Smart Structures and Systems, *Vol. 21, No. 1 (2018) 15-25*
DOI: https://doi.org/10.12989/sss.2018.21.1.015

A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium

Miloud Yazid[1], Houari Heireche[*1], Abdelouahed Tounsi[1,2,3],
Abdelmoumen Anis Bousahla[1,3,4] and Mohammed Sid Ahmed Houari[3,5]

[1]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[2]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia
[3]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria
[4]Centre Universitaire Ahmed Zabana de Relizane, Algeria
[5]Université Mustapha Stambouli de Mascara, Department of Civil Engineering, Mascara, Algeria

(Received May 7, 2017, Revised December 6, 2017, Accepted December 9, 2017)

Abstract. This work presents the buckling investigation of embedded orthotropic nanoplates such as graphene by employing a new refined plate theory and nonlocal small-scale effects. The elastic foundation is modeled as two-parameter Pasternak foundation. The proposed two-variable refined plate theory takes account of transverse shear influences and parabolic variation of the transverse shear strains within the thickness of the plate by introducing undetermined integral terms, hence it is unnecessary to use shear correction factors. Nonlocal governing equations for the single layered graphene sheet are obtained from the principle of virtual displacements. The proposed theory is compared with other plate theories. Analytical solutions for buckling loads are obtained for single-layered graphene sheets with isotropic and orthotropic properties. The results presented in this study may provide useful guidance for design of orthotropic graphene based nanodevices that make use of the buckling properties of orthotropic nanoplates.

Keywords: buckling; orthotropic nanoplates; nonlocal elasticity; Pasternak's foundations; HSDT

1. Introduction

Graphene (Geim and Novoselov 2007), the 2D counterpart of 3D graphite, has taken wide interest in solid-state physics, materials science and nanoelectronics since it was discovered in 2004 as the first free-standing 2D crystal. Graphene is becoming as a promising electronic material in post-silicon electronics. However, large scale synthesis of high quality graphene denotes a bottleneck for future generation graphene devices. Existing routes for graphene synthesis incorporate mechanical exfoliation of highly ordered pyrolytic graphite (HOPG) (Novoselov *et al.* 2004), removing Si from the surface of single crystal SiC (Ohta *et al*. 2006), depositing graphene at the surface of single crystal (Oshima and Nagashima 1997) or polycrystalline metals (Obraztsov *et al.* 2007), and various wet-chemistry-based approaches (Gomez-Navarro *et al.* 2007, Li *et al.* 2008). However, up to now no methods have provided high quality graphene with a large enough area as necessitates for application as a practical electronic material. In recent years, the study of mechanical response of graphene sheets has become an interesting topic and hence a few techniques and experimental methods have been employed for analyzing the characteristics of graphene sheets (Soldano *et al*. 2010).

Nonlocal elasticity theory (Eringen 1972, Eringen and Edelen 1972, Eringen 1996, Eringen 1983, 2002) was developed to consider the small scale impact in elasticity by supposing the stress at a reference point to be depending on strain field at every point in the body. In this way, the internal length scale could be simply introduced in constitutive equations as a material parameter. Only recently has the nonlocal elasticity model been incorporated to nanostructure applications. As the size scales are diminished, the effects of long-range interatomic and intermolecular cohesive forces on the static and vibration characteristics tend to be considerable and cannot be ignored. The classical theory of elasticity being the long wave limit of the atomistic theory neglects these influences. Thus, the conventional classical continuum mechanics would fail to capture the scale influences when dealing with nanostructures (Lu *et al.* 2006, Tounsi *et al.* 2013a). The small-size investigation via local theory over predicts the results. Thus, the incorporation of small influences is required to correct prediction of nanostructures. Chen *et al.* (2004) demonstrated that the nonlocal elasticity theory based models are physically reasonable from the atomistic viewpoint of lattice dynamics and molecular dynamics (MD) simulations. Peddieson *et al.* (2003) employed nonlocal elasticity to present the nonlocal Euler–Bernoulli beam model and concluded that nonlocal continuum mechanics could potentially play an important role in

∗Corresponding author, Professor
E-mail: heireche_h@yahoo.fr

ISSN: 1738-1584 (Print), 1738-1991 (Online)

nanotechnology applications. Several researchers applied the nonlocal elasticity theory to study the mechanical behaviors of nano plates/beams (Tounsi *et al*. 2013b, Benzair *et al*. 2008, Berrabah *et al.* 2013, Benguediab *et al.* 2014, Zemri *et al.* 2015, Larbi Chaht *et al.* 2015, Belkorissat *et al.* 2015, Kolahchi *et al*. 2015, Bounouara *et al.* 2016, Ghorbanpour Arani *et al*. 2016, Akbaş 2016a,b, Ahouel *et al.* 2016, Mouffoki *et al*. 2017, Bellifa *et al*. 2017a, Hanifi Hachemi Amar *et al*. 2017, Benadouda *et al.* 2017). Other works can be found on the effect of geometry and material properties of epoxy and carbon nanotubes on load transfer in carbon nanotube/epoxy composites under tension (Viet *et al.* 2017). Ebrahimi and Barati (2017) presented buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium. Karami and Shahsavari (2017) discussed the temperature-dependent flexural wave propagation in nanoplate-type porous heterogenous material subjected to in-plane magnetic field. Karami *et al*. (2017) examined the influences of triaxial magnetic field on the wave propagation behavior of anisotropic nanoplates. Nami *et al.* (2015) analyzed the thermal buckling response of functionally graded rectangular nanoplates based on nonlocal third-order shear deformation theory. Barati and Shahverdi (2016) proposed a four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions. Besseghier *et al.* (2017) investigated the free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory. Khetir *et al.* (2017) presented a new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates.

For the proper employment of graphene sheets, its behavior under in-plane or free in-plane force with small scale influences should be investigated. Hence, there is an important encouragement for obtaining proper understanding and theoretical modeling of the buckling of nanoplates. Since buckling of nanoscale plates is a considerable factor for proper design of nano-devices, numerous works are being proposed. Duan and Wang (2007) presented an exact closed-form solution for the axisymmetric bending of circular graphene sheets via the nonlocal continuum mechanics and the classical plate theory. Pradhan and Murmu (2009) utilized nonlocal elasticity model to study small-scale impacts on the stability of single-layered graphene sheets under biaxial compression. Furthermore, Pradhan and Murmu (2010) used the nonlocal elasticity model and differential quadrature procedure for the buckling behavior of rectangular single-layered graphene sheets under biaxial compression. Malekzadeh *et al.* (2011) studied the nonlocal parameter effect on the thermal stability of orthotropic nanoscale plates embedded in an elastic medium. They demonstrated that increasing the elastic medium coefficients, the influence of scale parameter on the thermal force ratio diminishes. Farajpour *et al.* (2012) investigated the stability of nanoplates under non-uniform compression with the nonlocal continuum mechanics model. Asemi *et al*. (2014) examined the influence of small scale on the thermal buckling of circular graphene sheets by employing the nonlocal elasticity theory. Sobhy (2014a) studied the natural frequency and buckling of orthotropic single-layered graphene sheets, resting on Pasternak’s elastic foundations with various boundary conditions. Zhang *et al*. (2015a) proposed a nonlocal continuum model for vibration of SLGSs based on the element-free kp-Ritz method. Zhang *et al.* (2015b) presented an investigation on the transient analysis of SLGSs by using the element-free kp-Ritz method. The classical plate theory is employed to describe the dynamic behavior of SLGSs. Nonlocal elasticity theory, in which nonlocal parameter is incorporated, is introduced to consider the small scale effect. Zhang *et al*. (2016a) investigated the vibration behavior of bilayer graphene sheets in a magnetic fields using classic plate theory combined with nonlocal elasticity theory to account for the small-scale effect. In addition, Zhang *et al.* (2016b) analyzed a geometrically nonlinear large deformation behavior of SLGSs is presented using the element-free kp-Ritz method. Zhang *et al*. (2017a) presented the mathematical modeling of the nonlinear vibration response of GSs using classic plate theory and nonlocal elasticity theory which accounts for the size effect. The numerical solutions are obtained through the element-free kp-Ritz method. Zhang *et al*. (2017b) discussed the vibration behavior of quadrilateral SLGSs in a magnetic field using classic plate theory and incorporating nonlocal elasticity theory. The element-free kp-Ritz method is utilized to perform the numerical simulation.

In the present article, the buckling properties of orthotropic single-layered graphene sheets (SLGSs) resting on two-parameter elastic foundations are studied using a new plate theory and nonlocal elasticity. The utilization of the integral term in the proposed kinematic led to a reducing in the number of variables and governing equations. Based on the nonlocal constitutive relations of Eringen, governing equations of nanoscale plates are obtained by employing using principle of virtual work. To prove the accuracy of the present formulation, the computed results are compared with reported by Sobhy (2014a). A detailed parametric investigation is presented to highlight the effects of the scale parameter, thickness-to-length ratio, and other parameters on the buckling of the nanoscale plates.

2. Theoretical formulation

2.1 Nonlocal elasticity theory: a review

In the case of the nonlocal linear elastic solids, the equations of motion take the following form (Eringen 1983, 2002)

$$t_{ij,j} + f_i = \rho\, \ddot{u}_i \tag{1}$$

in which ρ and f_i are, respectively, the mass density and the body (and/or applied) forces; u_i denotes the displacement vector; and t_{ij} is the stress tensor of the nonlocal elasticity expressed by

$$t_{ij}(X) = \int_V \alpha\left(\left|X'-X\right|\right)\sigma_{ij}(X')\,dv(X') \tag{2}$$

where X is a reference point in the body; $\alpha\left(\left|X'-X\right|\right)$ is the nonlocal kernel function; and σ_{ij} is the local stress tensor of local elasticity theory at any point X' in the body and respects the constitutive relations

$$\sigma_{ij}(X') = C_{ijkl}\,\varepsilon_{kl}(X') \tag{3}$$

$$\varepsilon_{kl}(X') = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right) \tag{4}$$

In the case of a general elastic material, where C_{ijkl} are the elastic modulus components with the symmetry characteristics $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$, and ε_{kl} is the strain tensor. It should be noted here that the boundary conditions involving tractions are based on the non-classical stress tensor t_{ij} and not on the classical stress tensor σ_{ij}.

The Characteristics of the nonlocal kernel $\alpha\left(\left|X'-X\right|\right)$ have been studied in detail by Eringen (1983). When $\alpha\left(\left|X\right|\right)$ takes on a Green's function of a linear differential operator $\&$, i.e.

$$\&\,\alpha\left(\left|X'-X\right|\right) = \delta\left(\left|X'-X\right|\right) \tag{5}$$

The nonlocal constitutive relation (2) is reduced to the differential equation

$$\& t_{ij} = \sigma_{ij} \tag{6}$$

and the integro-partial differential Eq. (1) is correspondingly reduced to the partial differential equation

$$\sigma_{ij} + \&\left(f_i - \rho\,\ddot{u}_i\right) = 0 \tag{7}$$

By matching the dispersion curves with lattice models, Eringen (1983, 2002) developed a non-classical model with the linear differential operator $\&$ expressed by

$$\& = 1 - \mu^2\nabla^2 \tag{8}$$

where $\mu = e_0 a$, a is an internal property length (lattice parameter, granular size or molecular diameters) and e_0 is a constant appropriate to each material. According to Eqs. (3), (4), (6) and (8), the constitutive relations may be written as

$$\left(1-\mu^2\nabla^2\right)t_{ij} = \sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \tag{9}$$

For simplicity and to avoid the use of the integro-partial differential relations, the model of nonlocal elasticity, presented by the Eqs. (6)-(9), has been widely employed for considering various problems of linear elasticity and mechanical behavior of micro-nanostructures.

2.1.1 Discussion of the nonlocal scaling parameter

The evaluation of the nonlocal parameter $e_0 = \mu / a$ in the nonlocal model has not been fully understood. The value of nonlocal parameter determined by various investigators is shown in the following Table 1.

Table 1 Values of nonlocal parameter available in lite rature

Investigators	$\mu = e_0 a$	Approach
Eringen (1983)	$0.39a$ *	Matching of the dispersion curves via nonlocal theory for plane wave and Born–Karman model of lattice dynamics at the end of the Brillouin zone
Eringen (1972)	$0.31a$ *	Comparison of the Rayleigh surface wave via nonlocal continuum mechanics and lattice dynamics
Hu *et al.* (2008)	$0.6a$ $	Comparison of transverse and torsional wave dispersion in CNTs via nonlocal elastic shell with molecular dynamics
Wang and Hu (2005)	$0.288a$ $	The flexural wave propagation in SWCNT via nonlocal Timoshenko beam model and molecular dynamic simulations
Zhang *et al.* (2005)	$0.82a$ $	Buckling analysis of SWCNT via Donnell shell theory and molecular mechanics simulations
Zhang *et al.* (2006)	$8.79a$ $	Elastic interactions between Stone-Wales and di-vacancy defects on carbon graphene sheet
Wang (2005)	< 2.1 nm	For a SWCNT if the measured wave propagation frequency value is assessed to be greater than 10THz
Sudak (2003)	$112.7a$ $	Critical buckling strain corresponding to various buckling modes for a double walled carbon nanotube
Wang *et al.* (2008)	0.7 nm	Comparing the length-dependent stiffness of SWCNT obtained from nonlocal elasticity with molecular simulation results
Duan *et al.* (2007)	$0 - 19\,a$ $	Using molecular dynamics results, for vibration of carbon nanotubes with nonlocal Timoshenko beam theory

* Here a is the lattice parameter

§ Here a is the carbon-carbon bond length in SWCNT or graphene (=1.42$A°$)

The table also indicates the approach used by the investigator(s) to assess the nonlocal coefficient. Duan and Wang (2007) have utilized the value of $e_0 a$ ranging from 0 to 2 nm for bending investigation of circular micro/nanoplates. Similar values are employed by many investigators for the investigation of nanostructures (Reddy 2007).

2.2 Kinematics and constitutive equations

The kinematic of the proposed theory is written as follows

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{10a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+k_2\, f(z)\int \theta(x,y)\,dy \qquad (10b)$$

$$w(x,y,z)=w_0(x,y) \qquad (10c)$$

(Bourada *et al*. 2016, Hebali *et al*. 2016, Bellifa *et al*. 2017b, El Haina *et al*. 2017, Menasria *et al*. 2017, Chikh *et al*. 2017)

where u_0, v_0, $w_0(x,y)$, and $\theta(x,y)$ are the 4 unknown displacement functions of mid-surface of the graphene sheet. Note that the integrals do not have limits. In the proposed model is considered terms with integrals instead of terms with derivatives. The constants k_1 and k_2 depends on the geometry.

In this research, the present higher-order shear deformation plate theory is used by setting

$$f(z)=z\left(\frac{5}{4}-\frac{5z^2}{3h^2}\right) \qquad (11)$$

It should be indicated that contrary to the FSDT, the proposed theory does not require shear correction coefficient. The kinematic relations can be written as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\ \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \qquad (12)$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix},$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}, \qquad (13a)$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_1\int\theta\,dy\\ k_2\int\theta\,dx\end{Bmatrix}$$

and

$$g(z)=\frac{df(z)}{dz} \qquad (13b)$$

The integrals considered in the above relations shall be resolved by a Navier type technique and can be written as follows:

where the coefficients A' and B' are considered according

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\qquad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y}, \qquad (14)$$

$$\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\quad \int\theta\,dy=B'\frac{\partial\theta}{\partial y}$$

to the type of solution utilized, in this case using Navier method. Therefore, A' and B' are written as follows

$$A'=-\frac{1}{\alpha^2},\quad B'=-\frac{1}{\beta^2},\ k_1=\alpha^2,\ k_2=\beta^2 \qquad (15)$$

where α and β are defined in expression (26).

2.3 Governing equations

Principle of virtual work can be used to determine the governing equations of the grapheme sheet. By employing the principle of virtual displacements (Bouderba *et al*. 2013, Tounsi *et al*. 2013c, Zidi *et al*. 2014, Ait Amar Meziane *et al*. 2014, Attia *et al*. 2015, Taibi *et al*. 2015, Al-Basyouni *et al*. 2015, Ait Yahia *et al*. 2015, Mahi *et al*. 2015, Bouderba *et al*. 2016, Boukhari *et al*. 2016, Beldjelili *et al*. 2016, Saidi *et al*. 2016, Bellifa *et al*. 2016, Bousahla *et al*. 2016, Houari *et al*. 2016, Zidi *et al*. 2017), the following governing equations can be determined

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \qquad (16a)$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0 \qquad (16b)$$

$$\frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+\overline{N}-\mu\nabla^2\overline{N}+k_w w-k_s\nabla^2 w=0 \qquad (16c)$$

$$-k_1 M_x^s-k_2 M_y^s-(k_1A'+k_2B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+k_1A'\frac{\partial S_{xz}^s}{\partial x}+k_2B'\frac{\partial S_{yz}^s}{\partial y}=0 \qquad (16d)$$

where k_w is Winkler's foundation stiffness and k_s is the shearing layer stiffness of the foundation.

with

$$\overline{N}=N_x^0\frac{\partial^2 w_0}{\partial x^2}+2N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+N_y^0\frac{\partial^2 w_0}{\partial y^2} \qquad (17)$$

The stress resultants N, M and S are defined by

$$\left(N_i,M_i^b,M_i^s\right)=\int_{-h/2}^{h/2}(1,z,f)\,\sigma_i\,dz,\quad (i=x,y,xy),$$
$$(i=x,y,xy)\ \text{and}\ \left(S_{xz}^s,S_{yz}^s\right)=\int_{-h/2}^{h/2}g\left(\tau_{xz},\tau_{yz}\right)dz \qquad (18)$$

2.4 Nonlocal stress resultants

The analysis is based on the fact that the elastic moduli of a graphene sheet in two perpendicular orientations are different, so the plate is taken to be anisotropic. In this present investigation, we emphasize much on orthotropicity of plates because nanoscale plates such as graphene sheets are reported to contain orthotropic properties (Reddy *et al*. 2006). The calculated elastic constants of the graphene sheet are found to conform to orthotropic material response (Reddy *et al*. 2006).

The nonlocal constitutive equations of an anisotropic grapheme sheet can be expressed as

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\\\tau_{yz}\\\tau_{xz}\end{Bmatrix}-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\\\tau_{yz}\\\tau_{xz}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\C_{12}&C_{22}&0&0&0\\0&0&C_{66}&0&0\\0&0&0&C_{55}&0\\0&0&0&0&C_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\\\gamma_{yz}\\\gamma_{xz}\end{Bmatrix}\tag{19}$$

where the stiffness coefficients, C_{ij}, can be defined by

$$C_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\; C_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\quad C_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},\\ C_{66}=G_{12},\quad C_{44}=G_{23},\quad C_{55}=G_{13},\tag{20}$$

Here, E_1, E_2 are Young's modulus, G_{12}, G_{23}, G_{13} are shear modulus, and ν_{12}, ν_{21} are Poisson's ratios. For the isotropic plate, these above material properties reduce to, $E_1=E_2=E$, $G_{12}=G_{23}=G_{13}=G$, $\nu_{12}=\nu_{21}=\nu$. The subscripts 1, 2, 3 correspond to *x*, *y*, *z* directions of Cartesian coordinate system, respectively.

By substituting Eq. (12) into Eq. (19) and the subsequent results into Eq. (18), the stress resultants are obtained as

$$\begin{Bmatrix}S^s_{xz}\\S^s_{yz}\end{Bmatrix}-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}S^s_{xz}\\S^s_{yz}\end{Bmatrix}=\begin{bmatrix}A^s_{55}&0\\0&A^s_{44}\end{bmatrix}\begin{Bmatrix}\gamma^0_{xz}\\\gamma^0_{yz}\end{Bmatrix}\tag{21a}$$

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\\M^b_x\\M^b_y\\M^b_{xy}\\M^s_x\\M^s_y\\M^s_{xy}\end{Bmatrix}-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}N_x\\N_y\\N_{xy}\\M^b_x\\M^b_y\\M^b_{xy}\\M^s_x\\M^s_y\\M^s_{xy}\end{Bmatrix}=\tag{21b}$$

$$\begin{bmatrix}A_{11}&A_{12}&0&B_{11}&B_{12}&0&B^s_{11}&B^s_{12}&0\\A_{12}&A_{22}&0&B_{12}&B_{22}&0&B^s_{12}&B^s_{22}&0\\0&0&A_{66}&0&0&B_{66}&0&0&B^s_{66}\\B_{11}&B_{12}&0&D_{11}&D_{12}&0&D^s_{11}&D^s_{12}&0\\B_{12}&B_{22}&0&D_{12}&D_{22}&0&D^s_{12}&D^s_{22}&0\\0&0&B_{66}&0&0&D_{66}&0&0&D^s_{66}\\B^s_{11}&B^s_{12}&0&D^s_{11}&D^s_{12}&0&H^s_{11}&H^s_{12}&0\\B^s_{12}&B^s_{22}&0&D^s_{12}&D^s_{22}&0&H^s_{12}&H^s_{22}&0\\0&0&B^s_{66}&0&0&D^s_{66}&0&0&H^s_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon^0_x\\\varepsilon^0_y\\\gamma^0_{xy}\\k^b_x\\k^b_y\\k^b_{xy}\\k^s_x\\k^s_y\\k^s_{xy}\end{Bmatrix}$$

and stiffness components are given as

$$\left\{A_{ij},B_{ij},D_{ij},B^s_{ij},D^s_{ij},H^s_{ij}\right\}=\\ \int_{-h/2}^{h/2}C_{ij}\left\{1,z,z^2,f(z),z\,f(z),f^2(z)\right\}dz,\quad i,j=1,2,6\tag{22a}$$

$$A^s_{ij}=\int_{-h/2}^{h/2}C_{ij}\left[g(z)\right]^2dz,\quad i,j=4,5\tag{22b}$$

2.5 Governing equations in terms of displacements

The nonlocal governing equations of the present model can be given in terms of displacements (w_0 , θ) by substituting stress resultants in Eq. (21) into Eq. (16) as

$$A_{11}\frac{\partial^2u_0}{\partial x^2}+A_{66}\frac{\partial^2u_0}{\partial y^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2v_0}{\partial x\,\partial y}=0,\tag{23a}$$

$$A_{22}\frac{\partial^2v_0}{\partial y^2}+A_{66}\frac{\partial^2v_0}{\partial x^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2u_0}{\partial x\,\partial y}=0,\tag{23b}$$

$$-D_{11}\frac{\partial^4w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4w_0}{\partial x^2\partial y^2}-\\ D_{22}\frac{\partial^4w_0}{\partial y^4}+\left(D^s_{11}k_1+D^s_{12}k_2\right)\frac{\partial^2\theta}{\partial x^2}\\ +2\left(D^s_{66}\left(k_1A'+k_2B'\right)\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+\left(D^s_{12}k_1+D^s_{22}k_2\right)\frac{\partial^2\theta}{\partial y^2}\\ +\left(1-\mu\nabla^2\right)\left(N^0_x\frac{\partial^2w_0}{\partial x^2}+2N^0_{xy}\frac{\partial^2w_0}{\partial x\partial y}+N^0_y\frac{\partial^2w_0}{\partial y^2}\right)\\ +\left(1-\mu\nabla^2\right)\left(K_ww_0-K_s\nabla^2w_0\right)=0\tag{23c}$$

$$\left(D^s_{11}k_1+D^s_{12}k_2\right)\frac{\partial^2w_0}{\partial x^2}+2\left(D^s_{66}\left(k_1A'+k_2B'\right)\right)\frac{\partial^4w_0}{\partial x^2\partial y^2}+\\ \left(D^s_{12}k_1+D^s_{22}k_2\right)\frac{\partial^2w_0}{\partial y^2}-H^s_{11}k_1^2\,\theta-H^s_{22}\,k_2^2\,\theta\\ -2H^s_{12}k_1k_2\theta-\left(\left(k_1A'+k_2B'\right)^2H^s_{66}\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+\\ A^s_{44}\left(k_2B'\right)^2\frac{\partial^2\theta}{\partial y^2}+A^s_{55}\left(k_1A'\right)^2\frac{\partial^2\theta}{\partial x^2}=0\tag{23d}$$

It is observed from Eq. (23) that the in-plane displacements (u_0, v_0) are uncoupled from the transverse displacements (w_0 and θ). Thus, the equations of motion for the transverse response of the plate are reduced to Eqs. (23(c))- (23(d)).

We consider the following relations for the present compression study

Here, the term ξ denotes the compression ratio $\xi = N_y^0 / N_x^0$.

3. Analytical solutions

A simply supported rectangular orthotropic nanoplate with length a and width b is considered here. Based on Navier method, the following expansions of generalized displacements are chosen to automatically satisfy the simply supported boundary conditions

$$\begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{25}$$

where W_{mn} and X_{mn} are arbitrary coefficients to be determined.

with

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{26}$$

By substituting equation (25) into equations (23), one obtains

$$[K]\{\Delta\} = \{0\} \tag{27}$$

where $\{\Delta\}$ denotes the columns

$$\{\Delta\} = \{W_{mn}, \quad X_{mn}\}^T \tag{28}$$

The elements K_{ij} of the symmetric matrix $[K]$, for the shear deformation plate theories, are given by

$$\begin{aligned} K_{33} &= -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right) - \\ &\quad k_w - \mu k_w\left(\alpha^2 + \beta^2\right) \\ &\quad + \left(N^T - k_s\right)\left(\alpha^2 + \beta^2\right) + \mu\left(N^T - k_s\right)\left(\alpha^2 + \beta^2\right)^2 \\ K_{12} &= -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\, D_{66}^s\alpha^2\beta^2 - \\ &\quad k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\ K_{22} &= -H_{11}^s k_1^2 - 2H_{12}^s k_1 k_2 - H_{22}^s k_2^2 - \\ &\quad (k_1 A' + k_2 B')^2 H_{66}^s\alpha^2\beta^2 - \\ &\quad \left(k_1 A'\right)^2 A_{55}^s\alpha^2 - \left(k_2 B'\right)^2 A_{44}^s\beta^2 \end{aligned} \tag{29}$$

4. Numerical results

Numerical results of the buckling loads of orthotropic SLGSs resting on Pasternak's elastic foundations are presented and discussed in this section using the proposed theory. Both isotropic and orthotropic material characteristics are illustrated in this work. For the stability problem, the SLGS is subjected to equal compressive in-plane loads in the x and y directions ($\xi = 1$). The orthotropic material characteristics are considered as (Sobhy 2014b)

$$\frac{E_1}{E_2} = 10, \quad \frac{G_{12}}{E_2} = 0.6, \quad \frac{G_{12}}{E_2} = 0.5, \quad G_{23} = G_{13}, \quad \nu_{23} = 0.3 \tag{30}$$

Many examples have been solved analytically by employing the following fixed data (unless otherwise stated) $a/h = 10$, $b/a = 1$, $m = n = 1$, $K_w = K_s = 100$. The employed non-dimensional quantities are

$$N_{cr} = \frac{N^0 a^2}{D\pi^2}, \quad K_w = \frac{k_w a^4}{D}, \quad K_s = \frac{k_s a^2}{D}, \quad D = \frac{h^3 E_2}{12(1 - \nu_{12}\nu_{21})} \tag{31}$$

Tables 2 and 3 explain the critical buckling load N_{cr} of isotropic and orthotropic SLGSs with or without elastic foundations for various values of nonlocal parameter μ and the side-to-thickness ratio a/h. The computed results are compared with those reported by Sobhy (2014a). It should be noted that when $\mu = 0$, we determine the natural frequency as that of nonlocal elasticity model. The critical buckling load of the local continuum model is higher than that of the nonlocal one. From Tables 2 and 3, one an see that excellent agreement exists between the results of this method involving only two unknowns and those of the sinusoidal shear deformation plate theory (SSDT) of Sobhy (2014a).

Fig. 1 depicts the small-scale effects on the non-dimensional buckling load for bi-axially compressed small-scale SLGSs. The results are compared with those generated using the SSDT proposed by Sobhy (2014a) and this for different values of the thickness ratio a/h. It can be seen that the two theories give the same results. However, it is noticed that the present theory involves only two governing equations, contrary to the SSDT of Sobhy (2014a) where three equations are needed. It can also be seen that as the nonlocal scale parameter increases the buckling load decreases.

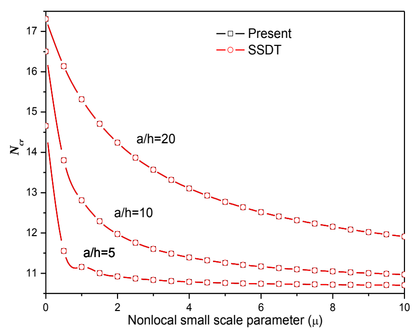

Fig. 1 A comparison of the non-dimensional buckling load for an orthotropic square SLGS ($K_w = 100$ and $K_s = 100$)

Table 2 Critical buckling load N_{cr} of isotropic single-layered graphene sheets with or without elastic foundations for various values of nonlocal parameter μ

μ	Theory	$(K_w = 0, K_s = 0)$				$(K_w = 100, K_s = 100)$			
		$a/h = 5$	10	20	30	$a/h = 5$	10	20	30
0	Sobhy (2014a)	4.01159	5.85591	6.66298	6.84003	14.65701	16.50133	17.30840	17.48544
	Present	4.00553	5.85401	6.66249	6.83981	14.65094	16.49943	17.30791	17.48522
1	Sobhy (2014a)	0.51232	2.16281	4.66960	5.74924	11.15774	12.80823	15.31502	16.39466
	Present	0.51155	2.16211	4.66925	5.74905	11.15697	12.80753	15.31467	16.39447
2	Sobhy (2014a)	0.27363	1.32634	3.59428	4.95850	10.91905	11.97176	14.23970	15.60392
	Present	0.27322	1.32591	3.59402	4.95834	10.91864	11.97133	14.23944	15.60376
3	Sobhy (2014a)	0.18667	0.95643	2.92152	4.35898	10.83209	11.60185	13.56694	15.00439
	Present	0.18639	0.95613	2.92130	4.35884	10.83180	11.60154	13.56672	15.00425
4	Sobhy (2014a)	0.14165	0.74786	2.46090	3.88879	10.78707	11.39328	13.10631	14.53421
	Present	0.14144	0.74762	2.46071	3.88866	10.78685	11.39304	13.10613	14.53408

Fig. 2 present the variation of the non-dimensional buckling load against the length of the orthotropic SLGS for different nonlocal parameter. It can be seen that the buckling load parameter increase for all values of nonlocal scale parameter by increasing of the length of the SLGS.

The effect of the elastic foundation parameters on the non-dimensional buckling load of orthotropic SLGS vs. the length of the SLGS is exhibited in Fig. 3.

It can be seen that as the length and the elastic foundation p arameters increase, buckling load increases. Further, it can b e observed that with the increasing of the length of SLGS, t he differences among the curves are almost fixed.

Finally, the non-dimensional buckling load orthotropic square SLGS vs. the thickness ratio a/h are plotted in Fig. 4 for different values of mode numbers m and n. From this figure, it can be observed that the buckling load increases monotonically as the ratio a/h increases and the mode numbers decrease.

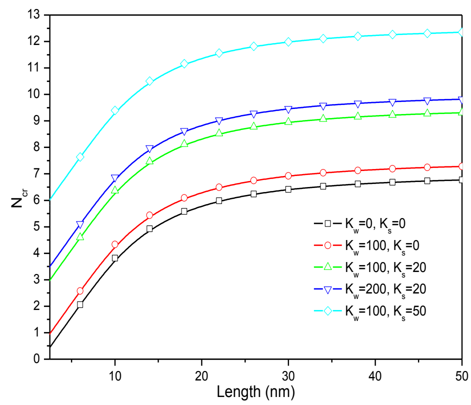

Fig. 3 Variation of critical buckling load against the length of orthotropic square SLGS for various values of the elastic foundation stiffness ($\mu = 4$)

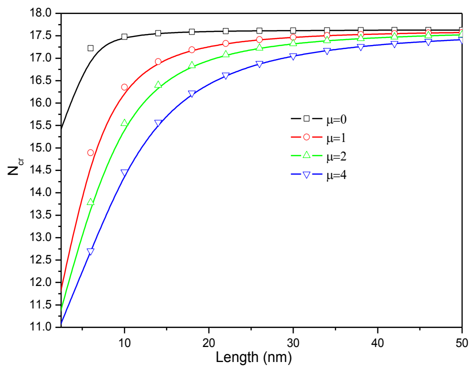

Fig. 2 Variation of the non-dimensional critical buckling load against the length of orthotropic square SLGS ($K_w = 100$ and $K_s = 100$)

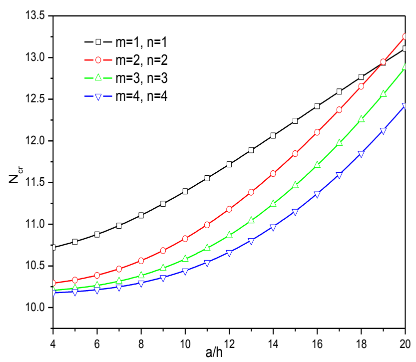

Fig. 4 Variation of buckling load versus the thickness ratio a/h of orthotropic square SLGS for various values of the mode numbers m and n ($\mu = 4$, $K_w = 100$ and $K_s = 100$)

5. Conclusions

In the present study, buckling response of orthotropic SLGS resting on tow parameter elastic foundations is investigated using continuum models. The model considers the effects of small scale and the parabolic variation of the transverse shear strains within the thickness of the SLGS and thus, it avoids the use of shear correction factors. The main advantage of the proposed novel nonlocal refined plate theory over the existing higher-order shear deformation theories is that the present ones involve fewer variables as well as governing equations. The computational cost can therefore be reduced. From the present study following conclusions are drawn:

- The obtained results are very agreement with those available in literature.
- The difference between the results of nonlocal elasticity theory and the local one diminishes as the ratio a/h increases.
- The buckling load is proportional to the elastic foundation stiffness and they are inversely proportional to the nonlocal scale parameter.
- Influences of mode numbers, thickness ratio and length of SLGS on the buckling load are investigated.
- As the nonlocal parameter (μ) increases, the non-dimensional buckling load obtained for the nonlocal theory become smaller than those for its local counterpart. The reduction may be explained as follows: the small scale effect makes the SLGS more flexible as the nonlocal model may be viewed as atoms linked by elastic springs (Boumia *et al.* 2014) while the local continuum model assumes the spring constant to take an infinite value.

Finally, an improvement of present work will be considered in the future work to take into account the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Larbi Chaht et al. 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016, Benbakhti *et al*. 2016, Benahmed *et al.* 2017, Sekkal *et al.* 2017, Bouafia *et al.* 2017, Benchohra *et al.* 2018, Abualnour *et al*. 2018).

References

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Ahouel, M., Houari, M.S.A., AddaBedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct*., **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater*., **16**(3), 293-318.

Akbaş, Ş.D. (2016a), "Wave propagation of a functionally graded beam in thermal environments", *Int. J. Eng. Appl. Sci.*, **8**(2), 30-39.

Akbaş, Ş.D. (2016b), "Forced vibration analysis of viscoelastic nanobeams embedded in an elastic medium", *Smart Struct. Syst.*, **18**(6), 1125-1143.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct*., **125**, 621-630.

Asemi, S.R., Farajpour, A., Borghei, M. and Hassani, A.H. (2014), "Thermal effects on the stability of circular graphene sheets via nonlocal continuum mechanics", *Lat. Am. J. Solids. Struct.*, **11**, 704-724.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct*., **18**(1), 187-212.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech*., **53**(6), 1143-1165.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech*., **60**(4), 707-727.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst*., **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J Braz. Soc. Mech. Sci. Eng*., **38**(1), 265-275.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017a), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech*., **62**(6), 695-702.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017b), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), "An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities", *Earthq. Struct.*, **13**(3), 255-265.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng.*, **12**(1), 9-34.

Benbakhti, A., Bachir Bouiadjra, M., Retiel, N. and Tounsi, A. (2016), "A new five unknown quasi-3D type HSDT for thermomechanical bending analysis of FGM sandwich plates", *Steel Compos. Struct*., **22**(5), 975-999.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech*., (In press).

Benguediab, S., Tounsi, A., Zidour, M. and Semmah, A. (2014), "Chirality and scale rffects on mechanical buckling properties of zigzag double-walled carbon nanotubes", *Compos. Part B*, **57**, 21-24.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct*., **23**(4), 423-431

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater*., **15**, 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst*., **19**(6), 601-614.

enzair, A., Tounsi, A., Besseghier, A., Heireche, H., Moulay, N. and Boumia, L. (2008), "The thermal effect on vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory", *J. Phys. D: Appl. Phys.*, **41**, 225404.

Berrabah, H.M., Tounsi, A., Semmah, A. and Adda Bedia, E.A. (2013), "Comparison of various refined nonlocal beam theories for bending, vibration and buckling analysis of nanobeams", *Struct. Eng. Mech*., **48**(3), 351-365.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst*., **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct*., **14**(1), 85-104.

Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech*., **58**(3), 397-422.

Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech*., **57**(5), 837-859.

Boumia, L., Zidour, M., Benzair, A. and Tounsi, A. (2014), "A Timoshenko beam model for vibration analysis of chiral single-walled carbon nanotubes", *Physica E*, **59**, 186–191.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "Anew simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct*., **18**(2), 409-423.

Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct*., **21**(6), 1287-1306.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth*., **11**(6), 1350082.

Chen, Y., Lee, J.D. and Eskandarian, A. (2004), "Atomistic viewpoint of the applicability of microcontinuum theories", *Int. J. Solids Struct.*, **41**, 2085-2097.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst*., **19**(3), 289-297.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng*., **11**(5), 671-690.

Duan, W.H. and Wang, C.M. (2007), "Exact solutions for axisymmetric bending of micro/nanoscale circular plates based on nonlocal plate theory", *Nanotechnology*, **18**, 385704.

Duan, W.H., Wang, C.M. and Zhang, Y.Y. (2007), "Calibration of nonlocal scaling effect parameter for free vibration of carbon nanotubes by molecular dynamics", *J. Appl. Phys.*, **101**, 024305.

Ebrahimi, F. and Barati, M.R. (2017), "Buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium", *J. Braz. Soc. Mech. Sci. Eng.*, **39**, 937-952.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech*., **63**(5), 585-595.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**, 795-810.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**, 4703-4710.

Eringen, A.C. (2002), "Nonlocal continuum field theories", Springer, New York, NY.

Eringen, A.C. and Edelen, D.G.B. (1972), "On non-local elasticity", *Int. J. Eng. Sci.*, **10**, 233.

Eringen, A.C. (1972), "Linear theory of non-local elasticity and dispersion of plane waves", *Int. J. Eng. Sci.*, **10**, 425.

Eringen, A.C. (1996), "Non-local polar field models", Academic, New York.

Farajpour, A., Shahidi, A.R., Mohammadi, M. and Mahzoon, M. (2012), "Buckling of orthotropic micro/nanoscale plates under linearly varying in-plane load via nonlocal continuum mechanics", *Compos. Struct.*, **94**, 1605-1615.

Geim, A.K. and Novoselov, K.S. (2007), "The rise of grapheme", *Nat. Mater.*, **6**, 183.

Ghorbanpour Arani, A., Cheraghbak, A. and Kolahchi, R. (2016), "Dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory", *Struct. Eng. Mech*., **60**(3), 489 - 505.

Gomez-Navarro, C., Weitz, R.T., Bittner, A.M., Scolari, M., Mews, A., Burghard, M. and Kern, K. (2007), "Electronic transport properties of individual chemically reduced graphene oxide sheets", *Nano Lett.*, **7**, 3499.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct*., **18**(1), 235-253.

Hanifi Hachemi Amar, L., Kaci, A. and Tounsi, A. (2017), "On the size-dependent behavior of functionally graded micro-beams with porosities", *Struct. Eng. Mech*., **64**(5), 527-541.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech. - ASCE*, **140**(2), 374-383.

Hebali, H., Bakora, A., Tounsi, A. and Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos. Struct*.,

22(3), 473-495.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct*., **22**(2), 257-276.

Hu, Y.G., Liew, K.M., Wang, Q., He, X.Q. and Yakobson, B.I. (2008), "Nonlocal shell model for elastic wave propagation in singleand double-walled carbon nanotubes", *J. Mech. Phys. Solids*, **56**(12), 3475.

Karami, B, Shahsavari, D. and Li, L. (2017), "Temperature-dependent flexural wave propagation in nanoplate-type porous heterogeneous material subjected to in-plane magnetic field", *J. Therm. Stresses*, 1-17. DOI: 10.1080/01495739.2017.1393781.

Karami, B., Janghorban, M. and Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel Compos. Struct.*, **25**(3), 361-374.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech.*, **64**(4), 391-402.

Kolahchi, R., Bidgoli, A.M.M. and Heydari, M.M. (2015), "Size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium", *Struct. Eng. Mech.*, **55**(5), 1001 - 1014.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442.

Li, X.L., Wang, X.R., Zhang, L., Lee, S.W. and Dai, H.J. (2008), "Chemically derived, ultrasmooth graphene nanoribbon semiconductors", *Science*, **319**, 1229.

Lu, P., Lee, H.P., Lu, C. and Zhang, P.Q. (2006), "Dynamic properties of flexural beams using a non-local elasticity model", *J. Appl. Phys.*, **99**, 073510.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model*., **39**(9), 2489-2508.

Malekzadeh, P., Setoodeh, A.R. and Alibeygi Beni, A. (2011), "Small scale effect on the thermal buckling of orthotropic arbitrary straight-sided quadrilateral nanoplates embedded in an elastic medium", *Compos. Struct.*, **93**, 2083-2089.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct.*, **25**(2), 157-175.

Mouffoki, A., AddaBedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst*., **20**(3), 369-383.

Nami, M.R., Janghorban, M. and Damadam, M. (2015), "Thermal buckling analysis of functionally graded rectangular nanoplates based on nonlocal third-order shear deformation theory", *Aerosp. Sci. Technol.*, **41**, 7-15.

Narendar, S., Roy Mahapatra, D. and Gopalakrishnan, S. (2011), "Prediction of nonlocal scaling parameter for armchair and zigzag single-walled carbon nanotubes based on molecular structural mechanics, nonlocal elasticity and wave propagation", *Int. J. Eng. Sci.*, **49**, 509-522.

Novoselov, K.S., Geim, A.K., Morozov, S.V., Jiang, D., Zhang, Y., Dubonos, S.V., Grigorieva, I.V. and Firsov, A.A. (2004), "Electric field effect in atomically thin carbon films", *Science*, **306**, 666.

Obraztsov, A.N., Obraztsova, E.A., Tyurnina, A.V. and Zolotukhin, A.A. (2007), "Chemical vapor deposition of thin graphite films of nanometer thickness", *Carbon*, **45**, 2017.

Ohta, T., Bostwick, A., Seyller, T., Horn, K. and Rotenberg, E. (2006), "Controlling the electronic structure of bilayer grapheme", *Science*, **313**, 951.

Oshima, C., Nagashima, A. (1997), "Ultra-thin epitaxial films of graphite and hexagonal boron nitride on solid surfaces", *J. Phys. Condens. Matter*, **9**, 1.

Peddieson, J., Buchanan, G.R. and McNitt, R.P. (2003), "Application of nonlocal continuum models to nanotechnology", *Int. J. Eng. Sci.*, **41**, 305.

Pradhan, S.C. and Murmu, T. (2009), "Small scale effect on the buckling of single-layered graphene sheets under biaxial compression via nonlocal continuum mechanics", *Comput. Mater. Sci.*, **47**(1), 268-274.

Pradhan, S.C. and Murmu, T. (2010), "Small scale effect on the buckling analysis of single-layered graphene sheet embedded in an elastic medium based on nonlocal plate theory", *Physica E*, **42**, 1293-1301.

Reddy, J.N. (2007), "Nonlocal theories for bending, buckling and vibration of beams", *Int. J. Eng. Sci.*, **45**, 288.

Reddy, C.D., Rajendran, S. and Liew, K.M. (2006), "Equilibrium configuration and elastic properties of finite grapheme", *Nanotechnology*, **17**, 864-870.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech*., **64**(6), 737-749.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng.*, **11**(2), 289-307.

Sobhy, M. (2014a), "Natural frequency and buckling of orthotropic nanoplates resting on two-parameter elastic foundations with various boundary conditions", *J. Mech.*, **30**(5), 443-453.

Sobhy, M. (2014b), "Generalized two-variable plate theory for multi-layered graphene sheets with arbitrary boundary conditions," *Acta Mechanica*, **225**(9), 2521-2538.

Soldano, C., Mahmood, A. and Dujardin, E. (2010), "Production, properties and potential of grapheme", *Carbon*, **48**(8), 2127-2150.

Sudak, L.J. (2003), "Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics". *J. Appl. Phys.*, **94**, 7281.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater*., **17**(2), 99 - 129.

Tounsi, A, Benguediab, S., Adda Bedia, E.A., Semmah, A. and Zidour, M. (2013a), "Nonlocal effects on thermal buckling properties of double-walled carbon nanotubes", *Adv. Nano Res.*, **1**(1), 1-11.

Tounsi, A., Semmah, A. and Bousahla, A.A. (2013b), "Thermal buckling behavior of nanobeam usin an efficient higher-order nonlocal beam theory", *J. Nanomech. Micromech. - ASCE*, **3**, 37-42.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013c), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol*., **24**(1), 209-220.

Viet, N.V., Wang, Q. and Kuo, W.S. (2017), "A studying on load transfer in carbon nanotube/epoxy composites under tension", *J. Model. Mech. Mater.*, **1**(1), 20160153.

Wang, L.F. and Hu, H.Y. (2005), "Flexural wave propagation in single-walled carbon nanotubes", *Phys. Rev. B*, **71**, 195412.

Wang, Q. (2005), "Wave propagation in carbon nanotubes via

nonlocal continuum mechanics", *J. Appl. Phys.*, **98**, 124301.
Wang, Q., Han, Q.K. and Wen, B.C. (2008), "Estimate of material property of carbon nanotubes via nonlocal elasticity", *Adv. Theor. Appl. Mech.*, **1**(1), 1-10.
Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.
Zhang, Y.Q., Liu, G.R. and Xie, X.Y. (2005), "Free transverse vibration of double-walled carbon nanotubes using a theory of nonlocal elasticity", *Phys. Rev. B*, **71**, 195404.
Zhang, X., Jiao, K., Sharma, P. and Yakobson, B.I. (2006), "An atomistic and non-classical continuum field theoretic perspective of elastic interactions between defects (force dipoles) of various symmetries and application to grapheme", *J. Mech. Phys. Solids*, **54**, 2304.
Zhang, Y, Lei, Z.X., Zhang, L.W., Liew, K.M. and Yu, J.L. (2015a), "Nonlocal continuum model for vibration of single-layered graphene sheets based on the element-free kp-Ritz method", *Eng. Anal. Bound. Elem.*, **56**, 90-97.
Zhang, Y, Zhang, L.W., Liew, K.M. and Yu, J.L. (2015b), "Transient analysis of single-layered graphene sheet using the kp-Ritz method and nonlocal elasticity theory", *Appl. Math. Comput.*, **258**, 489-501.
Zhang, Y, Zhang, L.W., Liew, K.M. and Yu, J.L. (2016a), "Free vibration analysis of bilayer graphene sheets subjected to in-plane magnetic fields", *Compos. Struct.*, **114**, 86-95.
Zhang, Y., Zhang, L.W., Liew, K.M. and Yu, J.L. (2016b), "Nonlocal continuum model for large deformation analysis of SLGSs using the kp-Ritz element-free method", *Int. J. Non-Linear Mech.*, **79**, 1-9.
Zhang, L.W., Zhang, Y. and Liew, K.M. (2017a), "Modeling of nonlinear vibration of graphene sheets using a meshfree method based on nonlocal elasticity theory", *Appl. Math. Model.*, **49**, 691-704.
Zhang, L.W., Zhang, Y. and Liew, K.M. (2017b), "Vibration analysis of quadrilateral graphene sheets subjected to an in-plane magnetic field based on nonlocal elasticity theory", *Compos. Part B: Eng.*, **118**, 96-103.
Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol*., **34**, 24-34.
Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153.

CC