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Application of couple sparse coding ensemble on structural damage detection

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Abstract. A method is proposed to detect structural damages in the presence of damping using noisy data. This method uses Frequency Response Function (FRF) and Mode-Shapes as the input parameters for a system of Couple Sparse Coding (CSC) to study the healthy state of the structure. To obtain appropriate patterns of FRF for CSC training, Principal Component Analysis (PCA) technique is adopted to reduce the full-size FRF to overcome over-fitting and convergence problems in machine-learning training. To verify the proposed method, a numerical two-story frame structure is employed. A system of individual CSCs is trained with FRFs and mode-shapes, and then termed ensemble to detect the health condition of the structure. The results demonstrate that the proposed method is accurate in damage identification even in presence of up to 20% noisy data and 5% unconsidered damping ratio. Furthermore, it can be concluded that CSC ensemble is highly efficient to detect the location and the severity of damages in comparison to the individual CSC trained only with FRF data.

Keywords: couple sparse coding; damage detection; frequency response function; principal component analysis; ensemble

1. Introduction

Damage identification of structures from changes in their vibrational characteristics is an inverse problem of important practical value. Every Structural Health Monitoring (SHM) program must necessarily deal with this concept. Noteworthy advances have been achieved on this topic in the last three decades, both from the theoretical and the applied points of view (Dilenaa *et al*. 2015).

Among all the various techniques of SHM, vibration based monitoring is a major one since vibration measurements can be used to detect concealed damages which cannot be observed by visual inspection. The basic idea of vibration-based health monitoring is the fact that damage frequently alters structural features such as stiffness and damping which can be revealed from the measured vibration responses of the structure (Khoshnoudian and Esfandiari 2011, Amezquita-Sanchez and Adeli 2016, Kaveh *et al.* 2016, Khoshnoudian *et al*. 2017).

Vibration-based methods inspect changes in the features extracted directly from measured data through signal processing methods and damage detection algorithms (Qiao *et al.* 2012). Pattern-recognition techniques are combined into the signal-based damage detection as an improvement for feature extraction, selection, and analysis. Several studies have implemented successful experimental applications of current procedure for structural damage detection (Jiang and Adeli 2007, Qiao *et al.* 2009).

Many dynamic characteristics of the structure could be used for damage detection, e.g. natural frequencies, mode-shapes and their derivative and frequency response functions. Comparing natural frequencies and mode-shapes it turns out that the mode-shapes perform better representing the damage occurrence despite of the fact that measurement of modal amplitude are usually affected by errors larger than those affecting natural frequencies. Mode-shape data contains direct information of the structural change character, although their capability for damage identifying requires the application of identification strategies (Dilenaa *et al.* 2015). Roy and Ray-Chaudhuri (2013) interconnected the effect of damage with the change in mode-shape, slope of mode-shape, and curvature of mode-shape. They used a numerical example to show the robustness of this change in mode-shape and its derivatives for damage identification. It was concluded that for SHM, mode-shapes are much more suitable than natural frequencies especially in cases in which different damages could similarly influence the natural frequencies, although using mode-shapes for damage detection has some difficulties and uncertainties (Roy and Ray-Chaudhuri 2013).

Modal identification can be time-consuming and the curve fitting process adds some inevitable errors. Also lots of information may be dismissed by using just mode-shapes. Thus, the direct use of measured data for damage detection (e.g., in the form of Frequency Response Functions, FRFs), may represent significant advantages. Furthermore, in some cases some of the mode-shapes do not sense any notable change when the damage is located close to a particular node. The use of FRFs can effectively overcome lots of these problems (Maia *et al*. 2003).

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Although, there are many other concerns working with FRF due to their large size of data especially in structures with high number of degrees of freedom.

Lee and Shin introduced a FRF based damage detection technique using spatially incomplete FRFs (Lee and Shin 2002). Esfandiari *et al*. developed a new method using FRFs and quasi-linear sensitive equations for model updating the structures (Esfandiari *et al.* 2009). Shadan *et al.* developed a method based on a sensitivity-based model updating approach which utilizes a pseudo-linear sensitivity equation. The experimental setup consists of a free-free aluminum beam which was employed to verify the accuracy of the developed approach (Shadan *et al.* 2016). Moreover, several researchers studied the structural damage identification by means of FRF derivatives such as FRF curvature method (Sampaio *et al*. 1999), FRF differences (Trendafilova and Heylen 2003) and PCA-compressed residual FRFs (Li *et al*. 2011).

In recent decades, beside sensitivity method which try to find a reliable straight relation between FRF changes and structural damage, there has been an increasing interest in using Artificial Intelligence (AI) make a reasonable connection between FRF data and damage properties. Based on recent investigations, different types of AI methods consisting of Artificial Neural Network (ANN), Genetic Algorithm (GA) and Support Vector Machine (SVM) are applied in damage detection problems (Wang 2015, He *et al.* 2013, Aydin and Kisi 2015). Among various artificial intelligent method, the Multi-Layer Perceptron (MLP) ANN is the most commonly used algorithm in these problems (Hakim and Abdul Razak 2013, Mehrjoo *et al*. 2008, Osama Abdeljaber *et al.* 2017, Abolbashari *et al.* 2014).

A major challenge in utilizing FRF data for SHM with the pattern recognition approach is the influence of noise, the large size of FRF matrixes and damping issues. The direct use of raw FRF data with their large size will consequently cause over-fitting during neural network training. This contributes to an impractical ANN in terms of its training and convergence stability (Zang and Imregun 2001). Therefore, a linear data compression method named Principal Component Analysis (PCA) is used to reduce dimensionality of the FRF data for feasible application of the ANNs. The PCA technique has been applied by several researchers. Dackermann *et al*. (2010) have surveyed a damage detection approach to determine damage in a two-story frame structure using FRF data as inputs of the ANNs. They applied the PCA approach to compress FRF data (Dackermann *et al.* 2010). Dackermann *et al.* (2011) applied ANN and FRF data for health state evaluation of timber bridge structures. Also an experimental four-girder timber-bridge and 12 different damage scenarios containing three different damage severities at four different locations was investigated (Dackermann *et al.* 2011).

To overcome the effects of highly noise polluted data and over-fitting issue in ANN, Dackermann *et al.* (2013) proposed an ANN training approach based on network ensemble to respect individual characteristics of different measurement locations. They developed their method for damage identification of member connections and localization of added mass in a two-story frame structure. They also concluded that the network ensemble approach is efficient in filtering poor results from underperforming networks. They considered up to 10% random noise for their numerical FRF data, while the damping was considered in their study (Dackermann *et al.* 2013).

Neural network ensemble is a learning paradigm in which many neural networks are jointly used to solve a problem (Zhou *et al.* 2002). Hansen and Salamon demonstrated that the generalization ability of machine-learning such as neural network system can be significantly improved through ensembling a number of networks (Hansen and Salamon 1990). Recently ensembling has more applications in both neural networks and machine learning communities. In this method several neural networks have been trained individually and then combined their predictions. The most principal approaches for ensemble networks are plurality or majority voting (Hansen L.K. and Salamon 1990), simple averaging (Opitz and Shavlik 1996) and weighted averaging (Perrone and Cooper 1993) to combine the predictions of each individual system.

Marwala and Hunt employed two-stage hierarchical neural network ensemble to identify damage severity in structures. In their proposed method, first, two individual networks were trained. The outcomes of the two individual networks were then jointly fed into a third ensemble network to predict the health condition of a cantilever beam (Marwala and Hunt 1999).

Moreover, Marwala conducted an experimental demonstration of a new neural network ensemble approach on a cylindrical shells. They fed their generated data separately into three neural networks, then combined them in an ensemble network. It was found that in contrast to many existing methods, the ensemble method identifies the damage cases better than any other individual network in practical works (Marwala 2000).

Along with and superior to the ANNs, sparse representation (SR) is one of the currently developed methods that attracted high attentions in pattern recognition and machine learning community (Liu *et al*. 2011). In sparse method, a dictionary is defined as a set of the active features. A few active features are expressed a data point adequately. Wright *et al.* (2009) showed that the SR is a powerful tool for many pattern recognition tasks, including face recognition and object classification (Wright *et al*. 2009). The advantages of SR is that the dictionary can be over-completed which will allow more flexibility in data representation. Huang *et al.* developed a sparse representation recovery method which is invariant to image plane transformation to deal with the misalignment and pose variation in face reorganization (Huang *et al*. 2008).

In scope of SHM, Wang et al. presented a comparison between the SR method with Fourier discriminant method. These approaches have been contributed to damage localization of a bridge laboratory-scale model, where it have been simulated the vehicle-structure interaction. The results demonstrated that SR method compared to Fourier discriminant method allows for higher classification accuracy (Wang *et al.* 2013). Zolfaghari et al. presented a couple sparse coding (CSC) based on the simple SR, which proposed a two-step approach that models the mapping from 2D human silhouettes to 3D human body pose configurations using training samples. It is concluded that the couple sparse coding algorithm significantly improves representations and has less estimation errors compared to simple sparse coding algorithm (Zolfaghari *et al.* 2014).

In previous investigations, the frequency range of the FRF needed to be selected with regards to the resonance and anti-resonance zones of the FRF (Ni *et al.* 2006, Nozarian and Esfandiari 2009, Shadan *et al.* 2015). This selection of the FRF data caused a significant data loss which could result in an oversight of damage. In this study, instead of specifying frequency range of FRF manually, the whole range of the FRF data from the signals can be used for sparse coding. Specifically, a dictionary learning process is employed to reveal the useful information in the resulting sparse representations while redundant information can be filtered. Thus, sparse coding with dictionary learning is used to feature extraction. To improve the efficiency of the SR method for damage identification under uncertainty conditions which appear as noise and damping in signal, the authors originally explored the general concept of using CSC ensemble for damage identification of structures with emphasis on noise and damping effects.

In this paper, a global vibration-based method for SHM is presented that uses measured FRFs data compressed by PCA and then utilizes CSC ensemble technique to investigate the health state of a frame structure. The efficiency of the novel developed method by means of CSC ensemble of mode shape and FRF in comparison to CSC method and also Artificial Neural Networks trained with FRF data is demonstrated.

Many researchers have investigated damage detection using changes in damping ratios even though calculation of such a parameter is facing with lots of uncertainties (Curadelli *et al.* 2008, Kang *et al*. 2012, Kuwabara *et al.* 2013). One of the most important issues is the fact that damping ratio do not only alter with stiffness reduction of the members but also are highly dependent on non-structural members and many other circumstantial parameters that have no relation with structural damages.

In the other hand, in most of the researches which ignore structural damping effects, another type of misleading could be occurred. Damping tends to diminish the amplitude of oscillations in an oscillatory system which is because of the fact that the energy dissipates the properties of a material or system under cyclic stress (Egba 2012). As the damping matrix is dependent on the stiffness matrix, the effects of damping change will be observed in vibration responses of the structure. When stiffness of the structural members reduces because of the environmental effects such as ageing decay, corrosion or loosening of connections, the damping ratio of the structure may alter. It may cause numerical errors with FRF data-set in comparison to real-time condition of the structure.

In this study damping errors are considered as environmental uncertainties just like measurement noise. Training data are considered to be noise free and with 0% damping ratio. Test data are polluted with up to 20% noise and up to 5% damping ratio. Robustness of the proposed method to both measurement noise and damping alteration is investigated by numerical examples.

In the presented study, a two-story frame structure is employed in order to verify the proposed damage identification method. Numerical models of the structure are subjected to several damage scenarios including single and multiple damage scenarios with up to four damaged elements. To simulate field-testing conditions, the numerical data is polluted with white Gaussian noise up to 20% noise-to-signal ratio while in the same studies the noise pollution is limited to 10% (Bandara *et al*. 2014, Dackermann *et al.* 2013). In order to apply the CSC ensemble technique, two individual CSC are employed. Also the first five mode-shapes of the structure and the FRF data compressed by PCA, are considered as inputs of a machine-learning training based on network ensemble to eliminate the effects of noise and damping in damage detection of the sample structure. The superior CSC ensemble functions as a screening mechanism that filters unreliable results, obtained from the two individual CSCs and delivers optimized damage prediction results.

2.1 Features extraction

The Frequency Response Function (FRF) of the structure, $H(\omega)$, is defined as the following:

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \tag{1}$$

Where, X (ω) is the system response and F(ω) stands for the system excitation function in the frequency-domain .

For a n-degree of freedom (nDOF) system, the frequency response function is a n×n square matrix that each of its elements is not an exact number but a function of frequency. Each of the FRFs rows represents the corresponding excitation DOF and each of its columns represents the corresponding measurement DOF. Since the frequency response function of the row i and the column j represents the measured response of the structure in the j^{th} DOF while the structure is excited in the i^{th} DOF.

The full-size FRF contained 20000 spectral lines and covered a frequency range of 0–200 Hz in this investigation. This large size of input data would cause severe difficulties in training convergence along with computational inefficiency. So, the PCA is proposed to reduce the size of the FRF data set that are appropriate input for pattern recognition (Pearson 1901). PCA is a numerical technique that linearly converts an original set of k variables into a smaller set of n independent variables ($n < k$), called PCs.

Each PC is a linear combination of the original variables. The PCs form an orthogonal basis for the space of the data since they are orthogonal to each other. The full set of PCs is equal to the original set of variables. By removing PCs of low power (corresponding to the lower eigenvalues), a dimensional reduction is performed without considerably affecting the original data.

To extract the mode-shapes from raw FRF data of a structure Peak Picking (PP) method is employed (Gentile and Saisi 2007). The PP method leads to trustworthy outcomes as long as the basic assumptions of low damping and well-separated mode-shapes are satisfied.

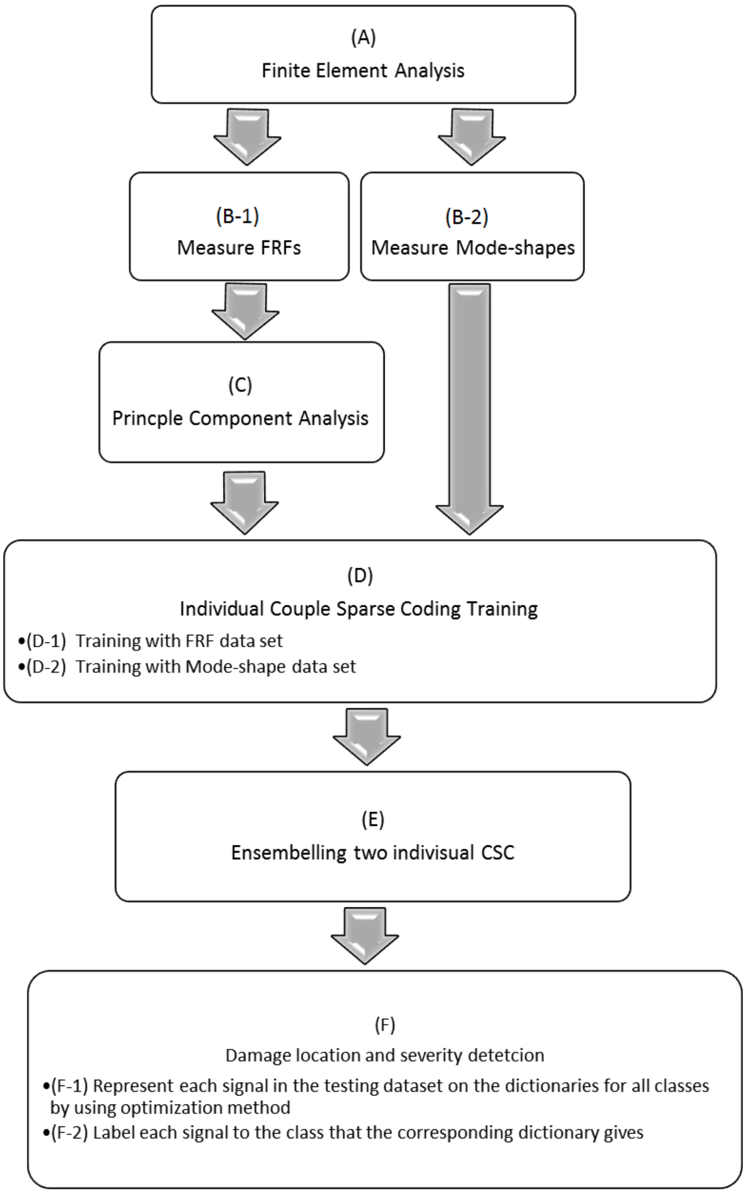


Fig. 1 The flow-chart of the proposed method

As a matter of fact, for a low damping FRFs reach a local maximum at the natural frequencies ω_n, the spectral matrix can be approximated as

$$H(\omega_n) = \alpha_n \phi_n \phi_n^T \tag{2}$$

Where α_n is calculated based on the damping ratio, the natural frequency, the modal participation factor and the excitation spectra. In the present application of the PP method, the natural frequencies were identified from resonant peaks in the FRFs.

2.2 Couple Sparse Coding (CSC)

In recent years, it was found that methods based on the (SR) have attracted many attentions in pattern recognition and machine learning community. It is clear that the (SR) is robust for representing noisy signals (Wright *et al*. 2009).

2.2.1 Sparse Representation (SR)

The concept of sparsity means that a set of data point of signals could be adequately presented by using only a few active features (atoms) without losing information due to the fact that a signal could have a low-dimensional space, even though it may have a high dimensionality.

A set of atoms is called the dictionary. In SR, signals have been demonstrated in an over-complete dictionary by using a few expansion coefficients, named sparse coding. Fitting data helps the dictionary to be updated. In order to have better understandings of SR theory, the following explanations are given:

The structure-obtained FRFs are presented as $x \in R^m$ by a linear combination of $D \in R^{m \times P}$ which is an over-complete dictionary

$$x = \alpha_1 d_1 + \alpha_2 d_2 + ... + \alpha_P d_P \tag{3}$$

Along with the minimization of $\|\alpha\|_0$, where P represents the number of atoms,

$\alpha = [\alpha_1, \alpha_2, ..., \alpha_P]^T \in R^P$ denotes the sparse code of x, and $\|\alpha\|_0$ is the l_0 norm, defined as the number of non-zero components of α.

There is no polynomial time solution for the above SR. However, recent studies have indicated that α can be obtained by the minimization of l_1 norm instead of l_0 norm (Wright *et al.* 2009). Moreover, l_1 norm is calculated by the summation of the absolute values of all entries of the coefficient vector α. This alternative algorithm leads to a convex optimization problem which can be solved in polynomial time using well-known methods.

K-SVD, as one of these methods, is a dictionary learning algorithm for creating a dictionary for sparse representations (Aharon *et al.* 2006). Orthogonal Matching Pursuit (OMP) algorithm is the other applicable method which is based on choosing the basis vector from an over-complete dictionary. The dictionary finds the basis vector to fit the given signal (Cai and Wang 2011).

2.2.2 Couple sparse coding formulation

This section describes the principals of incorporating FRF data into sparse coding formulation. This approach is based on the following assumptions:

FRF data obtained from the structure is represented as $x \in R^m$, while the structural damage properties (such as location and severity of damage) are represented as $y \in R^P$.

where $X = [x_1, x_2, ..., x_N]$ and $Y = [y_1, y_2,..., y_N]$ are the training input set of N FRF data and their corresponding damage scenarios, respectively. The problem is to display an unknown noise polluted FRF as a sparse linear combination of input features (X) by considering the local structure of the FRF data features and then retrieving the damage scenarios based on the corresponding sparse code.

The above-mentioned damage detection method uses two different dictionaries: the first one is $D_x \in R^{m \times N}$ which includes the training input FRF data and the other one is $D_y \in R^{k \times N}$ which includes the corresponding damage scenarios. The proposed optimization problem for damage detection of structure under the damage scenario y for a noise polluted measured FRF x can be expressed as

$$y = D_y \alpha \tag{4}$$

Where α is the sparse code of x that is computed as follows

$$\text{minimize}(\alpha): \|x - D_x \alpha\|_2^2 + \lambda_1 \|\alpha\|_1 + \lambda_2 \|\hat{y} - D_y \alpha\|_2^2 \tag{5}$$

Where λ_1 and λ_2 are the regularization parameters, and $\hat{y}$ is the corresponding damage feature of the FRF data having the minimum differences from x. It is important to mention that the proposed damage detection algorithm avoids under-fitting using multiple training samples as well as over-fitting using only the smallest number of training data points possible (Wright *et al.* 2009). The term $\|\hat{y} - D_y \alpha\|_2^2$ is based on the fact that neighboring FRF inputs are more likely to contain similar damage properties. Thus, the sparse vector α should not only reconstruct the FRF test input x from the columns of the dictionary D_x (FRF data dictionary) with minimum error, but also create the corresponding output from dictionary D_y (damage scenarios dictionary) having the minimum distance from $\hat{y}$.

2.2.3 Dictionary

Each signal has been characterized as a linear combination of the dictionary atoms. Dictionary learning updates the dictionary iteratively to reach the objective of expanding the given signal sparsely. One of the most important advantages of using dictionaries in SR with regard to traditional transforms, i.e., Fourier and wavelets, is its ability in learning by fitting itself to the given signals. Moreover, the other important feature of the dictionary that has recently attracted a lot of attention is its over-completeness. More atoms exist than the signal dimensions in the over-complete dictionaries. The completeness of the dictionary brings about more flexibility and more robustness against noise. Some well-known algorithms are k-svd and MOD methods or transformations, such as DCT, Bandelet or Contourlet (Rubinstein *et al.* 2010); however, many algorithms use the training data as columns (atoms) to have significant performance (Wright *et al.* 2009). The advantage of using the training data as dictionary atoms is due to the easy application of the model (i.e., small number of the model parameters that leads to less complexity and higher speed).

Two significant matters should be considered in the presented approach for damage detection: (1) Dictionary size should not be smaller than a certain size because of missing feature space information that can prevent achieving a reasonable reconstruction. The dictionary should be large enough to cover the possible variations. (2) Selected dictionary columns, samples, must span the activity space and also the number of the similar columns must be low. Donoho showed that the training samples should build an incoherent dictionary (Donoho 2006).

Considering these issues, the Random Selection Approach has been applied for selecting dictionary atoms in the present study. The Random Selection Approach selects the dictionary atoms randomly from the entire space.

2.2.4 Optimization

To describe the learning process of the sparse code, the proposed objective function $h(\alpha)$ for estimating the sparse code of the unknown input x is presented as follows

$$\text{h}(\alpha) = \|x - D_x \alpha\|_2^2 + \lambda_1 \|\alpha\|_1 + \lambda_2 \|d - D_y \alpha\|_2^2 \tag{6}$$

Where d is the corresponding mapped damage feature of the FRF data with the minimum variance from x. The

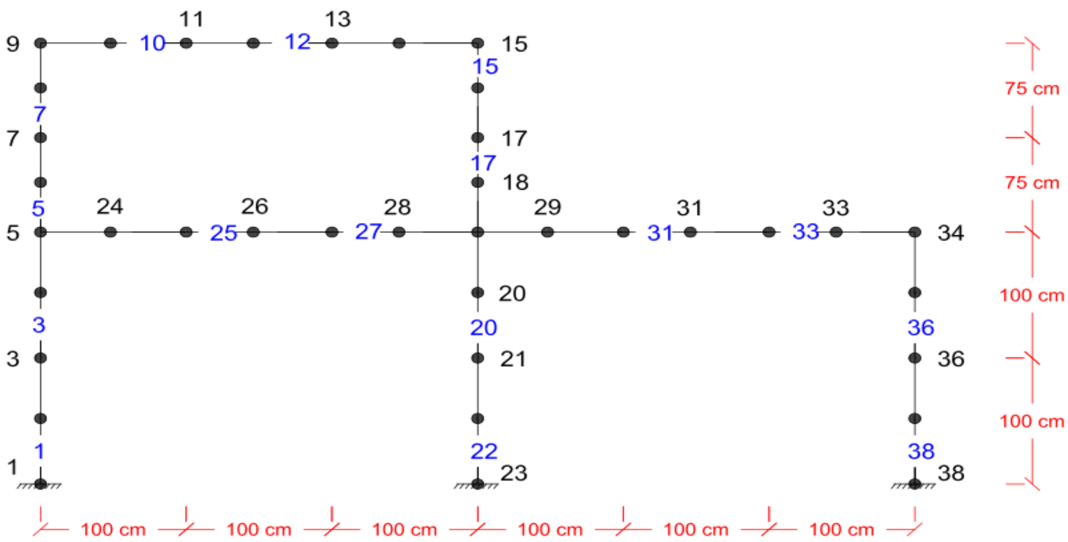


Fig. 2 Finite element model of the sample frame structure

function $h(\alpha)$ is convex with respect to the sparse code α, when the dictionaries D_x and D_y are fixed. Because of being convex, the $h(\alpha)$ is not continuously differentiable. As a result, the simple gradient-based approaches are difficult to apply (Lee *et al.* 2007). Therefore; in the current study, the feature-sign search algorithm (Lee *et al.* 2007) has been employed to solve the optimization problem. This algorithm proceeds in a series of feature-sign steps. In each step, the analytical solution is calculated α̂ new to the resulting unconstrained QP with respect to the current given presumption for the active set and the signs. Therefore, the solution is updated by the active set and the signs using an efficient discrete line search between the current solution and α̂ new (Wright *et al.* 2009).

3. Numerical verification

In order to verify the accuracy and robustness of the proposed method, a representative finite element model of a 2D frame structure is considered. The outcomes of the trained CSC ensemble are presented by various tables and figures. A numerical model of a two-span two-story frame structure with 38 elements and 105 DOFs is designed for damage identification in this section (Fig. 2). It is notable that damage is defined as the stiffness reduction of members which is corresponding to the member's moment inertia.

The FRF data is extracted from the structure by means of the excitation and measurement degrees of freedom which their corresponding points and directions are tabulated in Table 1. These DOFs are selected horizontally (*H*) on the columns and vertically (*V*) on the beams.

It is presumed that the exciter (such as shaker or hammer) is able to excite the structure in the frequency range of 0 to 200 Hz and the response data can be measured completely. Dividing the responses of the structure by the employed excitation in the frequency domain determines the FRF.

Table 1 Corresponding points and directions of excited and measured degrees of freedom

Excited DOFs	Measured DOFs
2 H	4 H
12 V	26 V
17 H	21 H
30V	29 V
36 H	37 H

The frame is supposed to be made of steel and the modulus of elasticity, Poisson's ratio, and density were considered as 200 GPa, 0.3 and 7850 $kg.m^{-3}$ respectively.

In order to simulate measurement errors in an experimental set up for FRFs extraction, up to 20% normal random noise is added to the extracted FRF data from the finite element model of the damaged structure. A sample noisy damped FRF with 20% noise and 5% damping ratio is illustrated in Fig. 3 and compared with undamped noise free FRF.

A modal damping ratio of 0% to 5% is applied to imitate the existing damping in the steel frame. It is noted that training data which are used to build the dictionary are established using the undamped model because it is assumed that the damping of the structure is unknown and can be treated as an error. But the test damaged samples are estimated using the damped FRFs of the structure and the effects of damping on damage detection is investigated.

In a low-damped structure the stiffness reduction affects mainly the natural frequencies, while the amplitude of the free vibration response is principally alters by damping. Thus damping errors cause difficulties and uncertainties in calculating mode-shapes of the structure (Zapico-Valle *et al.* 2014).

This effect can be observed in Fig. 4 which illustrates a damped FRF with 1% damping and the first five vibration modes along with their corresponding frequencies.

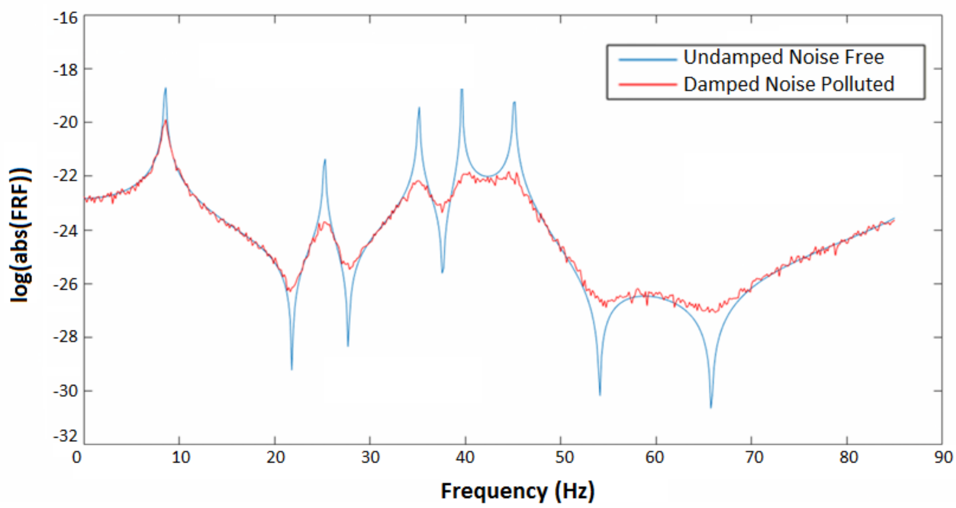


Fig. 3 Comparison of a FRF with 20% noise and 5% damping ratio and an undamped noise free FRF

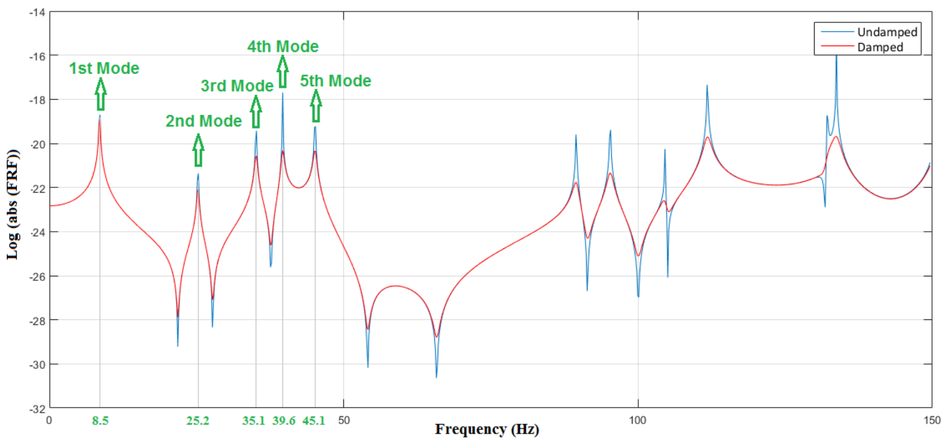


Fig. 4 A Sample FRF with ξ=0% and ξ=1% and showing the first 5 vibration modes

Since five measurement and five excitation degrees of freedom have been selected, 25 sets of FRF data are obtained. These data can be derived from the numerical calculation of FRF matrix, in which each row corresponds to an excitation and each column is referred to as measurement DOF.

One of the advantages of the proposed method is the fact that it doesn't need frequency ranges to be selected and all the measured FRF data can be used for damage identification. Due to the large size of input data a dimension reduction technique is needed to overcome the calculation limitations.

Ensemble sparse coding using both FRF data and the first 5 mode-shapes is utilized due to large errors and misidentifications of the sparse method using just FRF data especially in the presence of up to five percent damping ratio. The first five mode-shapes of the sample structure and their corresponding natural frequencies are shown in Fig. 5.

As discussed previously, for each of damage scenario there are 25 sets of FRF each containing response data from 0 to 200 Hz. Consequently there are 20000 input data points for each scenario. Such large numbers of input points can cause severe problems in training convergence in addition to computational inefficiency. Therefore, PCA is desirable in such applications to reduce the size. In this study, to compress the FRF data, matrices containing the data of FRFs of the structure condition in each damage scenario were formed and projected onto their PCs. Sample PCs for some of the damage scenarios and their sensitivities to noise pollution and damage severity are illustrated in Fig. 6.

A study on the sensitivity of the PCs was undertaken to determine the optimal number of PCs that contain sufficient data for damage identification. Fig. 7 represents the efficiency of PCA method with different PCs in damage detection and also highlights the superiority of the ensemble sparse technique over the approaches of simply using FRF data for couple sparse coding (CSC) and artificial neural networks (ANN). The performance is given for the training, validation and testing sets in Mean Square Error (MSE), which is defined as the difference between the predicted damage and actual damage for different damage scenario.

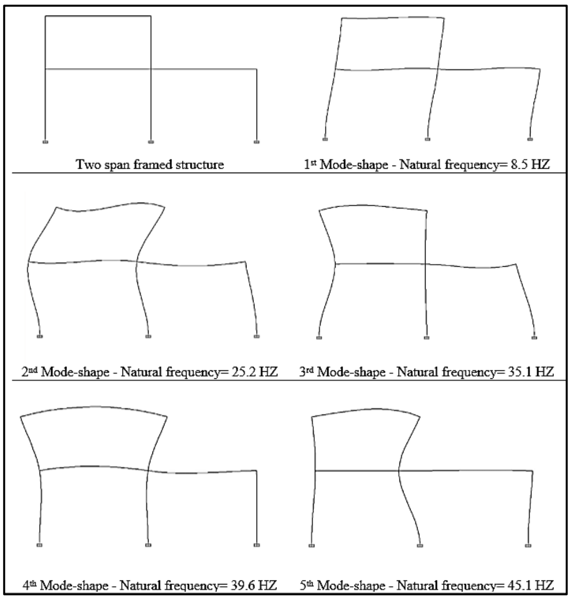


Fig. 5 The first five vibration modes along with their corresponding frequencies

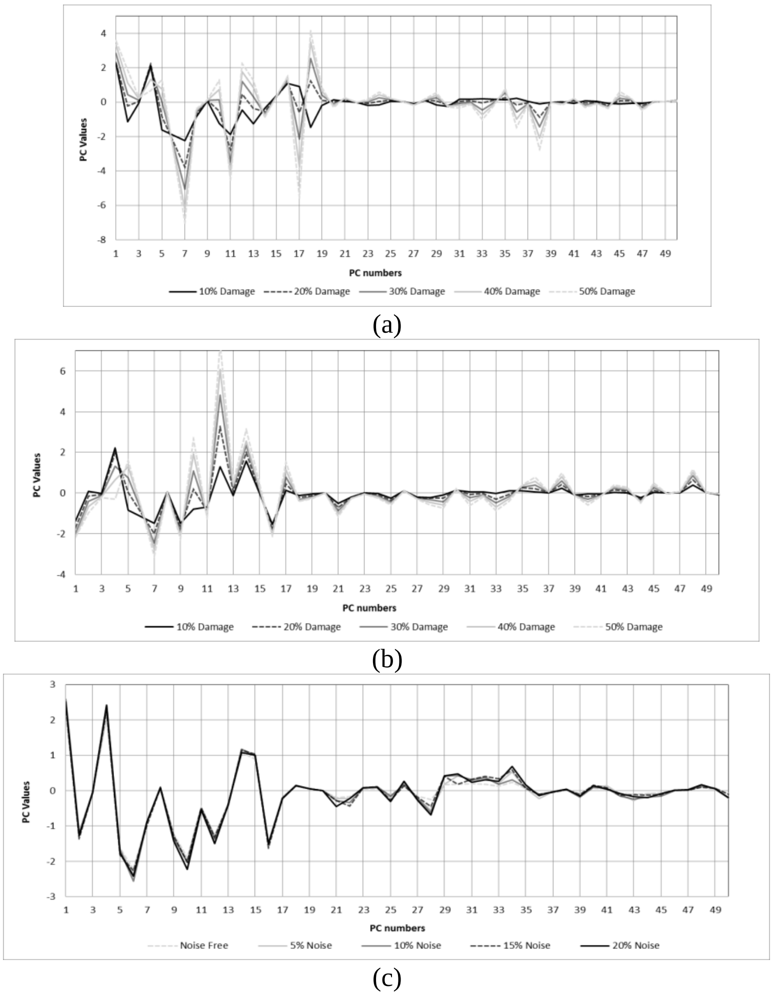
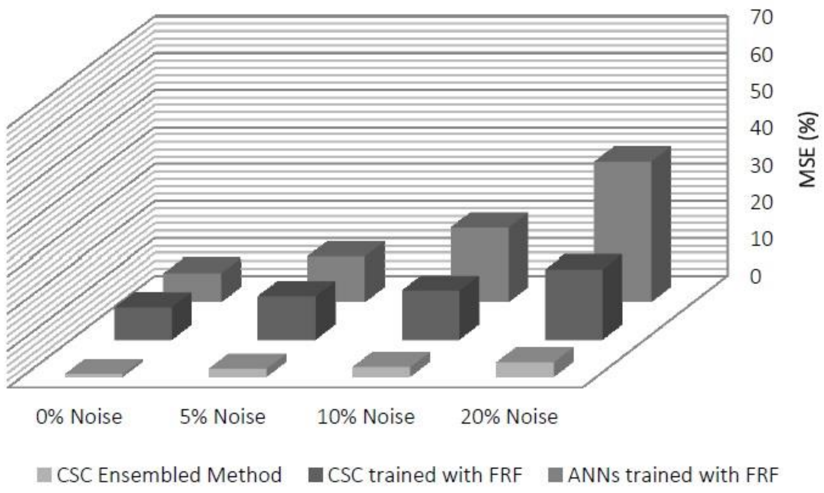
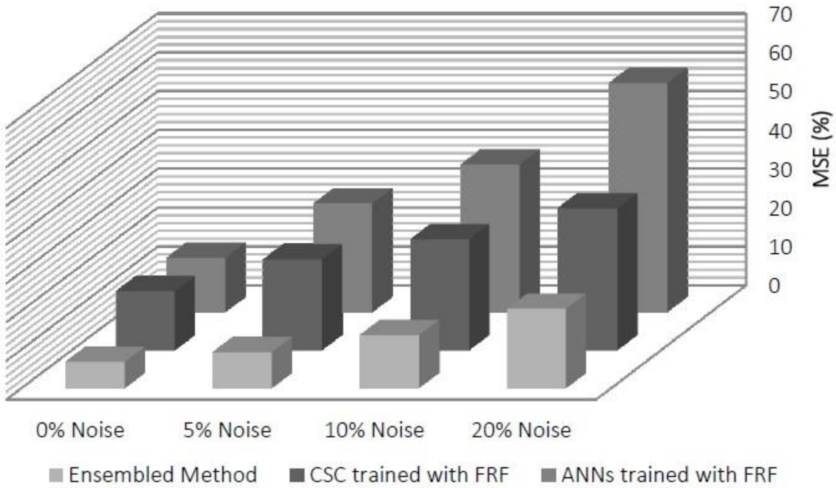


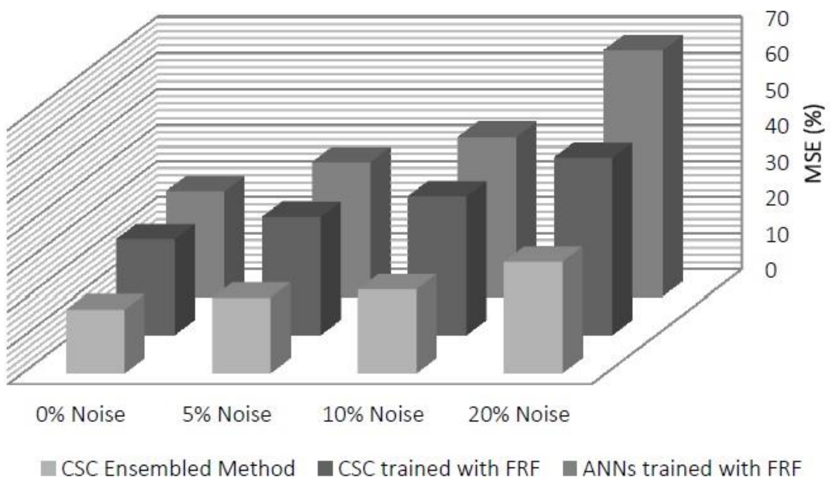
Fig. 6 Sample PCs for (a)different damage severities in element no.3, (b)different damage severities in element no.27 and (c)a sample scenario polluted by different amount of noise



(a)



(b)



(c)

Fig. 7 Comparison of Mean square error (MSE) of damage detection using the suggested ensemble method with CSC and ANN method using just FRFs data (a) FRF dimension reduced to 20 PCs, (b) FRF dimension reduced to 75 PCs and(c) FRF dimension reduced to 125 PCs

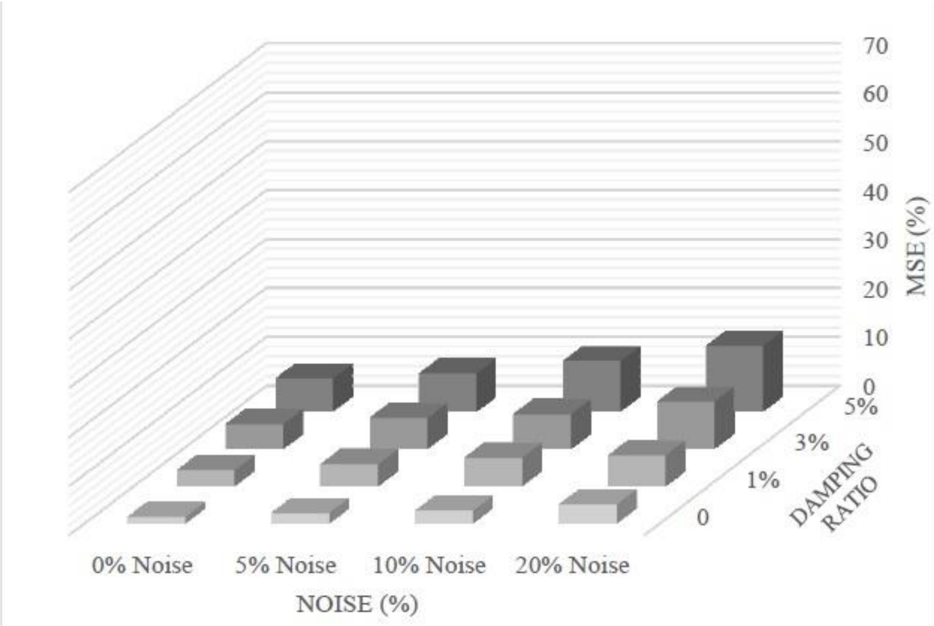


Fig. 8 Mean square error (MSE) of damage detection using the suggested ensemble method in presence of different amounts of noise and damping

$$MSE = \frac{1}{ne}\sum_{i=1}^{ne}\left|p_i^a - p_i^p\right|^2 \tag{7}$$

Where ne is the number of test damage scenarios and p_i^p is developed method perdition of actual damage. MSE value close to zero indicates that estimation error is close to zero.

Fig. 7 demonstrates that using 20 PCs leads to better results in damage detection especially in presence of noise (Fig. 7). Increase in the number of PCs will reduce the accuracy of the method and highlights the errors caused by noise pollution. Also ensemble sparse coding method using both FRF data and the first 5 mode-shapes leads to smaller errors and misidentifications compared to the sparse representation and a multilayer perceptron artificial neural network method. This ANN consists of three hidden layers each include 20 neurons. CSC and ANN are both just trained with FRF data. The misidentification increases significantly in the presence of high level noise pollution.

Besides, the sensitivity of the proposed ensemble method using both FRFs data and the first 5 mode-shapes of the sample structure method using 20 PCs (the most efficient number of PCs according to the previous study) to different amount of noise pollution and damping ratio errors which are caused by unwanted and unconsidered damping in the structure is shown in the Fig. 8.

It can be observed that in the worst case scenario with 20% noise pollution and 5% unconsidered damping ratio, the mean square error of the proposed method limits to 13.45%. It can be concluded that the proposed method is more or less insensitive to noise and damping uncertainties and can eliminate their effects on damage detection considerably.

3.1 Single damage scenarios

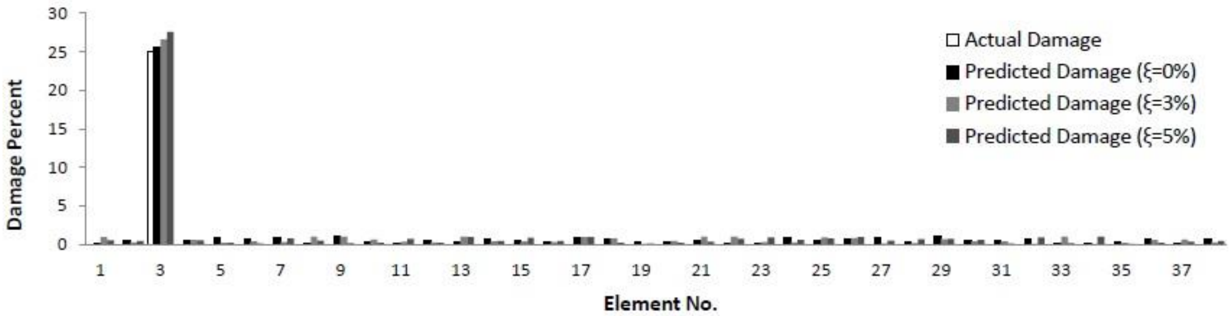
After evaluating the main factors affecting the detection accuracy, three single damage scenarios were investigated to verify the efficiency of this method. Single damage was simulated at certain elements to verify whether it could be detected by this method accurately. The two-span two-story sample structure is employed for this purpose (Fig. 2) and 10% white Gaussian noise and also three different levels of damping ratios (0%, 3% and 5%) were considered. Thus, three single damage cases were illustrated in Fig. 9 to show all the effects of noise and damping ratio uncertainties.

In the first damage scenario, 25% reduction in stiffness was introduced to element 3 in the middle of the left column. Damage detection result of this scenario is showed in Fig. 9a.

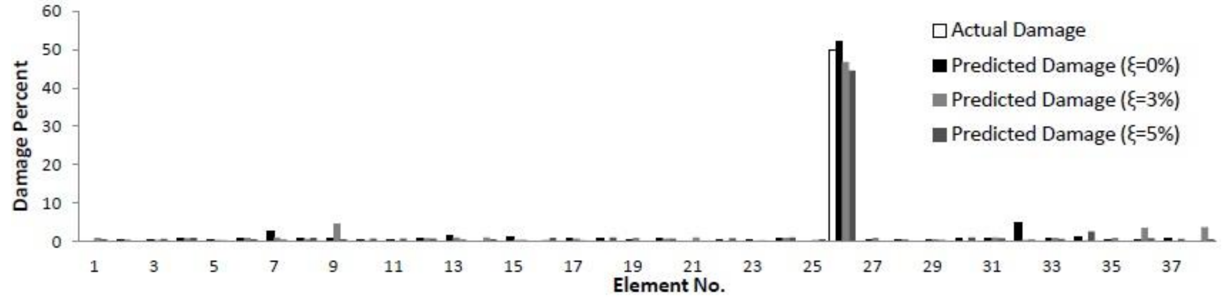
A high reliable damage locating was achieved. It can be demonstrated that uncertainties caused by damping errors leads to less than 4% error in severity detection but the damage location was predicted without any error.

The second scenario that is illustrated in Fig. 9b introduced 50% stiffness reduction in the vicinity of the middle of a beam and just like the first scenario both damage location and severity are achieved with high reliabilities even in presence of 5% damping ratio error. The damage severity is predicted to be 51.96%, 46.75% and 44.56% for damping ratios of 0%, 3% and 5% respectively.

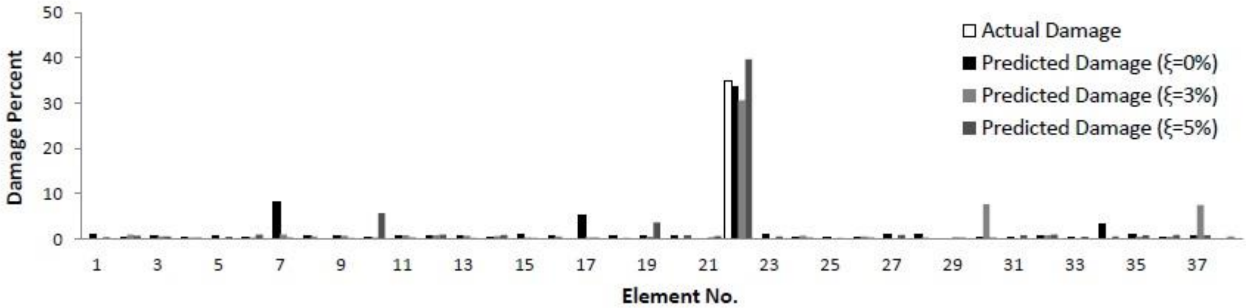
In the third scenario which is illustrated in Fig. 9(c), 40% stiffness reduction, was introduced to element 22 which is located in the vicinity of the middle fixed support. As shown in the figure, results present less precision than the first and second scenarios. This lack of accuracy is because of the fact that the elements near supports reveal very small response even when excited by large loads.



(a)



(b)



(c)

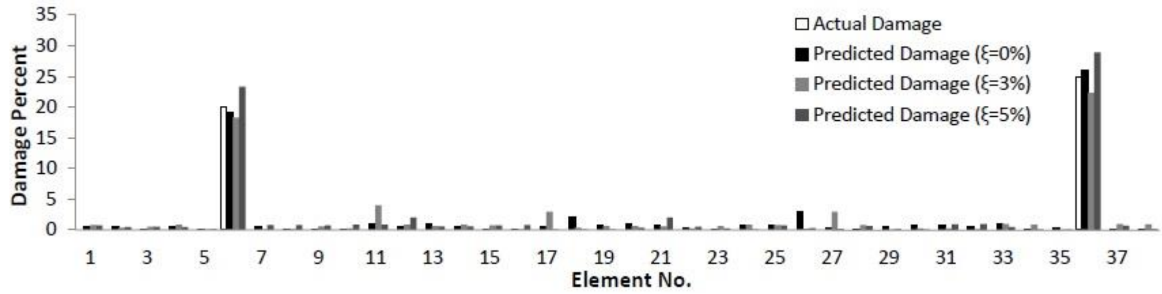
Fig. 9 damage detection of three sample single case studies in presence of damping (ξ=0 - 5%) and 10% noise (a) damage case 1, (b) damage case 2 and (c) damage case 3

3.2 Multiple damage scenarios

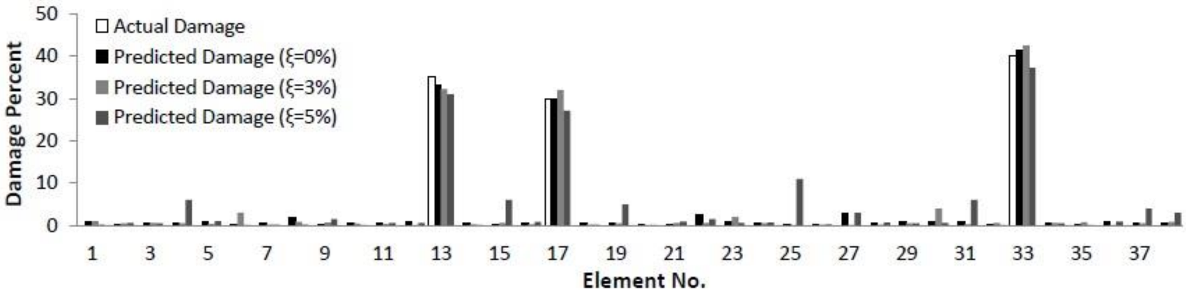
In this study, damage scenarios with two, three and four elements are investigated and the damage extent was also randomly defined for each damaged element within the range of 10–60% decrease of structural stiffness, assuming 5% increment. While the training data are noise free and without considering any damping, 10% noise is included in test FRF data and the robustness of the method against different amount of damping ratio from 0% to 5% is studied. In the training phase, different training damage scenarios are obtained by introducing in the finite element model with stiffness deterioration in the members. Damage is defined by reduction of the moment of inertia. After making the dictionary, the testing samples with damage extents different from the training ones are employed to evaluate the damaged member and the damage extent.

Fig. 10 shows the identification for three different scenarios in presence of 10% noise and with different damping ratios from 0% to 5%.

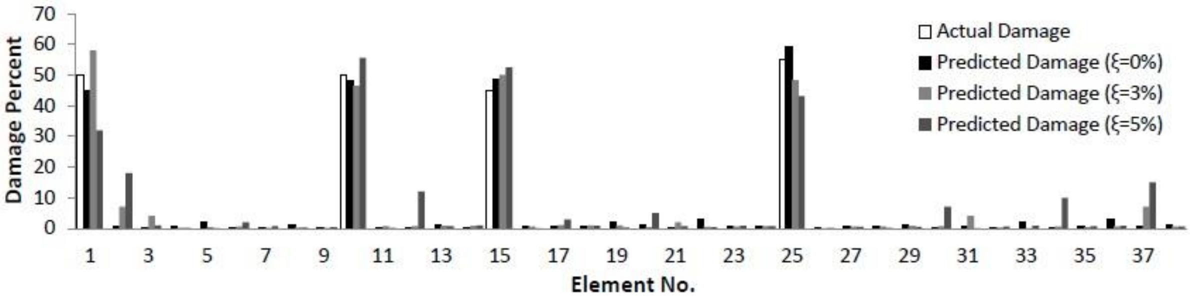
Although there are a few elements which are detected to be damaged by less than 10% stiffness reduction, it can be concluded that the identification accuracy of damage location and extent is highly acceptable in the first and second scenarios and the effect of damping errors are negligible. But in the third scenario (Fig. 10(c)) there is a large amount of error in damage severity detection especially in element number 1 which can be interpreted by the fact that one of the damages is occurred in the vicinity of fixed support and due to the reasons discussed in the previous section, damage detection errors are expected. Although the location of all the damages are predicted correctly while damage severities are predicted with up to 25% error in some elements. It can be concluded that in the scenarios with damages near the fixed supports the damages extent cannot be accurately identified for most cases. This is attributed to low modal sensitivities of the structure to near support damages.



(a)



(b)



(c)

Fig. 10 Damage detection of three sample multiple case studies in presence of damping (ξ=0 - 5%) and 10% noise (a) damage case 1, (b) damage case 2 and (c) damage case 3

4. Conclusions

The new damage detection method based on vibration characteristics of structures as inputs of Couple Sparse Coding (CSC) is proposed in this study. Frequency Response Function (FRF) compressed by Principal Component Analysis (PCA) and the first five Mode-Shapes were used to train and validate the system to develop its ability to identify structural damage. The reliability of the damage identification method with regard to damage location, damage severity, level of noise and various damping ratios was investigated via numerical simulations of a two-story frame structure. To take advantage of individual characteristic of FRF and mode-shapes, CSC ensemble was implemented. Two CSCs were trained and their damage predictions were combined in an ensemble system.

To simulate field-testing conditions and test the robustness of the proposed method against noise, white Gaussian noise of up to 20% noise-to-signal ratio was added to measured data by considering up to 5% damping ratio.

To detect the damage, the full range of FRF data (0 – 200 Hz) has been considered, while effects of damping and noise are significant on resonance and anti-resonance of FRF data. Therefore, CSC ensemble is employed to obtain more accuracy in the identification of damage in the structure.

The following conclusions are made:

(1) By adopting CSC ensemble, trained with FRF and mode-shapes, damages location and severity is detectable acceptably. It is worth nothing that in the vicinity of fixed supports, the location of damage can be detected, while the accuracy of the damage severity is reduced remarkably.

(2) The proposed method is acceptably robust against measurement noise and unpredicted damping, however the effect of damping ratio error on the results precision is larger in comparison to the noise pollution.

(3) The obtained results demonstrated the superiority of the CSC ensemble technique over the other approach which employs individual CSC just trained with FRF data-sets.

References

Abdeljaber, O., Avci, O., Kiranyaz, S. *et al.* (2017), "Real-time vibration-based structural damage detection using one-dimensional convolutional neural networks", *J. Sound Vib.*, **388**, 154-170.

Abolbashari, M.H., Nazari, F. and Rad, J.S. (2014), "A multi-crack effects analysis and crack identification in functionally graded beams using particle swarm optimization algorithm and artificial neural network", *Struct. Eng. Mech.*, **51**(2), 299-313.

Aharon, M., Elad, M. and Bruckstein, A. (2006), "K-svd: An algorithm for designing overcomplete dictionaries for sparse representation", *IEEE T. Signal Pr.*, **54**(11), 4311-4322

Amezquita-Sanchez, J.P. and Adeli, H. (2016), "Signal processing techniques for vibration-based health monitoring of smart structures", *Arch. Comput. Method. E.*, **23**(1), 1-15.

Aydin, K. and Kisi, O. (2015), "Damage detection in structural beam elements using hybrid neuro fuzzy systems", *Smart Struct. Syst.*, **16**(6), 1107-1132

Bandara R.P, Chan T.H. and Thambiratnam D.P. (2014), "Frequency response function based damage identification using principal component analysis and pattern recognition technique", *Eng. Struct.*, **66**(1), 116-128.

Cai, T.T. and Wang, L. (2011), "Orthogonal matching pursuit for sparse signal recovery with noise", *IEEE T. Inform. Theory*, **57**(7), 4680-4688

Curadelli, R.O., Riera, J.D., Ambrosini D., *et al*. (2008), "Damage detection by means of structural damping identification", *Eng. Struct.*, **30**(12), 3497-3504.

Dackermann U., Li J. and Samali B. (2010), "Quantification of notch-type damage in a two-storey framed structure utilising frequency response functions and artificial neural networks", *Proceeding of the 5th World Conference on Structural Control and Monitoring.*

Dackermann, U., Lim J. and Samali, B. (2013), "Identification of member connectivity and mass changes on a two-storey framed structure using frequency response functions and artificial neural networks", *J. Sound Vib.*, **332**(16), 3636-3653.

Dackermann, U., Li, J., Samali, B., *et al*. (2011), "Damage severity assessment of timber bridges using frequency response functions (FRFs) and artificial neural networks (ANNs) ", *Proceedings of the International Conference on Structural Health Assessment of Timber Structures (SHATIS 11).* Laboratorio Nacional de Engenharia Civil.

Dilenaa, M., Limongellib, M.P. and Morassi, A. (2015), "Damage localization in bridges via the FRF interpolation method", *Mech. Syst. Signal Pr.*, **52-53**, 162-180.

Donoho, D.L. (2006), "Compressed sensing", *IEEE T. Inform. Theory*, **52**(4), 1289-1306.

Egba, E.I. (2012), "Detection of structural damage in building using changes in modal dampong mechanism", *Int. J. Eng. Management Sci.*, **3**(3), 250.

Esfandiari, A., Bakhtiari-Nejad, F., Rahai, A., *et al.* (2009), "Structural model updating using frequency response function and quasi-linear sensitively equation", *J. Sound Vib.*, **326**(3-5), 557-573.

Gentile, M.C. and Saisi, A. (2007), "Ambient vibration testing of historic masonry towers for structural identification and damage assessment", *Constr. Buildi. Mater.*, **21**(6), 1311-1321.

Hakim, S.J.S. and Abdul Razak, H. (2013), "Adaptive neuro fuzzy inference system (anfis) and artificial neural networks (anns) for structural damage identification", *Struct. Eng. Mech.*, **45**(6), 779-802.

Hansen, L.K. and Salamon, P. (1990), "Neural network ensembles", *Pattern Anal. Machine Intelligence, IEEE T.*, **12**(10), 993-1001.

He, H., Yan, W. and Zhang, A. (2013), "Theoretical and experimental study on damage detection for beam string structure", *Smart Struct. Syst.*, **12**(3), 327-344.

Huang, J.Z., Huang, X.L. and Metaxas, D. (2008), Simultaneous image transformation and sparse representation recovery. *Computer Vision and Pattern Recognition.* Anchorage, AK IEEE

Jiang, X. and Adeli, H. (2007), "Pseudospectra, MUSIC, and dynamic wavelet neural network for damage detection of highrise buildings", *Int. J. Numer. Meth. Eng.*, **71**(5), 606-629.

Kang, F., Jun-Jie, L. and Qing, X. (2012), "Damage detection based on improved particle swarm optimization using vibration data", *Appl.Soft Comput.*, **12**(8), 2329-2335.

Kaveh, A., Vaez, S.R.H., Hosseini, P., *et al*. (2016), "Detection of damage in truss structures using simplified dolphin Echolocation algorithm based on modal data", *Smart Struct. Syst.*, **18**(5), 983-1004.

Khoshnoudian, F. and Esfandiari, A. (2011), "Structural damage diagnosis using modal data", *Scientia Iranica*, **18**(4), 853-860.

Khoshnoudian, F., Talaei, S. and Fallahian, M. (2017), "Structural damage detection using FRF data, 2D-PCA, artificial neural networks and imperialist competitive algorithm simultaneously", *Int. J. Struct. Stab. Dynam.*, **17**(7), 1750073.

Kuwabara, M., Yoshitomi, S. and Takewaki, I. (2013), "A new approach to system identification and damage detection of high-rise buildings", *Struct. Control Health Monit.*, **20**(5), 703-727.

Lee, H., Battle, A., Raina, R., *et al.* (2007), Efficient sparse coding algorithms. NIPS, Kolkata

Lee, U. and Shin, J. (2002), "A frequency response function-based structural damage identification method, Computers and Structures", *Comput. Struct.*, **80**(2), 117-132.

Li, J., Dackermann, U., Xu, Y.L., *et al*. (2011), "Damage identification in civil engineering structures utilising PCA-compressed residual frequency response functions and neural network ensembles", *Struct. Control Health Monit.*, **18**(2), 207–226.

Liu H., Liu C. and Huang Y. (2011), "Adaptive feature extraction using sparse coding for machinery fault diagnosis", *Mech. Syst. Signal Pr.*, **25**(2), 558-574.

Maia, N.M.M., Silva, J.M.M., Almas, E.A.M., *et al.* (2003), "Damage detection in structures: from mode shape to frequency response function methods", *Mech. Syst. Signal Pr.*, **17**(3), 489-498.

Marwala, T. (2000), "Damage Identification Using Committee of Neural Networks", *J. Eng. Mech.*, **126**(1), 43-50.

Marwala, T. and Hunt, H.E.M. (1999), "Fault identification using finite element models and neural networks", *Mech. Syst. Signal Pr.*, **13**(3), 475-490.

Mehrjoo, M., Khaji, N., Moharrami, H., *et al.* (2008), "Damage detection of truss bridge joints using artificial neural networks", *Expert Syst. Appl.*, **35**(3), 1122-1131.

Ni Y.Q., Zhou, X.T. and Ko, J.M. (2006), "Experimental investigation of seismic damage identification using PCA-compressed frequency response functions and neural networks", *J. Sound Vib.*, **290**(1), 242-263.

Nozarian, M.M. and Esfandiari, A. (2009), "Structural damage identification using frequency response function", *Mater. Forum*, **33**, 443-449.

Opitz, D.W. and Shavlik, J.W. (1996), "Actively searching for an effective neural network ensemble",*Connection Science*, **8**(3-4), 337-353.

Pearson, K. (1901), "On lines and planes of closest fit to systems of points in space", *Philos. Mag.*, **2**(11), 559-572.

Perrone, M.P. and Cooper, L.N. (1993), When networks disagree: Ensemble method for neural networks, (Ed., R.J. Mammone), *Artificial Neural Networks for Speech and Vision.* Chapman & Hall, New York.

Qiao, L., Esmaeily, A. and Melhem, H.G. (2009), "Structural damage detection using signal pattern-recognition", *Key Eng.Mater.*, **400**, 465-470.

Qiao, L., Esmaeily, A. and Melhem, H.G. (2012), "Signal pattern recognition for damage diagnosis in structures", *Comput. Aided Civil Infrastruct. Eng.*, **27**(9), 699-710.

Roy, K. and Ray-Chaudhuri, S. (2013), "Fundamental mode shape and its derivatives in structural damage localization", *J. Sound Vib.*, **332**(21), 5584-5593.

Rubinstein, R., Bruckstein, A. and Elad, M. (2010), "Dictionaries for sparse representation modeling", *Proceedings of the IEEE*, **98**(6), 1045-1057

Sampaio, R.P.C, Maia, N.M.M. and Silva, J.M.M. (1999), "Damage detection using the frequency-response-function curvature method", *J. Sound Vib.*, **226**(5), 1029-1042.

Shadan, F., Khoshnoudian, F. and Esfandiari, A. (2015), "A frequency response-based structural damage identification using model updating method", *Struct.Control Health Monit.*, **DOI: 10.1002/stc.1768**

Shadan, F., Khoshnoudian, F., Inman, D.J., *et al.* (2016), "Experimental validation of a FRF-based model updating method", *J. Vib. Control*.

Trendafilova, I. and Heylen, W. (2003), "Categorization and pattern recognition methods for damage localization from vibration measurements", *Mech. Syst. Signal Pr.*, **17**(4), 825-836.

Wang, Y. (2015), "Probabilistic-based damage identification based on error functions with an autofocusing feature", *Smart Struct. Syst.*, **15**(4), 1121-1137.

Wang, Z., Chen, S., Lederman, G., *et al*. (2013), "Comparison of sparse representation and fourier discriminant methods: damage location classification in indirect lab-scale bridge structural health monitoring", *Structures Congress*, 436-446.

Wright, J., Yang, A.Y., Ganesh, A., *et al.* (2009), "Robust face recognition via sparse representation", *IEEE T. Pattern Anal. Machine Intell.*, **3**(2), 210-227

Wright, J., Yi, M., Mairal, J., *et al*. (2009), "Sparse representation for computer vision and pattern recognition", *Proceedings of the IEEE*, **98**(6).

Zang, C. and Imregun, M. (2001), "Structural damage detection using artificial neural networks and measured FRF data reduced via principal component projection", *J. Sound Vib.*, **242**(5), 813-827.

Zapico-Valle, Luis J. and García-Diéguez M. (2014), "Dynamic modeling and identification of the Uniovi structure", *Int. J. Simul. Multidiscip. Des. O.*, **5,** 6.

Zhou, Z., Wu, J. and Tang, W. (2002), "Ensembling neural networks: Many could be better than all", *Artif. Intell.*, **137**, 239-263.

Zolfaghari, M., Jourabloo, A., Gozlo, S., et al. (2014), "3D human pose estimation from image using couple sparse coding", *Mach. Vision Appl.*, **25**(6), 1489-1499.

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