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A novel quasi-3D hyperbolic shear deformation theory for vibration analysis of simply supported functionally graded plates

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Abstract. An original quasi-3D hyperbolic shear deformation theory for simply supported functionally graded plates is proposed in this work. The theory considers both shear deformation and thickness-stretching influences by a hyperbolic distribution of all displacements within the thickness, and respects the stress-free boundary conditions on the upper and lower surfaces of the plate without using any shear correction coefficient. By expressing the shear parts of the in-plane displacements with the integral term, the number of unknowns and equations of motion of the proposed theory is reduced to four as against five in the first shear deformation theory (FSDT) and common quasi-3D theories. Equations of motion are obtained from the Hamilton principle. Analytical solutions for dynamic problems are determined for simply supported plates. Numerical results are presented to check the accuracy of the proposed theory.

Keywords: vibration; functionally graded plate; plate theory; thickness-stretching effect

1. Introduction

Functionally graded materials (FGMs) are a kind of non-homogeneous composites materials, in which the material characteristics change smoothly and continuously from one surface to another. A typical FGM is fabricated from a mixture of two material phases, for example a ceramic and a metal. An advantage of FGMs over laminated structures is that they eliminate the delamination mode of failure encountered in laminated structures. In addition, the physical and thermal properties of FGMs can be tailored to different applications and working environments. This makes FGMs preferable in many structural applications such as nuclear reactors, aerospace, mechanical, automotive, and civil engineering (Eltaher *et al*. 2013, Ait Amar Meziane *et al*. 2014, Ait Atmane *et al*. 2015, Kar *et al*. 2016, Janghorban 2016, Ahouel *et al.* 2016, Fahsi *et al*. 2017, Abdelaziz *et al*. 2017, Sekkal *et al*. 2017a).

Since the shear deformation influences are considerable in advanced composites like FGMs, shear deformation models such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be employed. The FSDT (Nguyen *et al*. 2008, Zhao *et al.* 2009, Hosseini-Hashemi *et al*. 2010, 2011, Irschik 1993, Nosier and Fallah 2008, Yang *et al*. 2009, Meksi *et al*. 2015, Bouderba *et al*. 2016, Bellifa *et al.* 2016, Youcef *et al.* 2018) provides acceptable results, but requires a shear correction coefficient that is difficult to determine out consistently due to dependent on many parameters considering geometry, boundary conditions, and loading conditions. To avoid the use of the shear correction coefficient and find a better prediction of the transverse shear deformation and normal strains in FG structures, HSDTs have been developed. In general, HSDTs can be constructed based on nonlinear variations of the in-plane displacements (Reddy 2000, Ferreira *et al*. 2005, Ait Atmane *et al*. 2010, Benyoucef *et al*. 2010, Mantari *et al.* 2012, Xiang *et al*. 2011, Xiang and Kang 2013, Thai and Kim 2013, Sobhy 2013, Bouderba *et al.* 2013, Ahmed 2014, Ait Yahia *et al*. 2015, Belkorissat *et al*. 2015, Al-Basyouni *et al.* 2015, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Baseri *et al*. 2016, Raminnea *et al*. 2016, Bousahla *et al*. 2016, Beldjelili *et al*. 2016, Janghorban 2016, Aldousari 2017, Bellifa *et al.* 2017a, b, El-Haina *et al*. 2017, Chikh *et al.* 2017, Besseghier *et al.* 2017, Benadouda *et al*. 2017, Rahmani *et al.* 2017, Menasria *et al*. 2017, Attia *et al.* 2018, Meksi *et al.* 2018, Bakhadda e*t al.* 2018, Yazid e*t al.* 2018) or both in-plane and transverse displacements (Chen *et al.* 2009, Fares *et al*. 2009, Talha and Singh 2010, Ferreira *et al.* 2011, Reddy 2011, Natarajan and Manickam 2012, Neves *et al.* 2012a, b, Neves *et al.* 2013, Jha *et al.* 2013, Swaminathan and Naveenkumar 2014, Bousahla *et al.* 2014, Hebali *et al.* 2014, Belabed *et al*. 2014, Fekrar *et al.* 2014, Bourada *et al*. 2015, Hamidi *et al*. 2015, Draiche *et al.* 2016, Akavci 2016, Bennoun *et al.* 2016, Bouafia *et al*. 2017, Sekkal *et al*. 2017b, Abualnour *et al*. 2018, Bouhadra *et al.* 2018, Benchohra *et al*. 2018) (i.e., quasi-3D theories). However, HSDTs are highly computational cost due to

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involving in many variables (e.g., theories by Neves et al. (2012a, b and 2013) with nine variables, Reddy (2011) with eleven variables, Jha *et al.* (2013) with twelve unknowns, Talha and Singh (2010) and Natarajan and Manickam (2012) with thirteen variables). Thus, needs exist for the development of quasi-3D HSDTs which are simple to use.

The proposed quasi-3D HSDT in this work, accounts for both the transverse shear deformation and thickness-stretching effects through the use of the integral term into the in-plane displacements. The present quasi-3D HSDT contains the same five variables as in the FSDT, but respects the traction-free boundary conditions on the upper and lower surfaces of the plate without requiring any shear correction coefficient. Equations of motion are obtained from Hamilton's principle. Analytical solutions of simply supported FG plates are presented. The computed results are compared with the existing solutions to verify the accuracy of proposed theory in predicting the dynamic response of FG plates.

2. Analytical modeling

The plate is graded from aluminum (lower surface) to alumina (upper surface) as presented in Fig. 1. The dimensions of the plates are $a \times b \times h$, where the length is "a", "b"is width and "h"is thickness of the plate. The gradation of material characteristics is in the thickness direction with metal and ceramic being the typical constituents. Aluminum/Alumina (Al/Al_2O_3), and Aluminum /Zirconia (Al/ZrO_2) are the examples of the FG plate.

2.1 Material variation laws

The constituent elements of FG plate are changing in thickness direction from bottom, where it is metal rich to the top, where the surface is ceramic rich. Macroscopically the plate is supposed homogenous and isotropic. This distribution is achieved by varying the volume fraction of the constituent elements. The volume fraction and hence material characteristics vary according to power law. Assume that, except constant Poisson’s ratio, the Young’s modulus E and the mass density ρ obey the power-law variation, namely (Tounsi *et al*. 2013, Zidi *et al.* 2014, Mahi *et al.* 2015, Taibi *et al.* 2015, Meradjah *et al*. 2015, Zemri *et al.* 2015, Mouffoki *et al*. 2017)

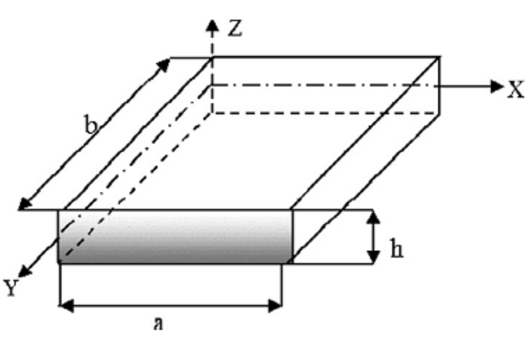

Fig. 1 Geometry of functionally graded plate

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^{p_1} \tag{1a}$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{1}{2} + \frac{z}{h}\right)^{p_2} \tag{1b}$$

Where (E_c, ρ_c) and (E_m, ρ_m) are the corresponding properties of the ceramic and metal, respectively, and p_1 , p_2 are constants. Poisson’s ratio is taken as $\nu = 0.3$ throughout the analyses. The value of p (p_1, p_2) equal to zero represents a fully ceramic plate and infinite p , a fully metallic plate. The distribution of the composition of ceramics and metal is linear for $p = 1$. Typical values for metal and ceramics used in the FG plate are listed in Table 1.

2.2 Displacement base field

In this article, further simplifying considerations are made to the conventional HSDTs with thickness stretching effect so that the number of unknowns is reduced. The displacement field of the classical HSDTs with thickness stretching effect is defined by

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y) \tag{2a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y) \tag{2b}$$

$$w(x,y,z) = w_0(x,y) + g(z)\varphi_z(x,y) \tag{2c}$$

Where u_0 ; v_0 ; w_0 , φ_x , φ_y , and φ_z are six unknown displacements of the mid-plane of the plate, and $f(z)$ represents shape function defining the variation of the transverse shear strains and stresses across the thickness.

Table 1 Material properties used in the FG plate

Properties	Metal	Ceramic	
	Aluminum (Al)	Alumina (Al_2O_3)	Zirconia (ZrO_2)
E (GPa)	70	380	200
ν	0.3	0.3	0.3
ρ (kg/m^3)	2702	3800	5700

In this article a novel displacement field with 5 unknowns is proposed, by considering that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field of the present theory can be written in a simpler form as

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{3a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{3b}$$

$$w(x,y,z) = w_0(x,y) + g(z)\varphi_z(x,y) \tag{3c}$$

The coefficients k_1 and k_2 depends on the geometry and expressed as follows

$$k_1 = \alpha^2\,, k_2 = \beta^2 \tag{4}$$

In this article, the present original HSDT is obtained by setting

$$f(z) = \frac{1}{2}\tanh\left(\frac{2\,z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2\cosh(1)^2} \quad \text{and}\; g(z) = \frac{df(z)}{dz} \tag{5}$$

The linear strain relations obtained from the displacement model of Eqs. (3(a)-3(c)), are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \quad \varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{6}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{7a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\int \theta\,dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta\,dy + \dfrac{\partial \varphi_z}{\partial y} \\ k_1\int \theta\,dx + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \varphi_z \tag{7b}$$

The integrals used in the above equations shall be resolved by a Navier type solution. The following relations can be obtained

$$\frac{\partial}{\partial y}\int \theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y},$$
$$\int \theta\,dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{8}$$

Where the coefficients A' and B' are adopted according to the type of solution employed, in this case by using Navier. Therefore, A' and B' are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \tag{9}$$

Where α and β are defined in expression (24).

For the FG plates, the stress–strain relation-ships for plane-stress state can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{10}$$

Where $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz})$ and $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the stress and strain components respectively. The C_{ij} expressions in terms of engineering constants are given below:

If $\varepsilon_z = 0$, then C_{ij} are the plane stress-reduced elastic constants

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \tag{11a}$$

$$C_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \tag{11b}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)}, \tag{11c}$$

If $\varepsilon_z \neq 0$, then C_{ij} are 3D elastic constants, given as follows

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)}{\nu}\lambda(z), \tag{12a}$$

$$C_{12} = C_{13} = C_{23} = \lambda(z), \tag{12b}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \mu(z) = \frac{E(z)}{2(1+\nu)}, \tag{12c}$$

With

$$\lambda(z) = \frac{\nu\, E(z)}{(1-2\nu)(1+\nu)}, \mu(z) = G(z) = \frac{E(z)}{2(1+\nu)} \tag{12d}$$

Where $\lambda(z)$ and $\mu(z)$ are the Lamé's coefficients.

The modulus $E(z)$ and the elastic coefficients $C_{ij}(z)$ vary through the thickness, according to Eq. (1).

2. Equation of motion

Hamilton's principle is used herein for the free vibration problem of FG plate. The principle can be stated in analytical form as (Attia *et al*. 2015, Larbi Chaht *et al.* 2015, Hachemi *et al*. 2017, Khetir *et al*. 2017, Klouche *et al*. 2017, Zidi *et al.* 2017, Kaci *et al*. 2018, Belabed *et al*. 2018, Zine *et al*. 2018)

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{13}$$

Where δU is the virtual strain energy and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}\right] dV \\ &= \int_A \Big[N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_z \delta \varepsilon_z^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ &\quad + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz} \delta \gamma_{yz}^s + S_{xz} \delta \gamma_{xz}^s\Big] dA = 0\end{aligned} \tag{14}$$

Where A is the top surface and the stress resultants $(N, M\ and\ S)$ are given by

$$\begin{Bmatrix} N_x, & N_y, & N_{xy}, \\ M_x^b, & M_y^b, & M_{xy}^b, \\ M_x^s, & M_y^s, & M_{xy}^s, \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{15a}$$

$$N_z = \int_{-h/2}^{h/2} \sigma_z g'(z) dz \tag{15b}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z) dz \tag{15c}$$

The variation of kinetic energy of the plate can be written as

$$\begin{aligned}\delta K &= \int_V \left[\dot{u}\, \delta \dot{u} + \dot{v}\, \delta \dot{v} + \dot{w}\, \delta \dot{w}\right] \rho(z)\, dV \\ &= \int_A \Big\{ I_0 \left[\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \dot{w}_0 \delta \dot{w}_0\right] \\ &- I_1 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_0}{\partial y} + \frac{\partial \dot{w}_0}{\partial y} \delta \dot{v}_0 \right) \\ &+ J_1 \left((k_1 A') \left(\dot{u}_0 \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \delta \dot{u}_0 \right) + (k_2 B') \left(\dot{v}_0 \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \delta \dot{v}_0 \right) \right) \\ &+ I_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) + K_2 \left((k_1 A')^2 \left(\frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} \right) + (k_2 B')^2 \left(\frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} \right) \right) \\ &- J_2 \left((k_1 A') \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} \right) + (k_2 B') \left(\frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) \right) \\ &+ j_1^s (\dot{w}_0 \delta \dot{\varphi} + \dot{\varphi} \delta \dot{w}_0) + K_2^s \dot{\varphi} \delta \dot{\varphi} \Big\} dA\end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; and $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, J_i^s K_i, K_i^s) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right) \rho(z) dz \tag{17a}$$

$$\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(f, z\, f, f^2\right) \rho(z) dz \tag{17b}$$

$$(J_1^s, K_2^s) = \int_{-h/2}^{h/2} (1, g(z)) g(z) \rho(z) dz \tag{17c}$$

Substituting Eq. (7) into Eq. (10) and the subsequent results into Eq. (15), the stress resultants can be expressed in terms of generalized displacements (u_0, v_0, w_0, θ, φ_z) as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23} \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23} \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \\ -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \\ k_1 \theta \\ k_2 \theta \\ (k_1 A' + k_2 B') \frac{\partial^2 \theta}{\partial x \partial y} \\ \varphi_z \end{Bmatrix} \tag{18a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2 B' \frac{\partial \theta}{\partial y} + \frac{\partial \varphi_z}{\partial y} \\ k_1 A' \frac{\partial \theta}{\partial x} + \frac{\partial \varphi_z}{\partial x} \end{Bmatrix} \tag{18b}$$

Where

$$\left(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij} \begin{pmatrix} 1, g^2(z), z, z^2, f(z), \\ z\, f(z), f^2(z) \end{pmatrix} dz \tag{19a}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^S, Z_{ij}\right) = \int_{-h/2}^{h/2} (1, z, f(z), g'(z)) g'(z) C_{ij} dz \tag{19b}$$

By employing the generalized displacement–strain expressions (Eqs. (6) and (7)) and stress–strain expressions (Eq. (10)), and integrating by parts and applying the fundamental lemma of variational calculus and collecting the coefficients of δu_0, δv_0, δw_0, $\delta \theta$, and $\delta \varphi_z$ in Eq. (13), the governing equations are determined as

$$
\begin{aligned}
&\delta u_0 : \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_0}{\partial x}+J_1k_1A'\frac{\partial \ddot{\theta}}{\partial x}\\
&\delta v_0 : \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial \ddot{w}_0}{\partial y}+J_1k_2B'\frac{\partial \ddot{\theta}}{\partial y}\\
&\delta w_0 : \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}=I_0\ddot{w}_0+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)\\
&\qquad +J_2\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+k_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)-I_2\nabla^2\ddot{w}_0+J_1^s\ddot{\varphi}\\
&\delta\theta : -k_1M_x^s-k_2M_y^s-(k_1A'+k_2B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+k_1A'\frac{\partial S_{xz}^s}{\partial x}+k_2B'\frac{\partial S_{yz}^s}{\partial y}\\
&\qquad =-J_1\left(k_1A'\frac{\partial \ddot{u}_0}{\partial x}+k_2B'\frac{\partial \ddot{v}_0}{\partial y}\right)+J_2\left(k_1A'\frac{\partial^2\ddot{w}_0}{\partial x^2}+k_2B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&\qquad -K_2\left((k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)\\
&\delta\varphi : -N_z+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}=J_1^s\ddot{w}_0+K_2^s\ddot{\varphi}
\end{aligned}
\tag{20}
$$

Substituting Eq. (18) into Eq. (20), the governing equations of the present quasi-3D hyperbolic shear deformation theory can be expressed in terms of displacements (u_0 , v_0 , w_0 , θ , φ_z) as

$$
\begin{aligned}
&A_{11}d_{11}u_0+A_{66}d_{22}u_0+(A_{12}+A_{66})d_{12}v_0+X_{13}d_1\varphi_z-B_{11}d_{111}w_0\\
&-(B_{12}+2B_{66})d_{122}w_0+\left(B_{66}^s(k_1A'+k_2B')\right)d_{122}\theta+\left(B_{11}^sk_1+B_{12}^sk_2\right)d_1\theta\\
&\qquad =I_0\ddot{u}_0-I_1d_1\ddot{w}_0+J_1k_1A'd_1\ddot{\theta}
\end{aligned}
\tag{21a}
$$

$$
\begin{aligned}
&A_{22}d_{22}v_0+A_{66}d_{11}v_0+(A_{12}+A_{66})d_{12}u_0+X_{23}d_2\varphi_z\\
&-B_{22}d_{222}w_0-(B_{12}+2B_{66})d_{112}w_0\\
&+\left(B_{66}^s(k_1A'+k_2B')\right)d_{112}\theta+\left(B_{22}^sk_2+B_{12}^sk_1\right)d_2\theta\\
&\qquad =I_0\ddot{v}_0-I_1d_2\ddot{w}_0+J_1k_2B'd_2\ddot{\theta}
\end{aligned}
\tag{21b}
$$

$$
\begin{aligned}
&B_{11}d_{111}u_0+(B_{12}+2B_{66})d_{122}u_0+(B_{12}+2B_{66})d_{112}v_0\\
&+B_{22}d_{222}v_0+Y_{13}d_{11}\varphi_z+Y_{23}d_{22}\varphi_z-D_{11}d_{1111}w_0\\
&-2(D_{12}+2D_{66})d_{1122}w_0-D_{22}d_{2222}w_0+\left(D_{11}^sk_1+D_{12}^sk_2\right)d_{11}\theta\\
&+2\left(D_{66}^s(k_1A'+k_2B')\right)d_{1122}\theta+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}\theta\\
&\qquad =I_0\ddot{w}_0+I_1(d_1\ddot{u}_0+d_2\ddot{v}_0)-J_2\left(k_1A'd_{11}\ddot{\theta}+k_2B'd_{22}\ddot{\theta}\right)\\
&\qquad -I_2(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0)+J_1^s\ddot{\varphi}
\end{aligned}
\tag{21c}
$$

$$
\begin{aligned}
&-\left(B_{11}^sk_1+B_{12}^sk_2\right)d_1u_0-\left(B_{66}^s(k_1A'+k_2B')\right)d_{122}u_0\\
&-\left(B_{66}^s(k_1A'+k_2B')\right)d_{112}v_0-\left(B_{12}^sk_1+B_{22}^sk_2\right)d_2v_0\\
&-k_1Y_{13}^s\varphi_z-k_2Y_{23}^s\varphi_z+\left(D_{11}^sk_1+D_{12}^sk_2\right)d_{11}w_0\\
&+2\left(D_{66}^s(k_1A'+k_2B')\right)d_{1122}w_0+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}w_0\\
&-H_{11}^sk_1^2\theta-H_{22}^sk_2^2\theta-2H_{12}^sk_1k_2\theta-\left((k_1A'+k_2B')^2H_{66}^s\right)d_{1122}\theta\\
&+A_{44}^s(k_2B')^2d_{22}\theta+A_{55}^s(k_1A')^2d_{11}\theta+A_{44}^s(k_2B')d_{22}\varphi_z+A_{55}^s(k_1A')d_{11}\varphi_z\\
&=-J_1\left(k_1A'd_1\ddot{u}_0+k_2B'd_2\ddot{v}_0\right)+J_2(k_1A'd_{11}\ddot{w}_0+k_2B'd_{22}\ddot{w}_0)\\
&\qquad +K_2((k_1A')^2d_{11}\ddot{\theta}+(k_2B')^2d_{22}\ddot{\theta})
\end{aligned}
\tag{21d}
$$

$$
\begin{aligned}
&-X_{13}d_1u_0-X_{23}d_2v_0-Z_{33}\varphi_z+Y_{13}d_{11}w_0+Y_{23}d_{22}w_0\\
&+(A_{44}^s-Y_{23}^s)(k_2B')d_{22}\theta+(A_{55}^s-Y_{13}^s)(k_1A')d_{11}\theta\\
&+A_{44}^sd_{22}\varphi_z+A_{55}^sd_{11}\varphi_z=J_1^s\ddot{w}_0+K_2^s\ddot{\varphi}
\end{aligned}
\tag{21e}
$$

Where d_{ij} , d_{ijl} and d_{ijlm} are the following differential operators

$$
d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\quad d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},
$$
$$
d_i=\frac{\partial}{\partial x_i},\ \ (i,j,l,m=1,2). \tag{22}
$$

3. Solution procedure

Consider a simply supported rectangular plate with length a and width b .The Navier solution procedure is employed to determine the analytical solutions for which the displacement variables satisfying the above boundary conditions and can be written in the following Fourier series

$$
\begin{Bmatrix} u_0\\ v_0\\ w_0\\ \theta\\ \phi_z \end{Bmatrix}=\begin{Bmatrix} U\,e^{i\omega t}\cos(\alpha\,x)\sin(\beta\,y)\\ V\,e^{i\omega t}\sin(\alpha\,x)\cos(\beta\,y)\\ W\,e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)\\ X\,e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)\\ \Phi\,e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y) \end{Bmatrix} \tag{23}
$$

Where (U , V , W , X , Φ) are unknown functions to be determined, ω is the frequency of free vibration of the plate, $\sqrt{i}=-1$ the imaginary unit.

Where

$$
\alpha=\pi/a\ \ \text{and}\ \beta=\pi/b \tag{24}
$$

Substituting Eq. (23) into Eq. (21), the following problem is obtained

$$
\left(\begin{bmatrix} S_{11}&S_{12}&S_{13}&S_{14}&S_{15}\\ S_{12}&S_{22}&S_{23}&S_{24}&S_{25}\\ S_{13}&S_{23}&S_{33}&S_{34}&S_{35}\\ S_{14}&S_{24}&S_{34}&S_{44}&S_{45}\\ S_{15}&S_{25}&S_{35}&S_{45}&S_{55} \end{bmatrix}-\omega^2\begin{bmatrix} m_{11}&m_{12}&m_{13}&m_{14}&m_{15}\\ m_{12}&m_{22}&m_{23}&m_{24}&m_{25}\\ m_{13}&m_{23}&m_{33}&m_{34}&m_{35}\\ m_{14}&m_{24}&m_{34}&m_{44}&m_{45}\\ m_{15}&m_{25}&m_{35}&m_{45}&m_{55} \end{bmatrix}\right)\begin{Bmatrix} U\\ V\\ W\\ X\\ \Phi \end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ 0\\ 0\\ 0 \end{Bmatrix} \tag{25}
$$

Where

$$
\begin{aligned}
&S_{11}=-(\alpha^2A_{11}+\beta^2A_{66})\\
&S_{12}=-\alpha\beta(A_{12}+A_{66})\\
&S_{13}=+\alpha^3B_{11}+\alpha\beta^2(B_{12}+2B_{66})\\
&S_{14}=+\alpha(k_1B_{11}^s+k_2B_{12}^s)-\alpha\beta^2B_{66}^s(k_1A'+k_2B')\\
&S_{15}=\alpha\,X_{13}\\
&S_{22}=-(\alpha^2A_{66}+\beta^2A_{22})\\
&S_{23}=+\alpha^2\beta(B_{12}+2B_{66})+\beta^3B_{22}\\
&S_{24}=+\beta(k_1B_{12}^s+k_2B_{22}^s)+\alpha^2\beta(k_1A'+k_2B')B_{66}^s
\end{aligned}
\tag{26}
$$

$$S_{25} = \beta X_{23}$$

$$S_{33} = -(\alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2(D_{12} + 2D_{66}))$$

$$S_{34} = -(\alpha^2 k_1 D_{11}^s + (k_2\alpha^2 + k_1\beta^2)D_{12}^s + \beta^2 k_2 D_{22}^s - 2\alpha^2\beta^2(k_1 A' + k_2 B')D_{66}^s)$$

$$S_{35} = -(\alpha^2 Y_{13} + \beta^2 Y_{23})$$

$$S_{44} = (-k_1^2 H_{11}^s + k_2^2 H_{22}^s + 2k_1 k_2 H_{12}^s + \alpha^2\beta^2(k_1 A' + k_2 B')^2 H_{66}^s + \alpha^2(k_1 A')^2 A_{55}^s + \beta^2(k_2 B')^2 A_{44}^s)$$

$$S_{45} = -(k_1 Y_{13}^s + k_2 Y_{23}^s + \alpha^2 k_1 A' A_{55}^s + \beta^2 k_2 B' A_{44}^s)$$

$$S_{55} = -(\alpha^2 A_{55}^s + \beta^2 A_{44}^s + Z_{33})$$

and

$$m_{11} = -I_0 \ , \quad m_{12} = 0, \quad m_{13} = \alpha I_1 \ ,$$

$$m_{14} = -J_1 k_1 A' \alpha \ , m_{15} = 0,$$

$$m_{22} = -I_0 \ , \quad m_{23} = \beta I_1 \ , \quad m_{24} = -k_2 B' \beta J_1 \ ,$$

$$m_{25} = 0,$$

$$m_{33} = -(I_0 + I_2(\alpha^2 + \beta^2)) \ ,$$

$$m_{34} = J_2\left(k_1 A' \alpha^2 + k_2 B' \beta^2\right), m_{35} = -J_1^s,$$

$$m_{44} = K_2\left((k_1 A')^2 \alpha^2 + (k_2 B')^2 \beta^2\right), m_{55} = -K_2^s,$$

4. Numerical results and discussion

In this part the accuracy of the present quasi 3D type HSDT with only five unknowns is evaluated, the free vibration analysis for simply supported functionally graded plate is studied. The theory is formulated in such way that the thickness stretching influence is considered. Various numerical examples for functionally graded and homogeneous plate with different values of the power indices, geometry ratios and aspects ratios for two types of functionally graded plate Al/Al_2O_3 and Al/ZrO_2.

Table 2 shows the fundamental frequencies parameters $\Omega = \omega h\sqrt{\rho_c/E_c}$ of the simply supported square aluminum/alumina plates in function of the thickness to length ratios $(a/h = 5, 10 \text{ and } 20)$ for different values of power law index p $(p = p_1 = p_2 = 0, 0.5, 1, 4, 10 \text{ and } \infty)$. The present results are compared with solutions based on the both FSDT and 2D HSDT developed by Matsunaga (2008), FSDT obtained by Zhao et al. (2009) and Hosseini-Hashemi *et al.* (2010) and RPT proposed by Benachour *et al*. (2011). The results shows that the present theory, which taking into account transverse normal deformation $(\varepsilon_z \neq 0)$, predicts the non-dimensional fundamental frequencies slightly large than the results obtained by other theories (Matsunaga 2008, Zhao *et al*. 2009, Hosseini-Hashemi *et al*. 2010, Benachour *et al*. 2011), which neglect the thickness stretching effect $(\varepsilon_z = 0)$, where the latter underestimate frequency parameter compared to the present theory.

Table 3 present the comparison of present frequency parameter $\Omega = \omega a^2\sqrt{\rho_c/E_c}/h$ with those given with FSDT (Zhao *et al*. 2009 and Hosseini-Hashemi *et al*. 2010) and with the theory based on HSDT developed by Benachour et al. (2011) for the both FGM $(FGM\ I: Al/Al_2O_3$ and FGM II$: Al/ZrO_2)$ squares plates with thickness ratio $(a/h = 10)$ for the different values of material index $(p = 0, 0.5, 1, 2, 5, 8, and\ 10)$, Results are in good agreement with the published of Benachour *et al.* (2011). As is indicated in the above section, the small difference noted between the results obtained by the present theory and Benachour *et al.* (2011) is due to the effect of thickness stretching which is omitted this latter (Benachour *et al.* 2011), It can be observed also that there is a remarkable difference between the non dimensional frequencies of Zhao *et al.* (2009) and those of high shear deformable plate theory (Benachour *et al.* 2011) and the present model. A reason of this difference is due to the fact that Zhao *et al.* (2009) utilized a numerical solution to determine the natural frequencies of the FG plates. It should be also noted that the difference between the present results and those reported by Zhao *et al.* (2009) and Hosseini-Hashemi *et al.* (2010) is due to the used theory in these later references which is FSDT. However, FSDT and HSDT (Benachour *et al*. 2011) neglect the thickness stretching effect. It can be concluded from Table 3 that the observed the difference between the tabulated results is due to the thickness stretching effect which is more pronounced in thick plates $(h/a = 0.1)$. Thus, the proposed theory is improved comparatively to the other theories FSDT (Zhao *et al*. 2009 and Hosseini-Hashemi *et al.* 2010) and HSDT (Benachour *et al.* 2011) because it considers the thickness stretching effect.

Table 4 shows a comparison of fundamental frequencies $\Omega = \omega h^2\sqrt{\rho_m/E_m}$ of simply supported Aluminum/Zirconia squares FG plates for $(h/a = 0.05, 0.1 \text{ and } 0.2)$ when $(p_1 = p_2 = 1)$ in the first part and $(p_1 = p_2 = p = 2, 3\ and\ 5)$ when $(h/a = 0.2)$ in the second part. The obtained results (quasi 3D) are compared with those given by the FSDT (Hosseini-Hashemi *et al.* 2010), 2D HSDT (Matsunaga 2008) and HSDT of (Pradyumna and Bandyopadhyay 2008 and Benachour *et al*. 2011). It can be seen that the present results are in good agreement with other theories. In addition, it should be indicated that the small difference observed is due to the effect of the transverse normal deformation included by the present theory.

Table 2 Comparison of fundamental frequency parameter $\Omega = \omega h\sqrt{\rho_c / E_c}$ simply supported Al / Al_2O_3 square plates

a/h	Theories	Power indices (p)					
		0	0.5	1	4	10	∞
20	Benachour *et al*. (2011)	0,01480	0,01254	0,01130	0,00980	0,00940	-
	Hosseini-Hashemi *et al.* (2010)	0,01480	0,01281	0,01150	0,01013	0,00963	-
	Zhao *et al.* (2009)	0,01464	0,01241	0,01118	0,00970	0,00931	-
	Present $(\varepsilon_z \neq 0)$	0.01485	0.01267	0.01151	0.01005	0.00953	-
10	Benachour *et al*. (2011)	0,05769	0,04900	0,04417	0,03804	0,03635	0,02936
	Matsunaga (2008)	0,05777	0,04917	0,04427	0,03811	0,03642	0,02933
	Hosseini-Hashemi *et al.* (2010)	0,05769	0,04920	0,04454	0,03825	0,03627	0,02936
	Zhao *et al.* (2009)	0,05673	0,04818	0,04346	0,03757	0,03591	-
	Matsunaga (2008)	0,06382	0,05429	0,04889	0,04230	0,04047	-
	Present $(\varepsilon_z \neq 0)$	0.05797	0.04953	0.04502	0.03901	0.03688	0.02950
5	Benachour *et al*. (2011)	0,2112	0,1806	0,1628	0,1375	0,1300	0,1075
	Matsunaga (2008)	0,2121	0,1819	0,1640	0,1383	0,1306	0,1077
	Hosseini-Hashemi *et al.* (2010)	0,2112	0,1806	0,1650	0,1371	0,1304	0,1075
	Zhao *et al.* (2009)	0,2055	0,1757	0,1587	0,1356	0,1284	-
	Matsunaga (2008)	0,2334	0,1997	0,1802	0,1543	0,1462	-
	Present $(\varepsilon_z \neq 0)$	0.2130	0.1832	0.1665	0.1413	0.1321	0.1084

Table 3 Comparison of fundamental frequency parameter ($\Omega = \omega a^2 \sqrt{\rho_c / E_c} / h$) for simply supported square FG plates when $h / a = 0.1$

FGMs	Theories	Power indices (p)						
		0	0.5	1	2	5	8	10
Al/Al_2O_3	Benachour *et al.* (2011)	5,7694	4,9000	4,4166	4,0057	3,7660	3,6831	3,6357
	Hosseini-Hashemi *et al*. (2010)	5,7693	4,9207	4,4545	4,0063	3,7837	3,6830	3,6277
	Zhao *et al.* (2009)	5,6763	4,8209	4,3474	3,9474	3,7218	3,6410	3,5923
	Present $(\varepsilon_z \neq 0)$	5.7967	4.9532	4.5015	4.1147	3.8524	3.7458	3.6883
Al/ZrO_2	Benachour *et al*. (2011)	5,7694	5,4380	5,3113	5,2923	5,3904	5,3950	5,3783
	Matsunaga (2008)	5,7769	-	5,3216	-	-	-	-
	Hosseini-Hashemi *et al*. (2010)	5,7693	5,3176	5,2532	5,3084	5,2940	5,2312	5,1893
	Zhao *et al.* (2009)	5,6763	5,1105	4,8713	4,6977	4,5549	4,4741	4,4323
	Present $(\varepsilon_z \neq 0)$	5.7967	5.4828	5.3761	5.3705	5.4520	5.4428	5.4208

Table 4 Comparison of fundamental frequency parameter $\Omega = \omega h^2 \sqrt{\rho_m / E_m}$ for simply supported square FG plates

Theories	$p=1$			$h/a=0.2$		
	$h/a=0.05$	$h/a=0.1$	$h/a=0.2$	$p=2$	$p=3$	$p=5$
Benachour *et al.* (2011)	0,0158	0,0618	0,2270	0,2249	0,2255	0,2266
Matsunaga (2008)	0,0158	0,0618	0,2285	0,2264	0,2270	0,2281
Pradyumna and Bandyopadhyay (2008)	0,0157	0,0613	0,2257	0,2237	0,2243	0,2253
Hosseini-Hashemi *et al.* (2010)	0,0158	0,0611	0,2270	0,2249	0,2254	0,2265
Pradyumna and Bandyopadhyay (2008)	0,0162	0,0633	0,2323	0,2325	0,2334	0,2334
Present $(\varepsilon_z \neq 0)$	0.0160	0.0626	0.2309	0.2293	0.2298	0.2302

The variation of the non dimensional fundamental frequency is shown in Figs. 2 and 3 for aluminum/alumina plate in function of geometry ratio (a/h). The results illustrate that the frequency parameter increase with increasing (a/h) ratio when $(a/h<20)$. It can be seen that the non-dimensional frequency found to be independent of the length thickness ratio (a/h) when $(a/h>20)$. As is indicated in the Figs. 2 and 3, the neglect of the transverse normal deformation underestimate the non-dimensional fundamental frequency.

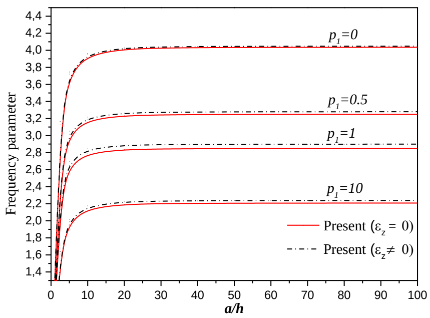

Fig. 2 Variation of frequency parameter of Al/Al_2O_3 plate with a/h ratio and p_1 index. ($a/b=0.5$ and $p_2=1$)

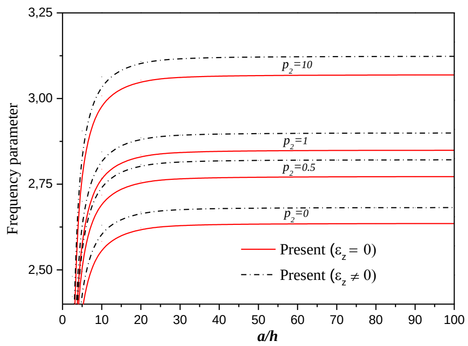

Fig. 3 Variation of frequency parameter of Al/Al_2O_3 plate with a/h ratio and p_2 index. ($a/b=0.5$ and $p_1=1$)

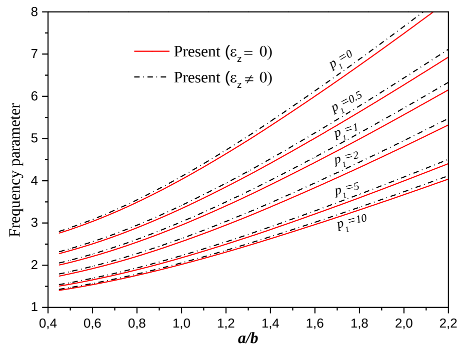

Fig. 4 Variation of frequency parameter of Al/Al_2O_3 plate with a/b ratio and p_1 index. ($a/h=2$ and $p_2=1$)

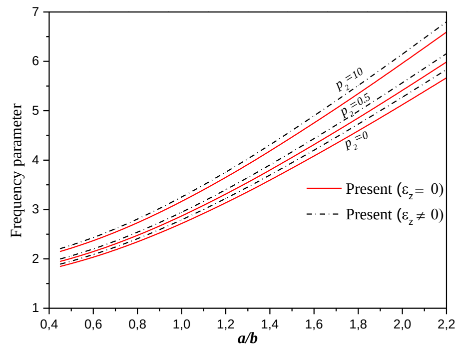

Fig. 5 Variation of frequency parameter of Al/Al_2O_3 plate with a/h ratio and p_1 index. ($a/h=2$ and $p_1=1$)

It is illustrate from Figs. 2 and 3 that the effect of p_1 is to make the plate stiffer when this gradient index is reduced. However, decreasing the second power law index p_2, make the plate soften as presented in Fig. 3.

The dynamic behavior of plate (Al/Al_2O_3) is shown in Figs. 4 and 5. It can be noted that the frequency parameter is in direct correlation relation with the aspect ratio (a/b).

For scale reason, only curves with $(p_2 = 0, 0.5\, and\ 10)$ are presented in Fig. 5. It should be noted that the non-dimensional frequency decreases with increasing the first power index p_1 (Fig. 4). However, it increases with increasing p_2 (Fig. 5).

5. Conclusions

This work present a free vibration analysis for simply supported functionally graded plate using an original quasi 3D HSDT with only five unknowns. The theory accounts for the stretching and shear deformation effects without requiring a shear correction factor. The equations of motion are derived by using the Hamilton's principle. These equations are solved via Navier's procedure. The results where compared with solutions of several theories such as FSDT and HSDT. In conclusion, it can be said that the present theory is not only accurate but also efficient in predicting fundamental frequency of functionally graded plates.

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