Smart Structures and Systems, *Vol. 22, No. 2 (2018) 151-159*
DOI: https://doi.org/10.12989/sss.2018.22.2.151

On determining a non-periodic preventive maintenance schedule using the failure rate threshold for a repairable system

Juhyun Lee[1a], Jihyun Park[1b] and Suneung Ahn[*2]

[1] *Department of Industrial and Management Engineering, Hanyang University, Seoul,15588, Republic of Korea*
[2] *Department of Industrial and Management Engineering, Hanyang University ERICA Campus, Ansan, 04763, Republic of Korea*

(Received May 8, 2017, Revised March 15, 2018, Accepted March 19, 2018)

Abstract. Maintenance activities are regarded as a key part of the repairable deteriorating system because they maintain the equipment in good condition. In practice, many maintenance policies are used in engineering fields to reduce unexpected failures and slow down the deterioration of the system. However, in traditional maintenance policies, maintenance activities have often been assumed to be performed at the same time interval, which may result in higher operational costs and more system failures. Thus, this study presents two non-periodic preventive maintenance (PM) policies for repairable deteriorating systems, employing the failure rate of the system as a conditional variable. In the proposed PM models, the failure rate of the system was restored via the failure rate reduction factors after imperfect PM activities. Operational costs were also considered, which increased along with the operating time of the system and the frequency of PM activities to reflect the deterioration process of the system. A numerical example was provided to illustrate the proposed PM policy. The results showed that PM activities performed at a low failure rate threshold slowed down the degradation of the system and thus extended the system lifetime. Moreover, when the operational cost was considered in the proposed maintenance scheme, the system replacement was more cost-effective than frequent PM activities in the severely degraded system.

Keywords: preventive maintenance; failure rate threshold; failure rate reduction factor; operational cost; minimal repair

1. Introduction

Most engineering systems and structures, such as aircraft, nuclear power plants, pipelines, and bridges, suffer increasing wear over time due to deterioration, which results in frequent failures. The performance deterioration can adversely affect the safety of these systems and reduce the operating life (Huang *et al.* 2016). In addition, once failures occur, they lead to high operating costs for the system due to the high repair costs. Hence, it is important to determine the timing of maintenance activities to keep the system in good condition. In practice, maintenance activities are considered a key part of the engineering system because they reduce unexpected failure and increase the functioning life of the system. Moreover, these maintenance activities restore the system to a good state by preventing degradation of system performance.

Maintenance activities are classified as corrective maintenance (CM) and preventive maintenance (PM). CM activities are unscheduled and performed when a system fails, and the system is restored from a failed state to a working state (Ahn and Kim 2011). PM activities are performed when system performance, such as time, reliability, failure rate, and availability, reaches a predetermined value, which reduces operational stress. Moreover, the performance of maintenance, such as the length of the PM interval, maintenance cost per unit time, and system lifetime, will vary depending on which condition variables are selected to perform PM. Therefore, determining the appropriate condition variable is important to establish optimal maintenance policies. However, establishing optimal maintenance policies may be difficult because they involve numerous uncertainties, such as status of the system, the level of system deterioration, maintenance costs, and repair effects.

Since the 1960s, research on maintenance policies has been widely performed to consider the numerous uncertainties. These maintenance policies can be classified into two types according to the criteria of performing PM activities (Ahmad and Kamaruddin 2012). One is a time-based PM policy in which PM activities are performed at a same time interval. Barlow and Hunter (1960) proposed a time-based periodic replacement policy with minimal repair. When failure occurs, minimal repair is immediately carried out and the system is restored to its prior state.

Many time-based preventive maintenance policies have been presented based on the concept of minimal repair. Nakagawa (1979) developed a periodic PM policy with the assumption that PM is imperfect. Canfield (1986) developed a hazard function that reflects the imperfect PM effect under a periodic maintenance policy. Park *et al.* (2000)

∗Corresponding author, Professor
E-mail: sunahn@hanyang.ac.kr
[a] Ph.D. student
E-mail: ljh812@hanyang.ac.kr
[b] Ph.D. student
E-mail: pjh3226@hanyang.ac.kr

http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

extended the model of Canfield (1986) and considered time-dependent CM costs. In addition, many periodic PM policies considering various decision variables, such as optimal frequency of PM actions, optimal recovery rate, and the optimal PM interval, have been presented (Jaturonnatee *et al*. 2006, Nakagawa and Mizutani 2009, Sheu and Chang 2009, Bouguerra *et al*. 2012, Toledo *et al.* 2016). However, periodic PM may not prevent frequently occurring breakdowns in the late period of system life because of system degradation, which also causes high operational costs (Huang *et al.* 2013).

The other is a condition-based PM policy (precisely non-periodic PM policy). In condition-based maintenance, the state of the condition variable is continuously monitored to determine the time to perform the PM activity before reaching the predicted state of failure, which improves system safety and reduces opportunity costs from severe failures (Roach 2009). It is important to determine the threshold value for these maintenance policies because performance of the maintenance policy depends on the threshold. Some researchers have studied the optimal condition-based PM model with a threshold value for system reliability (Zhao 2003, Zhou *et al.* 2009, Schutz and Rezg 2013) or failure rate (Dieulle *et al.* 2003, Yeh and Chang 2007).

Zhou *et al*. (2007) developed a reliability-centered predictive maintenance policy with a reliability threshold under situations where system reliability was continuously monitored. Park *et al.* (2008) developed the multi-objective optimum maintenance scenario for existing steel-girder bridges considering the minimizing life-cycle cost and the maximizing reliability. Doostparast *et al*. (2014) proposed an optimal PM schedule for coherent systems with consideration of a certain level of reliability. Lin *et al*. (2015) developed a non-periodic PM model that took into account various reliability constraints for complex systems. Lu *et al*. (2016) proposed a PM policy with a failure rate threshold for a single-machine manufacturing system by adding quality control. However, in traditional studies, establishing the cost-effectiveness of the maintenance policy may be difficult because the threshold value is predetermined. Therefore, this study focused on deriving the optimal threshold value that minimizes expected total costs.

The goal of this study was to develop the optimal non-periodic PM policy for a repairable system with a failure rate threshold and minimize the expected total cost per unit time. Whenever the failure rate of the system reaches a certain value, the imperfect PM activities are performed to restore the system. After imperfect PM, the failure rate of system is restored via failure rate reduction factors. If the system failure occurs before the imperfect PM, minimal repairs are carried out. This study not only considered the costs for replacement and related maintenance activities, but also considered operational costs. Actual engineering systems deteriorate with operating time and are hard to operate after a long period of use. When the state of such a system gets worse, replacing the system may be more economical rather than performing the maintenance activities, as operational costs are extremely high (Nguyen and Murthy 1981). Therefore, this study considered operational costs to develop a well-structured maintenance policy.

This study is organized as follows. Section 2 explains the assumptions to establish the non-periodic PM model, and derives the expected total cost function. Section 3 formulates the details of the proposed PM model and derives the optimal non-periodic PM models. It also shows the unique properties of the proposed PM model. Section 4 illustrates the optimal non-periodic PM model via a numerical example. Sensitivity analyses were conducted to investigate how changes in the parameters affect the proposed model. Finally, Section 5 discusses the conclusions of this research.

2. Sequential imperfect preventive maintenance model with failure rate threshold

2.1 Assumptions

With regard to how to model the imperfect PM, many studies have categorized maintenance activities into three types according to the restoration level of the system status (Nakagawa 1988, Wang 2002, Yanez, Joglar *et al*. 2002). Perfect repair restores the system to a state of "as good as new"; minimal repair restores the system to a state of "as bad as old"; and imperfect repair restores the system to a "better than old but worse than new" state, which is suitable for engineering systems because it represents the uncertainty of the deterioration process. Kijima (1989) developed a generalized renewal process to express all types of repair effects using the virtual age process. Martorell *et al.* (1999) developed a proportional age setback model by measuring the efficiency of PM activities using the age reduction factor. Lin *et al.* (2001) developed a hybrid model that combined the age reduction factor and the hazard-increasing factor. Doyen and Gaudoin (2004) proposed an arithmetic reduction of the age model (ARA) based on the age reduction factor and the arithmetic reduction of intensity model (ARI) based on the hazard reduction factor.

In this study, we consider the imperfect PM activity and situations in which minimal repairs are performed immediately after any failures. Moreover, this study does not consider perfect repair because it is not realistic to restore the system to a state of "as good as new" by any repairs. Hence, we assumed that the deterioration process of the system follows a non-homogeneous Poisson process (NHPP), as the above situations can be modeled via the NHPP. In addition, we assumed that the system described in this study undergoes the (N−1) imperfect PM activities and it is replaced at the Nth PM activity.

The detailed assumptions are as follows:

- The deterioration process of the system is assumed to be a NHPP that can be modeled by the Weibull power-law intensity given as $h(t)=\alpha\beta t^{\beta-1}$ where $t \geq 0$ is the elapsed time, $\alpha > 0$ is the scale parameter, and $\beta > 2$ is the deterioration parameter.
- When a system failure occurs before imperfect PM activities, minimal repairs are immediately carried out.

- All PM activities are performed when the system failure rate reaches a certain threshold value θ. After imperfect PM activities, the system is restored to a state of "better than old but worse than new" via the failure rate reduction factor.
- The duration for both minimal repair and imperfect PM is considered negligible.
- The system undergoes the (N–1) imperfect PM during its lifetime and is replaced at the Nth PM activity.
- The minimal repair cost C_m, the PM cost C_p, and the replacement cost C_r are assumed to be constants.

2.2 Imperfect PM activities and maintenance cost

In the present study, we considered the situation where the failure rate of the system can be restored via the failure reduction factor. This can be modeled via ARI model proposed by Doyen and Gaudoin (2004). The ARI model is appropriate to describe the effect of imperfect PM activities because of the assumption that the failure rate of a system is restored after performing imperfect PM activities.

The failure reduction factor ρ_i is used to describe the effect of imperfect PM in this study. If the first imperfect PM activity is performed at the time T_1, the failure rate of the system is restored from $h_0(T_1)$ to $\rho_1 h_0(T_1)$. The failure rate of the system is restored from $h_0(T_2)-(1-\rho_1)h_0(T_1)$ to $\rho_2(h_0(T_2)-(1-\rho_1)h_0(T_1))$ after the second imperfect PM activity. Based on these reduction processes, the reduction of the failure rate after the ith imperfect PM activity can be derived as

$$H_i = \rho_i\left(H_{i-1} + h_0(T_i) - h_0(T_{i-1})\right), \tag{1}$$

where $H_0 = 0$, i=1,2,…, N–1, and $0 < \rho_i < 1$. H_i indicates the reduction in the failure rate by the failure reduction factor after the ith PM activity. In addition, when $h_0(t)$ is the function of the failure rate for the initial state of system, the failure rate of the system after the ith imperfect PM activity can be determined as follows

$$\begin{aligned} h_i(t) &= h_0(t) - \left(h_0(T_i) - H_i\right) \\ &= h_0(t) - \sum_{j=1}^{i} h_0(T_j)(1-\rho_j)\left[\prod_{k=1}^{i}\rho_k \Big/ \prod_{k=1}^{j}\rho_k\right], \end{aligned} \tag{2}$$

where $t \in (T_i, T_{i+1})$, i=1,2,…,N. Fig. 1 shows the failure rate of the system based on the proposed PM model. In Fig. 1, θ represents the criterion for determining the schedule of imperfect PM activities. Therefore, the constraints for performing PM activities can be determined as

$$h_0(T_1) = h_1(T_2) = h_2(T_3) = \cdots = h_{N-1}(T_N) = \theta. \tag{3}$$

Solving Eq. (3) with respect to T_i, the operating time until the ith PM activities can be derived as

$$T_i = \left(i - \sum_{j=1}^{i-1}\rho_j\right)^{\frac{1}{\beta-1}} T_1. \tag{4}$$

The time interval at the ith PM cycle becomes

$$\begin{aligned} x_i &= T_i - T_{i-1} \\ &= \left[\left(i - \sum_{j=1}^{i-1}\rho_j\right)^{\frac{1}{\beta-1}} - \left(i - 1 - \sum_{j=1}^{i-2}\rho_j\right)^{\frac{1}{\beta-1}}\right] T_1, \end{aligned} \tag{5}$$

for i=1,2,…,N. Eqs. (4) and (5) show that the more PM activities are performed, the shorter the length the PM cycle becomes. In addition, they depend on the first time interval of the PM activities. Solving Eq. (3) with respect to T_1, the first time interval of the PM activities can be derived as

$$T_1 = \left(\frac{\theta}{\alpha\beta}\right)^{\frac{1}{\beta-1}}. \tag{6}$$

The system operating time until replacement can be calculated by the sum of the time interval up to the Nth PM cycle, which is given as

$$T_o = \sum_{i=1}^{N} x_i = T_N. \tag{7}$$

Because it is assumed that the deterioration process of the system follows the NHPP, the expected number of minimal repairs until replacement can be derived as

$$\begin{aligned} M_N &= \int_0^{T_1} h_0(t)dt + \int_{T_1}^{T_2} h_1(t)dt + \cdots + \int_{T_{N-1}}^{T_N} h_N(t)dt \\ &= H_0(T_N) + T_1 h_0(T_1) K(N), \end{aligned} \tag{8}$$

where

$$\begin{aligned} K(N) = \sum_{i=1}^{N-1}(1-\rho_i)\left(i - \sum\nolimits_{j=1}^{i}\rho_{j-1}\right)^{\frac{1}{\beta-1}} \\ -\left(N-1-\sum\nolimits_{k=1}^{N-1}\rho_k\right)\left(N - \sum\nolimits_{z=1}^{N-1}\rho_z\right)^{\frac{1}{\beta-1}}. \end{aligned} \tag{9}$$

Hence, the expected total cost per unit time can be constructed as

$$C_1(N,\theta) = \frac{C_m M_N + (N-1)C_p + C_r}{T_N}. \tag{10}$$

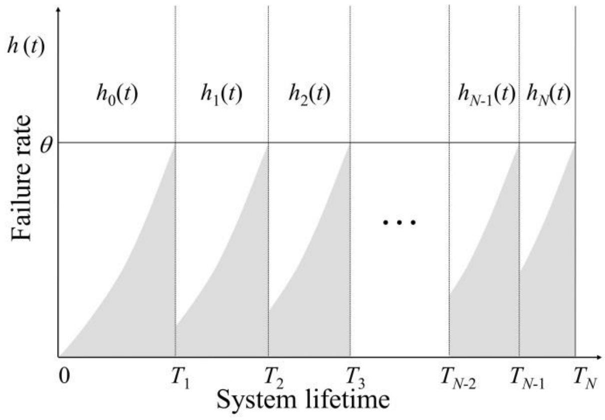

Fig. 1 The failure rate curve under the proposed model

Note that the expected total cost per unit time is composed of the cost for performing maintenance activities and the cost for replacing the system. It is assumed that these costs are constants and obtained from a field investigation. They may also be interpreted as the expected values for the corresponding random variables, so it is reasonable to assume that $max(C_m, C_p) < C_r$ (Lin *et al.* 2015).

2.3 Operational cost

The operational cost should be considered in order to illustrate deterioration process of the system and establish the realistic maintenance model. In practice, system failures increase as the operating time of the system increase, which leads to high operational costs. Hence, this study considered operational costs to illustrate the deterioration process of the system and construct the PM model. We used the operational cost proposed by Liao *et al.* (2010). This is composed of the fixed cost of operating, the cost that varies with the number of PM activities, and the cost that varies with the time interval of the PM cycles, which is given a

$$C_o(i,t) = c_{oc} + c_{op}i + c_{ot}t, \tag{11}$$

where c_{oc} denotes the fixed cost for the system operating, c_{op} denotes the relative changing cost depending on the number of PM activities, and c_{ot} denotes the relative changing cost depending on the time. The expected total cost per unit time considering the operational costs can be determined as follows

$$C_2(N,\theta) = C_1(N,\theta) + \frac{\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} C_o(j,t)dt}{T_N}, \tag{12}$$

where $C_1(N, \theta)$ represents the expected total cost per unit time.

3. Optimal preventive maintenance policies

Determining the optimal maintenance policy is important to decision makers. Hence, the decision maker should design an optimal PM policy that includes investigations on the mechanical condition, the state of the system, and the related costs of maintenance activities.

The basic assumptions are presented in section two, and two optimal non-periodic PM policies based on the failure rate threshold are established as follows

(1) *Policy* 1: This policy minimizes expected total cost per unit time, which is given in Eq. (10).

(2) *Policy* 2: This policy minimizes the expected total cost per unit time, which is given in Eq. (12) by using the operational cost in Eq. (11).

Note that the decision variables of the two policies are the optimal number of scheduled PM actions and the optimal failure rate threshold.

3.1 Optimization

Here, we tried to minimize $C_i(N, \theta)$ with (N, θ) for i = 1,2 to obtain the optimal proposed PM policies. The inequalities are

$$C_i(N-1,\theta) > C_i(N,\theta) \text{ and } C_i(N,\theta) \le C_i(N+1,\theta), \tag{13}$$

if and only if

$$Y_i(N-1,\theta) < C_r \text{ and } Y_i(N,\theta) \ge C_r, \tag{14}$$

for i = 1,2, and where Eq. (15) shown at the bottom of the page and where

$$S(N) = \left(N - \sum_{j=1}^{N-1}\rho_i\right)^{\frac{\beta}{\beta-1}} + \beta K(N). \tag{16}$$

Eq. (15) indicates a necessary condition to determine that Eq. (12) is a convex function with regard to N for the case where $\theta > 0$.

Through the following Proposition 1, we prove that there exists an optimal unique N^* (i.e., $\min_N C_i(N, \theta) = C_i(N^*, \theta)$) under certain conditions.

Proposition 1: If $\theta > 0$ and $Y_i(N, \theta)$ increases to infinity as $N \to \infty$ for i =1,2, there exists a finite and unique N^* that satisfies Eqs. (13)-(14).

The detailed proof Proposition 1 is in the Appendix.

For i = 1, 2, solving the first partial derivative of $C_i(N, \theta)$ with respect to θ to zero, respectively, $(dC_i(N, \theta) / d\theta) = 0$, if and only if

$$Z_i(\theta) = (N-1)C_p + C_r, \tag{17}$$

where

$$Y_i(N,\theta) = \begin{cases} \dfrac{C_m\alpha\left(\dfrac{\theta}{\alpha\beta}\right)^{\frac{\beta}{\beta-1}}\left(S(N+1)\left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}} - S(N)\left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}}\right) + C_p\left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}}}{\left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}} - \left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}}} - (N-1)C_p \\ \quad + (i-1)\left[c_{op}\sum_{i=1}^{N}\left(i-\sum_{j=0}^{i-1}\rho_j\right)\left(\dfrac{\theta}{\alpha\beta}\right)^{\frac{1}{\beta-1}} + \dfrac{c_{ot}}{2}\left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}}\left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}}\left(\dfrac{\theta}{\alpha\beta}\right)^{\frac{2}{\beta-1}}\right] & , N = 1,2,..., \\ 0 & , N = 0 \end{cases} \tag{15}$$

$$Z_i(\theta) = C_m S(N)\left(\frac{\theta}{\alpha\beta}\right)^{\frac{\beta}{\beta-1}} \alpha(\beta-1) + (i-1)\frac{1}{2}c_{ot}\left(N - \sum_{j=1}^{N-1}\rho_j\right)^{\frac{2}{\beta-1}}\left(\frac{\theta}{\alpha\beta}\right)^{\frac{2}{\beta-1}} \tag{18}$$

and for i = 1,2.

Additionally, let

$$\delta_i(\theta) = \alpha\beta C_m S(N)\left(\frac{\theta}{\alpha\beta}\right)^{\frac{\beta}{\beta-1}} + (i-1)c_{ot}\left(N - \sum_{j=1}^{N-1}\rho_j\right)^{\frac{2}{\beta-1}}\left(\frac{\theta}{\alpha\beta}\right)^{\frac{2}{\beta-1}}, \tag{19}$$

and

$$\gamma_i(\theta) = \alpha C_m S(N)\left(\frac{\theta}{\alpha\beta}\right)^{\frac{\beta}{\beta-1}} + (N-1)C_p + C_r + (i-1)\frac{1}{2}c_{ot}\left(N - \sum_{j=1}^{N-1}\rho_j\right)^{\frac{2}{\beta-1}}\left(\frac{\theta}{\alpha\beta}\right)^{\frac{2}{\beta-1}}, \tag{20}$$

for i = 1,2.

Through the following Proposition 2, we prove that there exists an optimal θ^* (i.e., $\min_\theta C_i(N, \theta) = C_i(N, \theta^*)$) under certain conditions.

Proposition 2: For i = 1,2, if $\lim_{\theta\to\infty}(\delta_i(\theta)/\gamma_i(\theta)) > 1$, then there exists a finite and unique θ^* that minimizes $C_i(N, \theta)$.

The detailed proof of Proposition 2 is in the Appendix.

Moreover, through the following Proposition 3, the optimal failure rate threshold of Policy 1 can be derived as Eq. (21).

Proposition 3: The optimal failure rate threshold of Policy 1 to perform each imperfect PM action is given as

$$\theta_1(N) = \alpha\beta\left(\frac{(N-1)C_p + C_r}{\alpha(\beta-1)C_m S(N)}\right)^{\frac{\beta-1}{\beta}}. \tag{21}$$

The detailed proof of Proposition 3 is in the Appendix.

The optimal failure rate threshold for Policy 2 can be obtained via the Newton-Rapson method. The optimal solutions for the proposed policies can be obtained via the following algorithm:

***Step* 1**: Set N = 1 and if Policy 1, go to step 1.1; otherwise go to step 1.2.

***Step* 1.1**: Compute the failure rate threshold $\theta_1(N)$ asgiven in Eq. (21), then go to Step 2.

***Step* 1.2**: Compute the failure rate threshold $\theta_2(N)$ as given in Eq. (17) using the Newton-Rapson method, then go to step 2.

***Step* 2**: If Policy 1, go to step 2.1; otherwise go to step 2.2.

***Step* 2.1**: Compute $C_1(N, \theta_1(N))$ as given in Eq. (10) and go to step 3.

***Step* 2.2**: Compute $C_2(N, \theta_2(N))$ as given in Eq. (12) and go to step 3.

***Step* 3**: Set N = N+1 and if *Policy* 1, go to step 3.1; otherwise go to step 3.2.

***Step* 3.1**: Compute $C_1(N+1, \theta_1(N+1))$ as given in Eq. (10) and go to step 3.

***Step* 3.2**: Compute $C_2(N+1, \theta_2(N+1))$ as given in Eq. (12) and go to step 3.

***Step* 4**: If Policy 1, go to step 4.1; otherwise go to step 4.2.

***Step* 4.1**: If $C_1(N, \theta_1(N)) < C_1(N+1, \theta_1(N+1))$ then attain the corresponding optimal solutions (N^*, θ_1^*), and stop the process, and go to step 5; otherwise go to step 3.

***Step* 4.2**: If $C_2(N, \theta_2(N)) < C_2(N+1, \theta_2(N+1))$ then attain the corresponding optimal solutions (N^*, θ_2^*), and stop the process, and go to step 5; otherwise go to step 3.

***Step* 5**: Compute the schedules of PM activities by using in Eq. (4).

4. Numerical example

A numerical example was used to illustrate the usefulness of the proposed PM policies. The parameters of the failure rate function were given as α = 1.8 and β = 2.6. The failure reduction factors were given as $\rho_i = i/(2i+1)$, i = 1,2,…, N–1, which increased as the number of imperfect PM activities increased (El-Ferik and Ben-Daya 2006, Lin and Huang 2010). The related costs for performing maintenance activities were considered as their ratios, which were given as C_m / C_p = 0.5 and C_r / C_p = 8. In addition, the costs associated with the operational cost were given as c_{ot} = 0.1, c_{op} = 0.05, and c_{ot} = 0.01. In this analysis, we considered that PM cost was set higher than the minimal repair cost.

For Policy 2, the tolerance of the Newton-Raphson method was set to $\varepsilon = 4.0\times10^{-6}$. The unit of operating time may be either years or thousands of hours. The results are summarized in Table 1. Fig. 2 represents the variation of the expected costs per unit time for each policy as the number of PM activities increase.

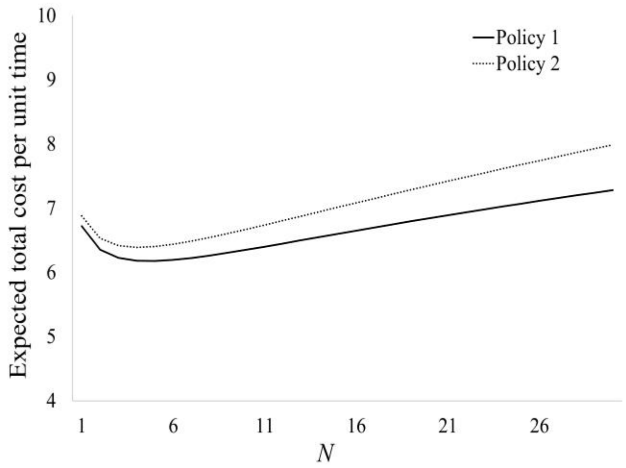

Fig. 2 The failure rate curve under the proposed model

Table 1 The optimal solutions and the system performance of optimal ones

	N^*	θ^*	$C(N^*, \theta^*)$	T_o	x_1	x_2	x_3	x_4	x_5
Policy 1	5	8.6752	6.1780	3.1564	1.4707	0.5532	0.4288	0.3700	0.3337
Policy 2	4	8.9938	6.3915	2.8870	1.5042	0.5658	0.4386	0.3785	

Table 2 Sensitivity analysis for the failure rate function parameters (given that C_m / C_p = 0.5, C_r / C_p = 8, c_{ot} = 0.1, c_{op} = 0.05, and c_{ot} = 0.01)

α	β	Policy 1				Policy 2			
		N^*	θ^*	$C(N^*, \theta^*)$	T_o	N^*	θ^*	$C(N^*, \theta^*)$	T_o
1.44	2.60	5	7.9618	5.6699	3.4392	4	8.2505	5.8844	3.1449
1.62	2.60	5	8.3307	5.9326	3.2869	4	8.6349	6.1466	3.0061
1.80	2.60	5	8.6752	6.1780	3.1564	4	8.9938	6.3915	2.8870
1.98	2.60	5	8.9992	6.4086	3.0425	4	9.3311	6.6219	2.7834
2.16	2.60	5	9.3054	6.6267	2.9426	4	9.6500	6.8397	2.6920
1.80	2.08	38	5.5165	3.9071	22.1820	11	5.7069	4.4622	8.4675
1.80	2.34	9	7.1286	5.2580	5.3138	7	7.3196	5.5517	4.6234
1.80	2.60	5	8.6752	6.1780	3.1564	4	8.9938	6.3915	2.8870
1.80	2.86	3	10.2895	6.8248	2.2530	3	10.2725	7.0114	2.2510
1.80	3.12	2	11.9420	7.2923	1.8163	2	11.9271	7.4621	1.8153

As shown in Table 1, the numerical example results for Policy 1 are better than optimal results obtained from Policy 2. However, it may not be reasonable to compare the numerical example results of the policies because the goals of the two policies are different. Therefore, we interpreted the numerical example results of both policies from an operational point of view. Table 1 shows that the length of the PM interval became longer when the operational cost was considered in the maintenance policy. This is because the optimal failure rate threshold of Policy 2 was higher than the optimal value obtained from Policy 1. Additionally, as a high failure rate threshold results in higher operational cost, the expected total cost per unit time increases. Hence, as shown in Table 1, the expected cost per unit time of Policy 2 was higher than that of Policy 1. In addition, because the operational cost increased as the operating time of the system increases, replacement of the system in Policy 2 was performed faster than that in Policy 1.

A sensitivity analysis was conducted for the situation in which α was adjusted from 1.44 to 2.16 and β was adjusted from 2.08 to 3.12 to determine the effect of the failure rate function parameters on the proposed PM policies. The results are summarized in Table 3. The results shows the change in the scale parameter had no effect on N^* of the two proposed PM policies. However, the length of the PM interval for both policies increased as α decreased; hence, it caused a lower than expected total cost per unit time and a longer operating time for the system. As the function of the optimal failure rate threshold is affected by the change in α, θ^* increased as α increased.

The change in β affects N^* of the two proposed PM policies unlike the results of the sensitivity analyses for α. In Policy 1, N^* increased from 2 to 38, as β decreased. In Policy 2, N^* increased from 2 to 11, as β decreased. In addition, the length of the PM interval increased as β decreased for both policies, resulting in a lower than expected maintenance cost and a longer operating time for the system. This is because β represents the level of system deterioration. When β is a low value, as deterioration of the system increases from the initial period, the failure rate of the system rapidly reaches the threshold value. Hence, the length of the PM interval becomes shorter, resulting in increased frequency of PM activities. However, although more PM activities are performed, a lower expected total cost per unit time is incurred because of the significantly longer PM interval length.

In contrast, if β is a high value, the failure rate of the system slowly reaches the threshold value rather than the case when β is a low value because the deterioration of the system rapidly increases during the wear-out period. Therefore, the length of the PM interval becomes longer, resulting in a low number of PM activities. However, the expected total cost per unit time becomes higher because of severe deterioration in the system, although the length of the PM interval is longer than when β is a low value. These results indicate that both policies are significantly sensitive to changes in β.

In addition, we conducted sensitivity analyses for related maintenance activity costs. The costs were given as C_m / C_p = 0.5 and C_r / C_p = 8. In this analysis, C_m increased from 0.40 to 0.60, C_r increased from 6.40 to 8.80, and C_p increased from 0.80 to 1.20. The results are summarized in Table 3. In the case when the cost for a minimal repair increased, the optimal number of PM activities does not change for both policies, but their imperfect PM activities were performed earlier. As shown in Table 3, the optimal failure rate threshold decreased as C_m increased. N^* decreased and θ^* increased, for both policies when the cost for PM increased because it is more economical to replace the system than to perform the PM activities.

Table 3 Sensitivity analysis for the costs related to maintenance activities (given that $\alpha = 1.8$, $\beta = 2.6$, $c_{ot} = 0.1$, $c_{op} = 0.05$, and $c_{ot} = 0.01$)

C_m	C_r	C_p	Policy 1				Policy 2			
			N^*	θ^*	$C(N^*, \theta^*)$	T_o	N^*	θ^*	$C(N^*, \theta^*)$	T_o
0.40	8.00	1.00	5	9.9522	5.6699	3.4392	4	10.3131	5.8844	3.1449
0.45	8.00	1.00	5	9.2564	5.9326	3.2869	4	9.5943	6.1466	3.0061
0.50	8.00	1.00	5	8.6752	6.1780	3.1564	4	8.9938	6.3915	2.8870
0.55	8.00	1.00	5	8.1811	6.4086	3.0428	4	8.4829	6.6219	2.7834
0.60	8.00	1.00	5	7.7545	6.6267	2.9426	4	8.0417	6.8397	2.6920
0.50	6.40	1.00	3	8.6057	5.5934	2.4404	3	8.5869	5.7844	2.4370
0.50	7.20	1.00	4	8.6055	5.9017	2.8085	3	9.0818	6.1069	2.5239
0.50	8.00	1.00	5	8.6752	6.1780	3.1564	4	8.9938	6.3915	2.8870
0.50	8.80	1.00	6	8.7734	6.4274	3.4890	5	9.0041	6.6555	3.2306
0.50	9.60	1.00	6	9.0830	6.6542	3.5654	6	9.0587	6.8999	3.5595
0.50	8.00	0.80	7	7.8651	5.8918	3.5303	6	8.0279	6.1421	3.3006
0.50	8.00	0.90	6	8.2552	6.0477	3.3587	5	8.4743	6.2771	3.1105
0.50	8.00	1.00	5	8.6752	6.1780	3.1564	4	8.9938	6.3915	2.8870
0.50	8.00	1.10	4	9.1653	6.2857	2.9213	4	9.1441	6.4949	2.9171
0.50	8.00	1.20	3	9.8144	6.3791	2.6493	3	9.7941	6.5711	2.6459

Additionally, the length of the PM interval in Policy 2 was shorter than that of Policy 1, as C_p increased. On the contrary, in the case when the cost for replacement increased, N^* increased although PM activities were performed with a longer PM interval. These results indicate that both policies are sensitive to changes in C_r and C_p. Therefore, the decision- maker should pay more attention to estimation of C_r and C_p.

5. Conclusions

In this study, two non-periodic PM policies were proposed for repairable deteriorating systems with a failure rate threshold. The function of expected total cost per unit time was determined, including the costs for a minimal repair, an imperfect PM activity, and a replacement. In the proposed PM policy, operational cost was considered a variable that changed based on the number of PM activities and the length of the PM interval to reflect the operational process of the system. The structural properties of the two proposed PM policies and their uniqueness were investigated and shown. An algorithm to determine the optimal threshold value of the failure rate and the optimal number of PM activities was provided. A numerical example was prepared and conducted to illustrate the usefulness of the two proposed PM policies. Sensitivity analyses revealed that the case when operating costs were not considered in the PM policy was sensitive to a change in β.

This study was devoted to obtaining the failure rate threshold that minimized the expected maintenance cost. Such a failure rate threshold can be a guideline at the planning stage to decide the level of system's failure rate. For example, if a decision maker prefers to lower maintenance cost rather than keep a system in good condition, the failure rate threshold obtained in this study may satisfy the decision maker. In addition, the predetermined PM schedule of this study can assist in arranging maintenance activities and reaching the inventory of the spare parts to near-zero. This study can be extended by health monitoring techniques. Nearly perfect information about the current state of the system can be provided using these techniques, which will allow more practical adjustments to pre-scheduled optimal PM policies.

Acknowledgments

This study was supported by the National Research Foundation of Korea(NRF) grant funded by the Korean government (MSIP) (No. NRF-2016R1A2B1008163).

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Appendix

A.1 Proof of proposition 1

The inequalities $C_i(N,\theta) \le C_i(N+1,\theta)$ and $C_i(N-1, \theta) > C_i(N, \theta)$ imply Eqs. (13) and (14). For respective i, to prove the proposition 1, we redefine Eq. (15) as

$$Y_1(N,\theta) = \frac{C_m \alpha (\theta/\alpha\beta)^{\beta/(\beta-1)}(B_N)}{A_N} + \frac{C_p(D_N)}{A_N} \tag{A1}$$

and

$$Y_2(N,\theta) = Y_1(N,\theta) + c_{op}E_N + \frac{c_{ot}}{2} F_N \left(\frac{\theta}{\alpha\beta}\right)^{\frac{2}{\beta-1}}, \tag{A2}$$

where

$$A_N = \left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}} - \left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}}, \tag{A3}$$

$$\begin{aligned} B_N = {} & S(N+1)\left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}} \\ & - S(N)\left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}}, \end{aligned} \tag{A4}$$

$$D_N = \left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}} - NA_N, \tag{A5}$$

$$E_N = \left(\frac{\theta}{\alpha\beta}\right)^{\frac{1}{\beta-1}} \sum_{i=1}^{N}(i-\sum_{j=0}^{i-1}\rho_j)^{\frac{1}{\beta-1}}, \tag{A6}$$

and

$$F_N = \left(N+1-\sum_{j=1}^{N}\rho_j\right)^{\frac{1}{\beta-1}}\left(N-\sum_{j=1}^{N-1}\rho_j\right)^{\frac{1}{\beta-1}}. \tag{A7}$$

For $i = 1$, B_N/A_N and D_N/A_N are increasing functions in N because A_N is a decreasing function in N; hence $Y_1(N, \theta)$ increases to infinity as $N \to \infty$. For $i = 2$, it is easily determined that $Y_2(N, \theta)$ increases as $N \to \infty$ because Eqs. (A6)-(A7) are the increasing function in N. Therefore, for each i, there exists a finite and unique N^* that satisfies Eqs. (13) and (14).

A.2 Proof of proposition 2

For each i, $(dC_i(N, \theta)/d\theta) = 0$ implies Eqs. (17) and (18). For $i = 1$, $J_1(\theta)$ is the increasing function in θ because $\lim_{\theta\to\infty} \delta_1(\theta)/\gamma_1(\theta) > 1$. For $i = 2$, $J_2(\theta)$ is the increasing function in θ because $\lim_{\theta\to\infty} \delta_2(\theta)/\gamma_2(\theta) > 1$. Therefore, for each i, because $J_i(\theta)$ is the increasing function in θ, there exists a finite and unique θ^* such that $J_i(0) < (N-1)C_p+C_r$ and $J_i(\infty) > (N-1)C_p+C_r$. In addition, θ^* satisfies Eqs. (17) and (18) and minimizes $C_i(N, \theta)$ for all $N \ge 1$.

A.3 Proof of proposition 3

For a given N and $i = 1$, a condition for θ to minimize Eq. (10) is $(dC_1(\theta|N)/d\theta) = 0$ because

$$\frac{d^2C_1(\theta\,|\,N)}{d^2\theta} > 0. \tag{A8}$$

Therefore, for the given N and $i = 1$, solving Eq. (17) with respect to θ, we obtain Eq. (21).