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A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory

Youcef Mokhtar[1], Houari Heireche[1], Abdelmoumen Anis Bousahla[1,2],
Mohammed Sid Ahmed Houari[3,4,5], Abdelouahed Tounsi[*4,6] and S.R. Mahmoud[7]

[1]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[2]Centre Universitaire Ahmed Zabana de Relizane, Algeria
[3]Université Mustapha Stambouli de Mascara, Department of Civil Engineering, Mascara, Algeria
[4]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria
[5]Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de génie civil, Algeria
[6]Civil and Environmental Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia
[7]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

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Abstract. In this paper, a novel simple shear deformation theory for buckling analysis of single layer graphene sheet is formulated using the nonlocal differential constitutive relations of Eringen. The present theory involves only three unknown and three governing equation as in the classical plate theory, but it is capable of accurately capturing shear deformation effects, instead of five as in the well-known first shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). A shear correction factor is, therefore, not required. Nonlocal elasticity theory is employed to investigate effects of small scale on buckling of the rectangular nano-plate. The equations of motion of the nonlocal theories are derived and solved via Navier's procedure for all edges simply supported boundary conditions. The results are verified with the known results in the literature. The influences played by Effects of nonlocal parameter, length, thickness of the graphene sheets and shear deformation effect on the critical buckling load are studied. Verification studies show that the proposed theory is not only accurate and simple in solving the buckling nanoplates, but also comparable with the other higher-order shear deformation theories which contain more number of unknowns.

Keywords: buckling; graphene; a simple 3-unknown theory; nonlocal elasticity theory; navier type solution

1. Introduction

Nowadays, nanostructures such as nanorods, nanobeams and nanoplates are being more and more used in micro/nano devices and systems such as biosensor, atomic force microscope, CNT-reinforced structures, micro-electro-mechanical systems (MEMS), and nano-electro-mechanical systems (NEMS), due to their high mechanical, thermal, chemical, and electronic characteristics (Ekinci and Roukes 2005, Zemri *et al*. 2015, Kolahchi *et al*. 2016a, Bilouei *et al*. 2016, Madani *et al*. 2016, Kolahchi 2017, Zamanian *et al*. 2017, Kolahchi *et al*. 2017a,b, Rahmani *et al.* 2017, Kolahchi and Cheraghbak 2017, Hajmohammad *et al*. 2017, Zarei *et al*. 2017, Shokravi 2017a,b,c, Bakhadda *et al*. 2018). Conducting experiments with nanoscale specimens is not only very complicated and difficult, but very expensive, because of the limitations in the mechanical analyses of nanostructures. Hence, theoretical modeling and numerical simulation becomes an important issue concerning its nanoengineering applications of nanostructures. In these applications, size effects often become very significant, the cause of which needs to be explicitly addressed with an important interest. It should be noted that conventional plate models based on classical continuum elasticity theories was widely used in a very long time and does not suitable for nanoplates due to neglecting size influence in nanostructures. This incited many researchers to establish plate models based on size-dependent continuum theories which account for the small scale effects. The nonlocal elasticity theory assumes that the stress at a reference point accounts for not only the strain at the reference point, but also on all other points in the domain (Eringen 1972, 1983). Thus, the small scale effects are included through the use of constitutive equations. The nonlocal elasticity theory has been widely used in small scale structures. In this context, a large number of studies have been performed to analyze the static bending (Aghababaei and Reddy 2007, Duan and Wang 2007, Reddy 2010, Zidi *et al*. 2014, Kolahchi *et al*. 2015, Yan *et al.* 2015, Liu *et al.* 2016, Mouffoki *et al.* 2017), dynamic (Pradhan and Phadikar 2009, Murmu and Pradhan 2009, Pradhan and Murmu 2011, Wang *et al.* 2011, Bessaim *et al.* 2015, Belkorissat *et al*. 2015, Larbi Chaht *et al.* 2015, Bounouara *et al.* 2016, Arani *et al*. 2016, Ahouel *et al*. 2016, Akbas 2016, Besseghier *et al*. 2017, Shen *et al.* 2017,

∗Corresponding author, Professor
E-mail: tou_abdel@yahoo.com

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Bouafia *et al.* 2017, Shokravi 2017d) and stability (Pradhan 2009, Pradhan and Phadikar 2010, Narendar 2011, Kolahchi and Bidgoli 2016, Shokrani *et al.* 2016, Kolahchi *et al.* 2016b, Khetir *et al.* 2017, Kolahchi *et al.* 2017c, Bellifa *et al.* 2017a, Yazid *et al.* 2018) responses of nanostructures. A critical review of more recent works on the development of nanobeams and plates models can be found in (Thai *et al.* 2017). All of these models were based on classical plate theory (CPT), first-order shear deformation plate theory (FSDT) and higher-order plate theory (HSDT). The CPT is only applicable for thin plates, ignores shear deformation effects and provides reasonable results for thin plates and gives acceptable results for thin structures (plates) only (Darilmaz 2015). However, it underestimates deflection and overestimates buckling load and frequency of moderately thick or thick plates (Ghugal and Shimpi 2002). The FSDT accounts for the transverse shear deformation effect and gives acceptable results for moderately thick and thin plates, but needs a shear correction to compensate for the difference between the actual stress state and the constant stress state due to a constant shear strain assumption through the thickness (Castellazzi *et al.* 2013, Al-Basyouni *et al.* 2015, Bellifa *et al.* 2016, Bouderba *et al.* 2016, Youcef *et al.* 2018). The HSDTs account for shear deformation effects by higher-order variations of in-plane displacements or both in-plane and transverse displacements through the thickness, provides a better prediction of response of thick plate and do not required any shear correction factor and satisfy zero shear stress conditions at top and bottom surfaces of plates (Bouderba *et al.* 2013, Tounsi *et al.* 2015, Bousahla *et al.* 2014, 2016, Fekrar *et al.* 2014, Ait Amar Meziane *et al.* 2014, Belabed *et al.* 2014, Hebali *et al.* 2014, Ait Atmane *et al.* 2015, Bourada *et al.* 2015, Ait Yahia *et al.* 2015, Mahi *et al.* 2015, Meradjah *et al.* 2015, Hamidi *et al.* 2015, Kar and Panda 2015, Taibi *et al.* 2015, Attia *et al.* 2015, 2018, Mehar and Panda 2016, Bennoun *et al.* 2016, Boukhari *et al.* 2016, Draiche *et al.* 2016, Kar *et al.* 2016, Beldjelili *et al.* 2016, Bellifa *et al.* 2017b, Chikh *et al.* 2017, Benadouda *et al.* 2017, Abdelaziz *et al.* 2017, El-Haina *et al.* 2017, Menasria *et al.* 2017, Sekkal *et al.* 2017a, b, Klouche *et al.* 2017, Zidi *et al.* 2017, Fahsi *et al.* 2017, Meksi *et al.* 2018, Benchohra *et al.* 2018, Abualnour *et al.* 2018, Zine *et al.* 2018, Kaci *et al.* 2018, Bouhadra *et al.* 2018). Recently, Tounsi and his co-workers (Houari *et al.* 2016, Tounsi *et al.* 2016, Hachemi *et al.* 2017, Belabed *et al.* 2018) developed a new simple shear deformation plate theory for bending response, buckling and vibration of simply supported FG plate with only three unknown functions.

The aim of this paper is to extend the new simple shear deformation theory of Houari *et al.* (2016), Tounsi *et al.* (2016), Hachemi *et al.* (2017) and Belabed *et al.* (2018) to the micro/nanoscale plates. The most interesting feature of this theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The proposed theory contains fewer unknowns and equations of motion than the first-order shear deformation theory. Indeed, unlike the previous mentioned theories, the number of variables in the present theory is same as that in the CPT. Equations of motion are derived from Hamilton's principle based on the nonlocal constitutive relations of Eringen. Analytical solutions for buckling load are presented for simply supported plates, and the obtained results are verified by comparing the obtained results with those reported in the literature to verify the accuracy of the present theory.

2. Theoretical formulation

Consider a SLGS of length l_a, width l_b and thickness h as indicated in Fig. 1. The origin of the coordinate system is considered at the center of the middle surface of the graphene sheet. Unlike the previous mentioned theories, the number of unknown functions involved in the present theory is only three as in CPT.

2.1 Kinematics of the present plate model

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point ($x, y, \pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} - f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u_0, v_0, and w_0 are three unknown displacement functions of midplane of the plate. $f(z)$ is a shape function representing the distribution of the transverse shear strains and shear stresses through the thickness of the plate and is given as

$$f(z) = \frac{2h^2\left(h\sinh\left(\frac{z}{h}\right)\cos\left(\frac{1}{2}\right)^2 - z\cosh\left(\frac{z}{h}\right)\right)}{\cosh\left(\frac{z}{h}\right)\cos\left(\frac{1}{2}\right)^2} \tag{2}$$

In this work, the shape function in Eq. (2) is expressed by a hyperbolic function and assures an accurate distribution of shear deformation through the nanoplate thickness and allows to transverse shear stresses vary as parabolic across the thickness as satisfying shear stress free surface conditions without using shear correction factors. Indeed, it should be mentioned that contrary to the first shear deformation theory (FSDT), the proposed theory does not require shear correction factors.

The non-linear von Karman strain–displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + f(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{4a}$$

$$\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^4 w_0}{\partial x^4} \\ -\dfrac{\partial^4 w_0}{\partial y^4} \\ -\dfrac{\partial^2 (\nabla^2 w_0)}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^3 w_0}{\partial y^3} \\ -\dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix} \tag{4b}$$

and

$$g(z) = f'(z), \quad \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \tag{5}$$

2.2 Stability equations

The governing differential equations of motion of the new plate theory in case of local form is as follows (Tounsi *et al.* 2016)

$$\begin{aligned} &\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta w_0: \ \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^4 S_x}{\partial x^4} + \frac{\partial^4 S_{xy}}{\partial x^3 \partial y} \\ &+ \frac{\partial^4 S_{xy}}{\partial y^3 \partial x} + \frac{\partial^4 S_y}{\partial y^4} - \frac{\partial^3 Q_{xz}}{\partial x^3} - \frac{\partial^3 Q_{yz}}{\partial y^3} + \overline{N} = 0 \end{aligned} \tag{6}$$

with

$$\overline{N} = N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2 N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} \tag{7}$$

N_x^0 and N_y^0 the in-plane loads perpendicular to the edges $x = 0$ and $y = 0$, respectively, N_{xy}^0 the distributed shear forces parallel to the edges $x = 0$ and $y = 0$, respectively.

The stress resultants N, M and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\, \sigma_i dz, \quad (i = x, y, xy) \tag{8a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{8b}$$

2.3 Constitutive relations

The nonlocal theory considers that the stress at a point is related not only on the strain at that point but also on strains at all other points of the body. Such dependencies are related to the inter-atomic bonds between an atom and its neighboring atoms (Kiani 2013). According to the nonlocal continuum theory (Eringen 1983), the nonlocal stress tensor

$$\left(1 - \mu \nabla^2 \sigma\right) = \tau \tag{9}$$

where ∇^2 is the Laplacian operator in two-dimensional Cartesian coordinate system; τ is the classical stress tensor at a point related to the strain by the Hooke's law; and $\mu = (e_0 a)^2$ is the nonlocal parameter which includes the small scale effect, a is the internal characteristic length and e_0 is a constant appropriate to each material.

For an isotropic micro/nanoscale plate, the nonlocal constitutive relation in Eq. (9) takes the following forms

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} - \mu \nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{10}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E}{1 - \nu^2}, \quad C_{12} = \nu C_{11} \tag{11a}$$

$$C_{44} = C_{55} = C_{66} = G = \frac{E}{2(1 + \nu)}, \tag{11b}$$

E and ν are the elastic modulus and Poisson's ratio, respectively. By utilizing Eqs. (3), (8) and (11), the stress resultants can be written in terms of displacements as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \\ S_{xy} \end{Bmatrix} - \mu \nabla^2 \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \\ \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} \tag{12a}$$

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{12b}$$

where

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{13a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{13b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{13c}$$

2.4 Equations of motion in terms of displacements

The nonlocal equations of motion of the present formulation can be written in terms of generalized displacements (u_0 , v_0 and w) by using the linear differential operator $\left(1-\mu\nabla^2\right)$ on Eq. (6)

$$\begin{aligned} &A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y} - B_{11}\frac{\partial^3 w_0}{\partial x^3} \\ &-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x \partial y^2} - \left(B_{66}^s\frac{\partial^5 w_0}{\partial x^3 \partial y^2} + \left(B_{12}^s + B_{66}^s\right)\frac{\partial^5 w_0}{\partial x \partial y^4} + B_{11}^s\frac{\partial^5 w_0}{\partial x^5}\right) = 0, \end{aligned} \tag{14a}$$

$$\begin{aligned} &A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y} - B_{22}\frac{\partial^3 w_0}{\partial y^3} - \left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2 \partial y} \\ &-\left(B_{66}^s\frac{\partial^5 w_0}{\partial x^2 \partial y^3} + \left(B_{12}^s + B_{66}^s\right)\frac{\partial^5 w_0}{\partial x^4 \partial y} + B_{22}^s\frac{\partial^5 w_0}{\partial y^5}\right) = 0, \end{aligned} \tag{14b}$$

$$\begin{aligned} &B_{11}\frac{\partial^3 u_0}{\partial x^3} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x \partial y^2} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2 \partial y} + B_{22}\frac{\partial^3 v_0}{\partial y^3} \\ &- D_{11}\frac{\partial^4 w_0}{\partial x^4} - 2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w_0}{\partial y^4} \\ &+\left[B_{11}^s\frac{\partial^5 u_0}{\partial x^5} + \left(B_{12}^s + B_{66}^s\right)\frac{\partial^5 u_0}{\partial x \partial y^4} + \left(B_{12}^s + B_{66}^s\right)\frac{\partial^5 v_0}{\partial x^4 \partial y}\right. \\ &+ B_{22}^s\frac{\partial^5 v_0}{\partial y^5} + B_{66}^s\frac{\partial^5 v_0}{\partial x^3 \partial y^2} + B_{66}^s\frac{\partial^5 v_0}{\partial x^2 \partial y^3} - 2D_{11}^s\frac{\partial^6 w_0}{\partial x^6} \\ &-2\left(D_{12}^s + 2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^2 \partial y^4} \\ &\left.-2\left(D_{12}^s + 2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^4 \partial y^2} - 2\, D_{22}^s\frac{\partial^6 w_0}{\partial y^6}\right] \\ &-\left[H_{11}^s\frac{\partial^8 w_0}{\partial x^8} + 2\left(H_{12}^s + H_{66}^s\right)\frac{\partial^8 w_0}{\partial x^4 \partial y^4}\right. \\ &\left.+ H_{66}^s\frac{\partial^8 w_0}{\partial x^6 \partial y^2} + H_{66}^s\frac{\partial^8 w_0}{\partial x^2 \partial y^6} + H_{22}^s\frac{\partial^8 w_0}{\partial y^8} - A_{44}^s\frac{\partial^6 w_0}{\partial x^6} - A_{55}^s\frac{\partial^6 w_0}{\partial y^6}\right] \\ &+\overline{N} - \mu\nabla^2\overline{N} = 0 \end{aligned} \tag{14c}$$

3. Analytical solution of simply supported nanoplate

Consider a simply supported rectangular plate with length l_a and width l_b under in-plane load in two directions $\left(N_x^0 = \gamma_1 N_{cr}, N_y^0 = \gamma_2 N_{cr}, N_{xy}^0 = 0\right)$. We are concerned with the exact solutions of Eq. (14) for a simply supported nanoplate. Based on Navier solution procedure, the displacements are assumed as follows

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{15}$$

where U_{mn}, V_{mn} and W_{mn} are arbitrary parameters to be determined, $\alpha = m\pi / l_a$ and $\beta = n\pi / l_b$.

Substituting Eq. (15) into Eq. (14), the analytical solutions can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} + k\,\lambda \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

where

$$\begin{aligned} &a_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right) \\ &a_{12} = -\alpha\,\beta\left(A_{12} + A_{66}\right) \\ &a_{13} = \alpha\left[B_{11}\alpha^2 + (B_{12} + 2B_{66})\,\beta^2 - \left(B_{11}^s\alpha^4 + B_{12}^s\beta^4 + B_{66}^s\alpha^2\beta^2 + B_{66}^s\beta^4\right)\right] \\ &a_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right) \\ &a_{23} = \beta\left[B_{22}\beta^2 + (B_{12} + 2B_{66})\,\alpha^2 - \beta\left(B_{22}^s\beta^4 + B_{12}^s\beta^4 + B_{66}^s\alpha^2\beta^2 + B_{66}^s\alpha^4\right)\right] \\ &a_{33} = -D_{11}\alpha^4 - 2(D_{12} + 2D_{66})\alpha^2\beta^2 - \beta\left[D_{22}\beta^4 - 2(D_{11}^s\alpha^6 + D_{22}^s\beta^6)\right. \\ &\qquad \left.-2\left(\alpha^4\mu^2 + \alpha^2\beta^4\right)\left(D_{12}^s + 2D_{66}^s\right)\right] - \\ &\left[H_{11}^s\alpha^8 + H_{22}^s\beta^8 + 2\alpha^4\beta^4(H_{12}^s + H_{66}^s)\right. \\ &\left.+\left(\alpha^6\mu^2 + \alpha^2\beta^6\right)H_{66}^s + A_{44}^s\alpha^6 + A_{55}^s\beta^6\right] \end{aligned} \tag{17}$$

$$k = N_{cr}\left(\gamma_1\alpha^2 + \gamma_2\beta^2\right), \quad \lambda = 1 + \mu\left(\alpha^2 + \beta^2\right)$$

4. Validation and comparison of results

In this section, the accuracy of the presented new plate theory for the buckling of single-layered graphene sheet (SLGS) is demonstrated by comparing the analytical solution with those of other available results in the literature. In addition, the influences of the nonlocal parameter and shear deformation on the mechanical buckling behaviors of the micro/nanoscale plates are investigated. The governing differential equations of motion of the nonlocal present new three variable plate theory are written in Eq. (14). By setting ($e_0 a = 0$) in Eq. (14) classical plate equations are obtained. In addition, by putting $D = E\,I$ and $b = \infty$ in Eq. (15), nonlocal solutions for buckling of beam of the present theory are obtained. A beam with the following material properties and geometrical dimensions are considered for comparison: elastic modulus $E_b = 30GPa$, length $l_{beam} = 10nm$, height h_{beam} varied, critical buckling loads are non-dimensionalised as $\overline{N} = N_{cr} l_a^2 / E\,I$.

Table 1 Non-dimensional critical buckling load

l_{beam}/h	$\sqrt{\mu}\,(nm)^2$	CPT Pradhan and Murmu (2009)	FSDT Hosseini Hashemi and Samaei (2010)	HSDT Pradhan (2009)	Present
100	0	9.8791	9.8671	9.8671	9.8669
	0.5	9.4156	9.4029	9.4031	9.4029
	1	8.9947	8.9803	8.9807	8.9806
	1.5	8.6073	8.5939	8.5947	8.5945
	2	8.2537	8.2393	8.2405	8.2403
20	0	9.8177	9.8067	9.8067	9.8027
	0.5	9.3570	9.3455	9.3455	9.3417
	1	8.9652	8.9527	8.9528	8.9222
	1.5	8.5546	8.5420	8.5421	8.5386
	2	8.2114	8.1898	8.1900	8.1867

In Table 1, a comparison of the first non-dimensional critical buckling loads $\overline{N}$ is carried out for the above mentioned beam, with the Euler-Bernoulli theory (EBT) obtained Pradhan and Murmu (2009), the first order shear deformation theory (FSDT) reported by Hosseini Hashemi and Samaei (2010) and also the higher order shear deformation theory (HSDT) obtained by Pradhan (2009). For the FSDT solutions the shear correction factor $k_s = 5/6$ is adopted. A good agreement is demonstrated between the present results for the beam and those of Pradhan (2009) and Hosseini Hashemi and Samaei (2010) and this whatever the value of the nonlocal parameter μ and various values of thickness ratio l_{beam}/h.

In this section, a simply supported square nanoplate made of single-layered graphene sheet (SLGS) is considered to illustrate the effects of size (length or breadth) of the graphene sheet, mode of buckling, nonlocal parameter, and thickness of the graphene sheet on the mechanical buckling respone of nanoplates. The geometric and mechanical properties of the SLGS are: $E = 1.02 TPa$, $\nu = 0.3$, $h = 0.34$nm. We define buckling load ratio as follows

Buckling Load Ratio:

$$\xi = \frac{\text{Buckling load calculated using nonlocal theory}}{\text{Buckling load calculated using local theory}} \tag{18}$$

Fig. 1 shows the variation of critical load ratio for the first few modes of buckling with respect to the length l of the graphene sheet and for different small scale coefficients. The length of the plate is varied from 5 to 30 nm. It can be seen that the buckling load ratio decreases with increase in nonlocal parameter and increases with increase in length of graphene sheet. This is more prominent in higher modes of buckling.

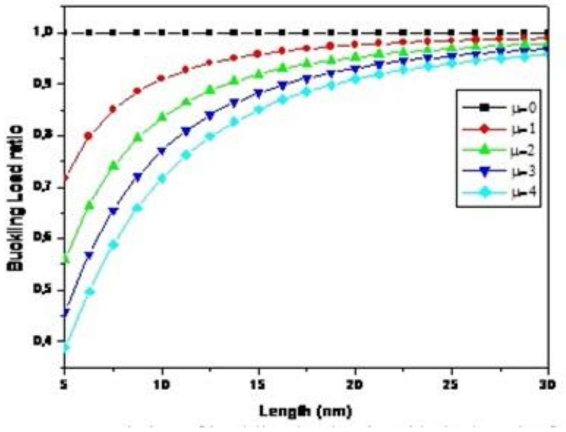

Fig. 1 Variation of buckling load ratio with the length of a square nanoplate for various nonlocal parameters

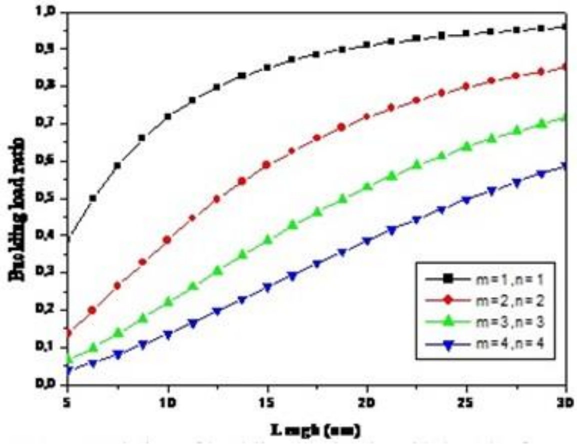

Fig. 2 Variation of buckling load ratio with the length of a square grapheme sheet for various modes of buckling for m = n

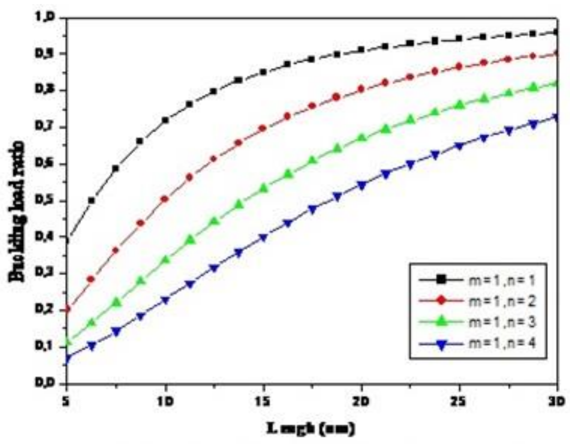

Fig. 3 Variation of buckling load ratio with the length of a square grapheme sheet for various modes of buckling for m $\neq$ n

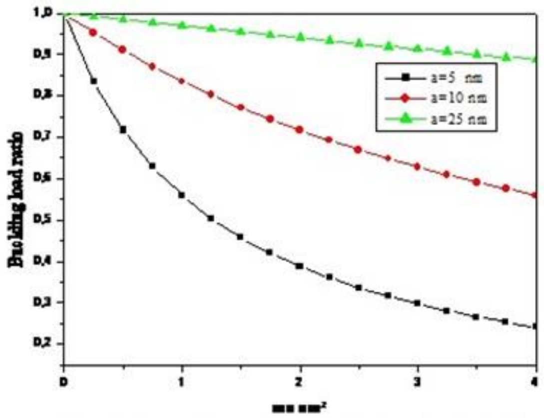

Fig. 4 Comparison of buckling load ratio of simply supported square nanoplate

To illustrate the influence of the high mode number on the buckling load of SLGS, variation of buckling load ratio with side length of SLGS and various mode of buckling are plotted in Figs. 2 and 3.

The SLGS is assumed to be a square plate and the length of the plate is varied from 5 to 30 nm and the value of nonlocal parameter (e_0a) is assumed to be $\sqrt{2}\,\mathrm{nm}^2$. It can be seen that the buckling load ratios decrease with increase in buckling modes.

Fig. 4 shown the effect of the size of the square nanoplate. As the size of the square nanoplate increases from $5\,\mathrm{nm} \times 5\,\mathrm{nm}$ to $25\,\mathrm{nm} \times 25\,\mathrm{nm}$ the buckling load ratio decreases drastically. The difference in buckling load ratio increases at higher values of the nonlocal scaling parameter.

5. Conclusions

A novel simple shear deformation theory of single layer graphene sheet is developed for buckling analysis of nanoplates. The present theory has only three unknown and three governing equation as in the classical plate theory. The present theory is capable of capturing small scale and shear deformation of nanoplates, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the nanoplates without considering the shear correction factor, instead of five as in the well-known higher-order shear deformation theory (HSDT). Based on the nonlocal differential constitutive relation of Eringen, the nonlocal equations of motion of the proposed theory are derived from Hamilton's principle. However, it is observed that the buckling load ratio decreases with increase in nonlocal small scale parameter and this variation is more prominent in higher modes of buckling. It can be concluded that the present theory, which does not require shear correction factor, is not only simple but also comparable to the first-order and higher order shear deformable theory.

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