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Influence of loose bonding, initial stress and reinforcement on Love-type wave propagating in a functionally graded piezoelectric composite structure

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Abstract. This present study investigates Love-type wave propagation in composite structure consists of a loosely bonded functionally graded piezoelectric material (FGPM) stratum lying over a functionally graded initially-stressed fibre-reinforced material (FGIFM) substrate. The closed-form expressions of the dispersion relation have been obtained analytically for both the cases of electrically open and electrically short conditions. Some special cases of the problem have also been studied and the obtained results are found in well-agreement with the classical Love wave equation. The emphatic influence of wave number, bonding parameter associated with bonding of stratum with substrate of the composite structure, piezoelectric coefficient as well as dielectric constant of the piezoelectric stratum, horizontal initial stresses, and functional gradedness of the composite structure on the phase velocity of Love-type wave has been reported and illustrated through numerical computation along with graphical demonstration in both the cases of electrically open and electrically short condition for the reinforced and reinforced-free composite structure. Comparative study has been carried out to analyze the distinct cases associated with functional gradedness of the composite structure and also various cases which reveals the influence of piezoelectricity, reinforcement and horizontal initial stress acting in the composite structure, and bonding of the stratum and substrate of the composite structure in context of the present problem which serves as one of the major highlights of the study.

Keywords: functionally graded; piezoelectric; loose bonding; reinforced; Love-type wave; initial stress; electrically open and short

1. Introduction

The concept of functionally graded materials (FGMs) was first suggested by Rabin and Shiota (1995) to address the needs of aggressive environment of thermal shock. FGMs have its wide application in various engineering fields including electronics, chemistry, optics, biomedicine, aerospace engineering etc. Ichinose *et al*. (2004) was able to fabricate ultrasonic transducers with functionally graded piezoelectric ceramics. On the macroscopic scale, FGMs are anisotropic, inhomogeneous and retain spatially continuous mechanical properties. Due to the absence of visible internal boundaries, internal stress peaks doesn’t occur in case of applied external loading and as a result interfacial debonding or stress concentration failure is preventable. In this respect, FGMs are more superior to the conventional laminated materials. Piezoelectric materials exhibit coupled effects between electric field and elastic deformation and integration with structures benefits to control deformation, vibration, acoustics, etc. The structures including FGMs bonded with piezoelectric actuators and sensors are smart in response to environmental changes. Cheng *et al.* (2000) discussed three-dimensional asymptotic approach to inhomogeneous and laminated piezoelectric plates. Lim and Lau (2005) studied a new two-dimensional model for electro-mechanical response of thick laminated piezoelectric actuator. Ootao and Tanigawa (2000) discussed the three-dimensional transient piezothermoelastic problem of an FGM rectangular plate bonded to a piezoelectric plate due to partial heat supply. Recently, Singh *et al.* (2015) investigated Love-type wave propagation in a piezoelectric structure with irregularity.

Wave propagation problems related to FGPM are attracting researchers now a days for theoretical and experimental contributions due to its extensive applications. In order to analyze characteristics and response of wave propagation in FGPM, HNM was proposed by Liu and Tani (1991) which is a combination of the finite element method and the Fourier transformation method. Liu *et al.* (2003) discussed the piezoelectricity effects on the dispersion and characteristics of waves in FGPM plates through the introduction of LIEs. Han and Liu (2003) investigated the frequency and group velocity dispersion behaviors and characteristic surface waves in FGPM cylinders. Recently, Du *et al.* (2007) obtained the analytical solutions of dispersion relations for electrically open or short circuit conditions when Love wave propagates in functionally graded piezoelectric material layer bonded to a semi-infinite homogeneous solid.

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The mechanical behavior of fiber-reinforced composite materials is modeled by the theory of linear elasticity for transversely isotropic materials. The characteristic property of a fiber-reinforced material is that in an elastic condition, fiber-reinforced material components act together as a single anisotropic unit. These materials are a family of composite materials, where the highly oriented polymer fibers derived from the same fiber are reinforced with each other Alumina or concrete is an example of such material. In such composites the fibers are arranged in parallel straight lines. Other configurations like circumferential reinforcement, for which the fibers are arranged in concentric circles, giving strength and stiffness in the tangential (or hoop) direction. Materials such as resins reinforced by strong aligned fibers exhibit highly anisotropic elastic behavior. To understand the mechanical property of fiber-reinforced materials inside the Earth, the study of the interactions between individual fibers and the surrounding matrix has become imperative. Fiber reinforcement mechanics is also relevant to the understanding of the performance and function of engineering fabrics or geotextiles. The idea of continuous theory in the fiber-reinforced material is developed by Adkins (1965), Spencer (1972), Maugin (1981) and many other authors on the theory of large deformations of elastic materials reinforced by inextensible cords. Hashin and Rosen (1964) studied the elastic moduli for fiber-reinforced materials. Belfield *et al.* (1983) studied the stress in elastic plates reinforced with fibers lying in concentric circles. Chattopadhyay and Singh (2012) investigated G-type seismic waves in fibre-reinforced media.

The study of Love-type waves in anisotropic and non-homogeneous medium are of great practical importance. It helps to design various acoustic device e.g., Love wave sensor, transducers, actuators etc. Through which the internal structure of the material can be characterized. Further, Love type waves are used in exploration of natural resources buried inside the Earth's surface viz. oils, gases, and other useful hydrocarbons and minerals. Love (2013) first introduced the mathematical model for the propagation of Love wave in a medium. Some physical factors involving manufacturing demand, overburdened layer, variation in temperature, slow process of creep etc. In the context of Earth science, it may be due to atmospheric pressure and gravitational field also which may give rise to a large amount of initial stress in a medium. This initial stress conceivably influences the propagation of waves through the mediums. A good amount of information can be gained from the disquisition given by Biot (1965). Many works considering pre-stressed media has been done by several authors. Dey *et al.* (1989) investigated the propagation of Love waves in an initially stressed anisotropic porous layer lying over a pre-stressed non-homogeneous elastic half-space. Chattopadhyay and Kar (1978) studied the propagation of Love waves in a pre-stressed homogeneous and isotropic crustal layer lying over a homogeneous and isotropic medium, having an irregular interface. Khurana (2001) showed the effect of initial stress on the propagation of Love waves. Motivated by these studies, initial stress has been considered in the considered composite structure.

However, loose bonding often occurs in surface acoustic wave (SAW) devices due to the aging of glue applied to two conjunct solids, microdefects, diffusion impurities, and other forms of damages. The elastic wave behavior across linear slip interface has been discussed by Schoenberg (1980). Termonia (1990) experimented on Fibre coating as a means to compensate for poor adhesion in fibre-reinforced materials. Nagy (1992) gave the Ultrasonic classification of imperfect interfaces. Chen *et al.* (2004) gave the exact solution of angle-ply piezoelectric laminates in cylindrical bending with interfacial imperfections. In the design and application of piezoelectric sensors, consideration of a possible imperfect interface is necessary. Singh *et al.* (2016) studied Love-type wave propagation in a corrugated piezoelectric structure whereas Kaur *et al.* (2016) investigated the influence of imperfectly bonded micropolar elastic half-space with non-homogeneous viscoelastic layer on propagation behavior of shear wave. Recently, Singh *et al.* (2017) studied the influence of imperfectly bonded piezoelectric layer with irregularity on propagation of Love-type wave in a reinforced composite structure. Till date, no study has been done to analyse the characteristics of Love-type wave propagation in a loosely bonded functionally graded piezoelectric material (FGPM) stratum lying over a functionally graded initially-stressed fiber-reinforced material (FGIFM) substrate revealing the impact of reinforcement, loose bonding and initial stress.

In the present study, the dispersion relations for the Love-type wave propagating in a composite structure comprised of a loosely bonded FGPM stratum lying over a FGIFM substrate in both the cases of electrically open and short conditions have been obtained in the closed-form analytically. The procured results are found to be in well-agreement with the classical Love wave equation as a special case of the problem. Numerical computation and graphical demonstration have been carried out to illustrate the significant influence of wave number, piezoelectric coefficient, dielectric constant, horizontal initial stresses acting in the composite structure, bonding parameter associated with the bonding of the stratum with substrate, and parameters associated with functional gradedness of the composite structure on the phase velocity of Love-type wave in electrically open and short conditions for reinforced and reinforced-free composite structure. Moreover, distinct cases associated with functional gradedness of the composite structure and also various cases revealing the influence of piezoelectricity, reinforcement and horizontal initial stress acting in the composite structure along with the bonding of the stratum and substrate of the composite structure on the dispersion curve of Love-type wave have been studied in a comparative manner which leads to some significant results

2. Basic assumption

A real parameter (bonding parameter) is defined by Murty (1975, 1976) to which numerical values can be assigned corresponding to a given degree of bonding between substrates. The particular cases of ideally smooth and fully bonded interfaces corresponding to the respective values 0 and ∞ of the bonding parameter have been discussed. The study is having three following basic assumptions:

(i) The stresses are continuous across the interface.

(ii) The microscopic structure of the material at the interface is such that a finite amount of slip can take place at the interface when a periodic wave is propagating.

(iii) There exist a linear relation between slip and shear stress at the interface which implies that different degrees of bonding correspond to different values of the constant of proportionality.

The first and second assumptions are the same as those involved in the study of wave motion at the smooth interface between two substrates (Chang 1971). The principle behind the third assumption is that there must exist some relation between local shearing stress and the slip at the interface.

The relation between shearing stress and slip is that if the shearing stress is zero, the interface behaves like an ideally smooth and slip is infinite whereas if the slip vanishes, the interface behaves as a fully bonded interface. We may assume

$$\text{Shearing stress} = K \times \text{slip} \tag{1}$$

at the interface of loosely bonded media. The intermediate values of K represent a loosely bonded interface. Therefore, vanishing of K corresponds to an ideally smooth interface and an infinitely large value of K corresponds to a welded interface.

In this problem we have considered a composite structure comprised of functionally graded piezoelectric stratum lying over a functionally graded initially-stressed Fibre-reinforced substrate. We see in Eq. (1), the effective bonding parameter, K can be redefined in a more useful form by examining the dependence of shearing stress, displacement component of the upper medium to the common interface (w_1) and displacement component of the lower medium to the common interface (w_2) on the wave number (k). It can be seen from equations 2.1-2.8 of Ewing *et al*. (1957) that shearing stress, varies as k^2 while (w_1) and (w_2) vary as k. This difference can be incorporated in the definition of K without loss of generality. Further, the dimensions on the two sides of Eq. (1) demand that K as it occurs there, cannot be a pure number. Keeping in view of all these considerations, we shall rewrite the Eq. (1) as

$$\text{Shearing stress} = ik\varepsilon\mu_T\left(w_1 - w_2\right) \tag{2}$$

where, ε is a pure number, $i=\sqrt{-1}$, μ_T is the rigidity of the lower medium to the common interface.

Now we introduced a new real variable Ω_1, such that $\varepsilon = \frac{c}{\beta_2}\frac{\Omega_1}{1-\Omega_1}$, where c is the phase velocity of wave and β_2 is the shear wave velocity of lower medium to the common interface.

Therefore

$$\text{shearing stress} = ik\mu_T\frac{c}{\beta_2}\frac{\Omega_1}{\left(1-\Omega_1\right)}\left(w_1 - w_2\right), \tag{3}$$

where the value of $\Omega_1 = 0$ corresponds to an ideally smooth interface and the value of $\Omega_1 = 1$ corresponds to a welded interface.

3. Formulation of the problem

Let us consider a functionally graded fibre-reinforced elastic substrate covered by loosely bonded FGPM stratum of finite thickness depicted in Fig. 1. Let us choose a rectangular co-ordinate system in such a way that the waves are propagating along y-direction and x-axis is vertically downwards. The functionally graded Fibre reinforced substrate is considered as initially stressed.

3.1 Dynamics of functionally graded piezoelectric material (FGPM) stratum

For the FGPM stratum, the equilibrium equations of elasticity without body forces and the Gauss' law of electrostatics without free charge are given as follows

$$\sigma_{ij,j} = \rho_1\ddot{u}_i \tag{4}$$

and

$$D_{i,i} = 0, \tag{5}$$

where $i,\ j = 1,\ 2,\ 3,\ \sigma_{ij}$ is the stress tensor, ρ_1 is the mass density of the FGPM stratum and function of x, and u_i and D_i denote the mechanical and electric displacements in the i^{th}-direction respectively. The dot $(.)$ denotes differentiation with respect to time (t), the comma $(,)$ followed by the subscript i indicates space coordinate differentiation and the repeated subscript index implies summation with respect to that index.

For the propagation of Love waves in the y-direction, displacement and electric potential are expressed as

$$u_1 = v_1 = 0,\ w_1 = w_1\left(x,y,t\right),\ \phi_1 = \phi_1\left(x,y,t\right), \tag{6}$$

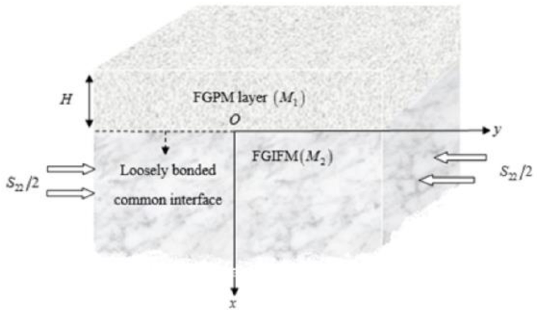


Fig. 1 Geometry of the problem

where (u_1, v_1, w_1) are the displacement components of x, y and z directions respectively of the FGPM stratum. Clearly, the constitutive equations for a transversely isotropic functionally graded piezoelectric material medium with z-axis being the symmetric axis of the material can be expressed as (Liu *et al.* 2001)

$$\left.\begin{aligned}
&\sigma_x = c_{11}(x)S_x + c_{12}(x)S_y + c_{13}(x)S_z - e_{31}(x)E_z,\\
&\sigma_y = c_{12}(x)S_x + c_{11}(x)S_y + c_{13}(x)S_z - e_{31}(x)E_z,\\
&\sigma_z = c_{13}(x)S_x + c_{13}(x)S_y + c_{33}(x)S_z - e_{33}(x)E_z,\\
&\tau_{yz} = c_{44}(x)S_{yz} - e_{15}(x)E_y,\\
&\tau_{zx} = c_{44}(x)S_{zx} - e_{15}(x)E_x,\\
&\tau_{xy} = \frac{1}{2}\left(c_{11}(x) - c_{12}(x)\right)S_{xy},\\
&D_x = e_{15}(x)S_{zx} + \varepsilon_{11}(x)E_x,\\
&D_y = e_{15}(x)S_{yz} + \varepsilon_{11}(x)E_y,\\
&D_z = e_{31}(x)S_x + e_{31}(x)S_y + e_{33}(x)S_z + \varepsilon_{33}(x)E_z,
\end{aligned}\right\} \tag{7}$$

where elastic constants c_{11}, c_{12}, c_{13}, c_{33} and c_{44}, piezoelectric constants e_{15}, e_{31} and e_{33} and dielectric constants ε_{11} and ε_{33} of the stratum are functions of x.

The relationship between mechanical displacement components and strain components of the stratum is given by

$$\begin{aligned}
&S_x = \frac{\partial u_1}{\partial x},\ S_y = \frac{\partial v_1}{\partial y},\ S_z = \frac{\partial w_1}{\partial z},\ S_{yz} = \frac{\partial w_1}{\partial y} + \frac{\partial v_1}{\partial z},\\
&S_{zx} = \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x},\ \text{and}\ S_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x}.
\end{aligned} \tag{8}$$

Also, electrical intensity and the electrical potential for the stratum are correlated as

$$E_x = -\frac{\partial \phi_1}{\partial x},\ E_y = -\frac{\partial \phi_1}{\partial y},\ \text{and}\ E_z = -\frac{\partial \phi_1}{\partial z}. \tag{9}$$

By means of (6), (8) and (9) lead to

$$S_x = 0,\ S_y = 0,\ S_z = 0,\ S_{xy} = 0,\ S_{yz} = \frac{\partial w_1}{\partial y},\ S_{zx} = \frac{\partial w_1}{\partial x}, \tag{10}$$

and

$$E_x = -\frac{\partial \phi_1}{\partial x},\ E_y = -\frac{\partial \phi_1}{\partial y},\ E_z = 0. \tag{11}$$

With aid of relations in (10) and (11), equations in (7) take the form

$$\begin{aligned}
&\sigma_x = 0,\ \sigma_y = 0,\ \sigma_z = 0,\ \tau_{xy} = 0,\\
&\tau_{yz} = c_{44}(x)\frac{\partial w_1}{\partial y} + e_{15}(x)\frac{\partial \phi_1}{\partial y},\\
&\tau_{zx} = c_{44}(x)\frac{\partial w_1}{\partial x} + e_{15}(x)\frac{\partial \phi_1}{\partial x},\\
&D_z = 0,\ D_x = e_{15}(x)\frac{\partial w_1}{\partial x} - \varepsilon_{11}(x)\frac{\partial \phi_1}{\partial x},\\
&\text{and}\ D_y = e_{15}(x)\frac{\partial w_1}{\partial y} - \varepsilon_{11}(x)\frac{\partial \phi_1}{\partial y}.
\end{aligned} \tag{12}$$

In view of Eq. (4), Eq. (5) and Eq. (12), the non-vanishing equation of motion expressed as

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho_1 \frac{\partial^2 w_1}{\partial t^2}, \tag{13}$$

and

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0. \tag{14}$$

With the help of equations in (12), Eq. (13) and Eq. (14) result in

$$\left.\begin{aligned}
&c_{44}(x)\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial c_{44}(x)}{\partial x}\frac{\partial w_1}{\partial x} + e_{15}(x)\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial \phi_1}{\partial x}\\
&+c_{44}(x)\frac{\partial^2 w_1}{\partial y^2} + e_{15}(x)\frac{\partial^2 \phi_1}{\partial y^2} = \rho_1 \frac{\partial^2 w_1}{\partial t^2},\\
&\frac{\partial e_{15}(x)}{\partial x}\frac{\partial w_1}{\partial x} + e_{15}(x)\frac{\partial^2 w_1}{\partial x^2} - \frac{\partial \varepsilon_{11}(x)}{\partial x}\frac{\partial \phi_1}{\partial x}\\
&-\varepsilon_{11}(x)\frac{\partial^2 \phi_1}{\partial x^2} + e_{15}(x)\frac{\partial^2 w_1}{\partial y^2} - \varepsilon_{11}(x)\frac{\partial^2 \phi_1}{\partial y^2} = 0.
\end{aligned}\right\} \tag{15}$$

Now, we shall assume that the material properties of the FGPM stratum have same exponential function distribution along the direction of $x-$ axis. In light of this, the material properties of the FGPM stratum are considered as

$$\begin{aligned}
&c_{44}(x) = c'_{44}e^{\nu_1 x},\ e_{15}(x) = e'_{15}e^{\nu_1 x},\\
&\varepsilon_{11}(x) = \varepsilon'_{11}e^{\nu_1 x},\ \text{and}\ \rho_1(x) = \rho'_1 e^{\nu_1 x},
\end{aligned} \tag{16}$$

where ν_1 is the material gradient of the FGPM stratum and $c'_{44}, e'_{15}, \varepsilon'_{11}$ and ρ'_1 are the values of $c_{44}, e_{15}, \varepsilon_{11},$ and ρ_1 respectively at $x = 0$.

Employing (16) in equations of (15), we instate

$$\left.\begin{aligned}
&c'_{44}\left(\nabla^2 w_1 + \nu_1 \frac{\partial w_1}{\partial x}\right) + e'_{15}\left(\nabla^2 \phi_1 + \nu_1 \frac{\partial \phi_1}{\partial x}\right)\\
&= \rho'_1 \frac{\partial^2 w_1}{\partial t^2},\\
&e'_{15}\left(\nabla^2 w_1 + \nu_1 \frac{\partial w_1}{\partial x}\right) = \varepsilon'_{11}\left(\nabla^2 \phi_1 + \nu_1 \frac{\partial \phi_1}{\partial x}\right).
\end{aligned}\right\} \tag{17}$$

Equations in (17) may be rewritten as

$$\left.\begin{aligned}
&\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} + \nu_1 \frac{\partial w_1}{\partial x} = \frac{1}{\beta_1^2}\frac{\partial^2 w_1}{\partial t^2},\\
&\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} + \nu_1 \frac{\partial \phi_1}{\partial x} = \frac{1}{\beta_1^2}\frac{e'_{15}}{\varepsilon'_{11}}\frac{\partial^2 w_1}{\partial t^2},
\end{aligned}\right\} \tag{18}$$

where $\beta_1 = \sqrt{\frac{\overline{c'_{44}}}{\rho'_1}}$, $\overline{c'_{44}} = \left(c'_{44} + \frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}}\right)$, and β_1 is the shear wave velocity in the FGPM stratum.

We assume the solutions of the Eq. (18) in the form

$$\left.\begin{aligned}
&w_1 = W_1(x)e^{ik(y-ct)},\\
&\phi_1 = \varphi_1(x)e^{ik(y-ct)},
\end{aligned}\right\} \tag{19}$$

where k is the wave number, c is the phase velocity, $i=\sqrt{-1}$ and $W_1(x)$ be the solution of the equation given by

$$\frac{d^2W_1}{dx^2}+\nu_1\frac{dW_1}{dx}+k^2b_1^2W_1=0, \tag{20}$$

where $b_1=\sqrt{c^2/\beta_1^2-1}$, c^2/β_1^2 is the phase velocity ratio and $\varphi_1(x)$ be the solution of the equation given by

$$\frac{d^2\varphi_1}{dx^2}+\nu_1\frac{d\varphi_1}{dx}-k^2\varphi_1=-\frac{k^2c^2}{\beta_1^2}\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}}W_1. \tag{21}$$

The solution of Eqs. (20) and (21) is written as

$$W_1(x)=\left\{A_1\cos(\gamma_1 x)+A_2\sin(\gamma_1 x)\right\}e^{-\nu_1 x/2},$$

and

$$\varphi_1(x)=\left[\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}}\left\{A_1\cos(\gamma_1 x)+A_2\sin(\gamma_1 x)\right\}e^{-\nu_1 x/2}+A_3e^{\eta_1 x}+A_4e^{\eta_2 x}\right].$$

where, $\gamma_1=\frac{\sqrt{4k^2b^2-\nu_1^2}}{2}$; $\eta_1=-\frac{\nu_1}{2}-\frac{\sqrt{\nu_1^2+4k^2}}{2}$

and $\eta_2=-\frac{\nu_1}{2}+\frac{\sqrt{\nu_1^2+4k^2}}{2}$.

Without loss of generality, it is assumed that the Love waves propagated along the positive direction of the x-axis, so the solution of the Eq. (18) can be written as

$$\left.\begin{aligned}
&w_1(x,y,t)=\left\{A_1\cos(\gamma_1 x)+A_2\sin(\gamma_1 x)\right\}e^{-\nu_1 x/2}e^{ik(y-ct)},\\
&\phi_1(x,y,t)=\left[\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}}\left\{A_1\cos(\gamma_1 x)+A_2\sin(\gamma_1 x)\right\}e^{-\nu_1 x/2}\right.\\
&\left.+A_3e^{\eta_1 x}+A_4e^{\eta_2 x}\right]e^{ik(y-ct)},
\end{aligned}\right\} \tag{22}$$

where A_1, A_2, A_3, A_4 are arbitrary constant.

3.2 Dynamics of the lower functionally graded initially stressed fibre-reinforced material (FGPM) half-space

The constitutive equation for a fibre-reinforced linearly elastic anisotropic medium with preferred direction $\vec{a}$ of reinforcement, is given by (Belfield *et al*. 1983)

$$\begin{aligned}
\sigma_{ij}&=\lambda\varepsilon_{kk}\delta_{ij}+2\mu_T\varepsilon_{kk}+\alpha\left(a_ka_m\varepsilon_{km}\delta_{ij}+a_ia_j\varepsilon_{kk}\right)\\
&+2(\mu_L-\mu_T)(a_ia_k\varepsilon_{kj}+a_ja_k\varepsilon_{ki})+\beta a_ka_m\varepsilon_{km}a_ia_j,
\end{aligned} \tag{23}$$

where $i,j,k,m=1,2,3,$ σ_{ij} are stress components, $\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ are components of infinitesimal strain, δ_{ij} is Kronecker delta, $\vec{a}=(a_1,a_2,a_3)$ is the preferred directions of reinforcement such that $a_1^2+a_2^2+a_3^2=1$. The vector $\vec{a}$ may be function of position. Indices take the values 1, 2, 3 and summation convention is employed. α,β and $(\mu_L-\mu_T)$ are reinforcement parameters. μ_T can be identified as the shear modulus in transverse shear across the preferred direction, and μ_L as the shear modulus in longitudinal shear in the preferred direction. α,β are specific stress components to take into account different layers for concrete part of the composite material, λ is Lame's constant of elasticity.

The authors assume

$$u_2=v_2=0,\ w_2=w_2(x,y,t), \tag{24}$$

and

$$\mu_T=\mu_T^{\iota}e^{\nu_2 x},\ \mu_L=\mu_L^{\iota}e^{\nu_2 x},\ \rho_2=\rho_2^{\iota}e^{\nu_2 x},\ S_{22}=S_{22}^{\iota}e^{\nu_2 x}, \tag{25}$$

where (u_2,v_2,w_2) are the displacement components of x, y and z directions respectively of fibre-reinforced half-space and ν_2 is considered as functional gradedness of fibre-reinforced half-space.

Also, μ_T^{ι}, μ_L^{ι}, ρ_2^{ι}, S_{22}^{ι} are the values of μ_T, μ_L, ρ_2, S_{22} respectively at $x=0$.

Clearly, the only non-vanishing equation of motion for the propagation of Love-type wave in the functionally graded initially stress fibre-reinforced half-space, as per Eqs. (23) and (24) is obtained as (Biot 1965)

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{s_{22}}{2}\frac{\partial^2 w_2}{\partial y^2}=\rho_2\frac{\partial^2 w_2}{\partial t^2}, \tag{26}$$

where $\frac{S_{22}}{2}$ is the initial stress, ρ_2 is the density of the fibre-reinforced half-space and

$$\left.\begin{aligned}
\sigma_{zx}&=e^{\nu_2 x}\left[\mu_T\frac{\partial w_2}{\partial x}+(\mu_L-\mu_T)a_1\left(a_1\frac{\partial w_2}{\partial x}+a_2\frac{\partial w_2}{\partial y}\right)\right],\\
\sigma_{zy}&=e^{\nu_2 x}\left[\mu_T\frac{\partial w_2}{\partial y}+(\mu_L-\mu_T)a_2\left(a_1\frac{\partial w_2}{\partial x}+a_2\frac{\partial w_2}{\partial y}\right)\right].
\end{aligned}\right\} \tag{27}$$

In view of Eq. (25), Eq. (26) leads to

$$\begin{aligned}
&P\left(\frac{\partial^2 w_2}{\partial x^2}+\nu_2\frac{\partial w_2}{\partial x}\right)+Q\left(\frac{\partial^2 w_2}{\partial y^2}+\nu_2\frac{\partial w_2}{\partial y}\right)\\
&+R\left(\frac{\partial^2 w_2}{\partial x\partial y}+\nu_2\frac{\partial w_2}{\partial y}\right)+\frac{S_{22}^{\iota}}{2}\frac{\partial^2 w_2}{\partial y^2}=\frac{1}{(\beta_2^{\iota})^2}\frac{\partial^2 w_2}{\partial t^2}
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
&P=1+\left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}}-1\right)a_2^2,\ Q=1+\left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}}-1\right)a_1^2,\\
&R=2a_1a_2\left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}}-1\right),\ \beta_2=\sqrt{\frac{\mu_T^{\iota}}{\rho_T^{\iota}}}.
\end{aligned} \tag{29}$$

Now, we assume the solution of the Eq. (28) of the form

$$w_2(x,y,t)=W_2(x)e^{ik(y-ct)}, \tag{30}$$

where k is the wave number and c is the common wave velocity. Eq. (28) when substituted upon by Eq. (30), takes the form

$$\begin{aligned}
&(P\nu_2+Rik)\frac{dW_2}{dx}+\left[-k^2\left(Q+\frac{S_{22}^{\iota}}{2}\right)+ik\frac{\nu_2}{2}R\right.\\
&\left.+\frac{k^2c^2}{(\beta_2^{\iota})^2}\right]W_2+P\frac{d^2W_2}{dx^2}=0,
\end{aligned} \tag{31}$$

The appropriate solution of Eq. (31) satisfying the condition $\lim_{x\to 0} W_2(x)=0$ is

$$W_2(x)=A_5 e^{-\gamma_2 x}, \tag{32}$$

Hence, the solution of Eq. (28) gives the expression for displacement in lower functionally graded initially stressed fibre-reinforced as

$$w_2(x,y,t)=A_5 e^{-\gamma_2 x} e^{ik(y-ct)}, \tag{33}$$

where A_5 is arbitrary constant and

$$\gamma_2=\frac{P\nu_2+\sqrt{P^2\nu_2^2-R^2k^2+4Pk^2Q+PS'_{22}k^2-4P\dfrac{k^2c^2}{(\beta'_2)^2}}}{2P}+i\frac{kR}{2P},$$

4. Boundary conditions and dispersion relation

The boundary conditions are as follows:

(i) The electrical boundary conditions at the free surface i.e., at $x=-H$ is

$$D_x(x,\ y)=0, \text{(Electrically open condition)} \tag{34}$$

$$\phi_1(x,\ y)=0. \text{(Electrically short condition)} \tag{35}$$

(ii) The mechanical traction-free condition at $x=-H$ is

$$\tau_{zx}^{(1)}(x,\ y)=0. \tag{36}$$

(iii) The continuous conditions at the interface i.e., at $x=0$ are

$$\tau_{zx}^{(1)}(x,\ y)=\tau_{zx}^{(2)}(x,\ y), \tag{37}$$

$$\phi_1(x,\ y)=0, \tag{38}$$

$$\tau_{zx}^{(2)}(x,\ y)=k_\sigma(w_1-w_2), \tag{39}$$

where $k_\sigma=ik\mu_T\dfrac{c}{\beta_2}\dfrac{\Omega_1}{(1-\Omega_1)}$ with Ω_1 as a bonding parameter between the upper layer and lower half-space.

Using boundary condition (34), we get

$$e'_{15}\left(A_3\eta_1 e^{-\eta_1 H}+A_4\eta_2 e^{-\eta_2 H}\right)=0, \tag{40}$$

Using boundary condition (35), we get

$$\begin{aligned}&\frac{e'_{15}}{\varepsilon'_{11}}\left\{A_1\cos(\gamma_1 H)-A_2\sin(\gamma_1 H)\right\}e^{\nu_1 H/2}\\&+A_3 e^{-\eta_1 H}+A_4 e^{-\eta_2 H}=0,\end{aligned} \tag{41}$$

Using boundary condition (36), we get

$$\begin{aligned}&\left[A_1\left(\gamma_1\sin(\gamma_1 H)-\frac{\nu_1}{2}\cos(\gamma_1 H)\right)\right.\\&\left.+A_2\left(\frac{\nu_1}{2}\sin(\gamma_1 H)+\gamma_1\cos(\gamma_1 H)\right)\right]\bar{c}'_{44}e^{\nu_1 H/2}\\&+e'_{15}\left(A_3\eta_1 e^{-\eta_1 H}+A_4\eta_2 e^{-\eta_2 H}\right)=0,\end{aligned} \tag{42}$$

Using boundary condition (37), we get

$$\begin{aligned}&\left(-A_1\frac{\nu_1}{2}+A_2\gamma_1\right)\bar{c}'_{44}+\left(A_3\eta_1+A_4\eta_2\right)e'_{15}\\&+A_5\left(\gamma_2 P-\left(\frac{\mu'_L}{\mu'_T}-1\right)a_1a_2ik\right)=0,\end{aligned} \tag{43}$$

Using boundary condition (38), we get

$$\frac{e'_{15}}{\varepsilon'_{11}}A_1+A_3+A_4=0, \tag{44}$$

Using boundary condition (39), we get

$$\begin{aligned}&A_1 ik\frac{c}{\beta_2}\Omega_1+A_5\left[ik\frac{c}{\beta_2}\Omega_1-(1-\Omega_1)\gamma_2 P\right.\\&\left.+(1-\Omega_1)\left(\frac{\mu'_L}{\mu'_T}-1\right)a_1a_2ik\right]=0,\end{aligned} \tag{45}$$

Eliminating arbitrary constants A_1, A_2, A_3, A_4 and A_5 from Eqs. (40), (42), (43), (44) and (45), we get following dispersion relation for electrically open condition

$$\tan(\gamma_1 H)=\frac{\bar{m}_1\bar{n}_1+\bar{m}_2\bar{n}_2}{\bar{m}_1^2+\bar{m}_2^2}, \tag{46}$$

where $\bar{m}_1, \bar{m}_2, \bar{n}_1$ and $\bar{n}_2$ are provided in Appendix I.

Eliminating arbitrary constants A_1, A_2, A_3, A_4 and A_5 from Eqs. (41)-(45), we obtain following dispersion relation for electrically short condition

$$\tan(\gamma_1 H)=\frac{m_1n_1+m_2n_2}{m_1^2+m_2^2}, \tag{47}$$

where m_1, m_2, n_1 and n_2 are provided in Appendix II.

5. Particular cases and validation

Case 5.1

If we consider both the mediums are without functional gradedness (i.e., $\nu_1=0$, $\nu_2=0$), the dispersion equations for electrically open and short case respectively take the following form.

Dispersion relation (46) in absence of functional gradedness in the composite structure for electrically open condition takes the form

$$\tan\left(\gamma_1' H\right)=\frac{\bar{m}_1'\bar{n}_1'+\bar{m}_2'\bar{n}_2'}{\bar{m}_1'^2+\bar{m}_2'^2}, \tag{48}$$

where $\bar{m}_1', \bar{m}_2', \bar{n}_1'$ and $\bar{n}_2'$ are provided in Appendix I.
Dispersion relation (47) in absence of functional gradedness in the composite structure for electrically short condition takes the form

$$\tan\left(\gamma_1' H\right)=\frac{m_1'n_1'+m_2'n_2'}{m_1'^2+m_2'^2}, \tag{49}$$

where m_1', m_2', n_1' and n_2' are provided in Appendix II.

Subcase 5.1.1

In addition to the condition stated in Case 5.1, if w**e** also consider the composite structure without reinforcement (i.e. $\mu_L'=\mu_T'=\mu_2$), the dispersion relations for electrically open condition (46) and short condition (47) respectively transform as follows

$$\tan\left(\gamma_1' H\right)=\frac{\bar{m}_1''\bar{n}_1''+\bar{m}_2''\bar{n}_2''}{\bar{m}_1''^2+\bar{m}_2''^2}, \tag{50}$$

and

$$\tan\left(\gamma_1' H\right)=\frac{m_1''n_1''+m_2''n_2''}{m_1''^2+m_2''^2}, \tag{51}$$

where $\bar{m}_1'', \bar{m}_2'', \bar{n}_1''$ and $\bar{n}_2''$ are provided in Appendix I with m_1'', m_2'', n_1'' and n_2'' being provided in Appendix II.

Case 5.2

If we consider the stratum without piezoelectricity (i.e., $e_{15}'=0$), both dispersion relations for electrically open condition (46) and short condition (47) reduces to

$$\tan\left(\gamma_1 H\right)=\frac{\bar{m}_1'''\bar{n}_1'''+\bar{m}_2'''\bar{n}_2'''}{\bar{m}_1'''^2+\bar{m}_2'''^2}, \tag{52}$$

where $\bar{m}_1''', \bar{m}_2''', \bar{n}_1'''$ and $\bar{n}_2'''$ are provided in Appendix III.

Subcase 5.2.1

In addition to the condition stated in Case 5.2, if we consider the composite structure without reinforcement, both the dispersion relations for electrically open condition (46) and short condition (47) transforms to

$$\tan\left(\gamma_1 H\right)=\frac{\grave{m}_1\grave{n}_1+\grave{m}_2\grave{n}_2}{\grave{m}_1^2+\grave{m}_2^2}, \tag{53}$$

where $\grave{m}_1, \grave{m}_2, \grave{n}_1$ and $\grave{n}_2$ are provided in Appendix III.

Case 5.3

If we consider that the stratum without piezoelectricity (i.e. $e_{15}'=0$), and substrate without horizontal initial stress $\left(\text{i.e. } \xi_2\left(=\frac{S_{22}'}{2}\right)=0\right)$, are in smooth contact (i.e., $\Omega_1\to 0$), both dispersion relations for electrically open condition (46) and short condition (47) reduces to

$$\tan\left(\gamma_1 H\right)=\frac{\bar{t}_1\bar{s}_1+\bar{t}_2\bar{s}_2}{\bar{t}_1^2+\bar{t}_2^2}, \tag{54}$$

where $\bar{t}_1, \bar{t}_2, \bar{s}_1$ and $\bar{s}_2$ are provided in Appendix III.

Subcase 5.3.1

In addition to the condition stated in Case 5.3, if we also consider the composite structure without reinforcement, both the dispersion relations for electrically open condition (46) and short condition (47) yield

$$\tan\left(\gamma_1 H\right)=\frac{\gamma_1\gamma_{22}'\mu_2}{\left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}'}, \tag{55}$$

where γ_{22}' is provided in Appendix III.

Case 5.4

If we consider the case when the stratum without piezoelectricity, and substrate without horizotal initial stress, are in welded contact, both the dispersion relations for electrically open condition (46) and short condition (47) reduces to

$$\tan\left(\gamma_1 H\right)=\frac{\bar{t}_1'\bar{s}_1'+\bar{t}_2'\bar{s}_2'}{\bar{t}_1'^2+\bar{t}_2'^2}, \tag{56}$$

where $\bar{t}_1', \bar{t}_2', \bar{s}_1'$ and $\bar{s}_2'$ are provided in Appendix III.

Subcase 5.4.1

In addition to the condition stated in Case 5.4, if we consider the composite structure without reinforcement, both the dispersion relations for electrically open condition (46) and short condition (47) takes the form

$$\tan\left(\gamma_1 H\right)=\frac{\mu_2\gamma_1\gamma_{22}'}{\left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}'}. \tag{57}$$

Case 5.5

For the purpose of validation of the present study, if we consider the composite structure is without functional gradednessness and the stratum without piezoelectricity, is in welded contact with substrate without reinforcement and without horizotal initial stress, then both the dispersion relations for electrically open condition (46) and short condition (47) is found to be in well-agreement to the following Classical Love wave equation.

$$\tan\left[kH\sqrt{c^2/\beta_1^2-1}\right]=\frac{\mu_2\sqrt{1-c^2/\beta_2^2}}{c_{44}^t\sqrt{c^2/\beta_1^2-1}} \tag{58}$$

6. Numerical discussion

The following data has been considered for numerical computation of phase velocity of Love-type wave propagating in a loosely bonded functionally graded piezoelectric material (FGPM) stratum with an underlying functionally graded initially-stressed fibre-reinforced material (FGIFM) substrate:

(i) Lithium Tantalate (LiTaO3) For FGPM stratum (Tiersten, 1969)

$$e_{15}^t=2.6C/m^2,\varepsilon_{11}^t=3.63\times10^{-10}C^2/Nm^2$$
$$c_{44}^t=9.4\times10^{10}N/m^2,\rho_1^t=7450kg/m^3$$

(ii) Crystalline graphite For FGIFM substrate (Gilis 1984, Pierson 2012)

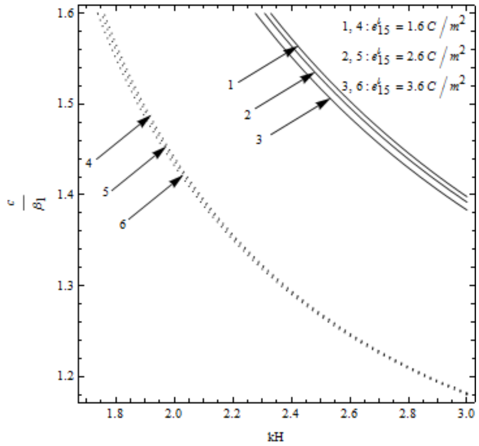
$$\mu_T^t=4.45\times10^{10}N/m^2,\mu_L^t=0.023\times10^{10}N/m^2,$$
$$\rho_2=2260kg/m^3,a_1=0.00316227,a_2=0.999995,a_3=0$$

Moreover, the values of the parameters used for the purpose of numerical computation are provided in Table 1.

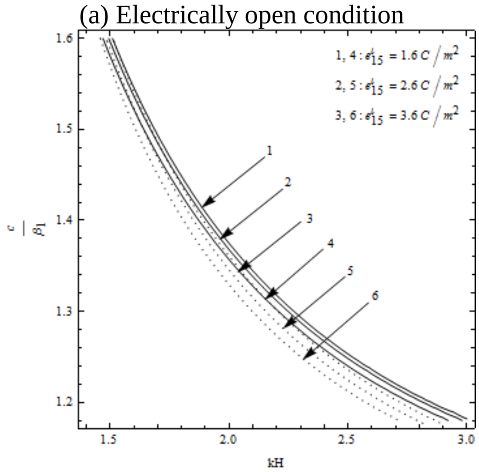
The graphical delineation of dimensionless phase velocity (c/β_1) against dimensionless wave number (kH) along with various affecting parameters in the composite structure has been demonstrated in Fig. 2(a) to Fig. 9(b).

Effect of parameters viz. piezoelectric coefficient, dielectric constant, horizontal initial stresses acting in the substrate of the composite structure, bonding parameter associated with the loose bonding of the stratum and substrate, and parameters associated with functional gradedness of the FGPM stratum and FGIFM substrate of the composite structure has been illustrated by means of graphs in both electrically open and short conditions for the reinforced and reinforced-free composite structure. In all these figures, the curves of solid line correspond to the case (reinforced composite structure) when FGPM stratum is lying over a FGIFM substrate while dotted line curves correspond to the case (reinforced-free composite structure) when FGPM stratum is lying over an initially-stressed functionally graded isotropic elastic substrate.

It is found that in all figures (Fig. 2(a) to Fig. 10) phase velocity of Love-type wave propagating in a loosely bonded FGPM stratum lying over a FGIFM substrate, decreases with increase in wave number in all the studied cases.



(a) Electrically open condition



(b) Electrically short condition

Fig. 2 Variation of dimensionless phase velocity (c/β_1) against dimensionless wave number (kH) for different values of piezoelectric coefficient (e_{15}^t) for both reinforced and reinforced-free composite structure

Table 1 Values of various dimensionless parameters

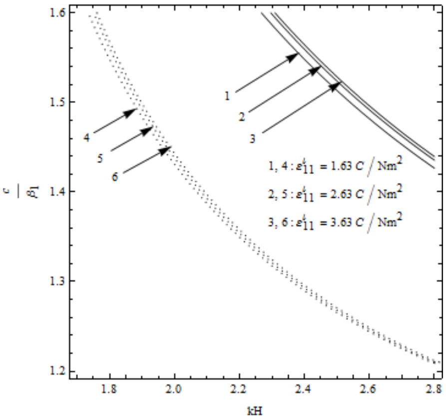
Parameters	Values	Parameters	Values
e_{15}	1.6, 2.6, 3.6, 5.6	$\nu_2 H$	0, 0.2, 1, 2
ε_{11}	1.63, 2.63, 3.63	Ω_1	0.6, 0.7, 0.8
$\nu_1 H$	0, 0.1, 1, 2	ξ_2	-0.4, -0.2, 0, 0.1, 0.2, 0.4

Specifically, Figs. 2(a), 3(a), 4(a), 5(a), 6(a), 7(a), 8(a) and 9(a) are associated with the case when Love-type wave is propagating in the reinforced and reinforced-free composite structure in electrically open condition. Figs. 2(b), 3(b), 4(b), 5(b), 6(b), 7(b), 8(b) and 9(b) are allied with the case when Love-type wave is propagating in the composite structure in electrically short condition with and without reinforcement.

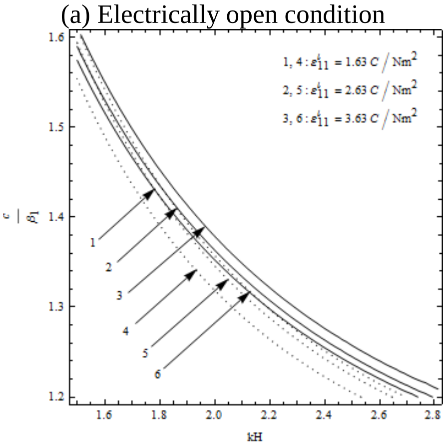
Clearly, Figs. 2(a) and 2(b) distinctly study the effect of piezoelectric coefficient (e_{15}^t) of FGPM stratum of the composite structure on the dispersion curve of the Love-type wave in both the cases of electrically open condition and electrically short condition respectively, each for the reinforced and reinforced-free composite structure. It is reported from these figures that e_{15}^{t} disfavors the phase velocity of Love-type wave propagating in the reinforced and reinforced-free composite structure. This disfavoring effect occurs in both the cases of electrically open and short condition irrespective of the presence or absence of reinforcement in the composite structure.

Fig. 3(a) and Fig. 3(b) thrashes out the effect of dielectric constant $\left(\varepsilon_{11}^{t}\right)$ of FGPM stratum on the dispersion curve of Love-type wave propagating in the composite structure in both the cases of electrically open and electrically short condition respectively, each for reinforced and reinforced-free cases. These figures reveal that the phase velocity of Love-type wave increases with increase in the value of dielectric constant for electrically open and short condition irrespective of the presence or absence of reinforcement in the composite structure.

A meticulous examination of Figs. 2(a) to 3(b) articulates that piezoelectricity of FGPM stratum has a decreasing effect of the phase velocity of Love-type wave propagating in the composite structure with or without reinforcement.



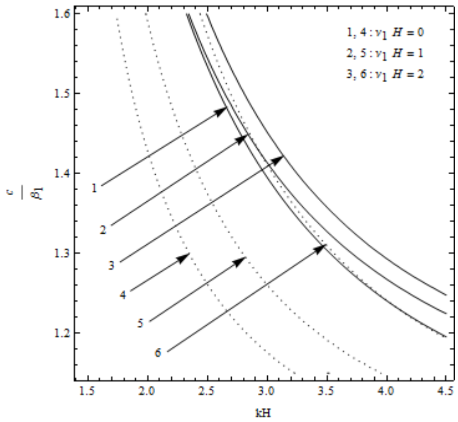
(a) Electrically open condition



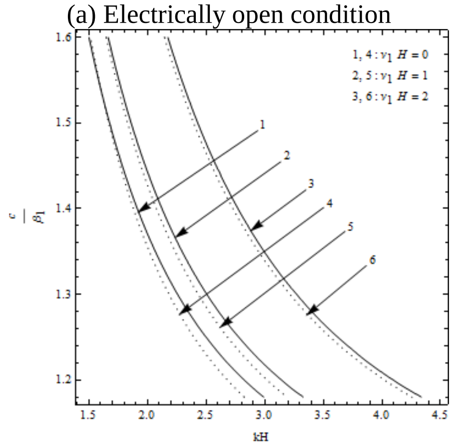
(b) Electrically short condition

Fig. 3 Variation of dimensionless phase velocity against dimensionless wave number for different values of dielectric constant $\left(\varepsilon_{11}^{t}\right)$ for both reinforced and reinforced-free composite structure

Figs. 4(a) and 4(b) depict the effect of parameter $\left(\nu_1 H\right)$ associated with functional gradedness of the FGPM stratum in the composite structure on the phase velocity of Love-type wave for both electrically open and short conditions respectively for reinforced and reinforced-free composite structure. Figs. 5(a) and 5(b) illustrate the effect of parameter $\left(\nu_2 H\right)$ associated with functional gradedness of the FGIFM substrate in the composite structure on the dispersion curve of Love-type wave for both electrically open and short conditions respectively, each for reinforced and reinforced-free composite structure. Curve 1 in Figs. 4(a) and 4(b) is associated to the case when the piezoelectric stratum of the reinforced composite structure is without functional gradedness for electrically open and short conditions respectively. In Figs. 4(a) and 4(b), curve 4 renders the case when the piezoelectric stratum of the reinforced-free composite structure is without functional gradedness for electrically open and short conditions respectively.



(a) Electrically open condition

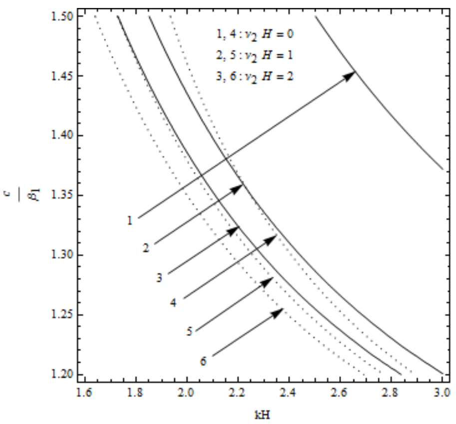


(b) Electrically short condition

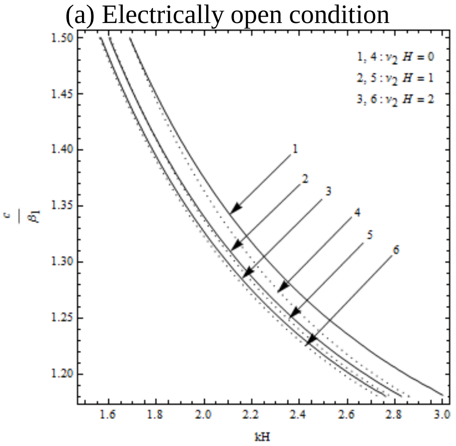
Fig. 4 Variation of dimensionless phase velocity against dimensionless wave number for different values of parameter $\left(\nu_1 H\right)$ associated with functional gradedness of the FGPM stratum for both reinforced and reinforced-free composite structure

Similarly, curve 1 in Figs. 5(a) and 5(b) portrays the case when the fibre-reinforced substrate of the composite structure is without functional gradedness for electrically open and short conditions respectively. Curve 4 in Figs. 5(a) and 5(b) represents the case when the initially-stressed isotropic elastic substrate of the composite structure is without functional gradedness for electrically open and short conditions respectively. In these figures (Figs. 4(a) to 5(b)), it is reported that parameter $(\nu_1 H)$ associated with functional gradedness of the FGPM stratum favors the phase velocity of Love-type wave in both electrically open condition and short condition for reinforced and reinforced-free cases. Parameter $(\nu_2 H)$ associated with functional gradedness of the composite structure disfavors the phase velocity of Love-type wave in both electrically open condition and short condition. Both the parametric effects ocurrs irrespective of the presence or absence of reinforcement in the composite structure. It is worthy to mention that the phase velocity of Love-type wave is found to be minimum in the case when the piezoelectric stratum is without functional gradedness and maximum when the initially-stressed substrate without functional gradedness in both electrically open and short condition.

The effect of the bonding parameter (Ω_1) associated with the bonding between stratum and substrate on the dispersion curve of Love-type wave has been delineated in Figs. 6(a) and 6(b) in both electrically open and short cases respectively, each for reinforced and reinforced-free composite structure. Curve 1 $(\Omega_1 = 0)$ in Figs. 6(a) and 6(b) indicates the case when the stratum and the substrate of the reinforced composite structure is in smooth contact for electrically open and short case respectively. Curve 4 $(\Omega_1 = 0)$ in Figs. 6(a) and 6(b) manifests the case when the stratum and the substrate of the reinforced-free composite structure is in smooth contact for electrically open and short case respectively. Curve 3 $(\Omega_1 = 1)$ in Figs. 6(a) and 6(b) represents the case of perfectly bonded (welded) FGPM stratum and FGIFM substrate for electrically open and short condition respectively. Curve 6 $(\Omega_1 = 1)$ in Figs. 6(a) and 6(b) demonstrates the case of welded contact between FGPM stratum and initially-stressed isotropic elastic substrate without functional gradedness for electrically open and short condition respectively. Curves 2 and 5 $(0<\Omega_1 < 1)$

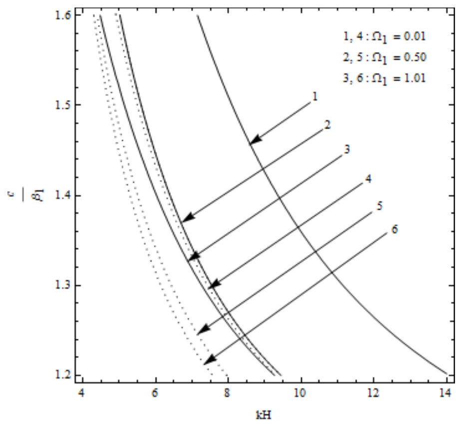


(a) Electrically open condition

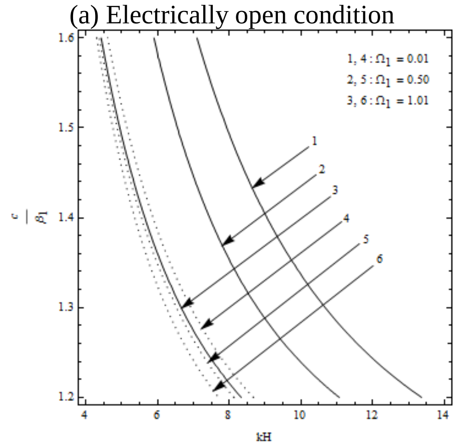


(b) Electrically short condition

Fig. 5 Variation of dimensionless phase velocity against dimensionless wave number for different values of parameter $(\nu_2 H)$ associated with functional gradedness of the FGIFM substrate for both reinforced and reinforced-free composite structure



(a) Electrically open condition



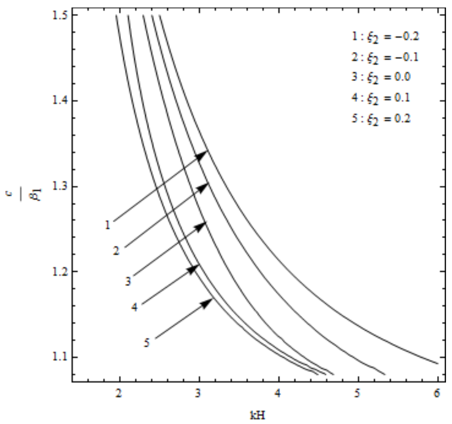
(b) Electrically short condition

Fig. 6 Variation of dimensionless phase velocity against dimensionless wave number for different values of bonding parameter (Ω_1) associated with bonding of FGPM stratum with FGIFM substrate for both reinforced and reinforced-free composite structure

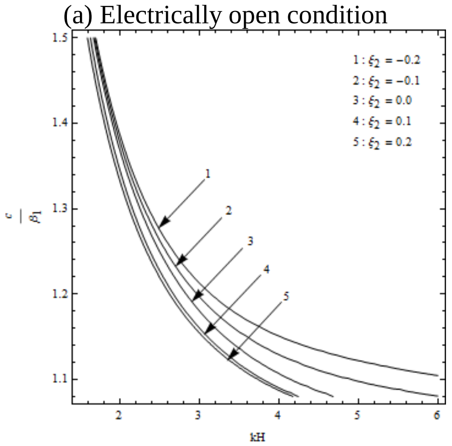
in both the figures correspond to the case when FGPM stratum is loosely bonded with initially-stressed substrate.
It is found from these figures that the bonding parameter has a decreasing effect on the dispersion curve of Love-type wave. This effect also occurs in the composite structure for both electrically open and short conditions irrespective of the presence or absence of reinforcement in the composite structure. It has also been reported that phase velocity is minimum for the case of perfect bonding between the stratum and substrate and maximum for the case of smooth bonding between the stratum and substrate of the composite structure.

The graphical characterization studying the impact of horizontal initial stress acting in the functionally graded fibre-reinforced substrate on the dispersion curve of Love-type wave has been plotted in Figs. 7(a) and 7(b) for electrically open and short condition respectively. The effect of horizontal initial stress acting in the isotropic elastic substrate without functional gradedness on the phase velocity of Love-type wave has been plotted in Figs. 8(a) and 8(b) for electrically open and short condition respectively. In Figs. 7(a) to 8(b), curves 1 and 2 represents the presence of horizontal tensile initial stress in the reinforced and reinforced-free composite structure for both electrically open and short condition. Curves 4 and 5 illustrate the presence of horizontal compressive initial stress in the reinforced and reinforced-free composite structure for both electrically open and short condition. Curve 3 renders the case of absence of the horizontal initial stress in the reinforced and reinforced-free composite structure for both electrically open and short condition.

It is established from these figures that the phase velocity of Love-type wave increases with increase in horizontal tensile initial stress whereas it decreases with the increase in horizontal compressive initial stress acting in lower substrate of the composite structure. Both the effects exhibits in electrically open and short case irrespective of the presence or absence of reinforcement in the composite structure.
Comparative study of the Figs. 2(a) to 8(b) manifest that the presence of reinforcement in the composite structure decrease the phase velocity of Love-type wave irrespective of the presence or absence of piezoelectricity, horizontal initial stress and functional gradedness of the composite structure along with the bonding parameter associated with the bonding between the stratum and substrate of the

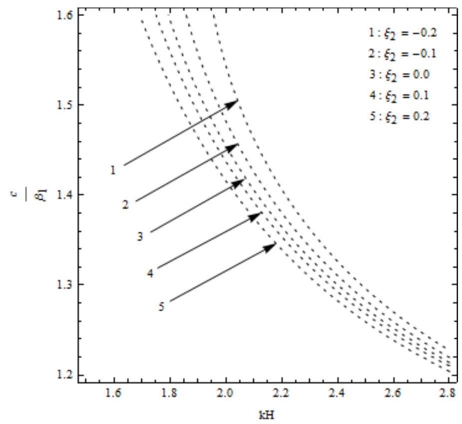


(a) Electrically open condition

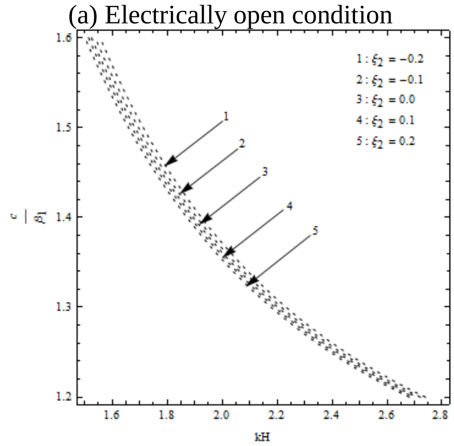


(b) Electrically short condition

Fig. 7 Variation of dimensionless phase velocity against dimensionless wave number for different values of dimensionless initial stress parameter $\left(\xi_2 = S_{22}^{\iota}/2\mu_T\right)$ associated with substrate in the reinforced composite structure



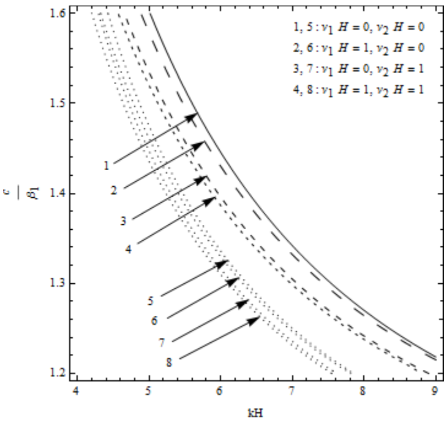
(a) Electrically open condition



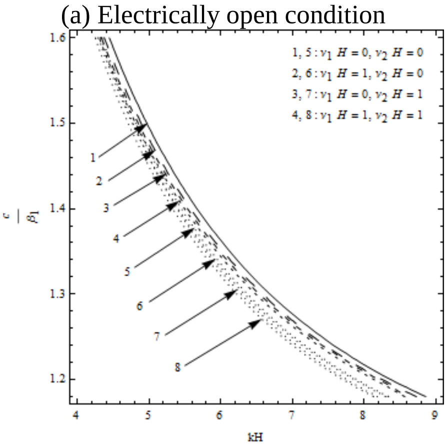
(b) Electrically short condition

Fig. 8 Variation of dimensionless phase velocity against dimensionless wave number for different values of dimensionless initial stress parameter $\left(\xi_2 = S_{22}^{\iota}/2\mu_T\right)$ associated with initially stressed substrate in the reinforced-free composite structure composite structure. It is also worthy to note that the phase velocity of Love-type wave is less in case of electrically short condition as compared to the case of electrically open condition for the composite structure with or without reinforcement.

In Figs. 9(a) and 9(b), distinct cases associated with the functional gradedness in the stratum and substrate of the composite structure has been demonstrated in a comparative manner. In both the figures, curve 1 (for reinforced composite structure i.e. Case 5.1) and curve 5 (for reinforced-free composite structure i.e. Subcase 5.1.1) represent when both the stratum and substrate of the composite structure are without functional gradedness for electrically open and short condition respectively; curve 2(reinforced composite structure) and curve 6 (reinforced-free composite structure) show the case when the piezoelectric stratum of the composite structure lying over a FGIFM substrate is without functional gradedness for electrically open and short condition respectively; curve 3 (reinforced composite structure) and curve 7 (reinforced-free composite structure) render the case when the FGPM stratum lying over an initially-stressed substrate of the composite structure is without functional gradedness for electrically open and short condition respectively; and curve 4 (reinforced composite structure) and curve 8 (reinforced-free composite structure) demonstrate the case when both the piezoelectric stratum and initially stressed substrate of the composite structure are functionally graded for electrically open and short condition respectively.



(a) Electrically open condition



(b) Electrically short condition

Fig. 9 Variation of dimensionless phase velocity against dimensionless wave number for distinct cases associated with functional gradedness of the stratum and substrate of reinforced and reinforced-free composite structure

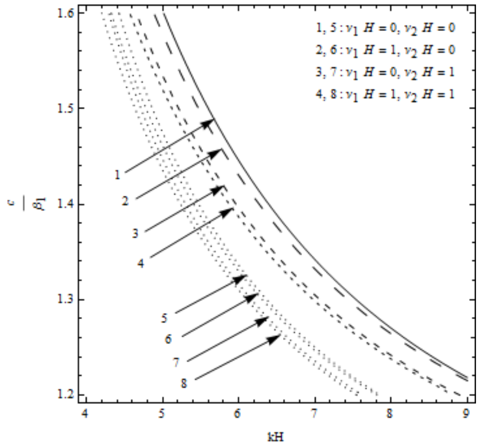


Fig. 10 Variation of dimensionless phase velocity against dimensionless wave number for distinct cases associated with piezoelectricity, reinforcement, initial stress and loose bonding of the composite structure

A meticulous examination of these figures reveals that the phase velocity of Love-type wave is maximum for the case when both the stratum and substrate of the reinforced and reinforced-free composite structure are without functional gradedness (i.e. Case 5.1 and Subcase 5.1.1 respectively). The phase velocity is found to be minimum for the case when both the stratum and substrate of the reinforced and reinforced-free composite structure are with functional gradedness. For the case when either the stratum or the substrate of the reinforced and reinforced-free composite structure is without functional gradedness, the phase velocity is found to be moderate. It is reported that the phase velocity is more for the case when the reinforced or reinforced-free composite structure is comprised of piezoelectric stratum without functional gradedness lying over a FGIFM substrate. The phase velocity is found to be less for the case when the reinforced or reinforced-free composite structure is comprised FGPM stratum lying over an initially-stressed substrate without functional gradedness

In a conclusive way, the effect of functional gradedness of the initially-stressed substrate on the phase velocity of Love-type wave dominates the effect of functional gradedness associated with the piezoelectric stratum in both electrically open and short conditions. This effect is also found to be irrespective of the presence or absence of reinforcement in the composite structure.

A comparative study of particular cases (Case 5.2 to 5.4) to unravel the effect of piezoelectricity, reinforcement and horizontal compressive initial stress along with bonding parameter associated with the bonding between the stratum and substrate of the composite structure on the dispersion curve of Love-type wave has been illustrated in Fig. 10. In

Fig. 10, curve 3 represents the case when Love-type wave is propagating in a loosely bonded FGPM stratum lying over a FGIFM substrate. Curve 2 manifests the case when the composite structure is comprised of loosely bonded functionally graded isotropic elastic stratum lying over a FGIFM substrate (i.e., Case 5.2). Curve 5 portrays the case when the composite structure is without piezoelectricity in the stratum and reinforcement in the substrate (i.e., Subcase 5.2.1). Curve 4 demonstrates the case when Love-type wave is propagating in a loosely bonded functionally graded isotropic elastic stratum lying over a functionally graded isotropic elastic substrate without horizontal initial stresses. Curve 6 illustrates the case when the functionally graded isotropic elastic stratum and functionally graded isotropic elastic substrate without horizontal initial stresses are in welded contact in the composite structure (i.e., Subcase 5.4.1) and curve 1 renders the case when the functionally graded isotropic stratum and functionally graded isotropic substrate without horizontal initial stresses are in smooth contact in the composite structure (i.e., Subcase 5.3.1).

It has been reported that the phase velocity of Love-type wave is maximum for the case when the functionally graded isotropic elastic stratum and the functionally graded isotropic elastic substrate without horizontal initial stresses are in smooth contact. On the other hand, the phase velocity is found to be minimum for the case when the composite structure is comprised of perfectly bonded functionally graded isotropic elastic stratum lying over a functionally graded isotropic elastic substrate without horizontal initial stresses. It has been found that when the composite structure is comprised of loosely bonded functionally graded isotropic elastic stratum lying over a functionally graded substrate, the combined effect of horizontal compressive initial stress and reinforcement associated with the substrate favours the phase velocity of Love-type wave. It has also been discovered that the when the composite structure is comprised of loosely bonded functionally graded stratum lying over a functionally graded initially-stressed substrate, the joint effect of piezoelectricity and reinforcement of the composite structure increase the phase velocity of Love-type wave.

7. Conclusions

This study deals with Love-type wave propagation in loosely bonded composite structure comprised of functionally graded piezoelectric material (FGPM) stratum lying over a functionally graded initially-stressed fibre-reinforced material (FGIFM) substrate. The closed form expressions of the dispersion relation have been obtained analytically for both the cases of electrically open and electrically short conditions. The study signifies the emphatic influence of wave number, piezoelectric coefficient, dielectric constant, horizontal initial stresses, bonding parameter and functional gradedness of the composite structure on the phase velocity of Love-type wave in both the cases of electrically open and electrically short conditions along with the presence and absence of reinforcement in the composite structure. Numerical computation and graphical demonstration have been executed to demonstrate the effects of these parameters on the phase velocity. In addition to this, comparative study has been carried out to analyze the distinct cases associated with functional gradedness, piezoelectricity, reinforcement and horizontal initial stress acting in the composite structure and bonding of the stratum and substrate on the dispersion curve of Love-type wave. In an absolutely precise way, the study bestows the following distinct peculiarities of the study:

- The established dispersion relations for Love-type wave propagating in a loosely bonded FGPM stratum lying over a FGIFM substrate for electrically open and electrically short conditions are in well-agreement with the classical Love wave equation.
- The phase velocity of Love-type wave decrease with the increase in wave number for electrically open and electrically short conditions irrespective of type of bonding, initial stress and functional gradedness in both reinforced and reinforced-free composite structure.
- Piezoelectric coefficient associated with FGPM stratum in reinforced and reinforced-free composite structure disfavors the phase velocity of Love-type wave for both electrically open and electrically short conditions. The phase velocity of Love-type wave increases with increase in the value of dielectric constant associated with FGPM stratum in reinforced and reinforced-free composite structure for both electrically open and electrically short conditions. Owing to this, it is established that piezoelectricity disfavors the phase velocity of Love-type wave propagating in the considered composite structure.
- It is reported that functional gradedness of the FGPM stratum of the composite structure favors the phase velocity of Love-type wave whereas functional gradedness of the FGIFM substrate of the composite structure disfavors the phase velocity of Love-type wave in both electrically open and short conditions irrespective of the presence or absence of reinforcement in the composite structure.
- The phase velocity of Love-type wave is maximum for the case when both the stratum and substrate of the reinforced and reinforced-free composite structure are without functional gradedness. On the other hand, it is found to be minimum for the case when both the stratum and substrate of the reinforced and reinforced-free composite structure are with functional gradedness. The phase velocity is found to be moderate for the case when either the stratum or the substrate of the reinforced and reinforced-free composite structure is without functional gradedness in both electrically open and short condition.
- The bonding of the stratum with the substrate in the composite structure with or without reinforcement has a significant effect on the dispersion curve of Love-type wave for both electrically open and short conditions. It is reported that phase velocity is minimum for the case of perfect bonding of stratum with substrate whereas it is maximum for the case of smooth bonding of stratum with substrate in both electrically open and short case irrespective of the presence or absence of reinforcement in the composite structure.

- The phase velocity of Love-type wave increases with increase in horizontal tensile initial stress and decreases with the increase in horizontal compressive initial stress acting in substrate in electrically open and short case irrespective of the presence or absence of reinforcement in the composite structure.
- The phase velocity of Love-type wave is found to be more in electrically open condition as compared to the electrically short condition irrespective of type of bonding, initial stress and functional gradedness in both reinforced and reinforced-free composite structure.
- It is reported that the presence of reinforcement in the composite structure increase the phase velocity of Love-type wave irrespective of the presence or absence of piezoelectricity, initial stress, loose bonding (of stratum and substrate) and functional gradedness (of the composite structure) in both electrically open and short conditions.
- Through meticulous examination it is observed that effect of reinforcement present in the composite structure is prominent on the phase velocity of Love-type wave as compared to the effect of piezoelectricity associated with the composite structure. Further, the effect of reinforcement dominates the effect of horizontal compressive initial stress acting in the composite structure.

The consequences of the theoretical study in the framework of the considered model can be employed in the field of acoustics, civil engineering, earth science engineering and seismology and may also find its possible application in acoustic devices, Love wave sensor, transducers, actuators or any other sensor devices to enhance the performance. The present study also may find its application in the field of health and usage monitoring providing permanent and integral monitoring of structural and functional integrity as well as implementing as a structural health monitoring system.

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Appendix I *(For electrically open case)*

$$\bar{m}_1 = -\left(1-\Omega_1\right)\gamma_2' P\left[\left(\eta_2 e^{-\eta_2 H} - \eta_1 e^{-\eta_1 H}\right)\left(\frac{\nu_1^2}{4} - \gamma_1^2\right)\bar{c}_{44}^{\iota} + \frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}\frac{\nu_1}{2}\eta_1\eta_2\left(e^{-\eta_1 H} - e^{-\eta_2 H}\right)\right]e^{\nu_1 H/2} + \left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)$$
$$\times\left(\frac{kR}{2} + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right)k\frac{c}{\beta_2}\Omega_1\frac{\nu_1}{2}e^{\nu_1 H/2},$$

$$\bar{m}_2 = \left[\left(\eta_2 e^{-\eta_2 H} - \eta_1 e^{-\eta_1 H}\right)\left(\frac{\nu_1^2}{4} - \gamma_1^2\right)\bar{c}_{44}^{\iota} + \frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}\frac{\nu_1}{2}\eta_1\eta_2\right.$$
$$\left.\times\left(e^{-\eta_1 H} - e^{-\eta_2 H}\right)\right]\left[\left\{k\frac{c}{\beta_2}\Omega_1 + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right\}\right.$$
$$\left.-\left(1-\Omega_1\right)\frac{kR}{2}\right]e^{\nu_1 H/2} + \left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)\gamma_2' Pk\frac{c}{\beta_2}\Omega_1\frac{\nu_1}{2}e^{\nu_1 H/2},$$

$$\bar{n}_1 = \left(1-\Omega_1\right)\gamma_2' P\left(e^{-\eta_1 H} - e^{-\eta_2 H}\right)\gamma_1\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}$$
$$-\left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)\left(\frac{kR}{2P} + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right)\gamma_1 k\frac{c}{\beta_2}\Omega_1 e^{\nu_1 H/2}.$$

$$\bar{n}_2 = \left[\left\{k\frac{c}{\beta_2}\Omega_1 + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right\} - \left(1-\Omega_1\right)\frac{kR}{2}\right]$$
$$\left(-e^{-\eta_1 H} + e^{-\eta_2 H}\right)\gamma_1\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}e^{\nu_1 H/2}$$
$$-\left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)\gamma_1\gamma_2' kP\frac{c}{\beta_2}\Omega_1 e^{\nu_1 H/2},$$

where

$$\gamma_2' = \frac{\left(P\nu_2 + \sqrt{P^2\nu_2^2 - R^2k^2 + 4Pk^2Q + PS_{22}^{\iota}k^2 - 4P\frac{k^2c^2}{\left(\beta_2^{\iota}\right)^2}}\right)}{2P}.$$

$$\bar{m}_1' = \left(1-\Omega_1\right)\gamma_2''' Pk\left(e^{kH} + e^{-kH}\right)\gamma_1'^2\bar{c}_{44}^{\iota},$$

$$\bar{m}_2' = k\left(e^{kH} + e^{-kH}\right)\gamma_1'^2\bar{c}_{44}^{\iota}\left[\left\{k\frac{c}{\beta_2}\Omega_1 + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right\} - \left(1-\Omega_1\right)\frac{kR}{2}\right],$$

$$\bar{n}_1' = \left(1-\Omega_1\right)\gamma_2' P\left(e^{-kH} - e^{kH}\right)\gamma_1\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}$$
$$-k\left(e^{-kH} + e^{kH}\right)\left(\frac{kR}{2P} + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right)\gamma_1' k\frac{c}{\beta_2}\Omega_1.$$

$$\bar{n}_2' = -\left[\left\{k\frac{c}{\beta_2}\Omega_1 + \left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)a_1 a_2 k\right\} - \left(1-\Omega_1\right)\frac{kR}{2}\right]$$
$$\times\left(e^{-kH} - e^{kH}\right)\gamma_1\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}$$
$$-\left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)\gamma_1'\gamma_2' kP\frac{c}{\beta_2}\Omega_1.$$

$$\bar{m}_1'' = \left(1-\Omega_1\right)\gamma_2''' k\left(e^{kH} + e^{-kH}\right)\gamma_1'^2\bar{c}_{44}^{\iota},$$

$$\bar{m}_2'' = k\left(e^{kH} + e^{-kH}\right)\gamma_1'^2\bar{c}_{44}^{\iota}k\frac{c}{\beta_2}\Omega_1,$$

$$\bar{n}_1'' = \left(1-\Omega_1\right)\gamma_2'\left(e^{-kH} - e^{kH}\right)\gamma_1'\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}},$$

$$\bar{n}_2'' = k\frac{c}{\beta_2}\Omega_1\left(-e^{-kH} + e^{kH}\right)\gamma_1'\eta_1\eta_2\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}$$
$$-\left(\eta_1 e^{-\eta_1 H} - \eta_2 e^{-\eta_2 H}\right)\gamma_1'\gamma_2' k\frac{c}{\beta_2}\Omega_1,$$

where

$$\gamma_2''' = \frac{\left(\sqrt{4k^2 + S_{22}^{\iota}k^2 - 4\frac{k^2c^2}{\left(\beta_2^{\iota}\right)^2}}\right)}{2},$$

$$\gamma_1' = k\sqrt{\left(c^2/\beta_1^2 - 1\right)}.$$

Appendix II *(For electrically short case)*

$$m_1 = L_1\left[\left(-\eta_2 e^{-\eta_2 H} + \eta_1 e^{-\eta_1 H}\right)\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}\frac{\nu_1}{2}e^{\nu_1 H/2}\bar{c}_{44}^{\iota} + \bar{c}_{44}^{\iota}e^{\nu_1 H/2}\right.$$
$$\left.\left(e^{-\eta_1 H} - e^{-\eta_2 H}\right)\left(\gamma_1^2\bar{c}_{44}^{\iota} - \frac{\nu_1^2}{2}\bar{c}_{44}^{\iota}\right) - \eta_2\eta_1\left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\frac{\left(e_{15}^{\iota}\right)^4}{\left(\varepsilon_{11}^{\iota}\right)^2}e^{\nu_1 H/2}\right]$$
$$+\bar{c}_{44}^{\iota}\frac{\nu_1}{2}\left(\eta_2 e^{-\eta_1 H} - \eta_1 e^{-\eta_2 H}\right)\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}e^{\nu_1 H/2} + k\frac{c}{\beta_2}\Omega_1\left(\frac{\mu_L^{\iota}}{\mu_T^{\iota}} - 1\right)$$
$$a_1 a_2 k\left\{\left(-\eta_2 e^{-\eta_2 H} + \eta_1 e^{-\eta_1 H}\right)\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}} + \left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\frac{\nu_1}{2}\bar{c}_{44}^{\iota}\right\}e^{\nu_1 H/2}$$
$$+k\frac{c}{\beta_2}\Omega_1\frac{kR}{2P}\left\{\left(-\eta_2 e^{-\eta_2 H} + \eta_1 e^{-\eta_1 H}\right)\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}} + \left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\frac{\nu_1}{2}\bar{c}_{44}^{\iota}\right\}e^{\nu_1 H/2},$$

$$m_2 = L_2\left[\left(-\eta_2 e^{-\eta_2 H} + \eta_1 e^{-\eta_1 H}\right)\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}\frac{\nu_1}{2}e^{\nu_1 H/2}\bar{c}_{44}^{\iota} + \bar{c}_{44}^{\iota}e^{\nu_1 H/2}\right.$$
$$\left.\left(e^{-\eta_1 H} - e^{-\eta_2 H}\right)\left(\gamma_1^2\bar{c}_{44}^{\iota} - \frac{\nu_1^2}{2}\bar{c}_{44}^{\iota}\right) - \eta_2\eta_1\left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\frac{\left(e_{15}^{\iota}\right)^2}{\varepsilon_{11}^{\iota}}\right]$$
$$+k\frac{c}{\beta_2}\Omega_1\gamma_2' P\left\{\left(-\eta_2 e^{-\eta_2 H} + \eta_1 e^{-\eta_1 H}\right)\frac{e_{15}^{\iota}}{\varepsilon_{11}^{\iota}} + \left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\frac{\nu_1}{2}\bar{c}_{44}^{\iota}\right\}e^{\nu_1 H/2}$$

$$n_1 = L_1\left[\left\{\left(\eta_2 e^{-\eta_2 H} - \eta_1 e^{-\eta_1 H}\right)\gamma_1 + \frac{1}{\cos\left(\gamma_1 H\right)}\left(\eta_1 - \eta_2\right)\gamma_1\right\}\bar{c}'_{44}\right.$$

$$\left. + \left\{\left(\eta_2 e^{-\eta_1 H} - \eta_1 e^{-\eta_2 H}\right) + \frac{1}{\cos\left(\gamma_1 H\right)}\left(\eta_1 - \eta_2\right)e^{-\left(\eta_1+\eta_2\right)H}\right\}\bar{c}'_{44}\gamma_1\right]$$

$$\times e^{\nu_1 H/2}\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}} + \left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\left(\frac{\mu'_L}{\mu'_T} - 1\right)a_1 a_2 k^2\frac{c}{\beta_2}\Omega_1\gamma_1\bar{c}'_{44}e^{\nu_1 H/2}$$

$$+ k\frac{c}{\beta_2}\Omega_1\frac{kR}{2}\left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\gamma_1\bar{c}'_{44}e^{\nu_1 H/2},$$

$$n_2 = L_2\left[\left\{\left(\eta_2 e^{-\eta_2 H} - \eta_1 e^{-\eta_1 H}\right)\gamma_1 + \frac{1}{\cos\left(\gamma_1 H\right)}\left(\eta_1 - \eta_2\right)\gamma_1\right\}\bar{c}'_{44}\right.$$

$$\left. + \left\{\left(\eta_2 e^{-\eta_1 H} - \eta_1 e^{-\eta_2 H}\right) + \frac{1}{\cos\left(\gamma_1 H\right)}\left(\eta_1 - \eta_2\right)e^{-\left(\eta_1+\eta_2\right)H}\right\}\left(\bar{c}'_{44}\right)^2\gamma_1\right]$$

$$e^{\nu_1 H/2}\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}} + k\frac{c}{\beta_2}\Omega_1\gamma'_2 P\left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\gamma_1\bar{c}'_{44}e^{\nu_1 H/2},$$

where

$$L_1 = -\mu'_T\gamma'_2 P\left(1 - \Omega_1\right),$$

$$L_2 = \mu'_T\left[k\frac{c}{\beta_2}\Omega_1 + \left(1 - \Omega_1\right)\left(\frac{\mu'_L}{\mu'_T} - 1\right)a_1 a_2 k\right]$$

$$-\mu'_T P\left(1 - \Omega_1\right)\frac{kR}{2P}.$$

$$m'_1 = L_1\bar{c}'_{44}\left(e^{-kH} - e^{kH}\right)\gamma_1'^2\bar{c}'_{44} - L_1\eta_2\eta_1\left(e^{kH} - e^{-kH}\right)\frac{\left(e'_{15}\right)^4}{\left(\varepsilon'_{11}\right)^2}$$

$$+ k^2\frac{c}{\beta_2}\Omega_1\left(\frac{\mu'_L}{\mu'_T} - 1\right)a_1 a_2 k\left(e^{kH} + e^{-kH}\right)\frac{e'_{15}}{\varepsilon'_{11}}$$

$$+ k\frac{c}{\beta_2}\Omega_1\frac{kR}{2}\left(e^{kH} + e^{-kH}\right)\frac{e'_{15}}{\varepsilon'_{11}},$$

$$m'_2 = L_2\bar{c}'_{44}\left(e^{kH} - e^{-kH}\right)\gamma_1'^2\bar{c}'_{44} - L_2\eta_2\eta_1\left(e^{kH} - e^{-kH}\right)\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}}$$

$$+ k^2\frac{c}{\beta_2}\Omega_1\mu'_T\gamma''_2 P\left(e^{kH} + e^{-kH}\right)\frac{e'_{15}}{\varepsilon'_{11}},$$

$$n'_1 = L_1\left[\left\{\left(e^{kH} + e^{-kH}\right)k\gamma_1 + \frac{2k}{\cos\left(\gamma_1 H\right)}\gamma'_1\right\}\bar{c}'_{44}\right.$$

$$\left. + \left\{k\left(e^{kH} + e^{-kH}\right) + \frac{2k}{\cos\left(\gamma_1 H\right)}\right\}\left(\bar{c}'_{44}\right)^2\gamma'_1\right]\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}}$$

$$+ \left(e^{kH} - e^{-kH}\right)\left(\frac{\mu'_L}{\mu'_T} - 1\right)a_1 a_2 k^2\frac{c}{\beta_2}\Omega_1\gamma'_1\bar{c}'_{44}$$

$$+ k\frac{c}{\beta_2}\Omega_1\frac{kR}{2}\left(e^{kH} - e^{-kH}\right)\gamma'_1\bar{c}'_{44},$$

where

$$L_1 = -\mu'_T\gamma'_2 P\left(1 - \Omega_1\right),$$

$$L_2 = \mu'_T\left[k\frac{c}{\beta_2}\Omega_1 + \left(1 - \Omega_1\right)\left(\frac{\mu'_L}{\mu'_T} - 1\right)a_1 a_2 k\right],$$

$$\gamma''_2 = \frac{\left(\sqrt{-R^2k^2 + 4Pk^2Q + PS'_{22}k^2 - 4P\frac{k^2c^2}{\left(\beta'_2\right)^2}}\right)}{2P},$$

$$\gamma'_1 = k\sqrt{\left(c^2/\beta_1^2 - 1\right)}.$$

$$n'_2 = L_2\left[\left\{-k\left(e^{kH} + e^{-kH}\right)\gamma'_1 + \frac{2k}{\cos\left(\gamma_1 H\right)}\gamma'_1\right\}\bar{c}'_{44}\right.$$

$$\left. + \left\{-\left(e^{kH} + e^{-kH}\right) + \frac{2k}{\cos\left(\gamma_1 H\right)}\right\}\left(\bar{c}'_{44}\right)^2\gamma'_1\right]$$

$$\times\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}} + k\frac{c}{\beta_2}\Omega_1\gamma''_2 P\left(e^{-\eta_2 H} - e^{-\eta_1 H}\right)\gamma'_1\bar{c}'_{44},$$

$$m''_1 = L_1\bar{c}'_{44}\left(e^{kH} - e^{-kH}\right)\gamma_1'^2\bar{c}'_{44} - L_1\eta_2\eta_1\left(e^{kH} - e^{-kH}\right)\frac{\left(e'_{15}\right)^4}{\left(\varepsilon'_{11}\right)^2}$$

$$+ k^2\frac{c}{\beta_2}\Omega_1\frac{kR}{2}\left(e^{kH} + e^{-kH}\right)\frac{e'_{15}}{\varepsilon'_{11}},$$

$$m''_2 = \mu'_T k\frac{c}{\beta_2}\Omega_1\left[\bar{c}'_{44}\left(e^{kH} - e^{-kH}\right)\gamma_1'^2\bar{c}'_{44} - \eta_2\eta_1\left(e^{kH} - e^{-kH}\right)\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}}\right]$$

$$+ k^2\frac{c}{\beta_2}\Omega_1\gamma''_2 P\frac{e'_{15}}{\varepsilon'_{11}}\left(e^{kH} + e^{-kH}\right),$$

$$n''_1 = L_1\left[\left\{-k\left(e^{kH} + e^{-kH}\right) + \frac{2k}{\cos\left(\gamma_1 H\right)}\right\}\gamma'_1\bar{c}'_{44}\right.$$

$$\left. + \left\{-k\left(e^{kH} + e^{-kH}\right) + \frac{2k}{\cos\left(\gamma_1 H\right)}\right\}\left(\bar{c}'_{44}\right)^2\gamma'_1\right]\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}}$$

$$+ k\frac{c}{\beta_2}\Omega_1\frac{kR}{2}\left(e^{kH} - e^{-kH}\right)\gamma'_1\bar{c}'_{44},$$

$$n''_2 = \mu'_T k\frac{c}{\beta_2}\Omega_1\left[\left\{-\left(e^{kH} + e^{-kH}\right)k\gamma'_1 + \frac{2k}{\cos\left(\gamma_1 H\right)}\gamma'_1\right\}\bar{c}'_{44}\right.$$

$$\left. + \left\{-\left(e^{kH} + e^{-kH}\right)k + \frac{2k}{\cos\left(\gamma_1 H\right)}\right\}\left(\bar{c}'_{44}\right)^2\gamma'_1\right]$$

$$\times\frac{\left(e'_{15}\right)^2}{\varepsilon'_{11}} + k\frac{c}{\beta_2}\Omega_1\gamma''_2\left(e^{kH} - e^{-kH}\right)\gamma'_1\bar{c}'_{44}.$$

Appendix III

$$\bar{m}_1''' = -\left(1-\Omega_1\right)\gamma_2' P\left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}' e^{\nu_1 H/2}$$

$$+\left(\frac{kR}{2}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)k\frac{c}{\beta_2}\Omega_1\frac{\nu_1}{2}e^{\nu_1 H/2},$$

$$\bar{m}_2''' = \left[\left\{k\frac{c}{\beta_2}\Omega_1+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right\}-\left(1-\Omega_1\right)\frac{kR}{2}\right]e^{\nu_1 H/2}$$

$$\times\left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}'+\gamma_2' Pk\frac{c}{\beta_2}\Omega_1\frac{\nu_1}{2}e^{\nu_1 H/2},$$

$$\bar{n}_1''' = \left(\frac{kR}{2P}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)\gamma_1 k\frac{c}{\beta_2}\Omega_1 e^{\nu_1 H/2},$$

$$\bar{n}_2''' = \gamma_1\gamma_2' kP\frac{c}{\beta_2}\Omega_1 e^{\nu_1 H/2},$$

where

$$\gamma_2' = \frac{\left(P\nu_2+\sqrt{P^2\nu_2^2-R^2k^2+4Pk^2Q+PS_{22}'k^2-4P\frac{k^2c^2}{\left(\beta_2'\right)^2}}\right)}{2P}.$$

$$\grave{\bar{m}}_1 = -\left(1-\Omega_1\right)\gamma_2'\mu_2'\left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}'+\frac{k^2\nu_1}{4}\frac{c}{\beta_2}\Omega_1,$$

$$\grave{\bar{m}}_2 = \left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\mu_2'\bar{c}_{44}' k\frac{c}{\beta_2}\Omega_1+\grave{\gamma}_2 k\frac{c}{\beta_2}\Omega_1\frac{\nu_1}{2},\ \grave{\bar{n}}_1 = 0,$$

$$\grave{\bar{n}}_2 = \gamma_1\grave{\gamma}_2 k\frac{c}{\beta_2}\Omega_1,$$

where $\grave{\gamma}_2 = \dfrac{\left(\nu_2+\sqrt{\nu_2^2+4k^2+S_{22}'k^2-4\frac{k^2c^2}{\left(\beta_2'\right)^2}}\right)}{2}.$

$$\bar{t}_1 = \left(\frac{kR}{2}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)k\frac{c}{\beta_2}\frac{\nu_1}{2},$$

$$\bar{t}_2 = \left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}' k\frac{c}{\beta_2}+\gamma_{22}Pk\frac{c}{\beta_2}\frac{\nu_1}{2},$$

$$\bar{s}_1 = \left(\frac{kR}{2}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)\gamma_1 k\frac{c}{\beta_2},$$

$$\bar{s}_2 = P\gamma_1\gamma_{22}k\frac{c}{\beta_2}.$$

where $\gamma_{22} = \dfrac{\left(P\nu_2+\sqrt{P^2\nu_2^2-R^2k^2+4Pk^2Q-4P\frac{k^2c^2}{\left(\beta_2'\right)^2}}\right)}{2P}.$

$$\gamma_{22}' = \left(\nu_2/2+\sqrt{\nu_2^2/4+k^2-\frac{k^2c^2}{\left(\beta_2'\right)^2}}\right),$$

$$\bar{t}_1' = +\left(\frac{kR}{2}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)k\frac{c}{\beta_2}\frac{\nu_1}{2}e^{\nu_1 H/2},$$

$$\bar{t}_2' = \left(\frac{\nu_1^2}{4}-\gamma_1^2\right)\bar{c}_{44}'\left[\left\{k\frac{c}{\beta_2}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right\}\right]e^{\nu_1 H/2},$$

$$\bar{s}_1' = \left(\frac{kR}{2P}+\left(\frac{\mu_L'}{\mu_T'}-1\right)a_1 a_2 k\right)\gamma_1 k\frac{c}{\beta_2}e^{\nu_1 H/2},$$

$$\bar{s}_2' = \gamma_1\gamma_{22}kP\frac{c}{\beta_2}e^{\nu_1 H/2}.$$