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SHM-based probabilistic representation of wind properties: Bayesian inference and model optimization

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Abstract. The estimated probabilistic model of wind data based on the conventional approach may have high discrepancy compared with the true distribution because of the uncertainty caused by the instrument error and limited monitoring data. A sequential quadratic programming (SQP) algorithm-based finite mixture modeling method has been developed in the companion paper and is conducted to formulate the joint probability density function (PDF) of wind speed and direction using the wind monitoring data of the investigated bridge. The established bivariate model of wind speed and direction only represents the features of available wind monitoring data. To characterize the stochastic properties of the wind parameters with the subsequent wind monitoring data, in this study, Bayesian inference approach considering the uncertainty is proposed to update the wind parameters in the bivariate probabilistic model. The slice sampling algorithm of Markov chain Monte Carlo (MCMC) method is applied to establish the multi-dimensional and complex posterior distribution which is analytically intractable. The numerical simulation examples for univariate and bivariate models are carried out to verify the effectiveness of the proposed method. In addition, the proposed Bayesian inference approach is used to update and optimize the parameters in the bivariate model using the wind monitoring data from the investigated bridge. The results indicate that the proposed Bayesian inference approach is feasible and can be employed to predict the bivariate distribution of wind speed and direction with limited monitoring data.

Keywords: structural health monitoring; wind properties; sequential quadratic programming algorithm; Bayesian inference; slice sampling; Markov chain Monte Carlo

1. Introduction

A large number of long-span bridges have been constructed in recent years and become crucial connection hinges to satisfy the requirement of transportation system development and social economic growth. However, with the increase of the span, the global stiffness of the bridge is reduced and it will be susceptible to the wind loading. Wind-induced vibration may lead to fatigue damage and even structural collapse which will cause a great adverse effect on the life-cycle cost of the bridge. The critical structural components of a long-span bridge are suffered from tremendous numbers of stress cycles induced by the traffic and wind loads, resulting in accumulated damage and eventually occurring structural failure. Especially for the bridges located in the coastal environment, wind-induced fatigue problem has received great attentions from the academic and industrial communities.

In wind engineering, many investigations have been conducted on the stochastic characterization of wind properties for the evaluation of wind-induced fatigue damage of the bridge during its operational stage. For this purpose, the joint probability distribution model of wind speed and direction needs to be constructed to characterize the wind properties over a long period. However, the traditional temporary measurement system only can obtain a small quantity of wind data around the bridge site, which are limited with regard to the whole design life of the bridge. In addition, most of the currently available probabilistic analysis methods just represent the features of wind measurement data by the descriptive statistics. In these methods, the wind parameters are estimated from a deterministic viewpoint, neglecting the uncertainty probably caused by the limited measurement data and measurement error. However, it is reasonable and necessary to take the effect of uncertainties of wind data for analysis of the wind properties around the bridge site (Alduse *et al.* 2015).

In comparison with the traditional methods, Bayesian approach considers the parameter as variable and facilitates the uncertainty analysis of the parameters. It realizes the parameter inference with small dataset by absorbing prior information from the other sources (Pardo-lguzquiza 1999). Bayesian approach has been applied in various research fields such as economics (Beck *et al.* 2012), statistics (Marin *et al.* 2012), and civil engineering (Lam *et al.* 2014). Smith and Naylor (1987) developed the Bayesian estimator for three-parameter Weibull distribution which was used to model the distribution of wind speed and compared with the maximum likelihood method. The comparative result indicated that Bayesian approach gave a better performance.

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Pang *et al.* (2001) employed the Bayesian estimation method to infer Weibull parameters using the wind speed data and concluded that Bayesian inference is practicable when wind data are limited. Erto *et al.* (2010) inferred the parameters of univariate wind speed distribution with one-month data and compared with the parameters evaluated by the maximum likelihood estimator based on one-year samples. Yang *et al.* (2017) used Bayesian approach to improve surface wind speed prediction under extreme weather conditions in order to reduce negative effects from civil infrastructure.

In this study, Bayesian inference approach is employed to update the wind parameters in the bivariate finite mixture distribution of wind speed and direction using limited monitoring data. For the proposed Bayesian inference approach, the selection of prior distribution, the definition of the likelihood function, and the calculation of posterior distribution are addressed in detail. The prior distributions of parameters are considered to follow normal distributions and the expectation and variance of distribution are determined in accordance with the results addressed in the companion paper. The likelihood function is formulated from the statistical model of the monitoring data. The multi-dimensional and complex posterior distribution is solved by the slice algorithm of MCMC method. The performance of the proposed Bayesian inference approach is evaluated by the numerical simulation examples for univariate and bivariate models. Furthermore, the proposed Bayesian inference approach is used to update and optimize the parameters in the bivariate model using the wind monitoring data from the investigated bridge.

2. Methodology

2.1 Bayes' theorem

In the statistical analysis, Bayes' theorem, firstly proposed by Thomas Bayes (Bayes 1763), describes the probability of an event based on the existing knowledge from specialists and new field observations (Box and Tiao 1992, Bernardo and Smith 2000). Form the Bayesian perspective, the error of the parameters should be considered by probability theory. Bayes' theorem is formulated mathematically by

$$f(\mathbf{\Theta}\mid\mathbf{y})=\frac{L(\mathbf{y}\mid\mathbf{\Theta})f(\mathbf{\Theta})}{\int L(\mathbf{y}\mid\mathbf{\Theta})f(\mathbf{\Theta})d\mathbf{\Theta}} \tag{1}$$

where $\mathbf{\Theta}$ is the target model parameters to be updated, $\mathbf{y}$ is the new field observations, $f(\mathbf{\Theta})$ is the prior density function representing previous cognition before the new field observations are acquired, $f(\mathbf{\Theta}|\mathbf{y})$ is the posterior density function with consideration of the new field observations, $L(\mathbf{y}|\mathbf{\Theta})$ is the conditional likelihood function, and the integration over $\mathbf{\Theta}$ in the denominator serves as a normalizing constant to make sure that the integration of posterior density function equals one. Based on the prior belief expressed by the prior density function $f(\mathbf{\Theta})$ and the field observations represented by the likelihood function $L(\mathbf{y}|\mathbf{\Theta})$, the posterior probability can be formed as the proportional to the prior probability and likelihood function.

One of the applications of Bayes' theorem is Bayesian inference which is a method of statistical inference for the dynamic analysis of a sequence of data. In Bayesian inference, Bayes' theorem is used to deduce and update the properties of an underlying probability distribution with more evidence and information available by computing the posterior probability. Bayesian inference is contrasted with descriptive statistics which is solely concerned with the properties of the observed data. With the assumption that the observed data are sampled from a large population, Bayesian inference can infer the properties of a population by analyzing the observed data drawn from the population. Bayesian inference acquires the posterior density function as a combination of two important parts: the prior density function and the likelihood function derived from a statistical model of the observed data.

In this study, the observed wind monitoring data $\mathbf{y}$ contains two parts: wind speed variable, v, and wind direction variable, θ. Bayesian inference is used to conduct parameter inference and model optimization for the joint probability distribution model of wind speed and direction based on the observed data and prior modeling experience. For the statistical model in the likelihood function, the joint PDF of wind speed and direction assumes that the wind speed and direction follow the finite mixture distribution with conditionally independent component densities, and is expressed by

$$f\left(v,\theta;N,\mathbf{\Theta}\right)=\sum_{i=1}^{N}w_i f_v\left(v;\alpha_i,\beta_i\right)f_\theta\left(\theta;\mu_i,\kappa_i\right) \tag{2}$$

where $f(v,\theta;N,\mathbf{\Theta})$ is the joint PDF of wind speed and direction, $f_v(v)$ is the PDF of wind speed, $f_\theta(\theta)$ is the PDF of wind direction, N is the number of components, and w_j is the weight of each mixture component and it satisfies that the summation equals one as

$$\sum_{j=1}^{N}w_j=1 \tag{3}$$

The Weibull distribution with two parameters is selected to represent the wind speed distribution function $f_v(v)$, which is written as (Weibull 1951)

$$f_v\left(v;\alpha,\beta\right)=\frac{\beta}{\alpha}\left(\frac{v}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right] \tag{4}$$

where $\alpha>0$ is the scale parameter, and $\beta>0$ is the shape parameter. For the wind direction distribution $f_\theta(\theta)$, the von Mises distribution is applied which can be written as

$$f_\theta\left(\theta;\mu,\kappa\right)=\frac{1}{2\pi I_0(\kappa)}\exp\left[\kappa\cos\left(\theta-\mu\right)\right] \tag{5}$$

where μ is the measure of the location, κ is similar to the variance in the normal distribution and represents the degree of concentration, and $I_0(\kappa)$ is the modified Bessel function of the first kind and order zero and can be expressed as

$$I_0(\kappa_i) = \frac{1}{\pi}\int_0^{\pi} e^{\kappa_i \cos(x)} dx \tag{6}$$

More specifically, the statistical model in the likelihood function, namely the joint PDF of wind speed and direction, can be presented as

$$\begin{aligned} f(v,\theta;N,\mathbf{\Theta}) = \sum_{i=1}^{N} w_i \frac{\alpha_i}{\beta_i}\left(\frac{v}{\beta_i}\right)^{\alpha_i - 1}\exp\left[-\left(\frac{v}{\beta_i}\right)^{\alpha_i}\right] \\ \cdot \frac{1}{2\pi I_0(\kappa_i)}\exp\left[\kappa_i \cos(\theta - \mu_i)\right] \end{aligned} \tag{7}$$

In the statistical model, $\mathbf{\Theta}$ represents the target model parameters to be updated, which contains the weight w_i, scale parameter α_i, shape parameter β_i, location parameter μ_i, and concentration parameter κ_i for each component. It is assumed that these parameters are mutually independent, and thus the prior distribution is expressed as

$$f(\mathbf{\Theta}) = \prod_{i=1}^{N} f(w_i)\, f(\alpha_i)\, f(\beta_i)\, f(\mu_i)\, f(\kappa_i) \tag{8}$$

The prior distributions of the parameters, except for the weight w, are considered to follow a Normal distribution. Taking the scale parameter α as an example, the probability density of the scale parameter α is referred to $N(\mu_\alpha, \sigma_\alpha^2)$ and can be expressed as

$$f(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_\alpha}\exp\left\{-\frac{(\alpha - \mu_\alpha)^2}{2\sigma_\alpha^{\,2}}\right\} \tag{9}$$

where μ_α is the expectation of the distribution, and σ_α is the standard deviation. Similarly, the prior distributions of the parameters β, μ and κ are $N(\mu_\beta, \sigma_\beta^2)$, $N(\mu_\mu, \sigma_\mu^2)$ and $N(\mu_\kappa, \sigma_\kappa^2)$ respectively. As for the weight parameter w, it is set to be constant.

With the observed wind monitoring data continuously collected by the SHM system, the likelihood function can be formulated from the statistical model of the observed data, and the expression of the likelihood function is shown as

$$L(\mathbf{y} \mid \mathbf{\Theta}) = \prod\left(\sum_{i=1}^{N} w_i f_v(\mathbf{v};\alpha_i,\beta_i) f_\theta(\mathbf{\theta};\mu_i,\kappa_i)\right) \tag{10}$$

The posterior distribution of each parameter can be calculated in accordance with Eq. (1) by multiplying the prior probability distribution with the likelihood function and then divided by the normalizing constant. Due to the posterior distribution is the arithmetic product of a series of distributions, the application of the statistical model to the observed data will lead to that the posterior distributions become the integration of complex and multi-dimensional distributions and it is difficult to calculate its analytical integration.

2.2 Markov Chain Monte Carlo

MCMC method is increasingly adopted in Bayesian analysis to solve the complicated, intractable and multi-dimensional posterior integration (Geyer 1992, Andrieu *et al.* 2003). The development of MCMC method has provided an insurance to compute large hierarchical models that require integrations over hundreds of unknown parameters. MCMC method is a general computational approach that supersedes analytical integration by summation over samples generated from the iterative algorithm, and it contains two main parts: Monte Carlo integration and Markov chain.

In MCMC method, Monte Carlo integration is a technique for numerical integration using randomly generated simulation data. The basic idea of Monte Carlo integration is to independently generate a set of samples $x^{(t)}$ from the target distribution $p(x)$ and use samples to approach the expectations of the complex distribution based on the summation of these samples, as expressed by

$$E[g(x)] = \int g(x)p(x)dx \underset{n\to\infty}{\approx} \frac{1}{n}\sum_{t=1}^{n} g(x^{(t)}) \tag{11}$$

where E is the mathematical expectation of the target distribution $p(x)$, and n is the number of samples. As shown in Eq. (11), the expectation can be approached with summation over a large set of samples based on the strong law of large numbers without analytical integration. It is obvious that the precision of the approximation largely depends on a large number of samples. Sampling can be carried out by several methods such as uniform sampling, stratified sampling, importance sampling, and rejection sampling. However, the random samples used in the Monte Carlo integration are statistically independent, thus a Markov chain is constructed to have the integrand as its equilibrium distribution.

A Markov chain is a stochastic process presenting a sequence of random variables in which the probability of each variable only depends on its previous variable. A sequence of stochastic variables $\mathbf{x}$ $(x_1,x_2,\ldots,x_t)$ is defined as Markov chain when it meets the following condition

$$q(x_{t+1} = s_{t+1} \mid x_1 = s_1, x_2 = s_2, \ldots, x_t = s_t) = q(x_{t+1} = s_{t+1} \mid x_t = s_t) \tag{12}$$

where x_1 is the starting point, and $q(x_{t+1}=s_{t+1}|x_t=s_t)$ is a transition function to determine the next state, i.e., x_2, x_3, …, x_t. The new state is only conditional on the last state and not affected by the initial value. The procedure for generating the Markov chains is: (i) set the state t=1, (ii) generate an initial value s_1 and set x_1=s_1, (iii) sample a new value s_{t+1} from the transition function and set x_{t+1}=s_{t+1}, and (iv) iterate step iii until t=T. In the procedure, the next state of the chain is only dependent with the previous state. More specifically, each Markov chain wanders around the state space and goes to the new state by the transition function.

As described in the previous part, Monte Carlo integration estimates the characteristics of target distribution and Markov chain generates a sequential process to provide some stationary distribution for sampling. Generally, based on these two main parts, MCMC method aims to compute the integration based on the samples from the stationary chain. Choosing a proper transition distribution $q(x)$, the samples from target distribution $p(x)$ can be generated. The fundamental

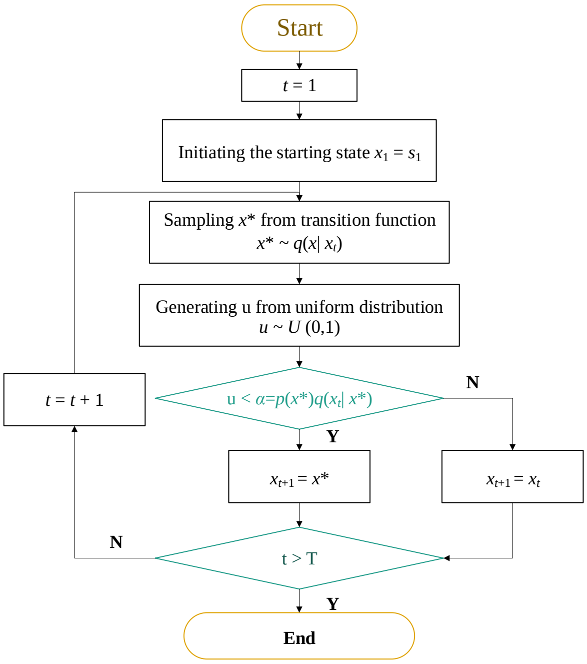


Fig. 1 Flowchart of MCMC method

flowchart of MCMC method is shown in Fig. 1. The change probability α of the current state plays an important role in the MCMC method. To avoid often rejecting the new value and falling into the local range, more solution space should be searched based on the superior acceptance probability to improve the efficiency of computation and accuracy of the result. There are several methods to reach this goal including the Metropolis-Hastings (MH) sampling (Metropolis *et al.* 1953, Hastings 1970), Gibbs sampling (Geman and Geman 1984, Gelfand and Smith 1990) and slice sampling (Neal and Radford 2003).

2.3 Slice sampling

Although the MH and Gibbs algorithms have been used in plenty of fields and make much contribution, there still exist some disadvantages and need suitable algorithms. The Gibbs algorithm needs to consider the sampling from non-standard distribution and the MH algorithm needs to find an appropriate distribution for efficient sampling. In this study, the slice sampling algorithm is used to overcome such problems (Wakefield *et al*. 1991, Higdon 1998, Damien *et al*. 1999, Neal and Radford 2003). The slice sampling algorithm reduces the degree of computation difficulty and there is no need to provide an appropriate distribution in comparison with the MH and Gibbs algorithms.

The slice sampling algorithm is selected to uniformly sampling a point from an arbitrary curve by drawing several thin uniform-height horizontal slices. It is processed by the following three-step procedure:

1. Set an initial value x_0 which satisfies $f(x_0)>0$ and get a vertical line $x=x_0$ within the region of $f(x)$;

2. A y value between 0 and $f(x_0)$ will be sampled, as $0<y<f(x_0)$, then draw a horizontal slice at the y height. Sample a point x_1 from the line segments of the curve within the interval $D=(x_L,x_R)$, where x_L and x_R are the intersections of the horizontal line and curve, i.e., $f(x_L)=f(x_R)=y$; and

3. Keep the process repeated with the new x until the assumed number of samples is achieved.

For the slice sampling algorithm, it is important to set the initial value because the sampling progress will be interrupted owing to the overflow mistake. In this study, the initial value is set as the expectation of prior distribution to guarantee the initial point is located in the domain of the target function $f(x)$. In order to sample from a distribution for a stochastic variable x with the density function proportional to $f(x)$, the joint distribution $p(x,y)$ of x and an auxiliary variable y is defined as a uniform distribution, which is expressed as

$$p(x,y)=\begin{cases} 1/M & 0<y<f(x) \\ 0 & \text{elsewhere} \end{cases} \tag{13}$$

where M is the integration of $f(x)$ over x. The marginal density of x is obtained by

$$p(x) = \int_0^{f(x)} (1/M)dy = f(x)/M \tag{14}$$

Finally, the stochastic variable x can be sampled by sampling the joint distribution $p(x,y)$ and ignoring the auxiliary variable y.

3. Numerical simulation

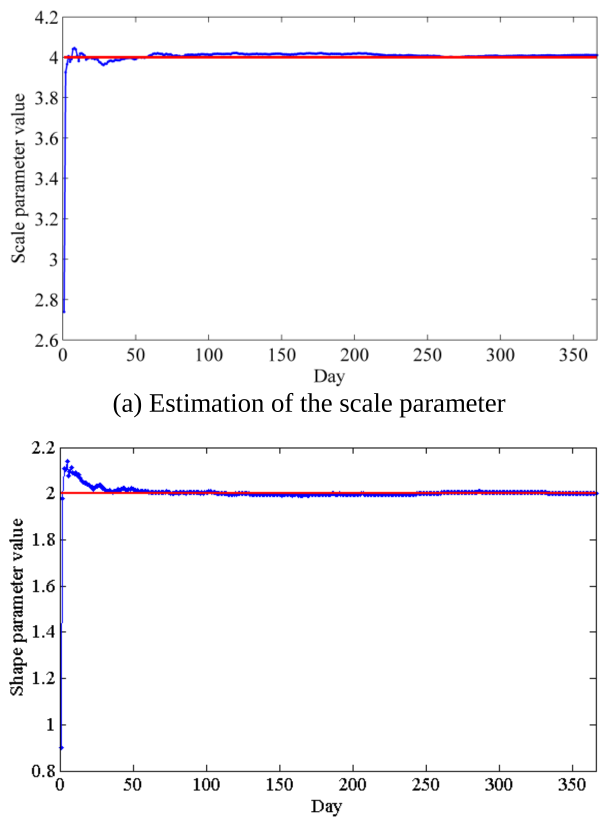
The numerical simulation is performed to investigate the effectiveness of the proposed method. The parameters of the univariate model and bivariate model are updated by Bayesian inference based on the simulated data.

3.1 Parameter inference for univariate model of wind speed

In this section, a numerical simulation example of parameter inference for a univariate model of wind speed is conducted. The univariate model of wind speed regardless of wind direction as expressed by Eq. (4) is employed. It is assumed that the distribution of wind speed follows a Weibull distribution with the scale parameter α=4 and shape parameter β=2, and its PDF can be written as $f(v;4,2)$. The one-year wind monitoring data include a total of 52,560 mean wind speed individuals simulated by the Monte Carlo method and are divided into 365 days with 144 mean wind speed individuals for each day. The setting of the Weibull parameters is considered to guarantee that the simulated wind data comply with the meteorological characteristics at the investigated bridge site to make sure the sampling data are close to the real situation.

The scale parameter α and the shape parameter β in wind speed distribution are supposed to be mutually independent and both follow the Normal distribution with non-zero expectation and the variance representing the error of the corresponding parameter. For the scale parameter α, the initial expectation of the distribution is randomly selected to make the simulated wind speed data between 0 and 14 m/s. For the variance of parameter α, the initial value is set as 10 being large enough without prior information. Similarly, the initial expectation of the shape parameter β is set from 1 to 8 and the variance of the parameter is set to be 10. The simulated wind data of the first day are selected as the new field observations and the MCMC method is used to calculate the posterior distribution of two parameters. The slice sampling algorithm is applied to efficiently sample in the target distributions and the sample size in each sample process is set as 4,000, which makes the Markov chain is stationary and the value fluctuates within a relatively narrow interval. The expectation and variance values of the Markov chain can be calculated for posterior distributions of parameters. The expectation and variance of posterior distribution serve as prior information and are combined with the wind speed data of the next day to generate a new posterior distribution. The process is sequentially repeated day by day. Fig. 2 shows the dynamic change of the parameters with the continuously added wind speed data.

As shown in Fig. 2, it can be found that the values of the scale parameter and shape parameter have a strong fluctuation at the beginning. It is obvious that the values have a sharp change and approaches to the true value with only a small quantity of iteration.



(a) Estimation of the scale parameter

(b) Estimation of the shape parameter

Fig. 2 Bayesian parameter inference for univariate model using simulated data

By the continuous iteration, the values of two parameters both tend to be steady and converge to the true value within a short time. With the newly added data, the curve changes nearby the real value. At the end of the curve, it nearly coincides with the line of the true value. The BIC value is used to evaluate the model based on two different estimation methods. Comparison results of the predicted distribution based on the proposed Bayesian parameter inference method and the SQP algorithm-based finite mixture modeling method (the detailed procedure of method is shown in the companion paper) are listed in Table 1. The initial values of scale parameter and shape parameter are 2.7393 and 0.8995 and approach to 4 and 2 within less than twenty iterations. It is found that the characteristic of 100-day data can be predicted by use of only 20-day data based on the proposed method. What's more, the proposed method provides a qualified uncertainty analysis of the parameter and a judgment about the credibility of the parameter. The BIC values illustrate that the parameter values inferred by the proposed method better represent the simulated wind speed data.

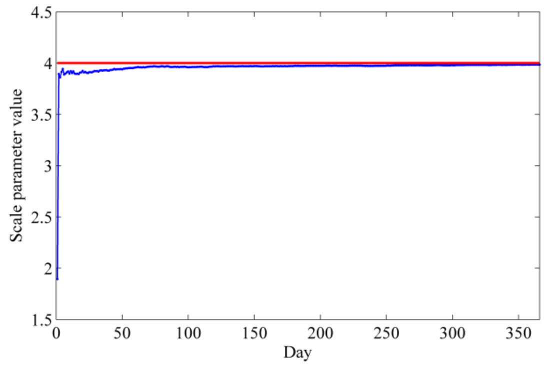
3.2 Parameter inference for bivariate model of wind speed and direction

The bivariate model of wind speed and direction considering the effect of wind direction better describes the characteristics of wind monitoring data and interprets the validity of the proposed method for the two-dimensional problem. In this case, the Weibull-von Mises mixture model, as presented in Eq. (7), with one component is utilized to produce the idealized data and its PDF can be expressed as

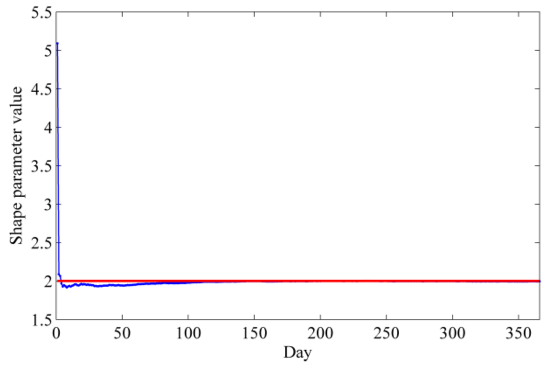
$$f(\mathbf{y};\mathbf{\Theta})=\frac{\alpha}{\beta}(\frac{v}{\beta})^{\alpha-1}\exp[-(\frac{v}{\beta})^{\alpha}]\cdot\frac{1}{2\pi I_0(\kappa)}\exp\left[\kappa\cos(\theta-\mu)\right] \tag{15}$$

where α is the scale parameter, β is the shape parameter for Weibull distribution, μ is the location parameter and κ is the concentration parameter for von Mises distribution.

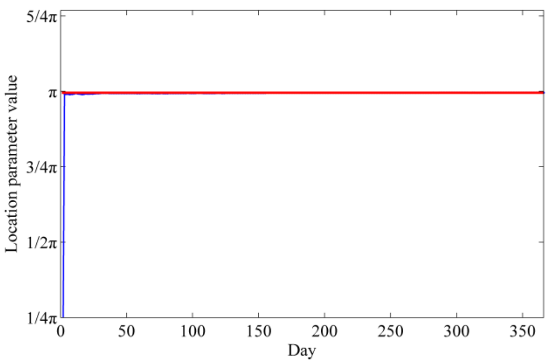
There are four parameters in the bivariate model need to be inferred, i.e., the scale parameter α, shape parameter β, location parameter μ, and concentration parameter κ. Considering the domain of the function and the climatological features of the site of the investigated bridge, the simulated values of scale parameter α and shape parameter β are selected as same as those in the univariate model. For the parameters in von Mises distribution, the simulated values of the location parameter μ and the concentration parameter κ are set as π and 10, respectively. The planar dataset including wind speed and direction from one year are produced based on the Weibull-von Mises mixture distribution $f(\mathbf{y};4,2,\pi,10)$ using the MCMC method. They are divided into 365 days with 144 mean wind speed and the corresponding direction individuals for each day. The histogram of the simulated wind data is shown in Fig. 4(a).



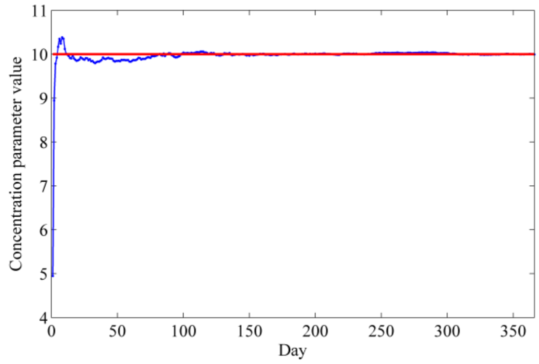
(a) Estimation of scale parameter



(b) Estimation of shape parameter

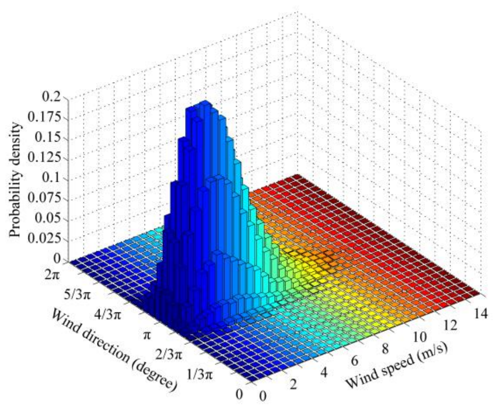


(c) Estimation of location parameter

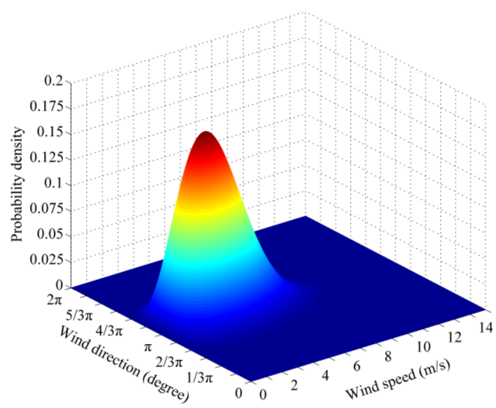


(d) Estimation of concentration parameter

Fig. 3 Bayesian parameter inference for bivariate model using simulated data



(a) Histogram of simulated data



(b) Predicted bivariate model of wind speed and direction

Fig. 4 Histogram and predicted joint PDF based on the proposed method

Table 1 Comparison results between the two methods

Parameter		SQP-based method	Proposed method		True value
			Initial	Final	
Weibull distribution	Scale parameter (α)	3.9823	2.7393	4.0056	4
	Shape parameter (β)	1.9461	0.8995	1.9903	2
BIC		208,875	\	208,843	\

As the same as the Weibull distribution in the univariate model, the scale parameter α and shape parameter β follow the normal distributions. For the parameters in von Mises distribution, the initial expectation of the location parameter μ is set in the range from 0 to 2π and the variance of the parameter is set as 10. Similarly, the initial expectation of the concentration parameter κ is set in the range from 0 to 15 and the variance of the parameter is set as 10. With the random expectations within their ranges, the prior of the scale parameter α, shape parameter β, location parameter μ,

Table 2 Comparison of the two methods

Parameter		SQP-based method	Proposed method Initial	Proposed method Final	True value
Weibull distribution	Scale parameter (α)	3.9746	1.8948	3.9802	4
	Shape parameter (β)	2.0016	5.0891	1.9936	2
von Mises distribution	Location parameter (μ)	3.1423	0.7852	3.1420	π
	Concentration parameter (κ)	10.0237	4.9459	10.0190	10
BIC		239,133		239,124	

and concentration parameter κ are N (1.89,10), N (5.09,10), N (0.78,10) and N (4.95,10), respectively. The process of parameter inference is repeated day by day with the same way for the univariate distribution.

The variations of the four parameters are shown in Fig. 3. The location parameter converges to the true value within the minimum quantity of iteration. The value of the concentration parameter has a relative higher fluctuation than the other three parameters. Through the iterative and continuous learning, the expectation values of the posterior distributions tend to be steady rapidly and become closer to the real value within a few days. The detailed results of parameter inference and the comparison between the two methods are listed in Table 2. It can be found that the values of parameters start from the random start values and finally converge to the true values which are closer than the parameters values based on SQP-based method. The BIC is applied to evaluate the performance of the predicted bivariate model based on the proposed method and SQP-based method. The BIC value of the predicted bivariate model based on the proposed method is lower than that based on the SQP-based method, which means that the bivariate model inferred by the proposed method can better represent the simulated wind data. The bivariate model of wind speed and direction based on the proposed method is shown in Fig. 4(b).

4. Application to the investigated bridge

The detailed information about the investigated bridge was shown in the companion paper as well as the SHM system and the anemometers measuring the wind data. The daily wind data are extracted from one-year monitoring wind data in 2015. After removing the abnormal and uncollected data, there are total 323 days within one year are employed for study in this section. Different from the numerical simulation study where the prior is noninformative, in this real application example, the parameters calculated by the SQP algorithm with the corresponding multi-start points and the distribution of each parameter serves as prior information. In the companion paper, the results indicated that the finite mixture model, namely Weibull-von Mises mixture model of wind speed and direction, fitted the wind data with three components.

Table 3 Prior information of parameters

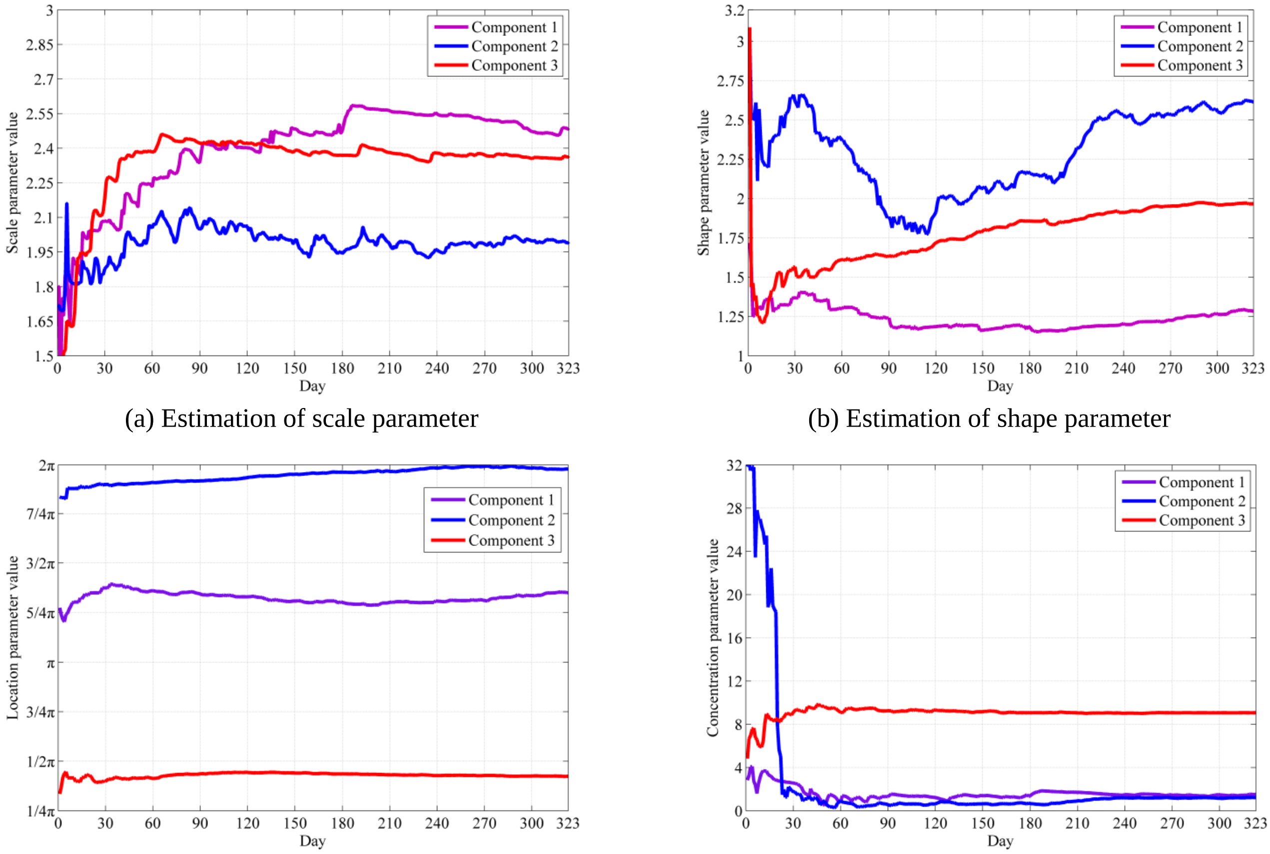
Prior information		Expectation 1	Expectation 2	Expectation 3	Variance 1	Variance 2	Variance 3
Weibull distribution	Scale parameter (α)	2.46	2.41	2.42	0.03	0.04	0.04
	Shape parameter (β)	2.25	2.17	2.20	0.11	0.14	0.14
von Mises distribution	Location parameter (μ)	2.35	2.40	2.46	4.02	4.56	4.78
	Concentration parameter (κ)	34.91	24.81	27.39	2.39	5.69	8.60

Table 4 Comparison of the two methods

Parameter		SQP-based method 1	SQP-based method 2	SQP-based method 3	Proposed method 1	Proposed method 2	Proposed method 3
Weight (w)		0.59	0.22	0.19	0.33	0.33	0.33
Weibull distribution	Scale parameter (α)	2.15	2.58	2.54	2.48	1.98	2.36
	Shape parameter (β)	2.15	2.58	2.54	1.28	2.61	1.96
von Mises distribution	Location parameter (μ)	5.83	1.58	0.98	4.25	6.41	1.34
	Concentration parameter (κ)	0.29	33.99	32.67	1.51	1.21	9.06
BIC			314,725			306,299	

Therefore, the proposed finite mixture model with three components is used as the likelihood function to depict the dataset with the given parameters. In the mixture model of three components, weights of each component are set as one third and the prior distribution of twelve parameters is determined by the parameter estimation calculated in the companion paper. There are total 7,053 parameter estimation results by the SQP algorithm with the corresponding multi-start points and 64 results with lower values of the fitness function are selected from all results to calculate the expectations and variances as prior information of the parameters. The prior expectations and variances of the scale parameters α, shape parameters β, location parameters μ, and concentration parameters κ in the mixture model are listed in Table 3.

Based on the calculated prior information of parameters and wind monitoring data, the joint probabilistic modeling of wind speed and direction is conducted by Bayesian inference. The value changes of four kinds of parameters are shown in Fig. 5. It can be found that all parameters change greatly at the beginning of the iteration and gradually become convergence after adding a number of daily wind monitoring data. Especially for the location parameter, concentration parameter, and scale parameter, the nine parameters in three components converge and become steady quickly. The shape parameters continuously change with more monitoring data, while tend to stable. The parameters after adding 323-day wind monitoring data are regarded as the final parameters for the joint probabilistic model of wind speed and direction. The fitness performance of the obtained model is compared with that of the optimal model in the companion paper based on the BIC.



(a) Estimation of scale parameter (b) Estimation of shape parameter

(c) Estimation of location parameter (d) Estimation of concentration parameter

Fig. 5 Bayesian parameter inference for bivariate model using monitoring data

Comparing with the BIC value of the optimal model in the companion paper, which is 314,725, the BIC value of the inferred model is 306,299, which means that the inferred model has a better performance. Through Bayesian inference, the joint probabilistic model can be updated by new monitoring data and better represents the distribution characteristics of wind speed and direction at the bridge site. The detailed result of parameter inference and comparison between the two methods are listed in Table 4.

5. Conclusions

This study addressed the Bayesian inference of the parameters in the bivariate finite mixture distribution of wind speed and wind direction considering the uncertainty caused by instrument error and limited monitoring data. The parameters of the univariate model and the bivariate model were inferred to illustrate the practicability of the proposed method. Based on the estimation results using the SQP algorithm, the slice algorithm was utilized to compute the complex integration, and the finite mixture distribution with three components served as the likelihood function to match the SHM data. The proposed method estimated the parameters effectively and precisely and provided a more suitable probability model to fit the data.

The results are summarized as follows: (i) the slice algorithm of MCMC can effectively solve the multi-dimensional and complicated posterior integration; (ii) the proposed Bayesian inference provides a dynamic description for the parameters in the bivariate model of wind speed and direction and provides a better model fitting the wind monitoring data to represent the wind stochastic characteristics at the bridge site; and (iii) the parameter estimated using limited and a small quantity of part of data can compete with the parameter estimated using the whole year data. The proposed Bayesian inference can contribute to predicting the future trend of wind speed and direction with available limited monitoring data.

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