In particular, it investigates a Kalman Filter implementation for the Heterogeneous Data Fusion of displacement and acceleration response signals of a structural system toward dynamic identification purposes. The procedure is perspectively aimed at enhancing extensive remote displacement measurements (commonly affected by high noise), by possibly integrating them with a few standard acceleration measurements (considered instead as noise-free or corrupted by slight noise only). Within the data fusion analysis, a Kalman Filter algorithm is implemented and its effectiveness in improving noise-corrupted displacement measurements is investigated. The performance of the filter is assessed based on the RMS error between the original (noise-free, numerically-determined) displacement signal and the Kalman Filter displacement estimate, and on the structural modal parameters (natural frequencies) that can be extracted from displacement signals, refined through the combined use of displacement and acceleration recordings, through inverse analysis algorithms for output-only modal dynamics identification, *based on displacements*.

**Keywords:** Structural Health Monitoring (SHM); Heterogeneous Data Fusion (HDF); displacement and acceleration measurements; Kalman Filter (KF); output-only structural identification

## 1. Introduction

In the wide field of civil engineering, Structural Health Monitoring (SHM) and the associated development of consistent simulation tools, such as model updating (see e.g., Lee *et al.* 2007, Wu and Wang 2014, Ferrari *et al.* 2015b, 2017, and references quoted therein), represent usual but non-trivial tasks. In the recent years, supported by the broad development of novel measurement technologies for structural identification purposes, Heterogeneous Data Fusion (HDF) approaches are increasingly adopted for supporting such activities (e.g., Xiao *et al*. 2005, Jiang *et al.* 2005, 2006, Su *et al.* 2009, Zhao *et al.* 2010, Cho *et al.* 2015a, Ferrari *et al.* 2015a, 2016, and cited references). Several pertinent contexts also concern the possible limitation and control of structural vibration, within different excitation regimes, by the insertion of appropriate damping devices (Salvi and Rizzi 2014, 2016, 2017, Salvi *et al.* 2015, and references quoted therein), which may need a fine tuning, as coupled to the same identification process (Wang and Lin 2015).

Data fusion procedures consist in integrating measurements acquired from different types of sensors, so that the resulting information may be characterized by a lower degree of uncertainty. In addition, if the measured data display a heterogeneous nature (for example, displacements and accelerations), an appropriate fusion is required for rendering a comprehensive description of the structure of interest (this can also alleviate errors of displacements computed on the basis of numerical integration from accelerometer records). In fact, while acceleration-based monitoring may detect variations on the structural condition, displacement records may alert for the presence of excessive service loads, as well as to enable for fatigue estimation. Also, they may be remotely acquired, in a convenient way.

While data fusion may represent a usual procedure in many research areas where signal analysis is commonly involved, its application to the civil engineering domain has not been deeply explored yet (first contributions in this field may be found in Smyth and Wu 2007, Chatzi and Smyth 2009, Chatzi and Fuggini 2012, 2015, Park *et al*. 2013, Ferrari *et al*. 2016). A crucial aspect is that, within this scenario, known difficulties commonly related to structural identification are augmented by issues connected to the necessary calibration of the filters employed within the data fusion procedure.

In this work, a Kalman Filter (KF) (Kalman 1960) implementation is developed for fusing simulated noise-added displacements and accelerations of a numerical structural system, for several types and levels of added noise. This aims at simulating measurements that may be extensively acquired "on field", through displacement sensors, and at exploring the perspective of their use for SHM and modal identification purposes, possibly complemented by the information coming from a few acceleration measurements.

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http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

Data fusion approaches between displacement and acceleration measurements involving KF are available in the very recent literature (see e.g., Kim *et al*. 2016, Lei *et al*. 2016, Lin and Luo 2016, Liu *et al*. 2016), through which KF should assist in estimating the condition of structures that undergo ambient vibrations, allowing for predicting potential damage and evaluating the residual performance capacity of a structure (Lei *et al*. 2015, Yuqing 2015). Appropriate damage detection may also require for an optimal design of the adopted sensor network, in terms of number, type and spatial deployment of sensors (Capellari *et al*. 2017).

Here, these goals may be pursued through modal dynamics identification techniques, which aim at determining the modal dynamics characteristics of a structure, primarily the natural frequencies. It is well-known that the variation of these quantities during the life-cycle of a structure may reveal potential changes in its performance characteristics. In the present identification perspective, main remote displacement signals are taken for the modal frequency extraction, possibly corroborated by a few acceleration response signals, in order to clear the frequency targeting, through effective data fusion based on Kalman filtering.

Papadimitriou *et al*. (2012) first suggested the possibility to adopt the structural dynamic response for fatigue damage identification purposes. In the last years, many other scientists have dealt with the topic of the dynamic response estimation of a structural system within a stochastic framework, and several algorithms have been developed to treat both linear models, e.g., Kalman Filter (Kalman 1960), and non-linear models, e.g., Particle Filter (Gordon *et al*. 1993) and Unscented Kalman Filter (Julier and Uhlmann 1998). The state of the system is represented in terms of displacements and velocities of the response at specific locations along the structure. In practice, however, it is not always possible to measure displacements and velocities of the considered structural system (Lee *et al*. 2007); thus, when the knowledge of such quantities may be required, KF represents an important tool for accurately reconstructing the whole dynamic response, starting from incomplete measurements (Lee and Yun 2008, Azam *et al*. 2015, Ding and Guo 2016, Eichstadt 2016, Kim and Sohn 2017).

In this paper, a Kalman Filter is implemented into a HDF process in order to combine numerically-determined data from heterogeneous sensors, i.e., displacement and accelerometer sensors, aiming at deriving accurate displacement estimates of a studied dynamical system, adopted then for modal dynamics identification purposes, based on displacement data. Both displacement and, to a lesser extent, acceleration measurements are considered to be affected by errors, represented by noise added to the signals, in order to simulate the difficulty in the "on field" detection of such a kind of data.

Then, thanks to the use of the enhanced KF estimated displacements, through the adoption of appropriate FDD inverse analysis algorithms for output-only modal dynamics identification (e.g., Zghal *et al*. 2014, Pioldi *et al*. 2015a, b, 2017, Chatzis *et al*. 2017, and wide reference frameworks discussed therein) it becomes possible to extract the natural frequencies of the benchmark structure, from displacement signals. Finally, from a comparison between these frequencies and the numerically-determined natural frequencies, it is possible to cross-evaluate the accuracy of the achieved KF estimates.

The main goal of the present research investigation is twofold:

- firstly, it aims at demonstrating the effectiveness of a KF implementation in civil contexts for SHM and identification purposes. Differently from what it has been previously shown in Ferrari *et al*. (2016), in which the efficacy of the fusion procedure has been preliminary demonstrated for a specific case study, concerning historic reinforced concrete Brivio bridge (1917), this paper provides a wider and more general treatment on the use of KF in HDF procedures, by exploring strengths and weaknesses of such a technique, and aiming at achieving a clarifying and comprehensive framework on the topic, within a controlled environment based on synthetic response signals;
- secondly, it extensively attempts to investigate the possibility of employing displacement data for modal identification purposes. Although modal properties may be conveniently extracted from acceleration measurements only, the perspective of alternatively using displacement recordings toward the same end may open up new scenarios in the signal acquisition stage, since it would make possible to monitor a specific structure (and to deduce its current modal properties) without directly acting on it (or only partially involving the structure through the placement of a few accelerometers), for example by using a total station.

The main achievements of the present research work may be resumed as follows:

- the adoption of remote displacement signals, possibly enriched by reliable acceleration recordings, through an original KF implementation included within the Data Fusion procedure, is shown to become effectively useful for structural monitoring purposes, as leading to a truthful reconstruction of the original structural response, despite for possible disturbances of various kinds;
- the maximum level of noise that may be tolerated on displacement and acceleration measurements is defined, to allow for a successful HDF and to achieve reliable estimates of the current structural dynamic response; within this process, filtered displacements (taking benefit from DF processing) reveal to be more sensitive to noise-affected accelerations than to noise-affected displacements (as raw displacement data);
- the multi-rate KF feature allowing for a relatively low sampling rate for the displacement measurements, fundamental to overcoming low-frequency integration errors, and for higher sampling rates for the accelerations (i.e., within the frequency range where accelerometers result more accurate), is revealed, enabling each sensor type to play its role on its inherent

strengths.

The paper is organized as follows. Section 2 provides a brief description of the benchmark dynamical system taken into consideration for the entire analysis, i.e., a 3-DOF shear-type frame. The main points followed during the analysis are outlined, and the multi-rate Kalman Filter scheme involved in the HDF procedure, as originally derived by Smyth and Wu (2007), is further elaborated through a dedicated numerical implementation. Section 3 presents various scenarios of numerical analysis, involving the Kalman filter algorithm, aiming at improving the displacement response of a linear dynamic system by complementary acceleration data. Different artificial random noise signals are added to the observed (numerically-determined) synthetic displacements of a 3-DOF frame under random force top-floor loading, simulating the error that may occur in the displacement sensors during the "on field" measurement acquisition stage. Also, the cases of noise-free accelerations and (slight) noise-affected accelerations are investigated through several numerical analyses, at increasing displacement noise levels, and for different inherent modal damping ratios of the underlying structural system. Limits of applicability of such a technique are explored and filter effectiveness is evaluated in terms of RMS error between originally uncorrupted displacements and filtered displacements, and on the basis of the modal parameters (natural frequencies) that can be extracted from the KF response estimation, based on displacement signals, through appropriate inverse analysis algorithms. The results related to the analyzed cases are presented and commented. It is worth noting that all the assertions provided within this paper hold true for synthetic signals generated from direct time integration prior to the filter and identification analysis. Conclusions and global remarks are finally outlined in Section 4, and a few future perspectives are disclosed.

## 2. Theory

### 2.1 Description of the benchmark structural system

The dynamic structural system taken into consideration for the present numerical simulation analyses is a 3-DOF shear-type building, as schematically represented in Fig. 1.

This choice is motivated by the simplicity of the geometrical and structural properties that characterize such a kind of system, thus allowing for easy analytical and numerical treatments. Furthermore, this structural modelization may be good enough to represent real benchmark structures, within the realm of dynamic response inspected here.

In order to make the analyses the more reliable as possible, plausible values of mass, stiffness, damping and geometrical structural characteristics are assumed. Mass $m$ of each floor is taken equal to 144 tons, and the stiffness of the columns is set as follows: $EJ_1 = 3.75 \cdot 10^7$ kN/m for the left columns and $EJ_2 = 1.56 \cdot 10^8$ kN/m for the right ones. Columns display a height $h$ of 3 m. Additionally, modal damping ratios $\zeta_i$ are assumed to be equal to 1%, 3% or 5% for all the modes, according to each analyzed case, spanning the whole dynamic response at increasing sub-critical realistic inherent structural damping.

Concerning input load $F(t)$ to be applied to the dynamical system, a common trend for KF estimation purposes (Kitanidis 1987, Hsieh 2000, Gillijns and DeMoor 2007) is to avoid using any *a-priori* knowledge of such an input force. In fact, structural systems are inherently characterized by uncertainty, relating to measurement errors, sensor noise, inefficacy of the numerical models, and lack of *a-priori* knowledge on both the system and the loading conditions. In order to comply with that, in the analyses presented later, a zero-mean random load $F(t)$ at around $1 \cdot 10^5$ kN is considered, as applied to the top floor of the building (Fig. 1), to compute the floor responses. Later on, these are output-only processed, without knowing source excitation $F(t)$.

Numerically-determined undamped modal natural frequencies $f_{n,i}$ of the benchmark structure are obtained as: $f_{n,1}$ = 2.658 Hz, $f_{n,2}$ = 7.448 Hz and $f_{n,3}$ = 10.763 Hz. Damped modal frequencies $f_{d,i} = f_{n,i} / \sqrt{1-\zeta_i^2}$ have also been calculated for the damped cases with $\zeta_i$ = 1%, 3% and 5%, for all the modes. Reference results are reported in Table 1. Damped natural frequencies $f_{d,i}$ will be later used as comparison terms for evaluating the accuracy of the Kalman Filter estimations.

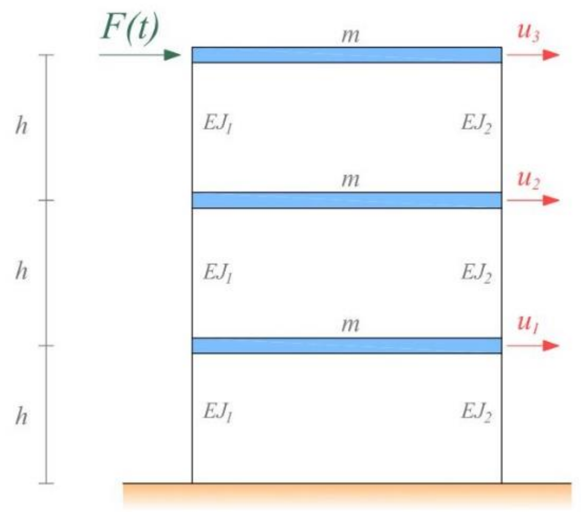


Fig. 1 Schematic view of the analyzed 3-DOF shear-type building under top-floor input force

Table 1 Frequencies $f_{d,i}$ of the damped structural system ($\zeta_i$ = 1%, 3% and 5%)

| | | $f_{d,1}$ [Hz] | $f_{d,2}$ [Hz] | $f_{d,3}$ [Hz] |
|---|---|---|---|---|
| $\zeta_i$ | **1%** | 2.658 | 7.448 | 10.762 |
| | **3%** | 2.657 | 7.445 | 10.758 |
| | **5%** | 2.655 | 7.439 | 10.749 |

### *2.2 Procedure of analysis*

The procedure of analysis is summarized as follows:

i. The dynamic response of the system is first determined in terms of displacements, velocities and accelerations, through the implementation of Newmark's method, consisting of a *step-by-step* direct time integration of the equations of motion. It is worth mentioning that only the third floor's kinematic response is going to be monitored, as this may resume the whole structural response and be suitable enough for first SHM purposes based on a single-channel recording.

ii. To simulate the measurement error that may occur in sensors during the data acquisition stage, a noise signal is selectively added to the original measurements. It is well-known that, for estimation purposes, KF provides the exact probability density function of the state of linear dynamical systems with a linear measurement model, with additive zero-mean Gaussian noise processes. Therein, the estimation capabilities of the filter are investigated for covering also non-Gaussian noises. In particular, a zero-mean random noise is employed to contaminate the data. During the analyses, both displacements (mainly) and accelerations are contaminated with different levels of noise.

iii. Moreover, in the numerical analyses, displacement and acceleration measurements are considered to be sampled at different rates, in accordance with the common capability of the employable instrumentation. In particular, displacements are sampled at lower frequencies than for accelerations. From the literature, a usual frequency range of acquisition has been observed as varying between 12.5 Hz and 100 Hz for displacements, and from 100 Hz up to 300 Hz for accelerations. Consequently, for the purposes of the present fusion procedure, a multi-rate Kalman filter scheme is adopted, as originally developed by Smyth and Wu (2007).

iv. KF effectiveness is measured in two different ways. The first one is based on the calculation of the Root Mean Square (RMS) error between the estimated displacements after KF application and the original (noise-free, numerically-determined) displacements; the second one is based on the comparison between the modal frequencies that can be extracted via appropriate inverse analysis algorithms from the original (noise-free, numerically-determined) displacement recordings and those that can be extracted from the filtered displacement estimations, possibly enriched by means of heterogeneous data fusion with a few reliable acceleration data, through appropriate output-only modal identification techniques applied on displacements.

### *2.3 Numerical Kalman Filter implementation*

Kalman Filter is an algorithm that can be used to estimate the health conditions of linear dynamical systems perturbed by a zero mean Gaussian white noise, through the fusion of data that may also be affected by measurement errors. The mathematical model used in the derivation of such a filter constitutes a reasonable representation for many problems of a practical interest, including control problems as well as estimation problems.

This section provides a schematic description of the linear multi-rate Kalman filter employed within the analyses for improving the estimation of measured displacements $x_m(t)$ (related here to SDOF top-floor displacement signal $u_3(t)$, see Fig. 1) by using measured accelerations $\ddot{x}_m(t)$, supposed to be acquired from acceleration sensors (here on the same monitored SDOF, i.e., related to $a_3(t) = \ddot{u}_3(t)$). According to Smyth and Wu (2007), in the case in which accelerations and displacements are available to be measured, the measurement process can be modeled in state-space form as follows

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ddot{x}_m(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \tag{1}$$

$$x_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + v(t) \tag{2}$$

where $\ddot{x}_m(t)$ denotes the exogenous input to the state transition function, which effectively coincides with the measured accelerations; $x_m(t)$ denotes the measured displacements; $w(t)$ and $v(t)$ are the process noise sources associated to accelerations and displacements, respectively (assumed to be Gaussian). By setting vector $\mathbf{x}(t) = [x(t); \dot{x}(t)]$, representing the system *state vector* (i.e. the unknown output from the filter), formulated via aggregation of filtered displacement $x(t)$ and velocity $\dot{x}(t)$ signals, Eqs. (1) and (2) may be compactly rewritten in matrix form as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{w}(t) \tag{3}$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + v(t) \tag{4}$$

where $u(t) = \ddot{x}_m(t)$, $y(t) = x_m(t)$, $\mathbf{w}(t) = [0; w(t)]$, and state matrix $\mathbf{A}$, input matrix $\mathbf{B}$ and output matrix $\mathbf{C}$ are defined as follows

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5}$$

Since these matrices are assumed to be known within the filtering analysis, this shall lead to a linear model-based (vs. model-free; Hamilton *et al*. 2016) Kalman filter, relying on minimizing the error between measured and filtered data, based on the availability of the above-mentioned matrices, governing corresponding state-space Eqs. (3) and (4), which represent the linear relationships between the states, the measurements and the associated measurement noises (Crawley and O'Donnell 1987). To implement the data fusion procedure, Eqs. (3) and (4) are then transformed into discrete form (zero-order hold assumption), as further indicated in the resuming flowchart of Fig. 2 (adapted from Ferrari *et al*. 2016).

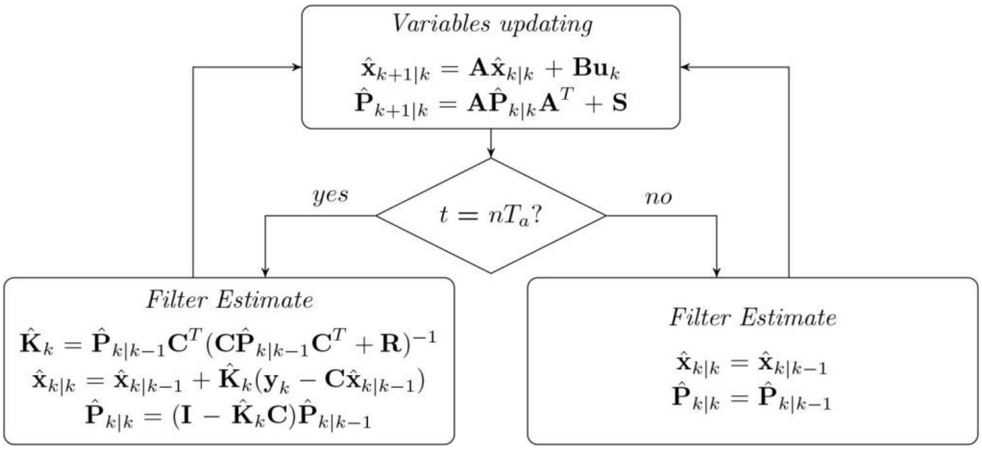


Fig. 2 Multi-rate Kalman filter flowchart (adapted from Ferrari *et al*. 2016)

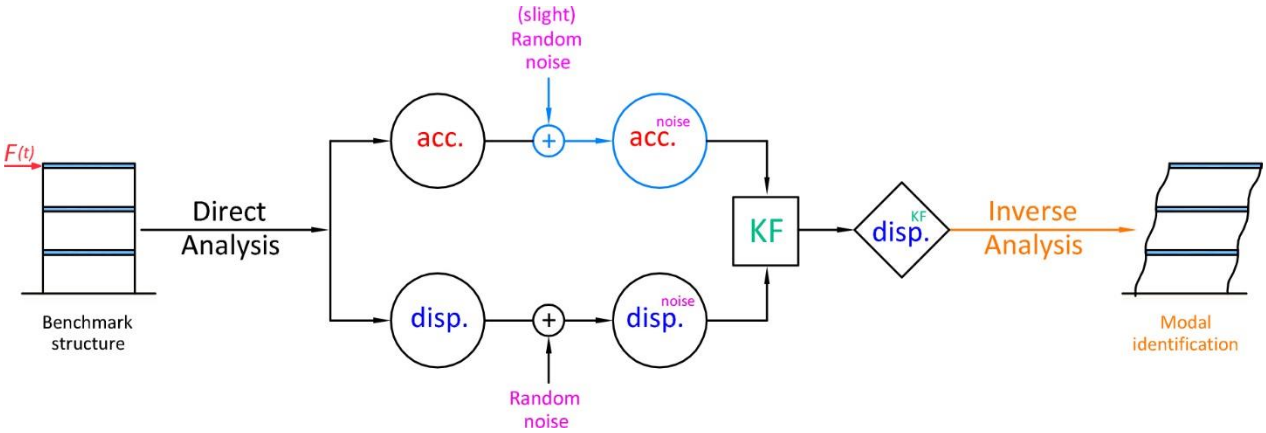


Fig. 3 Schematic general conception of the treated HDF cases: noise-free acceleration case (excluding the part in blue colour) and noise-affected acceleration case (considering the whole scheme). Determination of filtered displacements and possible subsequent phase of modal identification, based on displacements

In Fig. 2, $\hat{\mathbf{x}}_{j|i} = \mathbf{E}(\mathbf{x}_{j|i})$ represents the estimated expected value (mean) of state vector $\mathbf{x}_{j|i}$ at time instant *j*, using measurements up to (and including) time instant *i* (conditional expectation); similarly, $\hat{\mathbf{P}}_{j|i} = Var(\mathbf{x}_{j|i})$ is the variance of state vector $\mathbf{x}_{j|i}$; **S** and **R** represent the covariance of associated process noise sources $\mathbf{w}(t)$ and $v(t)$; $\hat{\mathbf{K}}$ is the so-called Kalman gain. Given matrices **A**, **B**, **C**, **S**, **R**, and supposing to know state mean $\hat{\mathbf{x}}$ and variance $\hat{\mathbf{P}}$ at time *k*, the recursive procedure summarized in the flowchart of Fig. 2 is based on the calculation of Kalman gain $\hat{\mathbf{K}}$ at same time step *k*, through which the knowledge about the state is updated, predicting quantities $\hat{\mathbf{x}}$ and $\hat{\mathbf{P}}$ at time *k+1*, using measurements up to time *k+1*.

The selection of assumed process noise (*p-noise*) covariance matrix **S**, usually mainly based on intuition, and of observation noise (*o-noise*) covariance matrix **R** displays a significant effect on the estimation performance of the Kalman filter. A basic way to think of matrices **S** and **R** is that they constitute weighting factors between the prediction (state) equations and the measurement (output) equations, and this ratio is expressed within the Kalman gain equation. Considering a larger **S** is equivalent to accounting for a larger uncertainty in the state equations, which is equivalent to less trusting the result of these equations, effectively meaning that the filter should more correct, by the measurement update. Similarly, considering a larger **R** is equivalent to accounting for a larger uncertainty in the measurements, which is equivalent to less trusting the measurements, effectively meaning that the filter should less correct, by the measurement update. On the basis of this, *p-noise* has been intuitively set equal to $10^{-12}$ for the whole analysis, since the model is considered to be very robust (the lower this is, the more accurate the model is considered to be). Instead, *o-noise*, which reveals the confidence given to the acquired measurements, has been set at around $10^{-3}$, so that the increasing level of noise on the displacements provides the filter with much of a freedom to adjust itself, with the smaller mean square error for the estimated acceleration signals (the lower the observation noise, the more severely the Kalman filter estimator is forced to fit the recorded accelerations). Such values of process noise and observation noise are then kept

constant during the performed analyses, regardless of the noise level affecting the displacement data. From the assumed values of *o-noise* and *p-noise*, matrices **S** and **R** are build up as follows (Smyth and Wu 2007): matrix **S** is a 2×2 matrix with zero entries, excepted for the *o-noise* level set in position $S_{22}$; matrix **R** reduces to a scalar, which is indeed set to the selected *p-noise* value.
Since accelerations and displacements are sampled at different time intervals, respectively named $T_a$ and $T_d$, where $T_a / T_d = n \in \aleph$ (Chatzi and Fuggini 2012), the filter has been implemented in a multi-rate configuration (Smyth and Wu 2007). This means that, at times $t = nT_a$, both the time and measurement update steps of the Kalman filter are performed, whereas when $nT_a < t < (n+1)T_a$ only the time update step (ignoring the observation innovation) is carried out. Despite that the outcomes of the filter represent the "updated" estimates of both the displacements and the velocities (derived from the acceleration input) of the benchmark structure, the present work plan focuses only on the displacement output.

## 3. Analysis results

In this section, a selection of the results from the numerical analyses performed by involving the multi-rate Kalman Filter algorithm are reported. Two main scenarios are presented. Initially, only the displacements are considered as affected by noise, while the accelerations are taken as noise-free. This might represent a quite realistic scenario, since accelerometers are able to capture signals with a higher level of accuracy than for displacement sensors (Smyth and Wu 2007). Secondly, also the acceleration data are *"noisified"* with a slight zero-mean random noise. Indeed, also data acquired via accelerometers may be subjected to measurement errors, albeit smaller than errors affecting displacements. Additional simulation results are available in Ravizza (2017).
All the analyses have been developed within an autonomous MATLAB implementation environment. A general resuming conceptual scheme of the treated HDF cases, with and without noise affecting the accelerations, also including the possible subsequent modal identification analysis performed on the filtered displacements, is synoptically depicted in Fig. 3.

### 3.1 Noise-free acceleration case

Acceleration data are here assumed as noise-free. In practice, it means to consider accelerometer measurements without errors, as illustrated in the synoptic scheme in Fig. 3 (i.e., without considering the part in blue colour).

In particular, 5%, 10%, 25%, 50%, 150% and 300% random noise levels are considered as applied to the displacements, for the different considered underlying modal damping ratios, namely $\zeta_i$ = 1%, 3% and 5%. From the analyses, it emerges that by involving KF into the fusion procedure it is possible to obtain refined displacement estimations, namely displacements endowed with an improved accuracy.

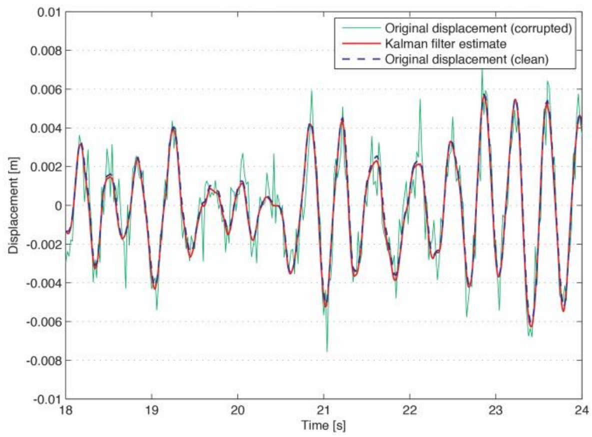


Fig. 4 KF response estimation for 25% noise level ($\zeta_i$ = 1%)

Table 2 RMS error [%] of KF response estimation for increasing displacement noise levels and damping ratios $\zeta_i$ (noise-free acceleration case)

| | | displacement noise | | | | | |
|---|---|---|---|---|---|---|---|
| | | **5%** | **10%** | **25%** | **50%** | **150%** | **300%** |
| | **1%** | 0.15 | 0.16 | 0.19 | 0.24 | 3.28 | 9.71 |
| $\zeta_i$ | **3%** | 0.16 | 0.18 | 0.20 | 0.25 | 3.21 | 9.22 |
| | **5%** | 0.13 | 0.17 | 0.18 | 0.37 | 3.75 | 9.91 |

In particular, from Fig. 4, which shows displacements before and after the fusion procedure, it is possible to appreciated how the curves, representing the original displacements (blue curve) and the KF estimated displacements (red curve), show a similar trend for a displacement random noise level of 25%. In Table 2, the RMS errors between the original displacements and the KF estimated displacements are reported for each examined scenario.

From Table 2, it is possible to observe that the proposed HDF technique is very robust, if accelerations are set as noise-free, despite for the very high noise levels on displacements. In fact, the RMS error is less than 1%, for displacement noise levels up to 50%, independently from the value of considered damping ratio $\zeta_i$.

Notice that RMS noise levels greater then 25% are considered to be quite unrealistic for civil engineering applications (Smyth and Wu 2007). RMS errors of about 3% and 9% refer, instead, to the cases characterized by a displacement noise level equal to 150% and 300%, respectively.

Fig. 5 shows the qualitative behavior of the RMS error for increasing random noise levels and damping ratios. From the graph, it clearly emerges how, up to a 50% of noise level, the RMS error lies well below 1%. It is interesting to note that the three depicted curves seem to present the same trend, with a very light influence of the inherent damping ratios, especially for noise levels below 50%. Consequently, it is possible to affirm that the Kalman Filter algorithm provides very consistent displacements for all the cases considered so far. This also demonstrates the effectiveness of the filter in case of smaller values of displacements, as derived from the adoption of greater damping ratios.

Effectiveness of the Kalman Filter is also tested based on an inverse analysis conducted for modal identification purposes from standard FDD, *based on displacement signals*. In particular, the procedure of modal identification has been applied to the displacements before (thus unprocessed by DF) and after (thus taking benefit from DF with accelerations) Kalman Filter application, and results have been reported for each of the analyzed cases.

From the comparison presented in Fig. 6, it is immediate to note how the peaks identified after the filtering procedure look much clearer than the same peaks identified from the unfiltered displacement data, especially when the noise level becomes considerable. This is primarily due to the beneficial effect induced by reliable acceleration data involved within the fusion procedure, which indeed display a relatively better information in the high-frequency regions, to enrich displacements.

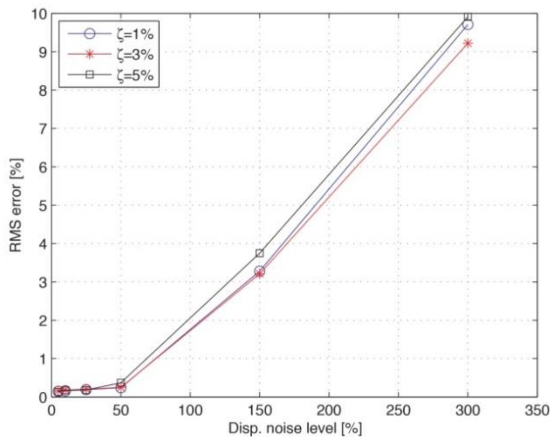


Fig. 5 RMS error trend of KF estimation at increasing displacement noise levels (noise-free acceleration case). A visible kink is recorded at 50% noise level

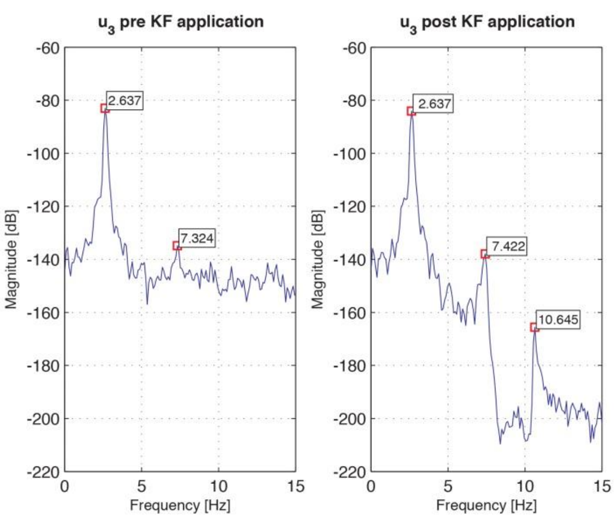


Fig. 6 FDD modal frequency identification on displacements for 25% noise level ($\zeta_i$ = 1%)

From the analyzed noise-free acceleration case, it can be asserted that the Kalman Filter algorithm is only slightly influenced by the amount of sub-critical structural damping, with a modal damping ratio in the order of a few percents, since in each case the RMS error remains at around 1%, for displacement noise levels up to 50% (see Fig. 5). It is worth to note that this may be considered as rather reliable for damping ratios typical of civil engineering structures; for higher damping ratios, further analyses may become necessary.

### *3.2 Noise-affected acceleration case*

The noise-affected acceleration case (slight noise) is now considered. This represents a further common scenario in practical cases, because in reality not only displacement sensors but also acceleration sensors may present measurement errors, though of a slight amount for the latter sensors. To take this into account, a slight random noise level has been added to the acceleration data, too. Damping ratios $\zeta_i$, instead, have been maintained as constant and all equal here to 1%, as representative of a slight sub-critical damping, in the whole analysis. The analysis follows again the scheme of Fig. 3, now also including the part highlighted in blue colour.

As previously stated, in this section the focus is now placed on the HDF scenario involving noise-affected acceleration measurements, even though here very low noise levels are applied.

In particular, 0%, 5%, 10%, 25% and 50% random noise levels are considered as applied to displacements, for simultaneous slight acceleration noise levels equal to 1%, 2% and 3%. From the results of the noise-affected acceleration case, it emerges that KF seems to be significantly affected by the level of slight noise added to the accelerations.

In fact, from Fig. 7, which shows the displacement histories before and after the fusion procedure with acceleration data, it is possible to appreciate how the curves, representing the original displacements (blue curve) and the KF estimated displacements (red curve), take different trends, already for low levels of acceleration random noise.

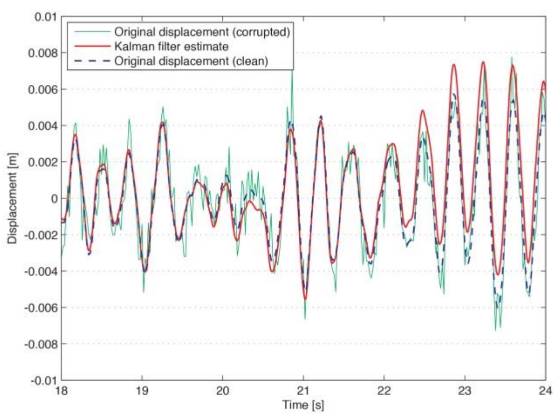


Fig. 7 KF response estimation for 25% noise level (1% acceleration noise)

Table 3 RMS error [%] of KF response estimation at increasing displacement noise level (noise-affected acceleration case)

| | | displacement noise | | | | |
|---|---|---|---|---|---|---|
| | | **0%** | **5%** | **10%** | **25%** | **50%** |
| **acc. noise** | **1%** | 1.17 | 1.97 | 2.72 | 4.01 | 6.67 |
| | **2%** | 8.72 | 11.03 | 12.41 | 13.08 | 16.98 |
| | **3%** | 19.67 | 25.93 | 27.51 | 32.77 | 35.24 |

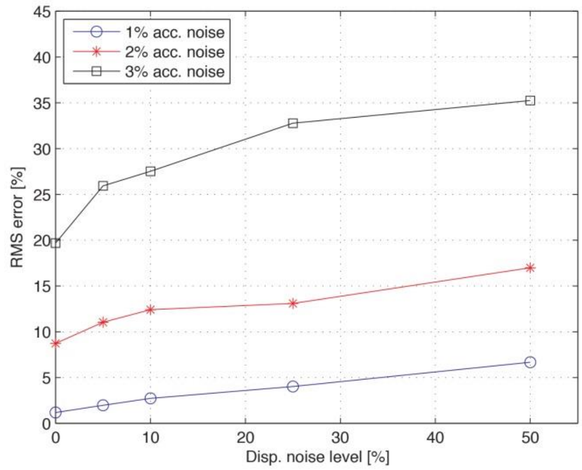


Fig. 8 RMS error of KF response estimation at increasing displacement noise levels (for 1%, 2% and 3% acceleration noise levels)

In Table 3, RMS errors are reported for each examined scenario. It is worth to note that only the 1% noise acceleration scenario may be considered to be acceptable for SHM purposes. In fact, the estimate errors related to the other two cases become too high; this could lead to an unreliable prediction of the dynamic response of the structural system.

From Table 3, it is possible to observe how the RMS error rapidly increases with the increase of the acceleration noise level, up to values over 30%, for an acceleration noise of 3%. However, in practice, considering the sensitivity of sensors, the more common scenario is to inherit a 1% noise level on accelerations and a displacement noise level between 5% and 10%. Within such a range, RMS errors below 3% have been recorded. This may also be graphically observed in Fig. 8, in which the three trends are depicted.

About the inverse identification method, from the FDD analyses *based on displacements* it emerges that the modal frequency identification has been anyway successful after Kalman Filter employment, as it can finally be appreciated from Fig. 9, in which a comparison between the frequencies identified from displacement signals before (DF unprocessed) and after (DF processed) KF application, through fusion of acceleration signals, is provided. This is probably due to the frequency content, which still remains good despite for the high levels of added noise.

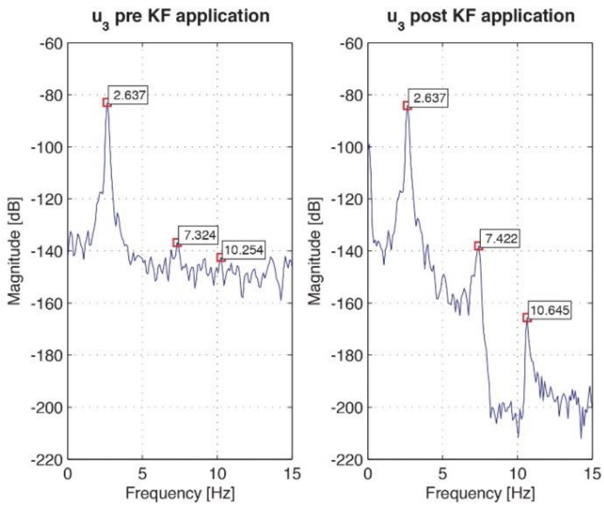


Fig. 9 FDD modal frequency identification on displacements for 25% noise level (1% acceleration noise)

An important assertion that can be derived from the analysis presented in this section is that Kalman Filter, differently from the modal identification procedure, is strongly affected by the level of noise added to the acceleration data.

The reason why filtered displacements seem to be very sensitive to noise-affected accelerations and, at the same time, rather insensitive to noise-affected displacements, has to be located in the adopted preliminary calibration of the Kalman filter. In fact, in the developed data fusion procedure, a crucial step is constituted by the *a-priori* definition of the degree of confidence to be given to the initial conditions of the model, stated in a probabilistic way. Such a degree of confidence has to be set within the state-space model in terms of process noise and observation noise, as previously stated in Section 2.3.

In practice, from the acquired understanding of the present data fusion analysis, acceleration measurements bring in a more powerful information than displacement records; thus, noise affecting acceleration data more readily affects the filtered displacements and then the identification outcomes that can be extracted from them.

So, adopting this configuration of the filter, it is advisable to maintain an acceleration noise level approximately below 1%, so that reliable response estimations may be reached. Consequently, in practical cases, a great attention must be given to the acquisition stage of such a data; in this sense, also the preliminary study of the appropriate collocation of the acceleration sensors shall play a key role for the success of the whole Heterogeneous Data Fusion process.

## 4. Conclusions

By means of the performed numerical analyses, this paper sheds light on several critical aspects inherent to the adoption of Kalman Filter in civil engineering implementations, including for Heterogeneous Data Fusion and modal identification purposes, *based on displacements*, possibly complemented by accelerations. This holds true in particular with reference to the maximum level of noise that may be tolerated on displacement and acceleration measurements, in order to allow for a successful HDF and thereby to achieve reliable estimates of the current structural dynamic response, perspectively based on extensive remote displacement response signals, corroborated by a few reliable acceleration response data.

The following salient issues from the performed analysis apply:

i. *Noise-free acceleration case*. From the analysis performed in this paper, it has been demonstrated that KF turns out able to provide reliable estimates of the structural dynamic response, for noise levels up to 50% in the displacement measurements. Indeed, in these cases, RMS errors below 1% between the estimated displacements after KF application and the original (noise-free, numerically-determined) displacements are recorded. This was made possible also by tuning the assumed noises of the filter, which usually constitutes a complex procedure to be made automatic. Further developments of the present study on these aspects may additionally be addressed in future work.
ii. *Noise-affected acceleration case*. Numerical results have shown that only the 1% acceleration noise level may lead to reliable estimates of the dynamic response after data fusion by KF adoption. This could be improved, e.g. by providing an adequate procedure for the fine tuning of the filter (in terms of process and observation noise levels) and may constitute the subject of future investigations. Consequently, in the present scenario, the positioning and setting of the accelerometers shall play a key role for a true success in the KF estimates. In the current analysis, if the intrinsic error characterizing accelerometer recordings becomes greater than 1%, RMS errors greater than 8% between the estimated displacements after KF application and the original (noise-free, numerically-determined) displacements, are observed. This constitutes, at present, a requirement of quality for the acceleration recordings, to be adopted within the current implementation settings, in the explored configuration.

In addition to the comparison between numerically-determined displacements and displacements obtained through KF application, integrating information from acceleration response signals, defined in terms of RMS error, an output-only FDD procedure of modal frequency identification in the Frequency Domain, *based on displacement recordings*, has been used to evaluate the KF effectiveness in terms of modal dynamics identification. It has been proven that the KF adoption has been useful also towards the modal identification purposes, considering the cases of noise-free and noise-affected accelerations.

The present paper brought forth some interesting aspects inherent to the employment of a model-based Kalman Filter in Heterogeneous Data Fusion toward SHM purposes. The filter's effectiveness has been successfully demonstrated, leading to the conclusion that KF shall constitute a strong tool for data fusion in the field of structural monitoring. Thanks to its robustness, a proper KF implementation may allow users to relax the acquisition rate of signals, shifting then the effort to the posterior stage of data processing. In this sense, KF could allow for handling practical situations in which, for different reasons, an optimal positioning of the sensors may not be feasible.

Another consideration that may be drawn from the present investigation is the perspective of cross-using extensive remote displacement measurements for the purposes of structural identification, through the support of selected reliable acceleration data, apt to enrich the available information. Thanks to the implemented KF Heterogeneous Data Fusion implementation, it has been shown that not only accelerations but also estimated displacements, embedding partial information from accelerations, may be successfully employed within a *modal identification procedure based on displacements*. This may point out to the perspective of adopting wide displacement acquisitions for SHM and identification purposes, possibly corroborated by a few reliable acceleration recordings, through appropriate and effective Heterogeneous Data Fusion, as outlined here.

This could be even explored further, in different structural contexts and application scenarios, eventually using *real* measurement data coming from true dynamical experimental campaigns (e.g., as those provided in Ferrari *et al.* 2015a, b, 2016, 2017), for a final assessment and validation.

## Acknowledgments

Public research support from *"Fondi di Ricerca d'Ateneo ex 60%"* and a ministerial doctoral grant and funds at the ISA Doctoral School, University of Bergamo, Department of Engineering and Applied Sciences (Dalmine) are gratefully acknowledged.

## References

Azam, S.E., Chatzi, E.N. and Papadimitriou, C. (2015), "A dual Kalman filter approach for state estimation via output-only acceleration measurements", *Mech. Syst. Signal Pr.*, **60-61**, 866-885.

Capellari, G., Chatzi, E.N., Mariani, S. and Azam, S.E. (2017), "Optimal design of sensor networks for damage detection, EURODYN 2017", *Proceedings of the 10th International Conference on Structural Dynamics, Faculty of Civil and Industrial Engineering*, Rome, Italy, 10-13 September 2017, Proceedia Engineering, 199, 1864-1869.

Chatzi, E.N. and Smyth, A.W. (2009), "The unscented Kalman filter and particle filter methods for non-linear structural system identification with non-collocated heterogeneous sensing", *Struct. Control Health Monit.*, **16**(1), 99-123.

Chatzi, E.N. and Fuggini, C. (2012), "Structural identification of a super-tall tower by GPS and accelerometer data fusion using a multi-rate Kalman filter", *Proceedings of the 3rd International Symposium on Life-Cycle Civil Engineering*, IALCCE 2012, Delft, Netherlands, October 3-6, 2012, 10, 144-151.

Chatzi, E.N. and Fuggini, C. (2015), “Online correction of drift in Structural Identification using Artificial White Noise observations and an Unscented Kalman Filter”, *Smart Struct. Syst.*, **16**(2), 296-328.

Chatzis, M.N., Chatzi, E.N. and Triantafyllou, S.P. (2017), “A Discontinuous Extended Kalman Filter for non-smooth dynamic problems”, *Mech. Syst. Signal Pr.*, **92**, 13-29.

Cho, S., Yun, C.B. and Sim, S.H. (2015), “Displacement estimation of bridge structures using data fusion of acceleration and strain measurement incorporating finite element model”, *Smart Struct. Syst.*, **15**(3), 645-663.

Cho, S., Park, J.W., Bae, M. and Sim, S.H. (2015), “Life-cycle of structural systems: design, assessment, maintenance and management”, *Proceedings of the 4rd International Symposium on Life-Cycle Civil Engineering*, IALCCE 2014, Tokyo, Japan, November 16-19, 2014, 144-151.

Crawley, E.F. and O’Donnell, K.J. (1987), “Force-state mapping identification of non-linear joints”, *AIAA J.*, **25**(7), 1003-1010.

Ding, Y. and Guo, L. (2016), “Structural identification based on incomplete measurements with iterative Kalman filter”, *Struct. Eng. Mech.*, **59**(6), 1037-1054.

Eichstadt, S., Makarava, N. and Elster, C. (2016), “On the evaluation of uncertainties for state estimation with the Kalman filter”, *Meas. Sci. Technol.*, **27**(12), 27-40.

Ferrari, R., Froio, D., Chatzi, E.N., Gentile, C., Pioldi, F. and Rizzi, E. (2015), “Experimental and numerical investigation for the structural characterization of a historic RC arch bridge”, *Proceedings of the COMPDYN 2015, 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Crete Island, Greece, 25-27 May 2015, 2337-2353, online available in Eccomas Proceedia.

Ferrari, R., Pioldi, F., Rizzi, E., Gentile, C., Chatzi, E.N., Klis, R., Serantoni, E. and Wieser, A. (2015), “Heterogeneous sensor fusion for reducing uncertainty in Structural Health Monitoring”, *Proceedings of the UNCECOMP 2015, 1st ECCOMAS Thematic Conference on International Conference on Uncertainty Quantification in Computational Sciences and Engineering*, Crete Island, Greece, 25-27 May 2015, 511-528, online available in Eccomas Proceedia.

Ferrari, R., Pioldi, F., Rizzi, E., Gentile, C., Chatzi, E.N., Serantoni, E. and Wieser, A. (2016), “Fusion of wireless and non-contact technologies for the dynamic testing of a historic RC bridge”, *Meas. Sci. Technol.*, **27**(12), 1-19.

Ferrari, R., Froio, D., Rizzi, E., Gentile, C. and Chatzi, E.N. (2018), “Model updating of a historic concrete bridge by sensitivity- and global optimization-based Latin Hypercube Sampling”, *Eng. Struct.*, **179**, 139-160.

Gillijns, S. and DeMoor, B. (2007), “Unbiased minimum-variance linear state estimation for linear discrete-time systems”, *J. Automatica*, **43**(1), 111-116.

Gordon, N.J., Salmond, D.J. and Smith, A.F.M. (1993), “Novel approach to non-linear/non-Gaussian Bayesian state estimation”, *Proceedings of Radar and Signal Processing*, **140**(2), 107-113.

Hamilton, F., Berry, T. and Sauer, T. (2016), “Ensemble Kalman filtering without a model”, *Phys. Rev. X*, **6**(1), 1-12.

Hsieh, C.S. (2000), “Robust two-stage Kalman filters for systems with unknown inputs”, *IEEE T. Automat. Contr.*, **45**(12), 2374-2378.

Jiang, S.F., Wang, L.S., Yin, X.Z. and Liu, M. (2005), “Data fusion technique in structural health monitoring”, *J. Shenyang Jianzhu University (Natural Science)*, **21**(1), 18-22.

Jiang, S.F., Chan, G.K. and Zhang, C.M. (2006), “Data fusion technique and its application in structural health monitoring”, Structural Health Monitoring and Intelligent Infrastructure - *Proceedings of the 2nd International Conference on Structural Health Monitoring of Intelligent Infrastructure*, 1125-1130.

Julier, S.J. and Uhlmann, J.K. (1998), “A non-divergent algorithm in the presence of unknown correlation”, *Proceedings of the American Control Conference, Albuquerque*, New Mexico, USA, June 1997, 2369-2373.

Kalman, R.E. (1960), “A new approach to linear filtering and prediction problems”, *J. Basic Eng.*, **82**(1), 35-45.

Kim, K., Choi, J., Koo, G. and Sohn, H. (2016), “Dynamic displacement estimation by fusing biased high-sampling rate acceleration and low-sampling rate displacement measurements using two-stage Kalman estimator”, *Smart Struct. Syst.*, **17**(4), 647-667.

Kim, K. and Sohn, H. (2017), “Dynamic displacement estimation by fusing LDV and LiDAR measurements via smoothing based Kalman filtering”, *Mech. Syst. Signal Pr.*, **82**, 339-355.

Kitanidis, P.K. (1987), “Unbiased minimum-variance linear state estimation”, *J. Automatica*, **23**(1), 775-778.

Lee, J.J., Fukuda, Y., Shinozuka, M., Cho, S. and Yun, C.B. (2007), “Development and application of a vision-based displacement measurement system for structural health monitoring of civil structures”, *Smart Struct. Syst.*, **3**(3), 373-384.

Lee, K.G. and Yun, C.B. (2008), “Parameter identification for non-linear behavior of RC bridge piers using sequential modified extended Kalman filter”, *Smart Struct. Syst.*, **4**(3), 319-342.

Lei, Y., Chen, F. and Zhou, H. (2015), “A two-stage and two-step algorithm for the identification of structural damage and unknown excitations: numerical and experimental studies”, *Smart Struct. Syst.*, **15**(1), 57-80.

Lei, Y., Luo, S. and Su, Y. (2016), “Data fusion based improved Kalman filter with unknown inputs and without collocated acceleration measurements”, *Smart Struct. Syst.*, **18**(3), 375-387.

Lin, X. and Luo, Z.C. (2016), “A new adaptive multi-rate Kalman filter for the data fusion of displacement and acceleration”, *Chine J. Geophysics*, **59**(5), 1608-1615.

Liu, L., Zhu, J., Su, Y. and Lei, Y. (2016), “Improved Kalman filter with unknown inputs based on data fusion of partial acceleration and displacement measurements”, *Smart Struct. Syst.*, **17**(6), 903-915.

Papadimitriou, C. and Lombaert, G. (2012), “The effect of prediction error correlation on optimal sensor placement in structural dynamics”, *Mech. Syst. Signal Pr.*, **28**, 105-127.

Park, J.W., Sim, S.H. and Jung, H.J. (2013), “Displacement estimation using multimetric data fusion”, *IEEE/ASME T. Mechatron*., **18**(6), 1675-1682.

Pioldi, F., Ferrari, R. and Rizzi, E. (2015), “Output-only modal dynamic identification of frames by a refined FDD algorithm at seismic input and high damping”, *Mech. Syst. Signal Pr.*, **68-69**, 265-291.

Pioldi, F., Ferrari, R. and Rizzi, E. (2015), “Earthquake structural modal estimates of multi-storey frames by a refined FDD algorithm”, *J. Vib. Control*, **23**(13), 2037-2063.

Pioldi, F., Ferrari, R. and Rizzi, E. (2017), “Seismic FDD modal identification and monitoring of building properties from real strong-motion structural response signals”, *Struct. Control Health Monit.*, **24**(11), 1-20.

Ravizza, G. (2017), "Dynamic response estimation by heterogeneous data fusion via Kalman filter adaptation, *Master Thesis in Building Engineering*, (Advisor: E. Rizzi, Co-advisor: R. Ferrari), School of Engineering, University of Bergamo, Italy, March 31, 2017, 186 pages.

Salvi, J. and Rizzi, E. (2014), “Optimum tuning of Tuned Mass Dampers for frame structures under earthquake excitation”, *Struct. Control Health Monit.*, **22**(4), 707-725.

Salvi, J., Rizzi, E., Rustighi, E. and Ferguson, N.S. (2015), “On the optimization of a hybrid Tuned Mass Damper for impulse loading”, *Smart Mater. Struct.*, **24**(8), 85-100.

Salvi, J. and Rizzi, E. (2016), “Closed-form optimum tuning formulas for passive Tuned Mass Dampers under benchmark excitations”, *Smart Struct. Syst.*, **17**(2), 231-256.
Salvi, J. and Rizzi, E. (2017), “Optimum earthquake-tuned TMDs: seismic performance and new design concept of balance of split effective modal masses”, *Soil Dyn. Earthq. Eng.*, **101**, 67-80.
Smyth, A. and Wu, M. (2007), “Multi-rate Kalman filtering for the data fusion of displacement and acceleration response measurements in dynamic system monitoring”, *Mech. Syst. Signal Pr.*, **21**(2), 706-723.
Su, Z., Wang, X., Cheng, L., Long, Y. and Chen, Z. (2009), “Erratum to On selection of data fusion schemes for structural damage evaluation”, *Struct. Health Monit.*, **8**(6), 573.
Wang, J.F. and Lin, C.C. (2013), “Extracting parameters of TMD and primary structure from the combined system responses”, *Smart Struct. Syst.*, **16**(5), 937-960.
Wu, B. and Wang, T. (2014), “Model updating with constrained unscented Kalman filter for hybrid testing”, *Smart Struct. Syst.*, **14**(6), 1105-1129.
Xiao, C., Qu, W. and Tan, D. (2005), “An application of data fusion technology in structural health monitoring and damage identification”, *Proceedings of SPIE - The International Society for Optical Engineering*, **5758**(55), 451-461.
Yuqing, Z., Xinfang, L., Fengping, L., Bingtao, S. and Wei, X. (2015), “An online damage identification approach for numerical control machine tools based on data fusion using vibration signals”, *J. Vib. Control*, **21**(15), 2925-2936.
Zghal, M., Mevel, L. and Del Moral, P. (2014), “Modal parameter estimation using interacting Kalman filter”, *Mech. Syst. Signal Pr.*, **47**(1-2), 139-150.
Zhao, X., Li, M., Song, G. and Xu, J. (2010), “Hierarchical ensemble-based data fusion for structural health monitoring”, *Smart Mater. Struct.*, **19**(4), 45-59.

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