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Dynamic analysis of nanoscale beams including surface stress effects

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Abstract. In this article, an analytic non-classical model for the free vibrations of nanobeams accounting for surface stress effects is developed. The classical continuum mechanics fails to capture the surface energy effects and hence is not directly applicable at nanoscale. A general beam model based on Gurtin-Murdoch continuum surface elasticity theory is developed for the analysis of thin and thick beams. Thus, surface energy has a significant effect on the response of nanoscale structures, and is associated with their size-dependent behavior. To check the validity of the present analytic solution, the numerical results are compared with those obtained in the scientific literature. The influences of beam thickness, surface density, surface residual stress and surface elastic constants on the natural frequencies of nanobeams are also investigated. It is indicated that the effect of surface stress on the vibrational response of a nanobeam is dependent on its aspect ratio and thickness.

Keywords: surface stress effects; nanoscales; free vibration; Gurtin-Murdoch

1. Introduction

The science and technology of nanostructure is a broad and interdisciplinary field of research and development that has exploded in the world over the last few years because of their excellent properties (Zhang 1998, Liu and Zhang 2004, Yan *et al*. 2007, Zhao *et al*. 2009, Qin *et al*. 2009, Belkorissat *et al*. 2015, Zemri *et al*. 2015, Al-Basyouni *et al.* 2015, Larbi Chaht *et al*. 2015, Ahouel *et al.* 2016, Bounouara *et al.* 2016, Bellifa *et al.* 2017a, Besseghier *et al.* 2017, Karami *et al*. 2017, Khetir *et al*. 2017, Hanifi Hachemi Amar *et al*. 2017, Mouffoki *et al.* 2017, Bouafia *et al*. 2017, Yazid *et al*. 2018) for their use in nanomechanical and nanoelectromechanical systems. It is possible to revolutionize the ways in which materials and products are created, as well as the scope and nature of the features that can be accessed. It already has an important commercial impact, which will surely increase in the future. Indeed, it is important to understand the static and dynamic mechanical behavior of these materials and advanced structures for the design and manufacture of nano-electromechanical (NEMS) systems. Due to their large surface/volume ratio, nanoscale structures exhibit significant size-dependent behavior (Chen *et al.* 2006, Wong *et al*. 1997, Miller and Shenoy 2000). Consequently, the surface effect must be considered for the analysis of materials and structures at the nanometric scale. In classical continuum mechanics, the effect of surface energy is ignored as it is small compared to the bulk energy. Nanoscale structures behave differently from their macroscale counterparts due to size effects. Some experimental researches and atomic calculations have proved that the mechanical properties to nanoscale depends on the size (Miller and Shenoy 2000, Xu *et al.* 2010). Since classical continuum theories are not capable of capturing size effects.

In nanostructures, the reliability of these theories in the analysis of the dynamic characteristics of nanostructures is doubtful. Consequently, various nonconventional continuum theories have been proposed to incorporate size effects into the governing equations of nanostructures.

The quantity of surface energy called free surface energy was first introduced by Gibbs (1906) into the thermodynamics of solid surfaces. When a material element has a characteristic length comparable to the intrinsic scale, free surface/interface energy can play an important role in its properties and behavior. There is another fundamental parameter, called surface stress, which was also defined by Gibbs (1906) for the first time. By analogy with the constitutive relation for the elastic body material, Miller and Shenoy (2000) suggested a constitutive equation of linear surface by introducing a set of elastic surface constants. Gurtin and Murdoch (1975, 1978) proposed a generic theoretical framework based on the concept of continuum mechanics that represent the surface energy/interface. In their Model, the surface is considered as a layer of zero

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thickness glued to the volume of the underlying material without sliding. The surface properties are different from those in the volume and are characterized by residual surface stress and surface Lamé constants. This theory has been widely used to study the mechanical response of structures at the nanometric scale and considers that the elastic responses of nanostructures significantly depend on surface elastic constants that could be determined by experiments or simulations.

Many theoretical approaches have been used to predict the properties of the surface. Surface stresses were evaluated using ab initio methods in semiconductors by Maede and Vanderbilt (1989) and in metals by Needs (Needs 1987). With the hypothesis of isotropy, Miller and Shenoy (2000) calculated the surface module of different surface orientations using the integrated atomic method. A systematic study of surface elastic constants using atom simulations was presented by Miller and Shenoy (2000). Recently, many researchers have used unconventional continuum theories including surface stress effects. Ould Youcef *et al.* (2015) studied the bending and buckling of nanowire with different high order shear deformation theories (HSDTs), By using the classical beam theory integrated with linear surface elasticity theory (He and Lilley 2008, He and Lilley 2008, Wang and Feng 2007, Wang and Feng 2009a, b, Rajapakse and Phani 2011, Song *et al*. 2011) studies the bending and buckling behavior of nanowires. Yan and Jiang (2011) used the Euler beam theory to study the buckling response of piezoelectric nanobeams with superficial stress. Ansari and Sahmani (2011) adopted different theories of beams for the analysis of the buckling of nanobeams with surface effect. Wang and Yang (2011) studied the buckling of nanobeams by considering geometric nonlinearity.

It should be noted that there is various HSDTs developed in literature (Bellifa *et al.* 2017b, Chikh *et al.* 2017, El-Haina *et al.* 2017, Abdelaziz *et al.* 2017, Benadouda *et al.* 2017, Draiche *et al.* 2016, Akavci 2016, Baseri *et al.* 2016, Barati and Shahverdi 2016, Becheri *et al.* 2016, Bouderba *et al.* 2016, Boukhari *et al*. 2016, Eltaher *et al.* 2016, Hamidi *et al.* 2015, Kar and Panda 2015, Hebali *et al*. 2014, Bouderba *et al*. 2013) and the need to propose other simple HSDTs is a topic for many recent works.

In this work, an analytical solution is developed for the dynamic analysis of nanoscale beams by introducing the surface effect. A refined higher shear deformation theory developed by Tounsi and his co-workers (Beldjelili *et al.* 2016, Bellifa *et al.* 2016, Bousahla *et al*. 2016) is used; the most important assumption used in the proposed beam theory is that the boom consists of bending and shearing components. The numerical results are illustrated to prove the difference between the responses of the nanoscale beams predicted by the conventional and unconventional solution which depends on the elastic constants of surface.

2. Formulation

Consider a nanobeam beam with a rectangular cross-section, length $L\,(0\le x\le L)$, width b $(0\le y\le b)$ and height h $(0\le z\le h)$ is modelled in Cartesian coordinate system (x, y, z) as shown in Fig. 1. The area and perimeter of the cross-section are A and s respectively. To incorporate the surface effects, it is assumed that the response of the beam is governed by the continuum theory proposed by Gurtin and Murdoch (1975, 1978).

2.1 Kinematics

The present two variable refined beam theory used by Benachour *et al*. (2011) and Hadji *et al.* (2011) is based on assumption that the in-plane and transverse displacements consist of bending and shear components in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The theory presented is variationally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions, are given as (Bessaim *et al.* 2013, Ait Amar Meziane *et al.* 2014, Belabed *et al.* 2014; Attia *et al*. 2015, Ait Yahia *et al.* 2015, Bennoun *et al*. 2016)

$$u(x,z,t)=-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{1a}$$

$$w(x,z,t)=w_b(x,t)+w_s(x,t) \tag{1b}$$

where t is time, u , w are displacements in the x, z directions, w_b and w_s are the bending part and the shear part of deflection, respectively.

The displacement fields of the third-order beam theory (TBT) based on Reddy (1984) can be determined from Eq. (1) by employing the shape functions $f(z)$

$$f(z)=\frac{4z^3}{3h^2} \tag{1c}$$

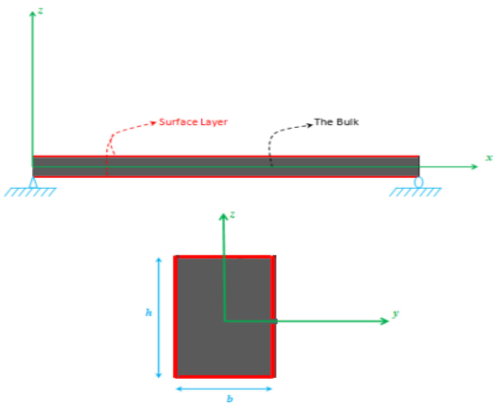

Fig. 1 Simply supported-simply supported straight uniform beam with rectangular cross section and its coordinate system

The strains associated with the displacements in Eq. (1) are

$$\varepsilon_x = -z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \quad \text{and} \quad \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \tag{2}$$

where

$$g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{3}$$

2.2 Surface elasticity model for nanowires and constitutive relations

The surface constitutive relations given by Gurtin and Murdoch (1975, 1978) can be simplified in present study as

$$\sigma_x^s = (2\mu^s + \lambda^s)\varepsilon_x + \tau^s \quad \text{and} \quad \tau_{xz}^s = \tau^s \frac{\partial w}{\partial x} \tag{4}$$

where τ^s is the residual surface stress under unconstrained conditions; μ^s and λ^s being the surface elasticity Lamé modulus.

Note that in a beam bending problem, the stress component σ_z is not zero. But it is small enough compared to axial stress σ_x to neglect in classical beam theory. However, in Gurtin-Murdoch model the surface is not in balance with the above assumption. To remedy this, following Lu *et al*. (2006) σ_z is assumed to vary linearly through the beam thickness and satisfy the equilibrium conditions on the surface. With this assumption, σ_z can be written as

$$\sigma_z = \frac{\left.\frac{\partial \tau_{xz}}{\partial x}\right|_{\text{at top}} + \left.\frac{\partial \tau_{xz}}{\partial x}\right|_{\text{at bottom}}}{2} + \frac{\left.\frac{\partial \tau_{xz}}{\partial x}\right|_{\text{at top}} + \left.\frac{\partial \tau_{xz}}{\partial x}\right|_{\text{at bottom}}}{h} z \tag{5}$$

According to this assumption, σ_z can be determined as

$$\sigma_z = \frac{2z\tau^s}{h}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) - \frac{2z\rho^s}{h}\left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}\right) \tag{6}$$

Substitution of Eqs. (2) into Eq. (4) yields

$$\sigma_{xx}^s = \left(2\mu^s + \lambda^s\right)\left(-z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2}\right) + \tau^s \tag{7a}$$

$$\tau_{xz}^s = \tau^s\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right) \tag{7b}$$

Since both the bulk and surfaces of the beam are assumed to be homogeneous and isotropic, the constitutive relations of the bulk material relating non-zero stresses σ_x^b and τ_{xz}^b can be expressed as

$$\sigma_x^b = E\varepsilon_x + \nu\sigma_z = E\left(-z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2}\right) + \frac{2z\nu\tau^s}{h}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) \tag{8a}$$

$$\tau_{xz}^b = G\gamma_{xz} = Gg(z)\frac{\partial w_s}{\partial x} \tag{8b}$$

where E is the elastic modulus, ν is Poisson's ratio and G is the shear modulus In this work, we consider a superposition between the quantities corresponding to the surface and the bulk and this summation is considered to facilitate only the mathematical formulation

$$\sigma_x = \sigma_x^b + \sigma_x^s \quad \text{and} \quad \tau_{xz} = \tau_{xz}^b + \tau_{xz}^s \tag{9}$$

2.3. Euler-Lagrange equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy 2002, Tounsi *et al*. 2013, Zidi *et al*. 2014, Bousahla *et al.* 2014, Mahi *et al*. 2015, Bourada *et al.* 2015, Taibi *et al.* 2015, Houari *et al.* 2016, Saidi *et al.* 2016, Zidi *et al.* 2017 ; Menasria *et al.*,2017; Sekkal *et al*. 2017, Abualnour *et al.* 2018)

$$0 = \int_{t_1}^{t_2}\left(\delta U + \delta V - \delta K\right)dt \tag{10}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy; and δK is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\delta U = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}\right)dz dx = \int_0^L \left(-M_b\frac{d^2\delta w_b}{dx^2} - M_s\frac{d^2\delta w_s}{dx^2} + Q\frac{d\delta w_s}{dx}\right)dx \tag{11}$$

where M_b, M_s and Q are the stress resultants defined as

$$(M_b, M_s) = \int_A (z, f)\,\sigma_x dA \quad \text{and} \quad Q = \int_A g\,\tau_{xz} dA \tag{12}$$

The variation of kinetic energy is expressed as

$$\delta K = \int_0^L \int_A \rho\left(\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}\right)dA dx = \int_0^L \left\{ \begin{aligned} & m_0\left(\dot{w}_b + \dot{w}_s\right)\delta\left(\dot{w}_b + \dot{w}_s\right) + m_2\frac{\partial \dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x} + \\ & m_5\frac{\partial \dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + m_4\left(\frac{\partial \dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x}\right) \end{aligned} \right\} dx \tag{13}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; ρ is the mass density of the bulk; ρ^s is the surface density of the surface layer and $(m_0, m_2, m_3, m_4, m_5)$ are mass inertias defined as

$$(m_0, m_2, m_3, m_4, m_5) = \rho \int_A \left(1, z^2, f, zf, f^2\right) dA + \rho^s \int_{\partial A} \left(1, z^2, f, zf, f^2\right) ds \tag{14}$$

Substituting Eqs. (11) and (13) into Eq. (10) and carrying out the integration by parts, the equations of motion of the proposed beam theory are determined as follows

$$\delta w_b : \frac{d^2 M_b}{dx^2} = m_0(\ddot{w}_b + \ddot{w}_s) - m_2 \frac{\partial^2 \ddot{w}_b}{\partial x^2} - m_4 \frac{\partial^2 \ddot{w}_s}{\partial x^2} \tag{15a}$$

$$\delta w_s : \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} = m_0(\ddot{w}_b + \ddot{w}_s) - m_4 \frac{\partial^2 \ddot{w}_b}{\partial x^2} - m_5 \frac{\partial^2 \ddot{w}_s}{\partial x^2} \tag{15b}$$

By substituting Eqs. (7) and (8) into Eq. (9), and the subsequent results into Eq. (12), the constitutive equations for the stress resultants are obtained as

$$M_b = \left(\frac{2I\nu\tau^s}{h} - D_{11} - \left(2\mu^s + \lambda^s\right) I_{p2}\right)\frac{\partial^2 w_b}{\partial x^2} + \left(\frac{2I\nu\tau^s}{h} - D_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p4}\right)\frac{\partial^2 w_s}{\partial x^2} - \frac{2I\nu\rho^s}{h}(\ddot{w}_b + \ddot{w}_s) \tag{16a}$$

$$M_s = \left(\frac{2I_1\nu\tau^s}{h} - D_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p4}\right)\frac{\partial^2 w_b}{\partial x^2} + \left(\frac{2I_1\nu\tau^s}{h} - H_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p5}\right)\frac{\partial^2 w_s}{\partial x^2} + I_{p3}\tau^s - \frac{2I_1\nu\rho^s}{h}(\ddot{w}_b + \ddot{w}_s) \tag{16b}$$

$$Q = \left[A_{55}^s + \mu^s J_{p1} + \frac{1}{2}\tau^s\left(J_{p3} - J_{p2}\right)\right]\frac{\partial w_s}{\partial x} \tag{16c}$$

where

$$\left(D_{11}, D_{11}^s, H_{11}^s\right) = \int_A E\left(z^2, z\, f(z), f^2(z)\right) dA \quad \text{and} \quad A_{55}^s = \int_A G[g(z)]^2 dA \tag{17a}$$

$$I = \int_A z^2 dA = \frac{bh^3}{12}; \quad I_1 = \int_A z f(z) dA \tag{17b}$$

$$J_{p1} = 2\int_{-h/2}^{h/2} [g(z)]^2 dz \quad ; \quad J_{p2} = \int_{-h/2}^{h/2} g(z) dz; \quad J_{p3} = \int_{-h/2}^{h/2} f'(z) g(z) dz; \tag{17c}$$

$$I_{p3} = \int_S f(z) ds \;\; ; \;\; I_{p4} = \int_S z f(z) ds \;\; ; \;\; I_{p5} = \int_S [f(z)]^2 ds \tag{17d}$$

By substituting Eq. (16) into Eq. (15), the governing equations can be expressed in terms of displacements (w_b, w_s) as

$$\begin{aligned}
&\left[\frac{2I\nu\tau^s}{h} - D_{11} - \left(2\mu^s + \lambda^s\right)\left(\frac{h^3}{6} + \frac{Ah}{2}\right)\right]\frac{\partial^4 w_b}{\partial x^4} + \\
&\left[\frac{2I\nu\tau^s}{h} - D_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p4}\right]\frac{\partial^4 w_s}{\partial x^4} \\
&+ H\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) = m_0(\ddot{w}_b + \ddot{w}_s) \\
&-\left(\frac{2I\nu\rho^s}{h} + m_2\right)\left(\frac{\partial^2 \ddot{w}_b}{\partial t^2}\right) \\
&-\left(\frac{2I\nu\rho^s}{h} + m_4\right)\left(\frac{\partial^2 \ddot{w}_s}{\partial t^2}\right)
\end{aligned} \tag{18a}$$

$$\begin{aligned}
&\left[\frac{2I_1\nu\tau^s}{h} - D_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p4}\right]\frac{\partial^4 w_b}{\partial x^4} + \\
&\left[\frac{2I_1\nu\tau^s}{h} - H_{11}^s - \left(2\mu^s + \lambda^s\right) I_{p5}\right]\frac{\partial^4 w_s}{\partial x^4} \\
&\quad + \left[A_{55}^s + \mu^s J_{p1} + \frac{1}{2}\tau^s\left(J_{p3} - J_{p2}\right)\right]\frac{\partial^2 w_s}{\partial x^2} \\
&+ q + H\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) = \\
&m_0(\ddot{w}_b + \ddot{w}_s) - \left(\frac{2I_1\nu\rho^s}{h} + m_4\right)\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2}\right) - \\
&\left(\frac{2I_1\nu\rho^s}{h} + m_5\right)\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2}\right)
\end{aligned} \tag{18b}$$

where H is the constant parameter which is determined by the residual surface tension τ^s (generally assumed as a positive number) and the shape of cross section. For rectangular beam cross sections, the surface elasticity tension is expressed by (Gao and Zhang 2015, Ansari *et al.* 2013)

$$H = 2b\tau^s \tag{19}$$

3. Closed-form solution for simply supported nanowires

Consider a simply supported beam with length L. The following expansions of displacements (w_b, w_s) are chosen to satisfy the simply supported boundary conditions of beam

Table 1 Material properties of Aluminum and Silicon

Material	E (GPa)	ν	μ^s (N/m)	λ^s (N/m)	τ^s (N/m)	ρ (kg/m^3)	ρ^s (kg/m^2)
Al	90	0.23	-3.493	-3.493	0.5689	2.7×10^{3}	5.46×10^{-7}
Si	107	0.33	-2.7779	-4.4939	0.6056	2.33×10^{3}	3.17×10^{-7}

$$w_b = \sum_{n=1}^{\infty} W_{bn}\sin(\alpha\, x)e^{i\omega t} \tag{20a}$$

$$w_s = \sum_{n=1}^{\infty} W_{sn}\sin(\alpha\, x)e^{i\omega t} \tag{20b}$$

where $i=\sqrt{-1}$, $\alpha = n\pi/L$, ω is the natural frequency, and W_{bn}, and W_{sn} are arbitrary parameters to be determined. Substituting Eq. (20) into Eq. (18), the following eigenvalue equation is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\right)\begin{Bmatrix} W_{bn} \\ W_{sn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$S_{11} = \alpha^4\left(H + \left(\frac{2I\nu\tau^s}{h} - D_{11} - \left(2\mu^s + \lambda^s\right)\left(\frac{h^3}{6} + \frac{Ah}{2}\right)\right)\right) \tag{22a}$$

$$S_{11} = \alpha^4\left(H + \left(\frac{2I\nu\tau^s}{h} - D_{11} - \left(2\mu^s + \lambda^s\right)\left(\frac{h^3}{6} + \frac{Ah}{2}\right)\right)\right) \tag{22b}$$

$$m_{11} = m_0 + \alpha^2\left(m_2 + \frac{2I\nu\rho^s}{h}\right), \quad m_{12} = m_0 + \alpha^2\left(m_4 + \frac{2I\nu\rho^s}{h}\right), \tag{22c}$$

$$m_{21} = m_0 + \alpha^2\left(m_4 + \frac{2I_1\nu\rho^s}{h}\right) \quad m_{21} = m_0 + \alpha^2\left(m_5 + \frac{2I_1\nu\rho^s}{h}\right) \tag{22d}$$

The natural frequencies of the nanobeam can be obtained by setting the determinant of the coefficient matrix in Eq. (21) to zero. For each choice of n, there is a corresponsive unique value of ω. The fundamental frequency is the smallest value of $\omega(n)$.

4. Numerical results and discussion

In this first example, the analytic solution for the free vibration behavior of the nanoscale beams with rectangular cross-section including surface stress effects is developed to investigate the effect of surface material properties, amplitude ratio and mode number on the natural frequency.

Beams made of aluminum (Al) and silicon (Si) are considered in the numerical study. The bulk and surface elastic constants have been acquired by Miller *et al*. (2000) and Shenoy (2005). The results are as shown in Table 1.

The dimensions for thin beams are L =120 *nm*, h = 6 *nm* and b = 3 *nm*, and those for thick beams are L= 50 *nm*, h = 6 *nm* and b = 3 *nm*. The solutions are shown in Tables 2 and 3. The corresponding solutions from classical thin and thick beam theories are also presented in parenthesis. The obtained results are compared with those computed independently by Chang *et al*. (2010) based on the Euler–Bernoulli beam theory (EBT), and first beam theory (FBT).

It can be seen that the results of present theories are in excellent agreement with those predicted by EBT and FBT. It is found that surface energy effects have a significant influence on the first natural frequency of thin and thick beams. However, the higher natural frequencies are not significantly affected as the bulk bending stiffness becomes the dominant factor controlling the higher modes.

It is worth pointing out that the natural frequencies with surface effects could increase or decrease compared with the classical results, depending on the signs of the surface elastic constants and wave number. As a result, for higher natural frequencies, the results from surface elastic model become smaller compared to the classical ones. This trend can be observed from Tables 2 and 3.

In fact for some higher modes, the classical solution overestimates the natural frequencies. It should be noted that thin beam theory is not generally accurate for higher modes and the thick beam theory should be used irrespective of the L/h ratio.

In the second example, the following material characteristics are used in computations as follows (Gurtin and Murdoch 1978)

$$E = 17.73\times10^{10}\ \mathrm{N/m^2}, \quad \nu = 0.27, \quad \rho = 7000\,Kg/m^3$$
$$\mu^s = 2.5\ \mathrm{N/m}, \quad \lambda^s = -8\ \mathrm{N/m}, \quad \tau^s = 1.7\ \mathrm{N/m},$$
$$\rho^s = 7\times10^{-6}\,Kg/m^2$$

It is supposed that $h=b=1\,\mathrm{nm}$ and L varies from $L/h=10$ to 50. Also, the non-dimensional natural frequency is defined as $\Omega = \omega L\sqrt{\rho/E}$. The non-dimensional natural frequencies corresponding to the first four modes of nanobeams are given in Table 4.

It is seen that the accuracy of the results enhances with increasing the number of nodes and converged results are obtained when the number of nodes becomes larger than a specific value. It can be seen that the results of present theories are in excellent agreement with those predicted by Ansari *et al.* (2013) for all values of thickness ratio L/h.

The frequency ratios of nanobeams with different thicknesses and aspect ratios are plotted in Fig. 2.

Table 2 Natural frequencies of aluminum beams

Beam type	1st (GHz)	2nd(GHz)	3rd (GHz)	4th (GHz)
		L* =120 *nm* , *h* = 6 *nm* and *b* = 3 *nm		
CPT [Chang Liu]	1.45 (1.09)	4.47 (4.36)	9.39 (9.82)	16.27 (17.45)
FSDT	1.45(1.09)	4.41 (4.29)	9.11(9.48)	15.41(16.44)
Present	1.45 (1.09)	4.41 (4.29)	9.10 (9.48)	15.37 (16.44)
		L* = 50 *nm* , *h* = 6 *nm* and *b* = 3 *nm		
CPT	6.21(6.28)	23.18(25.13)	51.47(56.55)	91.11(100.53)
FSDT [Chang Liu]	6.10 (6.14)	21.49 (23.13)	43.96 (47.84)	71.08 (77.43)
Present	6.08 (6.14)	21.38 (23.12)	43.63 (47.86)	70.26 (77.49)

Table 3 Natural frequencies of silicon beams

Beam type	1st (GHz)	2nd(GHz)	3rd (GHz)	4th (GHz)
		L* =120 *nm* , *h* = 6 *nm* and *b* = 3 *nm		
CPT [Chang Liu]	1.66 (1.28)	5.19 (5.12)	10.96 (11.53)	19.02 (20.49)
FSDT	1.66 (1.28)	5.12 (5.04)	10.62 (11.11)	17.98(19.23)
Present	1.66 (1.28)	5.12 (5.04)	10.61 (11.11)	17.96 (19.23)
		L* = 50 *nm* , *h* = 6 *nm* and *b* = 3 *nm		
CPT	7.22 (6.28)	27.12 (25.13)	60.27 (56.55)	106.72 (100.53)
FSDT [Chang Liu]	7.08 (7.20)	25.07 (27.02)	51.33 (55.70)	82.92 (89.76)
Present	7.08 (7.20)	25.02 (27.02)	51.15 (55.70)	82.46 (89.86)

Table 4 Non-dimensional natural frequencies of nanobeams corresponding to the four first mode-numbers

L/h	Ω_1		Ω_2		Ω_3		Ω_4	
	Ansari	Present	Ansari	Present	Ansari	Present	Ansari	Present
10	0.2936	0.2901	0.7870	0.7451	1.5504	1.3723	2.5914	2.1202
20	0.2624	0.2618	0.5870	0.5803	1.0155	0.9876	1.5688	1.4903
50	0.2530	0.2529	0.5168	0.5161	0.8013	0.7987	1.1148	1.1084

The frequency ratio is defined as the ratio of natural frequency predicted by the non-classical theory to the frequency given by the classical one. It can be seen that, as the aspect ratio increases, the natural frequencies obtained by the non-classical model become higher than those predicted by the classical theory. Also, it is observed that as the thickness of nanobeam increases, the non-classical natural frequencies tend to get closer to the classical natural frequencies. This indicates that the surface stress effect is more pronounced and must be taken into account when the thickness of the nanobeam is small. This kind of size effect diminishes for thick nanobeams.

Fig. 3 shows the effect of surface elastic constants on the fundamental frequencies of nanobeams with the assumptions of $\rho^s = \tau^s = 0$ and $h = b = 1$ nm. It can be seen that the positive surface elasticity increases the bending stiffness of nanobeam and thus, the natural frequency increases, while the negative one reduces the stiffness and natural frequency of nanobeam.

Fig. 4 reveals the effect of surface density on the fundamental frequency of nanobeams with the assumptions of $\mu^s = \lambda^s = \tau^s = 0$ and $h = b = 1$ nm. The results for the classical nanobeam are also plotted for comparison. It can be seen that, the fundamental frequencies of nanobeams decrease with an increase in the value of ρ^s, especially for lower aspect ratios. Furthermore, it can be seen that for all values of surface density, the non-classical fundamental frequencies are smaller than those predicted by the classical beam model.

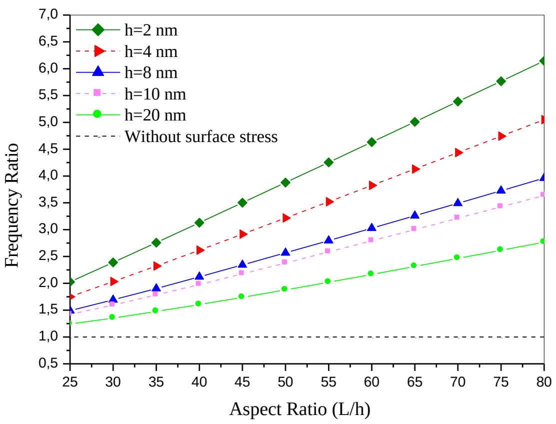

Fig. 2 Frequency ratios of a nanobeam as a function of aspect ratio (*L*/*h*) for different values of thickness (*b* = 1 nm)

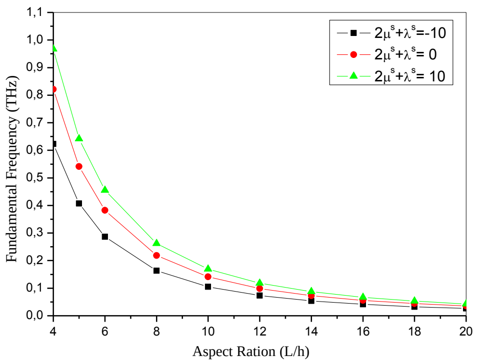

Fig. 3 Effect of the surface elastic constants on the fundamental frequency of nanobeams at various aspect ratios ($\rho^S = \tau^S = 0$)

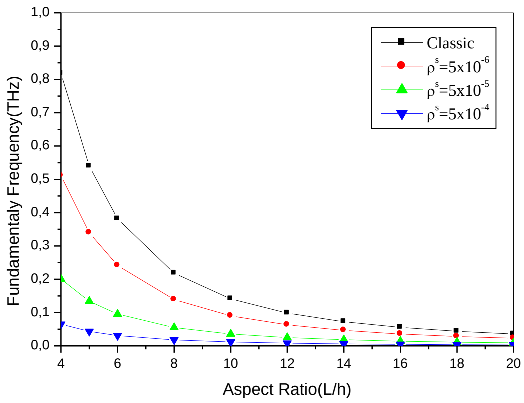

Fig. 4 Effect of the surface density on the fundamental frequency of nanobeams with different boundary conditions at various aspect ratios ($\mu^S = \lambda^S = \tau^S = 0$)

The influence of surface residual stress on the fundamental frequencies of nanobeams is presented in Fig. 5 with assuming $\mu^S = \lambda^S = \rho^S = 0$ and $b = h = 1$ *nm*. It can be concluded from the figure that the positive and negative surface residual stresses increases and decreases the bending stiffness of nanobeams, respectively. Therefore, the non-classical fundamental frequencies for positive τ^S are larger than those given by the classical beam model, while the completely reversed behavior is observed for the negative values of τ^S.

5. Conclusions

In this work, a non-classical solution for the free vibrations of nanobeams including surface stress effects was presented. Based on the Gurtin–Murdoch elasticity theory, the influence of surface stress was incorporated into the classical high-order surface stresses. It is found that surface energy effects have a significant influence on the first natural frequency of thin and thick beams. It can be seen that the positive surface elasticity increases the bending stiffness of nanobeam and thus, the natural frequency increases, while the negative one reduces the stiffness and natural frequency of nanobeam. It was indicated that the effect of surface stress on the vibrational response of nanobeams is dependent on the aspect ratio and thickness of beams. It was also indicated that vibrational response of nanobeams strongly depends on the magnitudes of surface elasticity constants.

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