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Motion-based design of TMD for vibrating footbridges under uncertainty conditions

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Abstract. Tuned mass dampers (TMDs) are passive damping devices widely employed to mitigate the pedestrian-induced vibrations on footbridges. The TMD design must ensure an adequate performance during the overall life-cycle of the structure. Although the TMD is initially adjusted to match the natural frequency of the vibration mode which needs to be controlled, its design must further take into account the change of the modal parameters of the footbridge due to the modification of the operational and environmental conditions. For this purpose, a motion-based design optimization method is proposed and implemented herein, aimed at ensuring the adequate behavior of footbridges under uncertainty conditions. The uncertainty associated with the variation of such modal parameters is simulated by a probabilistic approach based on the results of previous research reported in literature. The pedestrian action is modelled according to the recommendations of the Synpex guidelines. A comparison among the TMD parameters obtained considering different design criteria, design requirements and uncertainty levels is performed. To illustrate the proposed approach, a benchmark footbridge is considered. Results show both which is the most adequate design criterion to control the pedestrian-induced vibrations on the footbridge and the influence of the design requirements and the uncertainty level in the final TMD design.

Keywords: footbridge; passive structural control; tuned mass damper; uncertainty; probabilistic approach; constrained single-objective optimization; genetic algorithms

1. Introduction

A tuned mass damper (TMD) is a passive control device which has been widely used to mitigate both wind-induced (Bortoluzzi *et al.* 2015) and pedestrian-induced vibrations on footbridges (Caetano *et al.* 2010, Dallard *et al.* 2001, Soong and Costantinou 1994, Van Nimmen *et al.* 2016). The TMD device is composed by three elements: (i) a mass, (ii) a viscous damper and (iii) a spring. The TMD mass is linked to the structure via the spring and the viscous damper (Fig.1a). The movement of the TMD mass and the structure must be adjusted via the tuning between the natural frequency of the damping device and the natural frequency of the vibration mode which needs to be controlled (Connor, 2003). Different methods have been proposed to obtain the most adequate TMD parameters depending on the nature of the excitation (Asami *et al*. 2002, Salvi and Rizzi 2016). Among the different proposals, the current trend (Casciati *et al*. 2014, Nagarajaiah and Jung 2014) for the design of TMD aimed at mitigating the walking pedestrian-induced vibrations on footbridges, proposes the use of some variant of the performance-based design method (Liang 2007). According to this method, the TMD design problem is transformed into a structural optimization problem (Arora 2007). The objective of this optimization problem is to obtain the TMD parameters (design variables) that, minimizing the cost of the TMD (objective function), ensure the compliance of the design requirements (constraints) established by the designer/owner/ manufacturer. As optimization method, a nature-inspired computational algorithm is usually employed, due to both the nonlinear relations between the constraints and the design variables and in order to guarantee that a global optimum solution is reached (Bekdas and Nigdeli 2011, Mirzai *et al*. 2017). Although these methods allow obtaining a more accurate estimation of the TMD parameters, the performance of TMDs installed on real footbridges is still not as good as it is expected (Caetano *et al*. 2010). The main factor, which causes the reduction of the TMD performance, is the variability observed in the different parameters that characterize the pedestrian-structure interaction model while the footbridge is in service (Casciati 2016, Venuti *et al*. 2016). In order to shed some light on this issue, the stochastic character of the pedestrian action was initially taken into account in the TMD design (Marano *et al.* 2010). Subsequently, the attention was focused on considering the uncertainty associated with the modification of the modal parameters of the structure (Jiménez-Alonso and Sáez 2017b, Lievens *et al.* 2016) induced by the changes of the operational and

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environmental conditions (Hu *et al.* 2013, Soria *et al.* 2016). Two approaches have been usually employed to simulate this uncertainty: (i) a probabilistic approach (Jiménez-Alonso and Sáez 2017b) and (ii) a fuzzy logic approach (Lievens *et al.* 2016, Ramezani *et al.* 2017). Regardless of the considered approach, the importance and necessity of considering this factor in the TMD design have been verified numerically (Jiménez-Alonso and Sáez 2017b, Lievens *et al.* 2016). Notwithstanding, a sensibility study is needed in order to analyse the performance of the different design criteria together with the effect of the different design requirements and the different uncertainty levels on the TMD parameters. The main objective of this paper is precisely to perform such sensitivity study under a probabilistic approach.

For this purpose, a robust optimization design method (Zang *et al.* 2005), based on a multi-objective optimization problem (Jiménez-Alonso and Sáez 2017b), is transformed herein into a constrained single-objective optimization problem with the aim to determine the most adequate TMD parameters under uncertainty conditions. The objective function is defined in terms of the mass of the TMD. As design variables, the TMD parameters are selected. Two types of constraints may be considered: (i) equality constraints, which allow considering different design criteria in order to constrain the form of the frequency response function of the structure (Asami *et al.* 2002); and (ii) inequality constraints, which allow guaranteeing the compliance of the design requirements established by the designer/owner/manufacturer (Butz *et al.* 2007, Setra 2006, Caetano *et al.* 2009). As optimization method, genetic algorithms are adopted.

The uncertainty of the modal parameters of the footbridge is simulated via a probabilistic approach (Engen *et al.* 2017). The modal parameters of the footbridge (namely, the natural frequencies and associated damping ratios) are assumed as random variables (Hu *et al.* 2013, Soria *et al.* 2016). The probabilistic distribution function of the maximum dynamic response of the structure is estimated via the application of the Monte Carlo simulation method (Bucher 2009). This probabilistic distribution function allows defining the inequality constraints of the problem in terms of the considered uncertainty level. This probabilistic approach presents two advantages when compared to the methods based on fuzzy logic (Lievens *et al.* 2016, Marano and Cuaranta 2009, Ramezani *et al.* 2017): (i) the method allows obtaining a direct relationship between the uncertainty level and the TMD parameters; and (ii) the probabilistic method is simpler and its use is more widespread among design and structural engineers.

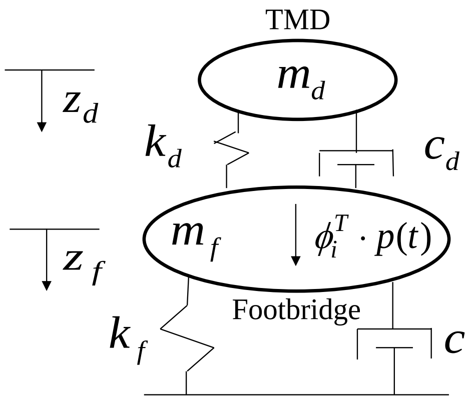
In order to validate numerically the proposed motion-based design optimization method and to perform the sensitivity analysis, a steel footbridge, as reported in the French standard (Setra 2006), is considered as benchmark. A finite element model of the footbridge is performed. Its modal parameters are obtained from a numerical modal analysis. As the first vertical vibration mode of the footbridge is prone to vibrate due to the walking pedestrian-induced excitation in vertical direction, the vibration serviceability limit state of the footbridge needs to be checked. The pedestrian action is defined according to the recommendations of the Synpex guidelines (Butz *et al.* 2007). Two design scenarios are established considering two different design requirements (comfort and fatigue) and a high pedestrian traffic. The uncertainty associated with the value of its first vertical natural frequency and associated damping ratio is simulated considering them as uncorrelated normal random variables. The probabilistic distribution of the maximum vertical acceleration of the footbridge is obtained numerically via a Monte Carlo simulation (Wang and Chen 2017). A convergence analysis is performed to establish the sample size of the simulation. Provided that even the average maximum vertical acceleration of the footbridge is greater than the allowable vertical acceleration for each design scenario, a TMD is installed at the mid-span of the footbridge to reduce the pedestrian-induced vibration. The motion-based design optimization method has been implemented to determine the TMD parameters under uncertainty conditions. Finally, a comparison among the TMD parameters obtained considering three different design criteria (the conventional H_∞ and H_2 criteria (Asami *et al.* 2002) and a new criterion, $\mathrm{H}_{\mathrm{opt}}$, proposed by the authors (Jiménez-Alonso and Sáez 2017b)), different design scenarios and different uncertainty levels is performed.

The main outcomes of this study are: (i) the H_{opt} criterion allows obtaining the most adequate TMD parameters to control the pedestrian-induced vibrations on footbridge under uncertainty conditions; (ii) an increase of the design requirements (comfort and fatigue requirements) originates a reduction of the height of the peaks of the frequency response function of the footbridge; and (iii) the proportional relationship existing between the uncertainty level and the TMD parameters.

This paper is organized as follows. The formulation of the TMD-footbridge interaction model is presented in the second section. Additionally, in the same section, the main assumptions of the interaction model are enumerated, and the pedestrian action, simulated according to the recommendations of the Synpex guidelines (Butz *et al.* 2007), is described. In the third section, a motion-based design optimization method is proposed and implemented to obtain the TMD parameters, that minimizing the weight of the damping device, ensure the compliance of the comfort and fatigue requirements of the TMD-footbridge system under uncertainty conditions. In the fourth section, the numerical validation of the proposed design method is performed. A benchmark footbridge is utilized for this purpose. A comparison between the TMD parameters obtained considering the different design criteria, the different design requirements and the different uncertainty levels is provided. Finally, some concluding remarks are drawn to close the paper.

2. TMD-footbridge interaction model under walking load

The TMD-footbridge interaction model may be formulated from the application of the principle of dynamic equilibrium to the balance of two sub-models (Clough and Penzien, 1993): (i) the TMD device and (ii) the footbridge. A scheme of the TMD-footbridge interaction model is illustrated in Fig. 1(a). The formulation of the TMD-footbridge interaction model, considered in this study, is based on the following assumptions: (i) there is a dominant vibration mode which needs to be controlled; (ii) the TMD is located at the position with the maximum vertical displacement (Fig. 1(b)) of the considered vibration mode; and (iii) the vibration modes are normalized to unity. The TMD is modelled as a single degree of freedom system characterized by three design parameters: the TMD mass, $\mathrm{m_d}$ [kg], the TMD damping, $\mathrm{c_d}$ [sN/m], and the TMD stiffness, $\mathrm{k_d}$ [N/m]. On the other hand, the dynamic response of the footbridge is simulated via its modal decomposition, so that its dynamic behavior is characterized by the modal mass of the considered vibration mode, $\mathrm{m_f}$ [kg], its modal damping, $\mathrm{c_f} = 4 \cdot \pi \cdot \mathrm{m_f} \cdot \mathrm{f_f} \cdot \zeta_\mathrm{f}$ [sN/m], and its modal stiffness, $\mathrm{k_f} = \mathrm{m_f} \cdot (2 \cdot \pi \cdot \mathrm{f_f})^2$ [N/m] (being $\mathrm{f_f}$. the natural frequency of the considered vibration mode and ζ_f its associated damping ratio). For the sake of simplicity, this interaction model is only developed here for the vertical direction, although it could be easily generalized to any other spatial directions.



(a)

(b)

Fig. 1 (a) TMD-footbridge interaction model under vertical walking load (Caetano *et al*. 2009) and (b) TMD installed under the deck of a footbridge (Butz *et al*. 2007)

Therefore, the following system of coupled equations follows from dynamic equilibrium

$$m_f \cdot \ddot{z}_f + c_f \cdot \dot{z}_f + c_d \cdot (\dot{z}_f - \dot{z}_d) + k_f \cdot z_f + k_d \cdot (z_f - z_d) = \emptyset_i^T \cdot p(t) \tag{1}$$

$$m_d \cdot \ddot{z}_d + c_d \cdot (\dot{z}_d - \dot{z}_f) + k_d \cdot (z_d - z_f) = 0 \tag{2}$$

where $\ddot{z}_f$, $\dot{z}_f$, z_f are the vertical acceleration, velocity and displacement of the considered vibration mode of the footbridge [m/s^2, m/s, m]; $\ddot{z}_d$, $\dot{z}_d$, z_d are the vertical acceleration, velocity and displacement of the TMD [m/s^2, m/s, m]; $z_r = z_d - z_f$ is the relative displacement between the TMD and the footbridge [m]; $\emptyset_i^T$ is the transpose of the considered vibration mode of the structure and $p(t)$ is the ground reaction force generated by pedestrian flows [N/m].

Eqs. (1) and (2) may be reorganized in matrix form to yield

$$\mathrm{M} \cdot \ddot{\mathrm{z}}(\mathrm{t}) + \mathrm{C} \cdot \dot{\mathrm{z}}(\mathrm{t}) + \mathrm{K} \cdot \mathrm{z}(\mathrm{t}) = \mathrm{F}(t) \tag{3}$$

where

$$\mathrm{M} = \begin{bmatrix} m_f & 0 \\ 0 & m_d \end{bmatrix} \ \mathrm{C} = \begin{bmatrix} c_f + c_d & -c_d \\ -c_d & c_d \end{bmatrix}$$
$$\mathrm{K} = \begin{bmatrix} k_f + k_d & -k_d \\ -k_d & k_d \end{bmatrix} \ \mathrm{F(t)} = \begin{bmatrix} \emptyset_i^T \cdot p(t) \\ 0 \end{bmatrix} \tag{4}$$

$$\ddot{\mathrm{z}}(\mathrm{t}) = \begin{bmatrix} \ddot{z}_f(t) \\ \ddot{z}_d(t) \end{bmatrix} \ \dot{\mathrm{z}}(\mathrm{t}) = \begin{bmatrix} \dot{z}_f(t) \\ \dot{z}_d(t) \end{bmatrix} \ \mathrm{z}(\mathrm{t}) = \begin{bmatrix} z_f(t) \\ z_d(t) \end{bmatrix} \tag{5}$$

The resulting set of governing equations may be solved either in frequency or in time domain. Although the resolution in frequency domain may be advantageous, since it allows reducing the simulation time for this particular case, its implementation may become complex when the proposed formulation is generalized to other scenarios as, for instance, if multiple TMDs are provided to control the dynamic response of the structure. For this reason, the above system of equations will be herein integrated in time domain using the β-Newmark method, with parameters $\beta = 1/4$ and $\gamma = 1/2$ in order to ensure an unconditionally stable solution (Clough and Penzien 1993).

The ground reaction force, $p(t)$, is simulated herein, according to the recommendations of the Synpex guidelines (Butz *et al.* 2007), as a distributed harmonic force that represents the equivalent effect of a walking pedestrian flow

$$p(t) = 280 \cdot \cos(2\pi \cdot f_s \cdot t) \cdot n' \cdot \psi / L_f \ \text{[N/m]} \tag{6}$$

where L_f is the length of the footbridge [m], $280 \cdot \cos(2\pi \cdot f_s \cdot t)$ is the harmonic force due to a single pedestrian, being 280 the vertical dynamic load factor of the ground reaction force and f_s the step frequency (it is assumed that it equals the natural frequency of the footbridge, f_f, (Butz *et al.* 2007)); n' is the equivalent number of pedestrians on the footbridge (i.e., number of synchronized pedestrians that originate the same response of the footbridge as n arbitrary pedestrians with a randomly distributed step frequency (Butz *et al.* 2007)), whose value may be determined from

$$n' = \begin{matrix} 10.8 \cdot \sqrt{\zeta_f \cdot n} \\ 1.85 \cdot \sqrt{n} \end{matrix} \quad \text{if} \quad \begin{matrix} d < 1.00\ P/m^2 \\ d \geq 1.00\ P/m^2 \end{matrix} \tag{7}$$

with d being the pedestrian density [d=Person/m^2], ζ_f being the damping ratio of the considered vibration mode. In Eq. (6) ψ is the reduction coefficient, which takes into account the probability that the footfall frequency approaches the natural frequency under consideration (Butz *et al.* 2007). Its value may be obtained in vertical direction from (for pedestrians walking)

$$\psi = \begin{cases} 0 \\ \frac{1}{0.45}(f_f - 1.25) \\ 1 \\ 1 - \frac{1}{0.20}(f_f - 2.10) \\ 0 \\ \frac{0.25}{0.90}(f_f - 2.50) \\ 0.25 \\ 0.25 - \frac{0.25}{0.40}(f_f - 4.20) \\ 0 \end{cases} \quad \text{if} \quad \begin{matrix} f_f < 1.25 \\ 1.25 \leq f_f < 1.70 \\ 1.70 \leq f_f < 2.10 \\ 2.10 \leq f_f < 2.30 \\ 2.30 \leq f_f < 2.50 \\ 2.50 \leq f_f < 3.40 \\ 3.40 \leq f_f < 4.20 \\ 4.20 \leq f_f < 4.60 \\ 4.60 \leq f_f \end{matrix} \ [\text{Hz}] \tag{8}$$

First, the different design scenarios must be established according to the expected pedestrian traffic class (Table 1), the location and the relevance of the footbridge. Subsequently, the design requirements established by the designer/owner/manufacturer of the structure must be checked.

Table 1 Traffic classes and pedestrian densities according to the Synpex guidelines (Butz *et al.* 2007)

Traffic Class	Density d	Description
TC1	15 P	Very weak traffic
TC2	<0.20 P/m^2	Comfortable and free walking
TC3	<0.50 P/m^2	Unrestricted walking, significantly dense traffic
TC4	<1.00 P/m^2	Uncomfortable situation, obstructed walking
TC5	<1.50 P/m^2	Unpleasant walking, very dense traffic

Table 2 Comfort classes according to the Synpex guidelines (Butz *et al.* 2007)

Comfort class	Degree	Allowable vertical acceleration ($\ddot{z}_{lim}$)
CL1	Maximum	<0.50 m/s^2
CL2	Medium	0.50-1.00 m/s^2
CL3	Minimum	1.00-2.50 m/s^2
CL4	Discomfort	>2.50 m/s^2

Two design requirements are usually considered: (i) a comfort requirement, which establishes the comfort class (Table 2) that the footbridge must satisfy and (ii) a fatigue requirement, which limits the maximum relative displacement between the TMD and the footbridge in order to avoid fatigue problems in the TMD (Caetano *et al.* 2009, Weber *et al.* 2006).

3. Motion-based design of TMD under uncertainty conditions

The design of the TMD parameters may be carried out via the application of a performance-based design optimization method (Connor 2003, Liang 2007). The main purpose of this design method is to obtain the TMD parameters that, minimizing the cost of the damping device, ensure the compliance of the design requirements established by the designer/owner/manufacturer. As the design requirements, which need to be accomplished, are defined in terms of both the accelerations of the structure, $\ddot{z}_f$, and the relative displacement, z_r, between the TMD and the structure, this design process may be understood as a motion-based design optimization method (Jiménez-Alonso and Sáez 2017a).

For the TMD design, the following equivalent parameters are normally defined: the mass ratio, $\mu = m_d/m_f$; the frequency ratio, $\delta_d = f_d/f_f$ (where f_d is the natural frequency of the TMD); and, ζ_d, the damping ratio of the TMD

$$m_d = \mu \cdot m_f \tag{9}$$

$$c_d = 4 \cdot m_d \cdot \pi \cdot \delta_d \cdot f_f \cdot \zeta_d \tag{10}$$

$$k_d = m_d \cdot (2 \cdot \pi \cdot \delta_d \cdot f_f)^2 \tag{11}$$

The proposed approach is based on the general scheme of a constrained single-objective optimization problem (Nocedal and Wright 1999), which may be expressed as

$$\text{Minimize } f(\theta_i) \tag{12}$$

$$\text{Subject to} \quad \begin{matrix} g_{eq,j}(\theta_i) = g^*_{eq,j} & j = 1,2,\dots,s \\ g_j(\theta_i) \leq g^*_j & j = 1,2,\dots,k \end{matrix} \tag{13}$$

$$\theta_i^l \leq \theta_i \leq \theta_i^u \; i = 1,2,\ldots,n_d \tag{14}$$

where $f(\theta_i)$ is the objective function, $g_{eq,j}(\theta_i)$ is the *jth* equality constraint, $g_{eq,j}^*$ is the threshold of the *jth* equality constraint, *s* is the number of equality constraints, $g_j(\theta)$ is the *jth* inequality constraint, g_j^* is the threshold of the *jth* inequality constraint, k is the number of inequality constraints, θ_i^l are the lower and θ_i^u the upper bounds of the design variables, θ_i, and n_d is the total number of design variables.

This general approach is particularized herein to take into account four key aspects: (i) the minimization of the weight of the TMD (objective function); (ii) the nonlinear relation between the TMD parameters (the design variables) and the constraints (optimization algorithm); (iii) the consideration of the uncertainties associated with the modification of the modal parameters of the structure (inequality constraints); and (iv) the selection of a design criterion to constrain the form of the frequency response function of the structure, $\mathrm{H_a}$ (in terms of accelerations), according to the considered design criterion.

The first aspect is taken into account via the definition of the objective function in term of the TMD mass ratio, μ, since the cost of the TMD depends mainly on this parameter (Arora 2007).

The second aspect may be addressed by selecting as optimization method a nature-inspired computational (NIC) algorithm. Concretely, genetic algorithms have been chosen here for this purpose (Koh and Perry 2010). It is well-known that gradient-based optimization methods may have difficulties to solve accurately nonlinear optimization problems, since they may present several local minimums. However, NIC algorithms have been widely applied to obtain the global minimum in different structural engineering applications (Wang *et al*. 2015). Among these NIC algorithms, genetic algorithms have proven their effectiveness to solve nonlinear optimization problems. Genetic algorithms (Nocedal and Wright 1999) minimize the considered objective function with the aim of obtaining a global solution, using for this purpose a cooperative population which is iteratively modified, according to several random rules (initialization, crossover, reproduction and mutation).

The third aspect, the consideration of the uncertainties of the modal parameters of the structure, is undertaken via a probabilistic approach (Jiménez-Alonso and Sáez 2017b). Herein, it is assumed that the change of the modal parameters of the footbridge is only originated by the modification of the operational and environmental conditions (Hu *et al.* 2012, Soria *et al.* 2016). Therefore, we assume that the numerical model used to simulate the dynamic response of the structure has been updated based on the experimental identification of the modal parameters of the structure (Zivanovic *et al.* 2007). According to this approach, both the natural frequency and the damping ratio of the vibration mode, which needs to be controlled, may be modelled as uncorrelated random variables which follow a predetermined probabilistic distribution function. In consequence, the response of the structure is equally governed by a probabilistic distribution function. In order to estimate this function, the Monte Carlo method may be used (Wang and Chen 2017, Bucher, 2009). For this purpose, a sample of the possible states of the structure (random sets of natural frequencies and associated damping ratios) must be generated. The sample size is determined via a convergence study, in order to guarantee that the sample size is large enough to ensure an accurate estimation of the dynamic response of the structure under a preselected significance level. The probabilistic distribution function of the response allows defining the inequality constraints under a probabilistic approach. In this manner, the TMD-footbridge system must meet the design requirements according to the confidence level established by the designer/owner/manufacturer.

Regarding the fourth aspect, a design criterion must be established in order to constrain the form of the frequency response function of the structure. Two types of design criteria may be considered depending on whether equality constraints are included or not in the optimization problem.

In the first case, which is adopted conventionally by the most recent design guidelines (Butz *et al*. 2007, Setra 2006), equality constraints are included in the formulation. These equality constraints are imposed to force the frequency response function of the structure, $\mathrm{H_a}$, to adopt a pre-established shape. As the form of the frequency response function, $\mathrm{H_a}$, is strongly conditioned by the frequency ratio, δ_d, and damping ratio, ζ_d, of the TMD, the equality constraints normally act directly on these two parameters, establishing their values. Among the different proposals, two criteria are usually considered to define the equality constraints (Asami *et al*. 2002): (i) the H_∞ criterion and (ii) the H_2 criterion. Under the H_∞ criterion (Den Hartog 1956), the form of the frequency response function of the structure is modified in order to minimize the dynamic response of the structure under a harmonic excitation; whilst, in accordance with the H_2 criterion (Crandall and Mark 1963), the frequency response function is adapted to reduce the dynamic response of the structure under a random excitation.

In this way, the formulation of the design optimization algorithm based on the H_∞ and H_2 criteria may be written in a unified fashion as

$$\begin{gathered}\text{find } (\mu,\ \delta_\mathrm{d},\ \zeta_\mathrm{d}),\text{ optimize } f(\mu)=\mu,\text{ subject to}\\ g_{eq,1}(\mu,\delta_\mathrm{d}) = \delta_\mathrm{d} - h_1\big(\mathrm{H_i}(\mu)\big) = 0\\ g_{eq,2}(\mu,\zeta_\mathrm{d}) = \zeta_\mathrm{d} - h_2\big(\mathrm{H_i}(\mu)\big) = 0\\ g_1(\mu,\delta_\mathrm{d},\zeta_\mathrm{d}) = \frac{\ddot{z}_{f,\alpha}}{\ddot{z}_{lim}} - 1 \leq 0\\ g_2(\mu,\delta_\mathrm{d},\zeta_\mathrm{d}) = \frac{z_{r,\alpha}}{z_{r.lim}} - 1 \leq 0\end{gathered} \tag{15}$$

where $h_1(\cdot)$ is a function (Table 3) that constrains the frequency ratio in terms of the considered design criterion, $\mathrm{H_i}(\mu)$ (being *i* a subscript that reflects the selected design criterion: H_∞ or H_2), $h_2(\cdot)$ is a function (Table 3) that constrains the damping ratio of the TMD in terms of the considered design criterion, $\ddot{z}_{f,\alpha}$ is the percentile α^{th} of the probability distribution function of the maximum vertical

Table 3 Functions ($h_1(\cdot)$ and $h_2(\cdot)$) to define the equality constraints in terms of the two conventional criteria, H_∞ and H_2 (Asami *et al*. 2002)

	H_∞	H_2
h_1	$\frac{1}{1+\mu}$	$\frac{1}{1+\mu}\sqrt{1+\frac{\mu}{2}}$
h_2	$\sqrt{\frac{3\mu}{8(1+\mu)}}$	$\sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$

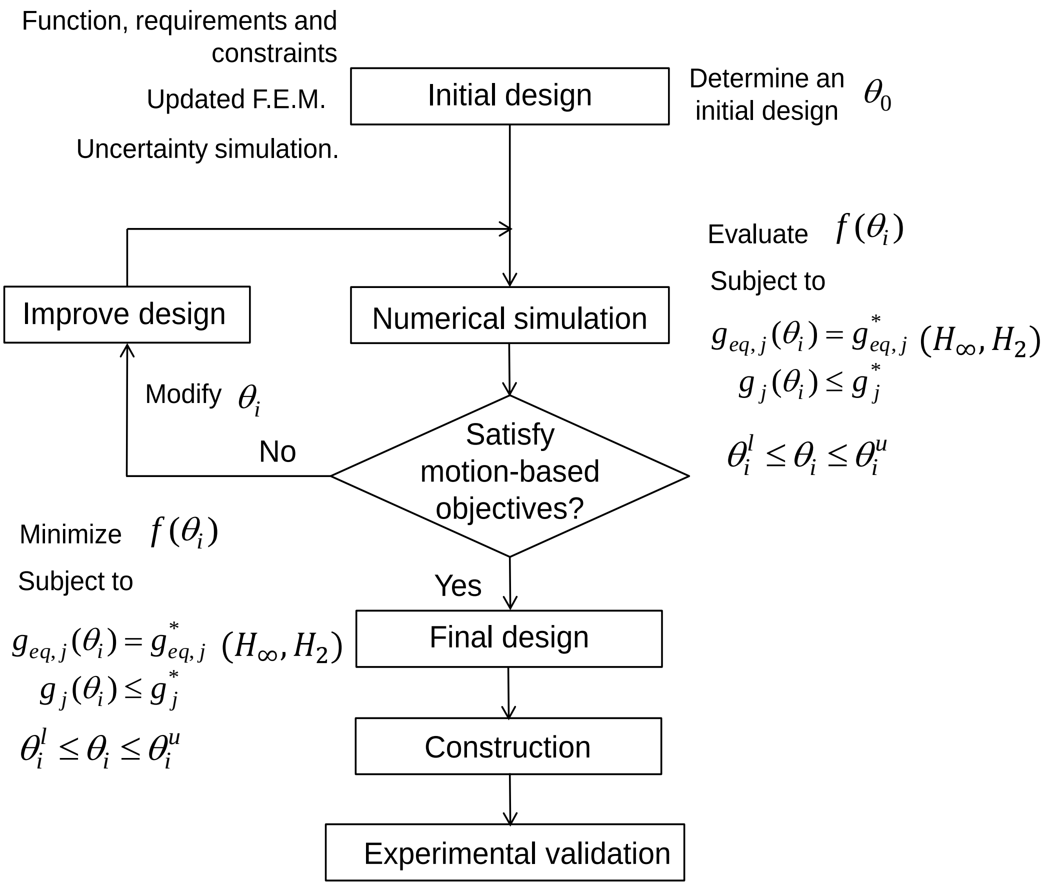


Fig. 2 Layout of the motion-based design optimization method under uncertainty conditions

accelerations of the structure; $\ddot{z}_{lim}$ is the allowable acceleration required by the designer/owner (Table 2), $z_{r,\alpha}$ is the percentile α^{th} of the probability distribution function of the maximum vertical relative displacements between the TMD and the footbridge and $z_{r,lim}$ is the allowable relative displacement between the TMD and the footbridge recommended by the manufacturer.

In the second case, only inequality constraints are applied to the optimization problem. In this manner, the form of the frequency response function, H_a, is freely adjusted to the particular conditions of each design problem. This second design method may be advantageous under the uncertainty conditions associated with the change of the modal parameters of the footbridge. Under these circumstances, the relaxation of the optimization problem may favour finding solutions (TMD designs) which better match the particular requirements of the problem, reducing, as a consequence, the cost of the damping device (Jiménez-Alonso and Sáez 2017b). Herein, this third design criterion is denominated as the H_{opt} criterion. Thus, the formulation of the motion-based design optimization algorithm may be expressed as

$$\begin{gathered}\text{find } (\mu,\ \delta_d,\ \zeta_d),\ \text{optimize } f(\mu) = \mu,\ \text{subject to} \\ g_1(\mu, \delta_d, \zeta_d) = \frac{\ddot{z}_{f,\alpha}}{\ddot{z}_{lim}} - 1 \leq 0 \\ g_2(\mu, \delta_d, \zeta_d) = \frac{z_{r,\alpha}}{z_{r.lim}} - 1 \leq 0\end{gathered} \tag{16}$$

The main steps of the proposed motion-based design optimization method are illustrated in Fig. 2. It is important to point out here that the objective of this paper is to

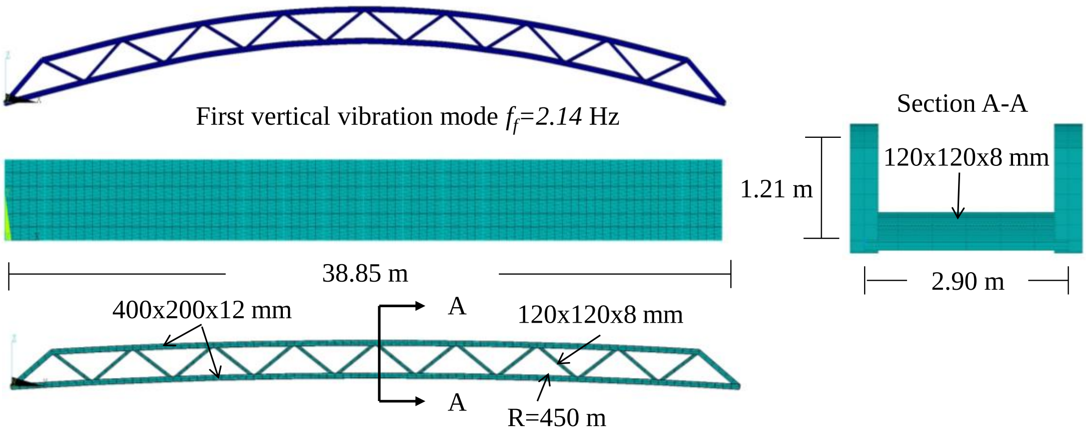


Fig. 3 Finite element model of the benchmark footbridge and first vertical vibration mode (Setra 2006)

analyze the performance of the approach based on the $\mathrm{H_{opt}}$ criterion, so that the formulations based on the $\mathrm{H_\infty}$ and $\mathrm{H_2}$ criteria are only included for comparison purposes.

Once the final design variables are obtained and after the TMD is built and installed on the footbridge, designers must carry out experimental tests to verify that all the comfort and fatigue requirements are fulfilled. For this purpose, two experimental tests are usually conducted (Caetano *et al.* 2010): (i) ambient vibration tests in order to estimate experimentally the modal properties of the footbridge and (ii) pedestrian tests in order to correlate numerically and experimentally the dynamic response of the footbridge under one or more pedestrians at controlled step frequencies.

4. Numerical validation.

4.1 Description of the benchmark footbridge and preliminary numerical modal analysis

For our purposes, a numerical footbridge, which is used by the French standard (Setra 2006) to illustrate the procedure to check the vibration serviceability limit will be adopted as benchmark. The footbridge is configured by a simple span of 38.85 m of length. The structural system is formed by two lateral steel Warren-trusses, braced transversally between their lower chords by strut elements and a reinforced concrete slab of 0.10 m of thickness. The trusses are curved with a vertical curvature radius of 450 m. The height of the trusses is 1.21 m and their lateral separation is 2.90 m. The width of the concrete slab is 2.50 m. The slab is supported by the strut elements configuring a composite steel-concrete section. Both the upper and lower chords of the trusses consist of rectangular hollow section 400x200x12 mm, and the diagonal and strut elements consist of rectangular hollow section 120x120x8 mm. Its supports are pinned at one side and vertically simple supported at the other side (Fig. 3).

The finite element model of the structure is built with both beam elements, BEAM188 (2 nodes per element, 6 d.o.f. in each node), and shell elements, SHELL181 (4 nodes per element, 6 d.o.f. in each node). The finite element (FE) package Ansys (Ansys 2017) has been used for this purpose. The footbridge is modelled using a mesh of 646 beam elements and 540 shell elements (Fig. 3). A linear behavior is considered for the constitutive law of the two materials, reinforced concrete and steel. The mechanical properties adopted are: (i) for the reinforced concrete, a Young's modulus, $E_c = 31000$ MPa, a Poisson's ratio, $\upsilon_c = 0.20$ and a density, $\rho_c = 2500$ kg/m^3; and (ii) for the steel, a Young's modulus, $E_s = 210000$ MPa, a Poisson's ratio, $\upsilon_s = 0.30$, and a density, $\rho_s = 7850$ kg/m^3. The structural damping ratio of the structure, ζ_f, is 0.6%, according to the recommendations of the Synpex guidelines (Butz *et al.* 2007).

The numerical modal parameters of the footbridge have been obtained via a numerical modal analysis based on the finite element model of the structure. For the purpose of this study, it is assumed that this numerical model reflects adequately the dynamic behavior of the structure, so it can be considered an "updated" finite element model of the structure. The natural frequency ($\mathrm{f_f} = 2.14$ Hz) of the first vertical vibration mode (Fig. 3) is within the range ($1.25 - 2.30$ Hz) that characterizes the pedestrian-structure interaction in vertical direction (Butz *et al.* 2007), so that it is necessary to check the vibration serviceability limit state of the structure. The recommendations of the Synpex guidelines (Butz *et al.* 2007) have been followed herein for this purpose. Additionally, the modal mass of the considered vibration mode, $\mathrm{m_f} = 34706$ kg, has been determined via the numerical modal analysis.

According to these guidelines, two design scenarios have been taken into account. For the determination of the ground reaction force, $p(t)$, a walking pedestrian density of 1 P/m^2 has been considered for both design scenarios (Table 1). In order to ensure the compliance of the comfort requirement established by the designer/owner, in each design scenario, the maximum vertical acceleration of the footbridge, $\ddot{z}_f$, must be lower than an allowable vertical acceleration, $\ddot{z}_{lim}$, which depends on the required comfort class (Table 2). In the first design scenario (D.S. I), the

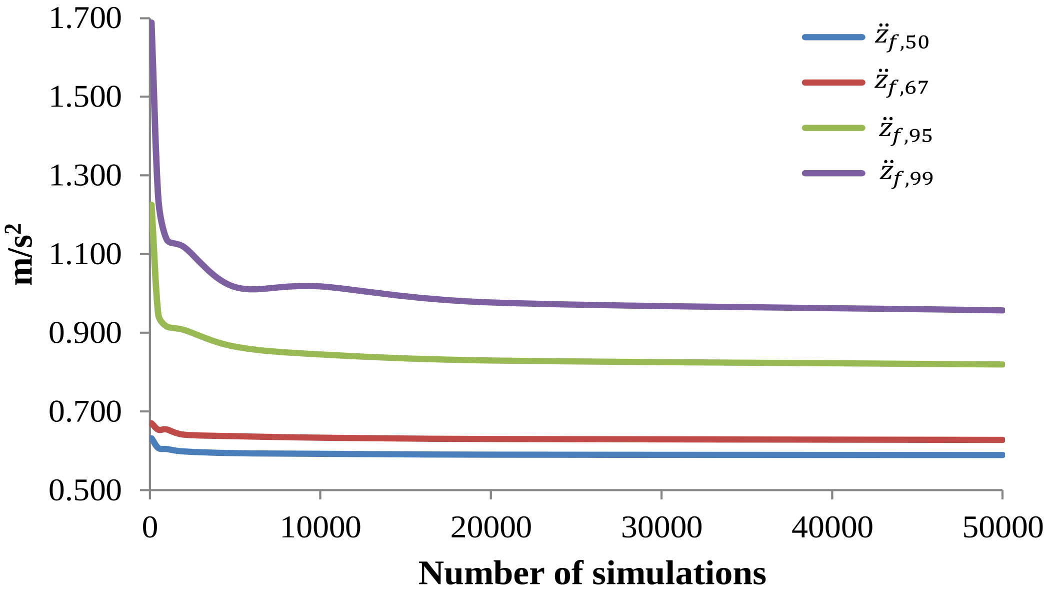


Fig. 4 Convergence analysis of the response of the TMD-footbridge interaction model under uncertainty conditions considering the H_∞ criterion (where $\ddot{z}_{f,\propto}$ is the percentile α^{th} of the probability distribution function of the maximum vertical acceleration of the footbridge)

allowable vertical acceleration is established in 1.00 m/s^2, whist, in the second design scenario (D.S. II), the allowable vertical acceleration is established in 0.50 m/s^2. If these requirements are not met, a TMD must be installed to reduce the amplitude of the pedestrian-induced vibrations below the mentioned threshold. The TMD design will be performed considering the proposed motion-based design optimization method. As requirement for the TMD design, the maximum vertical relative displacement between the TMD and the footbridge, z_r, is limited to 20 mm to avoid fatigue problems in the spring of the damping device (Caetano *et al.* 2009). Finally, the variation of the modal parameters of the structure due to the changes in the operational and environmental conditions is modelled via a probabilistic approach.

In summary, the main steps of the procedure to ensure the compliance of the vibration serviceability limit state are:

(i) to check if any vibration mode of the benchmark footbridge is prone to vibrate due to pedestrian-induced excitations.

(ii) if so, to determine the modal mass, modal damping and modal stiffness of the affected vibration mode and the ground reaction force for each considered design scenario.

(iii) to simulate numerically the variation of the modal parameters of the benchmark footbridge associated with the modification of the operational and environmental conditions.

(iv) to assess numerically the vibration serviceability limit state of the benchmark footbridge under uncertainty conditions.

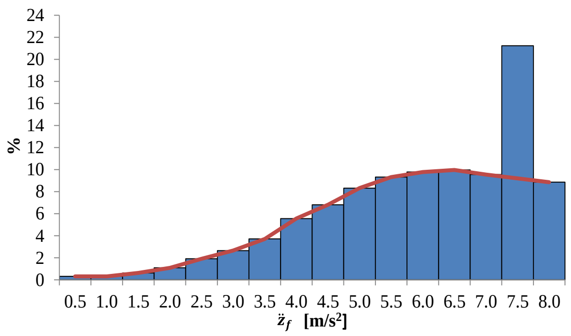
(v) if the comfort requirements are not met, the dynamic response of the structure is controlled by the implementation of a TMD.

4.2 Numerical simulation of the uncertainty: a probabilistic approach

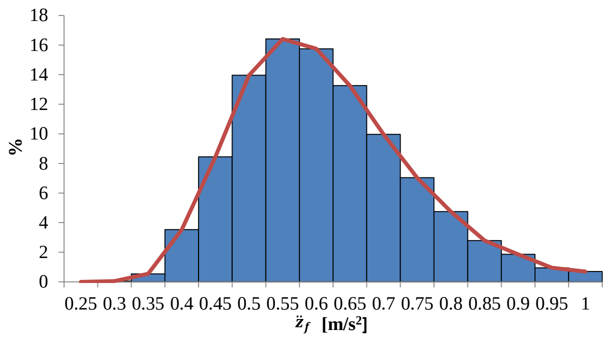
As it was previously mentioned, the uncertainty of the modal parameters of the structure is simulated via a probabilistic approach. Concretely, in this study, the first vertical natural frequency of the footbridge, f_f, and its associated damping ratio, ζ_f, are considered as uncorrelated random variables. According to the results provided by several researchers, these variables are assumed to follow a normal probabilistic distribution function with a range of variation of ±10% (Hu *et al*. 2012, Soria *et al.* 2016). In order to obtain the probabilistic distribution function of the maximum dynamic response of the structure, the Monte Carlo method has been used (Bucher 2009). In each simulation the first vertical natural frequency of the footbridge, f_f, its associated damping ratio, ζ_f, and –accordingly- the ground reaction force, $p(t)$, have been modified.

The selection of the sample size is one of the key points in the Monte Carlo simulations. For this aim, a convergence analysis has been performed. The mathematical package Matlab (2017) has been used for this purpose. The probabilistic distribution function of the maximum dynamic response of the structure (maximum vertical acceleration) has been estimated considering different sample sizes. The variation of this probabilistic distribution function in terms of the sample size (number of simulations that need to be calculated to define the probabilistic distribution function) has been analyzed for four characteristic percentiles (50^{th}, 67^{th}, 95^{th} and 99^{th}). The convergence analysis has been conducted for two systems: (i) the benchmark footbridge without TMD and (ii) the benchmark footbridge with TMD. For the second system, the TMD parameters have been determined, for this convergence analysis, considering the H_∞ criterion, the 99th percentile of the uncertainty level and the first design scenario (D.S.I). Fig. 4 illustrates the convergence analysis performed for the second system. The four considered percentiles α^{th} of the probabilistic distribution function of the vertical maximum acceleration of the footbridge, $\ddot{z}_{f,\alpha}$, are represented versus the number of simulations of each sample. As Fig. 4 shows the four curves stabilize their response if the sample size (number of simulations) is greater than 40000. This sample size ensures a significance level of 0.01 and an accuracy of 0.01 m/s^2 for the estimation of the probabilistic distribution function of the vertical accelerations of the structure and it has been considered as sample size for this study.

Finally, the histogram of a sample (with 40000 simulations) of the maximum vertical acceleration of the benchmark footbridge is shown in Fig. 5. Concretely, Fig. 5.a shows the histogram of a sample of the original benchmark footbridge without TMD, and Fig. 5.b shows its counter for the footbridge with TMD designed according to the H_∞ criterion. As it is shown in Fig. 5, the effect of the installation of the TMD is clear, reducing not only the maximum vertical acceleration of the footbridge but also modifying the shape of the histogram. For this particular case, the histogram of the benchmark footbridge without TMD is not well fitted by any conventional probabilistic function; whilst the histogram of the benchmark footbridge with TMD is fitted adequately by a log-normal probabilistic function. The TMD reduces not only the maximum vertical acceleration but also its range of variation.



(a)



(b)

Fig. 5 Histograms of the dynamic response (vertical accelerations) of the benchmark footbridge a) without TMD and b) with TMD considering the H_∞ criterion

Table 4 Percentile α^{th} of probabilistic distribution function of the maximum vertical acceleration of the benchmark footbridge without TMD

α^{th}	50	67	95	99
$\ddot{z}_{f,\propto}$ [m/s^2]	4.64	6.47	7.71	7.92

4.3 Numerical assessment of the vibration serviceability limit state of the footbridge under uncertainty conditions.

As indicated above in order to assess numerically the vibration serviceability of the footbridge, the probabilistic distribution function of the maximum vertical acceleration of the footbridge without TMD is estimated via a Monte Carlo simulation. The values corresponding to four percentiles (50th, 67th, 95th and 99th) of this probabilistic distribution function are shown in Table 4. Even the average value of the probabilistic distribution of the maximum vertical acceleration, $\ddot{z}_{f,50}$, is greater than the allowable acceleration, $\ddot{z}_{lim}$, established for each considered design scenario ($\ddot{z}_{lim} = 1.00$ m/s^2 for D.S. I and $\ddot{z}_{lim} = 0.50$ m/s^2 for D.S. II). For this reason, the dynamic response of the benchmark footbridge must be controlled via the installation of a TMD at the mid-span.

4.4 Motion-based design of the TMD parameters of the benchmark footbridge under uncertainty conditions.

In order to meet the comfort requirement of the footbridge, a TMD is installed at its mid-span. The TMD is designed based on the proposed motion-based design optimization method. The three previously mentioned design criteria (H_∞, H_2 and H_{opt}) have been compared in order to find out which is the one that better adapts to the particular variability of this problem. Additionally, the effect of the comfort requirements (different design scenario) and uncertainty level (different percentile) on the TMD parameters is analyzed.

A search domain for each TMD parameter has been established in order to guarantee that the TMD parameters obtained maintain an adequate engineering significance: (i) mass ratio, $\mu \in [0.00 - 0.07]$, (ii) frequency ratio, $\delta_d \in [0.85 - 1.00]$, and (iii) damping ratio, $\zeta_d \in [0.02 - 0.20]$.

As optimization algorithm, genetic algorithms have been used. A population of 100 design vector has been selected. As stop criteria two conditions have been included: (i) the maximum number of iterations has been set to 100 and (ii) the tolerance of the maximum variation of the objective function has been set to 10^{-5}.

Table 5 summarizes the detailed TMD parameters in terms of the uncertainty level for each design criteria and design scenario.

Finally, the frequency response function (in terms of accelerations), H_a, of the benchmark footbridge is obtained considering the three design criteria, the two design scenarios and the different confidence levels. Fig. 6 illustrates the different frequency response functions obtained by assuming that the modal parameters of the

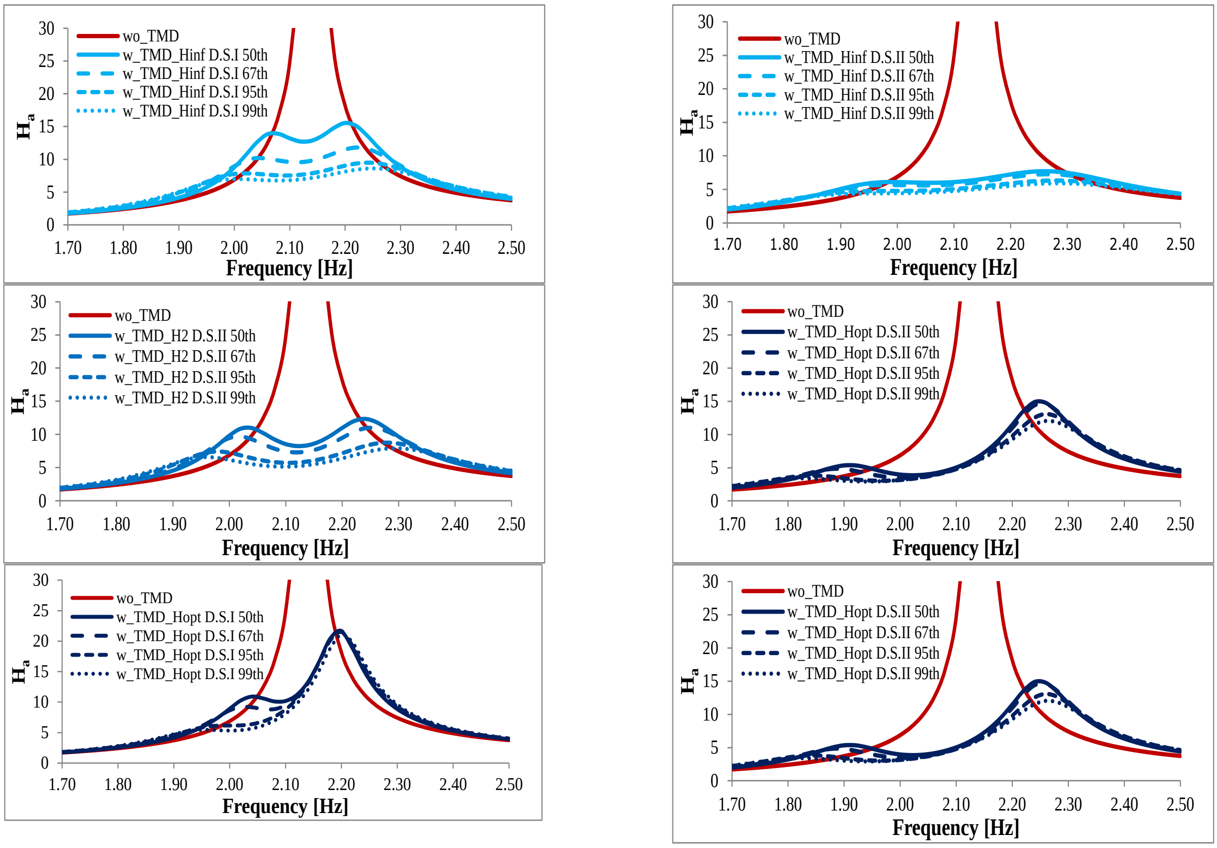


Fig. 6 Frequency Response Function, H_a, for the different design criteria, uncertainty levels and design scenarios (f_f =2.14 Hz and $\zeta_f = 0.6$ %)

footbridge adopt their average values (f_f =2.14 Hz and $\zeta_f = 0.6\%$) and varying the TMD parameters in terms of the design criteria, uncertainty levels and design scenarios. As it is clearly shown in Fig. 6, the motion-based design optimization method modifies the shape of the frequency response function in order to meet the different constraints of the problem.

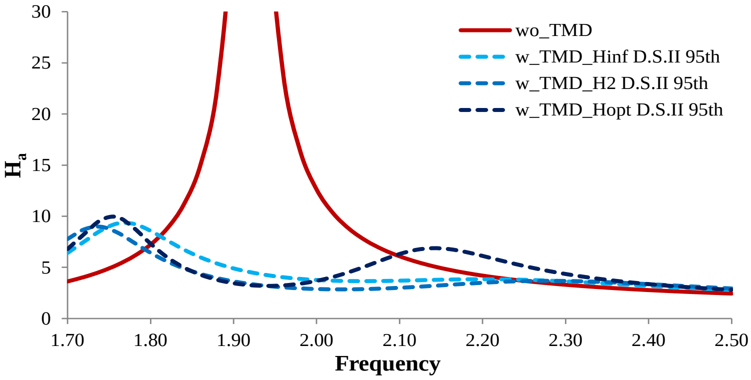
Some relevant conclusions may be obtained from the analysis of the Fig. 6: (i) the increase of the comfort requirement leads to a reduction in the peaks of the frequency response function and to an increase in the distance between the original natural frequency of the structure and the new two natural frequencies generated by the installation of the TMD; (ii) the increase of the uncertainty level generates the same effect but at lower scale; and (iii) the shape of the frequency response function depends strongly on the considered design criteria.

Finally, a comparative study is performed to analyze the effect of the variation of the first vertical natural frequency of the footbridge on the form of its frequency response function, when considering the different design criteria. The frequency response function of the structure, with and without TMD, is obtained considering three characteristic values of its first vertical natural frequency (a minimum value, $f_f = 1.92$ Hz; an average value, $f_f = 2.14$ Hz; and a maximum value, $f_f = 2.35$ Hz), a fixed uncertainty level (95^{th} percentile) and the design scenario II.

Fig. 7 illustrates the comparison among the frequency response functions (in terms of accelerations) obtained for the different cases. As Fig. 7 illustrates, the form of the frequency response function varies significantly among the three considered design criteria. This variation is especially remarkable for the H_{opt} criterion, and it is originated by the different frequency ratio, δ_d, obtained for the TMD design according to the different design criteria. The H_{opt} criterion takes advantage of the fact that the ground reaction force, $p(t)$, is reduced significantly when the vertical natural frequency of the footbridge becomes greater than 2.10 Hz. According to this criterion, the TMD is tuned around the lower bound of the variation range of the natural frequency; so that, when the natural frequency of the structure varies (increasing its value), the frequency response function of the structure presents a peak located in the range of frequencies where the ground reaction force is minimum. On the other hand, the conventional criteria (H_∞ and H_2) lead to a more averaged value of the frequency ratio, which implies that if the TMD is de-tuned, the frequency response function may present a peak located in the range of frequencies where the ground reaction force is maximum, making necessary an increase in the damping ratio of the TMD in order to guarantee an adequate dynamic behavior of the structure.

Table 5 TMD parameters versus the uncertainty level for each design criteria and design scenario

		μ [%]			δ_d [-]			ζ_d [%]		
D.S.	th	H_∞	H_2	H_{opt}	H_∞	H_2	H_{opt}	H_∞	H_2	H_{opt}
I	50	0.65	1.20	0.61	0.994	0.991	0.976	4.92	5.45	3.86
	67	1.28	1.57	0.73	0.987	0.988	0.968	6.88	6.23	4.19
	95	2.16	2.67	1.14	0.979	0.980	0.943	8.90	8.09	5.22
	99	2.72	3.36	1.38	0.974	0.976	0.935	9.96	9.05	5.26
II	50	3.52	4.46	2.58	0.966	0.968	0.938	11.29	10.39	5.55
	67	4.05	5.15	2.99	0.961	0.963	0.929	12.08	11.13	5.78
	95	5.58	7.15	4.02	0.947	0.950	0.909	14.08	13.03	6.91
	99	6.52	8.39	4.61	0.939	0.942	0.899	15.15	14.05	7.74



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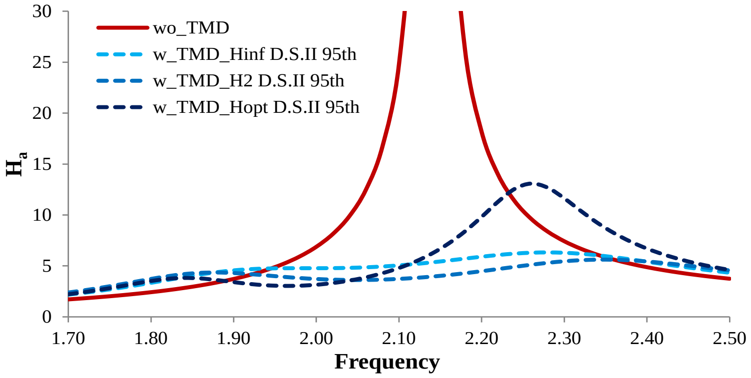

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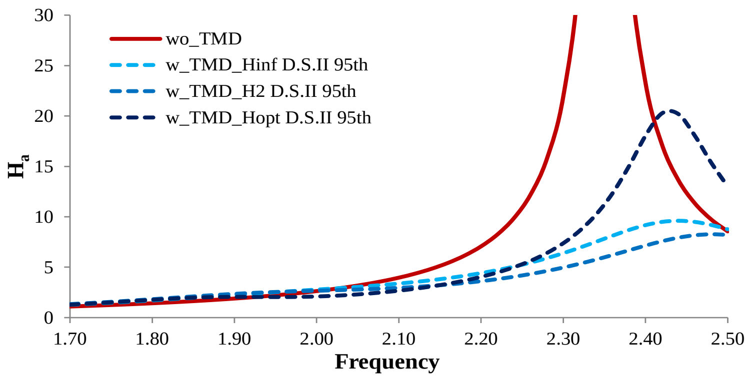


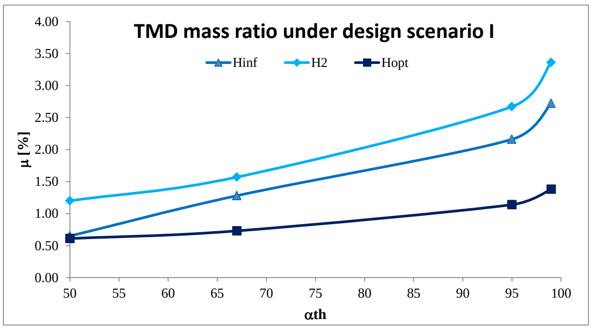
(c)

Fig. 7 Comparison of the Frequency Response Function, H_a, for different values of the first vertical natural frequency of the structure, f_f, considering the different design criteria, a fixed uncertainty level (95th percentile) and the design scenario II. (a) $f_f = 1.92$ Hz, (b) $f_f = 2.14$ Hz and (c) $f_f = 2.35$ Hz

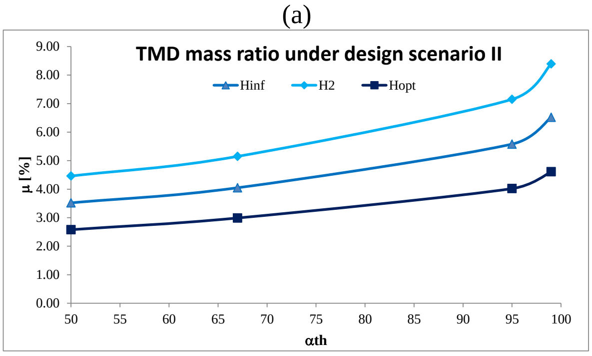
4.5 Discussion of the results.

Following the previous comparative study, the differences among the performance of the different design criteria under the different requirements and uncertainty conditions are next highlighted. In particular as the H_{opt} criterion is concerned, Fig. 8 illustrates the variation of the TMD mass ratio for the three design criteria in terms of the uncertainty level. The two design scenarios have been depicted separately (Figs. 8(a) and 8(b)). From the analysis of Fig. 8, it may be concluded that the H_{opt} criterion allows obtaining the best TMD parameters. This criterion reduces the value of the objective function, the TMD mass ratio, μ, with respect to the other two criteria. This reduction is strongly influenced by the comfort requirements, reaching an averaged reduction coefficient of 45.56% for the first design scenario and 35.81% for the second. The other TMD parameters follow the same trend.

Finally, the effect of the uncertainty in the TMD parameters is clear. An increase of the uncertainty level leads to an increase of the TMD parameters. According to Fig. 8 the relationship between the TMD mass ratio, μ, and the uncertainty is quasi-linear in the range from the 50^{th} to the 95^{th} percentile. However, if the uncertainty level is greater, the increase of the TMD parameters follows an exponential function. In consequence, it may be an adequate criterion to consider this value as the upper limit of the uncertainty in real applications, in order to reach a balance between the cost of the TMD and the safety.



(a)



(b)

Fig. 8 TMD mass ratio, μ, versus de uncertainty level for each design criterion and design scenario (a) D.S. I and (b) D.S. II)

5. Conclusions

The main contributions of this paper are: (i) the proposal and implementation of a motion-based design optimization method to determine the best TMD parameters which allow mitigating the pedestrian-induced vibrations on footbridges under uncertainty conditions and (ii) to analyze the influence of both the design requirements and the uncertainty level in the TMD parameters.

The motion-based optimization design method is formulated as a constrained single-objective optimization problem, where the objective function is defined in terms of the TMD mass ratio, μ, the design variables are the equivalent TMD parameters, (μ, δ_d, ζ_d), and equality and inequality constraints are included to constrain the form of the frequency response function of the structure and to guarantee the compliance of the comfort and fatigue requirements of the TMD-footbridge system under uncertainty conditions. Three design criteria have been considered: (i) the H_∞ criterion, (ii) the H_2 criterion and (iii) the H_{opt} criterion. As optimization method, genetic algorithms have been used. The uncertainty associated with the variation of the modal parameters of the structure has been simulated herein by a probabilistic approach, assuming that the modal parameters of the structure are random variables. The Monte Carlos simulation method has been used to estimate numerically the probabilistic distribution function of the maximum dynamic response of the structure under pedestrian action.

As benchmark, a footbridge, which is prone to vibrate due to walking pedestrian-induced excitation in vertical direction, has been chosen. A TMD has been installed at the mid-span of the structure to mitigate the pedestrian-induced vibrations. The pedestrian load is defined as an equivalent harmonic load according to the recommendations of Synpex guidelines. Two design scenarios have been considered in terms of the comfort requirements. The TMD parameters have been obtained via the implementation of the motion-based design optimization method considering the uncertainty conditions. A sensibility study has been performed to analyze the influence of the three mentioned design criteria, the design requirements and the uncertainty level on the TMD parameters.

This study shows that the H_{opt} criterion provides the best cost-effective TMD parameters under uncertainty conditions. The frequency response function obtained using this criterion adapts better to the pedestrian-structure interaction problem. Additionally, this study shows how the TMD mass increases quasi-linearly with the uncertainty level and illustrates the effect of the design requirements on the peak of the frequency response function of the footbridge.

Nevertheless, further studies are needed, both to better characterize the probabilistic distribution function that define the change of the modal parameters of the structure during its overall life-cycle and to assess experimentally the performance of the TMD designed according to the proposed method.

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